CFD 入门练习 4

1.对一维线性波动方程 $u_t + u_x = 0, x \in [0,1], t \geq 0$ **,满足以下初始条件** $u(x,0) = u_0(x)$ **,及周期边界条件** $u(0,t) = u(1,t) = 0$ **。其中**

$$u_0(x) = \sin(2\pi x)$$

考虑均匀网格,并定义 CFL= $\Delta t / \Delta x$ **,编写程序试着使用不同 CFL 数，不同终止时间，不同时间推进格式(显式欧拉方法，TVDRK3)时候的数值解。并试着探索以下方法的空间精度。**

(1) DG(P0)

(2) rDG(P0P1)

(3) DG(P1)

(4) rDG(P1P2)

注：(1)本题所考虑的均匀网络均以 $\Delta x = 0.01$ 为空间步长。

(2)对于每一个数值方法，均分别用显式欧拉与 TVDRK3 进行时间推进。

(3)CFL 取值分别为 0.001，1。(DGP0 中 CFL 额外取值 0.01，0.1)

(4)终止时间 ENDT 分别取 0.35，10。(DGP0 中 ENDT 额外取值 1，10；DG(P1)与 rDG(P1P2)取 0.35，2)

(5)计算空间精度均采取高斯勒让德积分(3 次)。

解：这里仅展示数值解与解析解的对比图以及空间精度值 e 。

(1) DG(P0)显式欧拉时间推进：

表 1：显式欧拉推进格式下 DG(P0)的空间精度值 e

CFL / ENDT	0.001	0.01	0.1	1
0.35	0.048	0.0476	0.0436	0.0074
1	0.127	0.126	0.1156	0.0074
2	0.2308	0.2291	0.2118	0.0074
10	0.6088	0.607	0.5875	0.0074

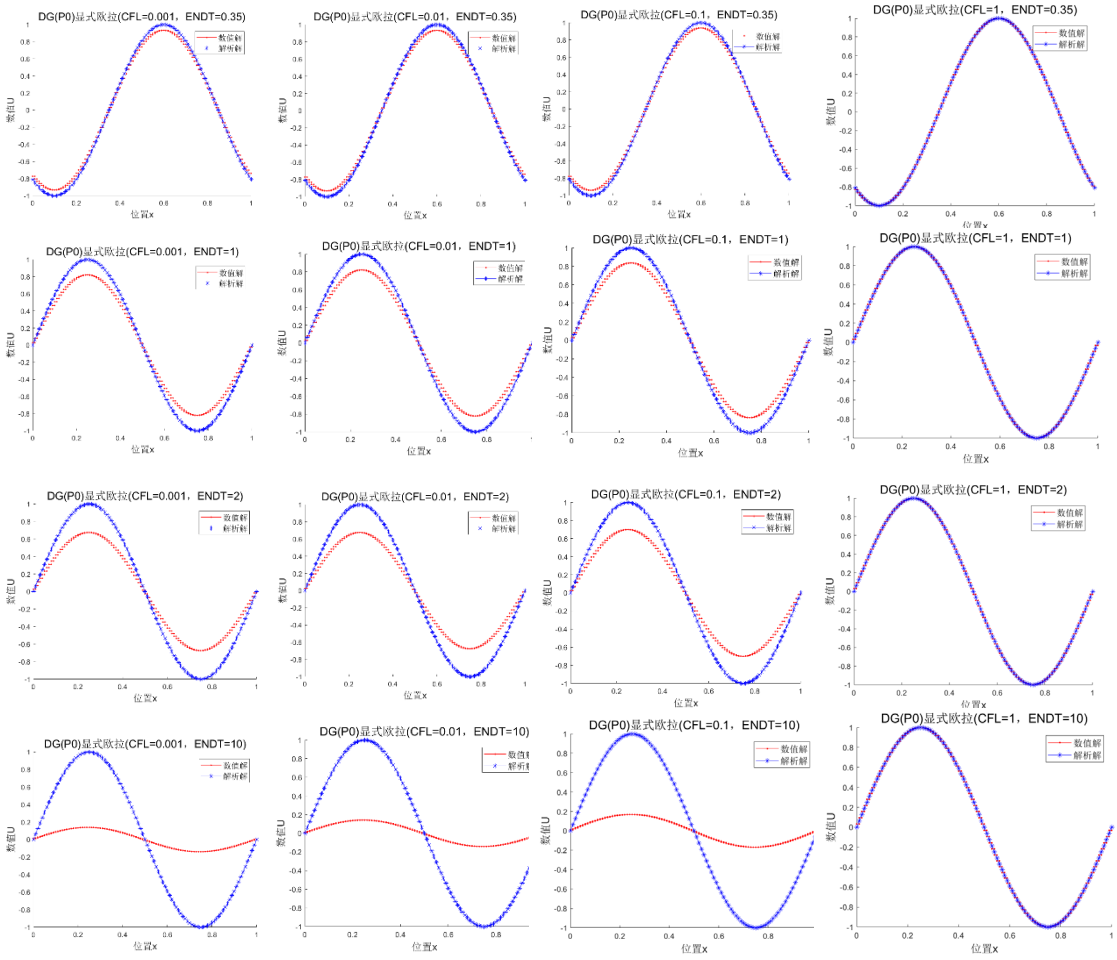

图 1：显式欧拉推进格式下 DG(P0)的数值解与解析解比较图

DG(P0)-TVDRK3 时间推进：

表 2：TVDRK3 推进格式下 DG(P0)的空间精度值 e

CFL / ENDT	0.001	0.01	0.1	1
0.35	0.0481	0.0481	0.0481	0.0481
1	0.1271	0.1271	0.1271	0.1272
2	0.2309	0.2309	0.2309	0.2309
10	0.609	0.609	0.609	0.609

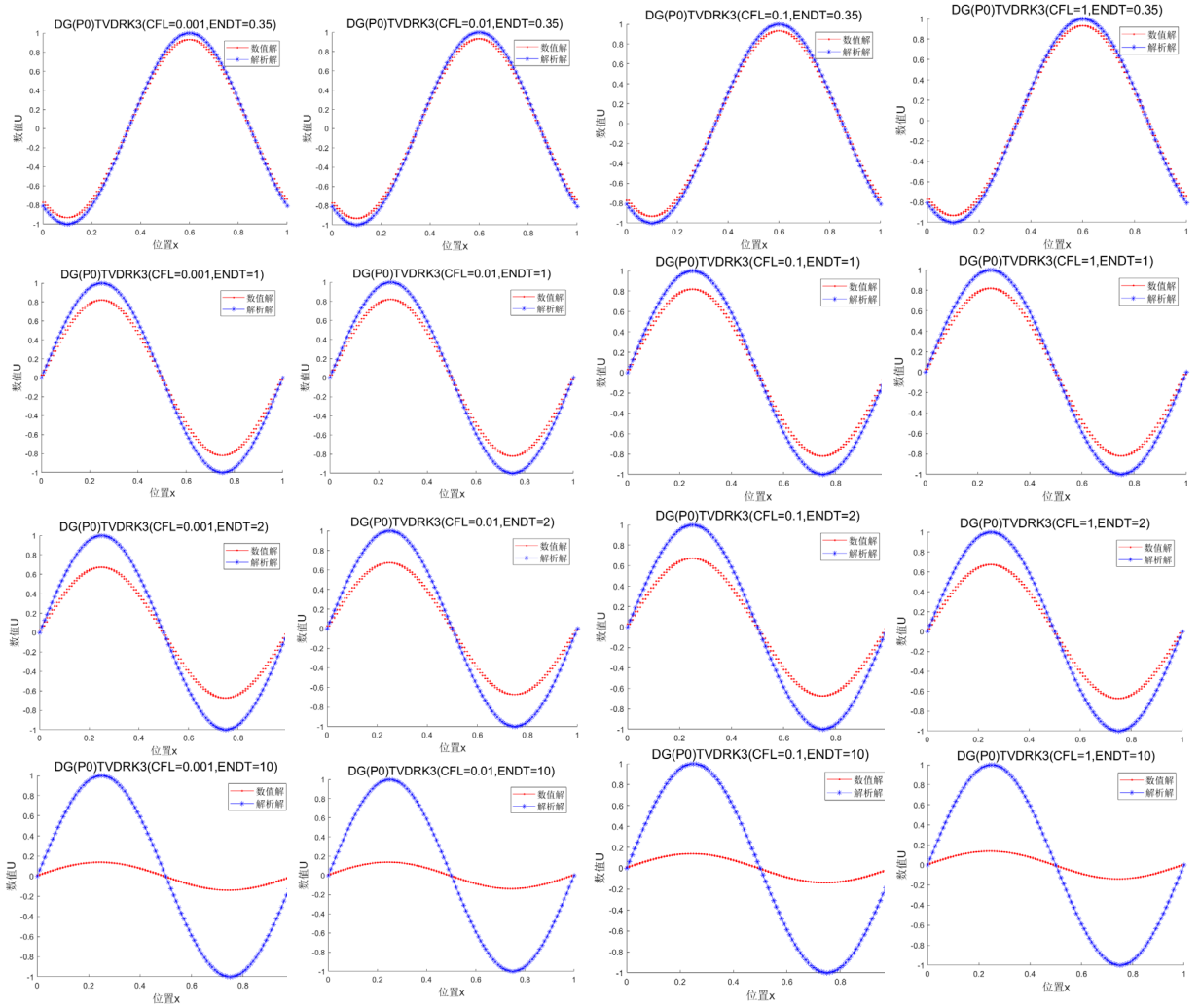

图 2：TVDRK3 推进格式下 DG(P0)的数值解与解析解比较图

(2)rDG(P0P1)显式欧拉时间推进

表 3：显式欧拉推进格式下 rDG(P0P1)的空间精度值 e

CFL / ENDT	0.001	1
0.35	0.0074	0.0507
10	0.0164	Inf

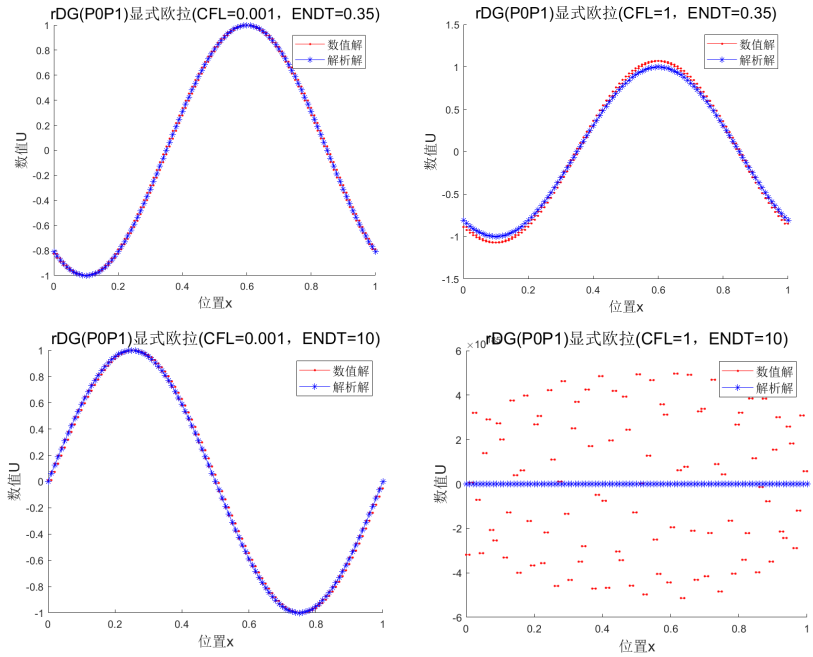

图 3：显式欧拉推进格式下 rDG(P0P1)的数值解与解析解比较图

rDG(P0P1)-TVDRK3 时间推进

表 4：TVDRK3 推进格式下 rDG(P0P1)的空间精度值 e

CFL / ENDT	0.001	1
0.35	0.0496	1.04e+04
10	0.8604	Inf

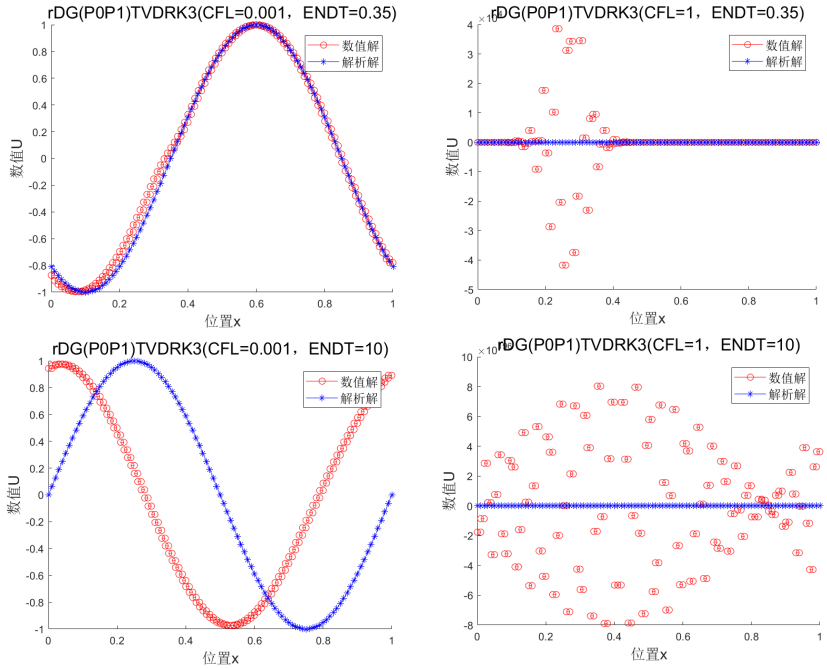

图 4：TVDRK3 推进格式下 rDG(P0P1)的数值解与解析解比较图

(3)DG(P1)显式欧拉时间推进

表 5：显式欧拉推进格式下 DG(P1)的空间精度值 e

CFL / ENDT	0.001	1
0.35	0.1200	6.84e+20
2	0.1200	1.4438e+136

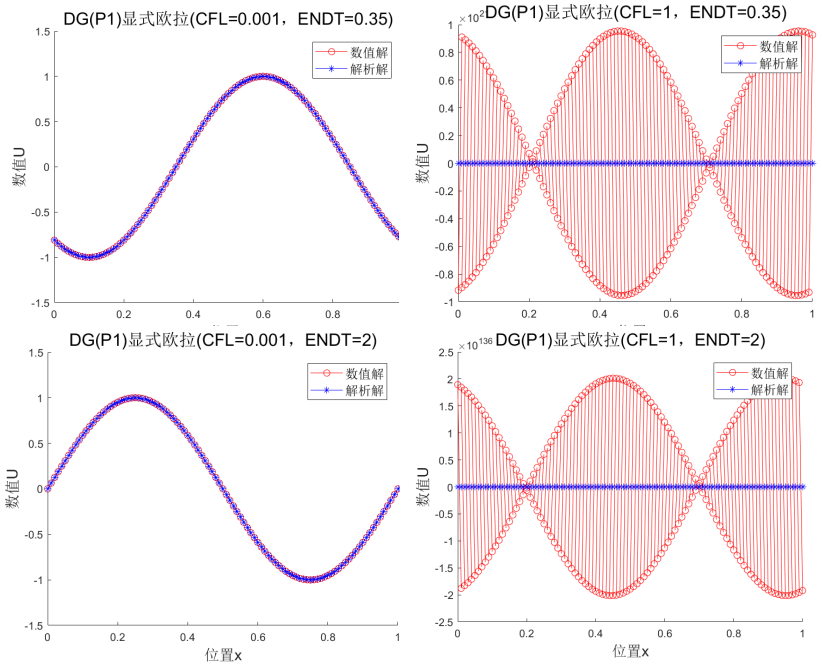

图 5：显式欧拉推进格式下 DG(P1)的数值解与解析解比较图

DG(P1)-TVDRK3 时间推进

表 6：TVDRK3 推进格式下 DG(P1)的空间精度值 e

CFL / ENDT	0.001	1
0.35	0.1200	6.84e+20
2	0.1200	1.4438e+136

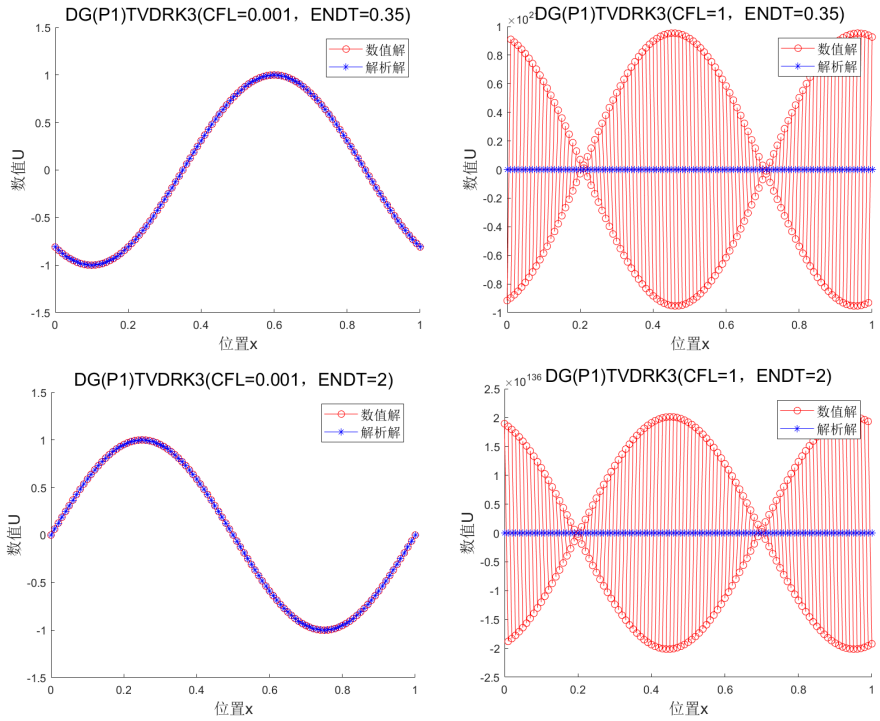

图 6：TVDRK3 推进格式下 DG(P1)的数值解与解析解比较图

(4)rDG(P1P2)显式欧拉时间推进

表 7：显式欧拉推进格式下 rDG(P1P2)的空间精度值 e

CFL / ENDT	0.001	1
0.35	0.1200	6.84e+20
2	0.1201	1.3946e+136

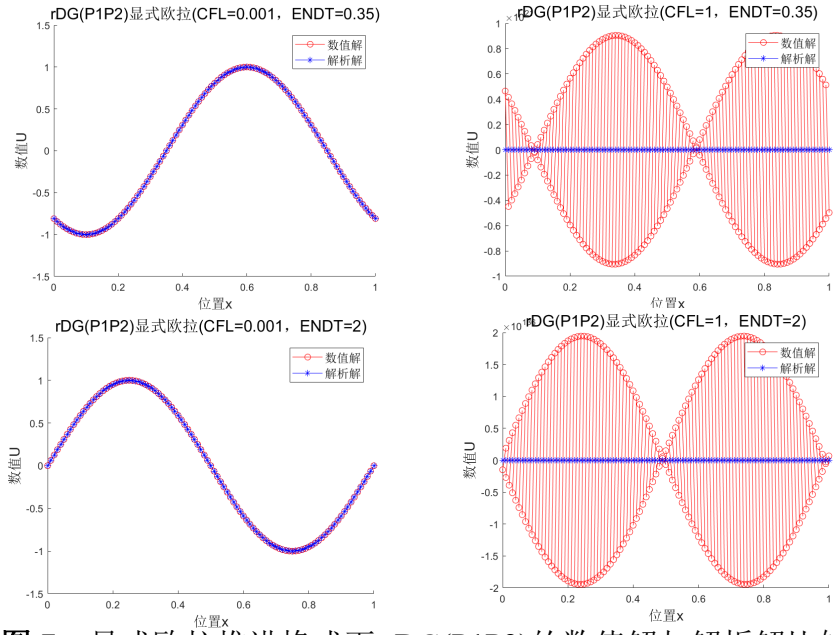

图 7：显式欧拉推进格式下 rDG(P1P2)的数值解与解析解比较图

rDG(P1P2)-TVDRK3 时间推进

表 8：TVDRK3 推进格式下 DG(P1)的空间精度值 e

CFL / ENDT	0.001	1
0.35	0.1200	6.84e+20
2	0.1201	1.3946e+136

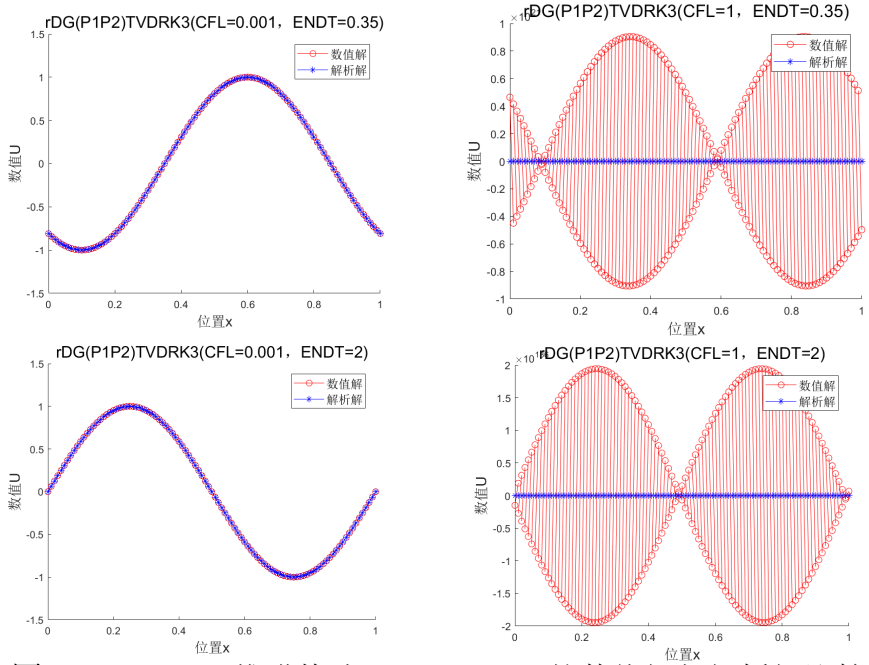

图 8：TVDRK3 推进格式下 rDG(P1P2)的数值解与解析解比较图

附录

DGP0-显式欧拉

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(1,numberx-1);
Unext=zeros(1,numberx-1);
Unumsolution=zeros(1,numberx-1);
Uexasolution=zeros(1,numberx);
Unumsolution1=zeros(1,2);
B=zeros(2,numberx-1);

%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(1,k)=0.5*(sin(2*pi*x)+sin(2*pi*(x+deltx)));
    k=k+1;
end

%solve the numsolution
for n=deltt:deltt:endt
    for k=2:numberx-1
        Unext(1,k)=Ucurrent(1,k)+CFL*(Ucurrent(1,k-1)-Ucurrent(1,k));
    end
    Unext(1,1)=Ucurrent(1,1)+CFL*(Ucurrent(1,numberx-1)-Ucurrent(1,1));Ucurrent=Unext;
end
Unumsolution=Ucurrent;

%solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(2*pi*(x-endt));
    k=k+1;
end

%% post-processing
%calculate the exact value
 figure
```

```
 hold on
 x=0*deltx:deltx:1*deltx;
 Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(1,1);
 plot(x,Unumsolution1,'-r.');hold on
 H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;
      Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(1,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0)显式欧拉(CFL=1，ENDT=0.35)','fontsize',16)
hold off

%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-Unumsolution(1,m))^2;k=k+1;
            end
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```

DGP0-TVDRK3

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
```

```
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(1,numberx-1);
Unext=zeros(1,numberx-1);
Unumsolution=zeros(1,numberx-1);
Uexasolution=zeros(1,numberx);
Unumsolution1=zeros(1,2);
Unk=zeros(2,numberx-1);
Uhold=zeros(1,numberx-1);
afa=[0,1/4,2/3];beta=[1,1/4,2/3];gama=[1,3/4,1/3];
%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(1,k)=0.5*(sin(2*pi*x)+sin(2*pi*(x+deltx)));
    k=k+1;
end
Unk(1,:)=Ucurrent(1,:);Uhold(1,:)=Ucurrent(1,:);

%solve the numsolution DRK3
for n=deltt:deltt:endt
    for istage=1:3

Unk(2,1)=gama(istage)*Unk(1,1)+afa(istage)*Uhold(1,1)+beta(istage)*deltt*(Uhold(1,numberx-1)-
Uhold(1,1))/deltx;
    for k=2:numberx-1
        Unk(2,k)=gama(istage)*Unk(1,k)+afa(istage)*Uhold(1,k)+beta(istage)*deltt*(Uhold(1,k-1)-
Uhold(1,k))/deltx;
    end
    Uhold(1,:)=Unk(2,:);
    end
    Unext(1,:)=Uhold(1,:);Ucurrent=Unext;Unk(1,:)=Unext(1,:);Uhold(1,:)=Unext(1,:);
end
Unumsolution=Ucurrent;

%solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(2*pi*(x-endt));
    k=k+1;
end

%% post-processing
%calculate the exact value
```

```
 figure
 hold on
 x=0*deltx:deltx:1*deltx;
 Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(1,1);
 plot(x,Unumsolution1,'-r.');hold on
 H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;
      Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(1,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0)TVDRK3(CFL=1,ENDT=0.35)','fontsize',16)
hold off

%calculate the accuracy of space "高斯勒让德积分"
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-Unumsolution(1,m))^2;k=k+1;
            end
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```

rDGP0P1-显式欧拉

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(1,numberx-1);
Unext=zeros(1,numberx-1);
Unumsolution=zeros(1,numberx-1);
Uexasolution=zeros(1,numberx);
Uxr=zeros(1,numberx-1);
Unumsolution1=zeros(1,2);
B=zeros(2,numberx-1);

%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(1,k)=0.5*(sin(2*pi*x)+sin(2*pi*(x+deltx)));
    k=k+1;
end
for k=2:numberx-2
    Uxr(1,k)=(Ucurrent(1,k+1)-Ucurrent(1,k-1))/(2*deltx);
end
Uxr(1,1)=(Ucurrent(1,2)-Ucurrent(1,numberx-1))/(2*deltx);Uxr(1,numberx-1)=(Ucurrent(1,1)-
Ucurrent(1,numberx-2))/(2*deltx);

%solve the numsolution
for n=deltt:deltt:endt
    for k=2:numberx-1
        f2=Ucurrent(k)+0.5*Uxr(k)*deltx;
        f1=Ucurrent(k-1)+0.5*Uxr(k-1)*deltx;
        R=f1-f2;
       Unext(1,k)=Ucurrent(1,k)+CFL*R;
    end
    Unext(1,1)=Ucurrent(1,1)+CFL*(Ucurrent(numberx-1)+0.5*Uxr(numberx-1)*deltx-Ucurrent(1)-
0.5*Uxr(1)*deltx);Ucurrent=Unext;
    for k=2:numberx-2
    Uxr(1,k)=(Ucurrent(1,k+1)-Ucurrent(1,k-1))/(2*deltx);
    end
    Uxr(1,1)=(Ucurrent(1,2)-Ucurrent(1,numberx-1))/(2*deltx);Uxr(1,numberx-1)=(Ucurrent(1,1)-
Ucurrent(1,numberx-2))/(2*deltx);
```

```
end
Unumsolution=Ucurrent;

%solve the exasolution
k=1;
for x=0:deltx:endx
      Uexasolution(1,k)=sin(2*pi*(x-endt));
      k=k+1;
end

%% post-processing
%calculate the exact value
 figure
 hold on
 x=0*deltx:deltx:1*deltx;
 Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(1,1);
 plot(x,Unumsolution1,'-r.');hold on
 H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;
      Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(1,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('rDG(P0P1)显式欧拉(CFL=1，ENDT=0.35)','fontsize',16)
hold off

%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-Unumsolution(1,m))^2;k=k+1;
            end
```

```
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```

rDGP0P1- TVDRK3

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(1,numberx-1);
Unext=zeros(1,numberx-1);
Unumsolution=zeros(1,numberx-1);
Uexasolution=zeros(1,numberx);
Uxr=zeros(1,numberx-1);
Unumsolution1=zeros(1,2);
Unk=zeros(2,numberx-1);%store the intermediate quantity of TVDRK3
Uhold=zeros(1,numberx-1);
afa=[0,1/4,2/3];beta=[1,1/4,2/3];gama=[1,3/4,1/3];

%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(1,k)=0.5*(sin(2*pi*x)+sin(2*pi*(x+deltx)));
    k=k+1;
end
for k=2:numberx-2
    Uxr(1,k)=(Ucurrent(1,k+1)-Ucurrent(1,k-1))/(2*deltx);
end
Uxr(1,1)=(Ucurrent(1,2)-Ucurrent(1,numberx-1))/(2*deltx);Uxr(1,numberx-1)=(Ucurrent(1,1)-
Ucurrent(1,numberx-2))/(2*deltx);
Unk(1,:)=Ucurrent(1,:);Uhold(1,:)=Ucurrent(1,:);

%solve the numsolution TVDRK3
for n=deltt:deltt:endt
    for istage=1:3
```

```
Unk(2,1)=gama(istage)*Unk(1,1)+afa(istage)*Uhold(1,1)+beta(istage)*deltt*(Ucurrent(numberx-
1)+0.5*Uxr(numberx-1)*deltx-Ucurrent(1)+0.5*Uxr(1)*deltx)/deltx;
      for k=2:numberx-1
            f2=Ucurrent(k)+0.5*Uxr(k)*deltx;
            f1=Ucurrent(k-1)+0.5*Uxr(k-1)*deltx;
            R=f1-f2;
          Unk(2,k)=gama(istage)*Unk(1,k)+afa(istage)*Uhold(1,k)+beta(istage)*deltt*R/deltx;
      end
      Uhold(1,:)=Unk(2,:);
      end
      Unext(1,:)=Uhold(1,:);Ucurrent=Unext;Unk(1,:)=Unext(1,:);Uhold(1,:)=Unext(1,:);
      for k=2:numberx-2
      Uxr(1,k)=(Ucurrent(1,k+1)-Ucurrent(1,k-1))/(2*deltx);
      end
      Uxr(1,1)=(Ucurrent(1,2)-Ucurrent(1,numberx-1))/(2*deltx);Uxr(1,numberx-1)=(Ucurrent(1,1)-
Ucurrent(1,numberx-2))/(2*deltx);
end
Unumsolution=Ucurrent;

%solve the exasolution
k=1;
for x=0:deltx:endx
      Uexasolution(1,k)=sin(2*pi*(x-endt));
      k=k+1;
end

%% post-processing
%calculate the exact value
 figure
 hold on
 x=0*deltx:deltx:1*deltx;
 Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(1,1);
 plot(x,Unumsolution1,'-ro');hold on
 H1=plot(x,Unumsolution1,'-ro');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;
      Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(1,i);
      plot(x,Unumsolution1,'-ro')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
```

```
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('rDG(P0P1)TVDRK3(CFL=1，ENDT=0.35)','fontsize',16)
hold off

%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-Unumsolution(1,m))^2;k=k+1;
            end
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```

DGP1-显式欧拉

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(1,numberx);
M=[deltx,0;0,deltx/12];
R=zeros(2,1);R1=zeros(2,1);
Unumsolution1=zeros(1,2);
%% solve the question
%initial    condition set up(dimensionless)
%Uc
k=1;
for x=0:deltx:endx-deltx
   Ucurrent(1,k)=sin(2*pi*(x+deltx/2));
      k=k+1;
end
%Uxc
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=2*pi*cos(2*pi*(x+deltx/2))*deltx;
     k=k+1;
end

%solve the numsolution
for n=deltt:deltt:endt
      for k=2:numberx-1
            f1=Ucurrent(1,k-1)+Ucurrent(2,k-1)/2;
            f2=Ucurrent(1,k)+Ucurrent(2,k)/2;
            R(1,1)=f1-f2;
            R(2,1)=-0.5*(f1+f2)+Ucurrent(1,k);
            Unext(:,k)=Ucurrent(:,k)+M\R*deltt;
      end
      R1(1,1)=Ucurrent(1,numberx-1)+Ucurrent(2,numberx-1)/2-Ucurrent(1,1)-Ucurrent(2,1)/2;
      R1(2,1)=-0.5*(Ucurrent(1,numberx-1)+Ucurrent(2,numberx-
1)/2+Ucurrent(1,1)+Ucurrent(2,1)/2)+Ucurrent(1,1);
      Unext(:,1)=Ucurrent(:,1)+M\R1*deltt;
   Ucurrent=Unext;
```

```
end

for i=1:numberx-1
Unumsolution(1,i)=Ucurrent(1,i)+Ucurrent(2,i)*(-1/2);
Unumsolution(2,i)=Ucurrent(1,i)+Ucurrent(2,i)*(1/2);
end

%solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(2*pi*(x-endt));
    k=k+1;
end


%% post-processing
figure
hold on
x=0*deltx:deltx:1*deltx;
Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(2,1);
plot(x,Unumsolution1,'-ro');hold on
H1=plot(x,Unumsolution1,'-ro');hold on
for i=2:numberx-1
    x=(i-1)*deltx:deltx:i*deltx;
    Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(2,i);
    plot(x,Unumsolution1,'-ro')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P1)显式欧拉(CFL=1，ENDT=0.35)','fontsize',16)
hold off


%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
for x=0:deltx:endx-deltx
  for i=1:3
      xi=deltx/2*t(i)+0.5*(2*x+deltx);
```

```
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-((m-
1)*deltx+deltx/2))))^2;
            end
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```

DGP1-TVDRK3

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=0.001;deltt=CFL*deltx;
endx=1;endt=5;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(1,numberx);
M=[deltx,0;0,deltx/12];
R=zeros(2,1);R1=zeros(2,1);
Unumsolution1=zeros(1,2);
Unk=zeros(4,numberx-1);%store the intermediate quantity of TVDRK3
Uhold=zeros(2,numberx-1);
afa=[0,1/4,2/3];beta=[1,1/4,2/3];gama=[1,3/4,1/3];
%% solve the question
%initial    condition set up(dimensionless)
%Uc
k=1;
for x=0:deltx:endx-deltx
   Ucurrent(1,k)=sin(2*pi*(x+deltx/2));
      k=k+1;
end
%Uxc
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=2*pi*cos(2*pi*(x+deltx/2))*deltx;
      k=k+1;
end
```

```
Unk([1,2],:)=Ucurrent;Uhold=Ucurrent;

%solve the numsolution TVDRK3
for n=deltt:deltt:endt
    for istage=1:3
        R1(1,1)=Ucurrent(1,numberx-1)+Ucurrent(2,numberx-1)/2-Ucurrent(1,1)-Ucurrent(2,1)/2;
        R1(2,1)=-0.5*(Ucurrent(1,numberx-1)+Ucurrent(2,numberx-
1)/2+Ucurrent(1,1)+Ucurrent(2,1)/2)+Ucurrent(1,1);

Unk([3,4],1)=gama(istage)*Unk([1,2],1)+afa(istage)*Uhold([1,2],1)+M\R1*beta(istage)*deltt;
    for k=2:numberx-1
        f2=Ucurrent(1,k)+Ucurrent(2,k)/2;
        f1=Ucurrent(1,k-1)+Ucurrent(2,k-1)/2;
        R(1,1)=f1-f2;
        R(2,1)=-0.5*(f1+f2)+Ucurrent(1,k);
       Unk([3,4],k)=gama(istage)*Unk([1,2],k)+afa(istage)*Uhold([1,2],k)+M\R*beta(istage)*deltt;
    end
    Uhold([1,2],:)=Unk([3,4],:);
    end

Unext([1,2],:)=Uhold([1,2],:);Ucurrent=Unext;Unk([1,2],:)=Unext([1,2],:);Uhold([1,2],:)=Unext([1,2]
,:);

end

for i=1:numberx-1
Unumsolution(1,i)=Ucurrent(1,i)+Ucurrent(2,i)*(-1/2);
Unumsolution(2,i)=Ucurrent(1,i)+Ucurrent(2,i)*(1/2);
end

%solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(2*pi*(x-endt));
    k=k+1;
end


%% post-processing
figure
hold on
x=0*deltx:deltx:1*deltx;
Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(2,1);
plot(x,Unumsolution1,'-ro');hold on
```

```
H1=plot(x,Unumsolution1,'-ro');hold on
for i=2:numberx-1
    x=(i-1)*deltx:deltx:i*deltx;
    Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(2,i);
    plot(x,Unumsolution1,'-ro')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P1)TVDRK3(CFL=1，ENDT=10)','fontsize',16)
hold off


%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-((m-
1)*deltx+deltx/2))))^2;
            end
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```

rDGP1P2-显式欧拉

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Uxx=zeros(1,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(1,numberx);
M=[deltx,0;0,deltx/12];
R=zeros(2,1);R1=zeros(2,1);
Unumsolution1=zeros(1,2);
A=[1/12;1/2;1/12;-1/2];
%% solve the question
%initial    condition set up(dimensionless)
%Uc
k=1;
for x=0:deltx:endx-deltx
   Ucurrent(1,k)=(cos(2*pi*x)-cos(2*pi*(x+deltx)))/(2*pi*deltx);
      k=k+1;
end
%Uxc
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=2*pi*cos(2*pi*(x+deltx/2))*deltx;
      k=k+1;
end

%Uxxc
for k=2:numberx-2
      b=[Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1)-Ucurrent(1,k)-0.5*Ucurrent(2,k);
            Ucurrent(2,k+1)-Ucurrent(2,k);
            Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1)-Ucurrent(1,k)+0.5*Ucurrent(2,k);
            Ucurrent(2,k-1)-Ucurrent(2,k)];
      Uxx(k)=A\b;
end
b=[Ucurrent(1,2)-0.5*Ucurrent(2,2)-Ucurrent(1,1)-0.5*Ucurrent(2,1);
            Ucurrent(2,2)-Ucurrent(2,1);
            Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-1)-Ucurrent(1,1)+0.5*Ucurrent(2,1);
            Ucurrent(2,numberx-1)-Ucurrent(2,1)];
```

```
Uxx(1)=A\b;
b=[Ucurrent(1,1)-0.5*Ucurrent(2,1)-Ucurrent(1,numberx-1)-0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,1)-Ucurrent(2,numberx-1);
        Ucurrent(1,numberx-2)+0.5*Ucurrent(2,numberx-2)-Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,numberx-2)-Ucurrent(2,numberx-1)];
Uxx(numberx-1)=A\b;

%solve the numsolution
for n=deltt:deltt:endt
    for k=2:numberx-1
        f1=Ucurrent(1,k-1)+Ucurrent(2,k-1)/2+1/12*Uxx(k-1);
        f2=Ucurrent(1,k)+Ucurrent(2,k)/2+1/12*Uxx(k);
        R(1,1)=f1-f2;
        R(2,1)=-0.5*(f1+f2)+Ucurrent(1,k);
        Unext(:,k)=Ucurrent(:,k)+M\R*deltt;
    end
    R1(1,1)=Ucurrent(1,numberx-1)+Ucurrent(2,numberx-1)/2+1/12*Uxx(numberx-1)-Ucurrent(1,1)-
Ucurrent(2,1)/2-1/12*Uxx(1);
    R1(2,1)=-0.5*(Ucurrent(1,numberx-1)+Ucurrent(2,numberx-1)/2+1/12*Uxx(numberx-
1)+Ucurrent(1,1)+Ucurrent(2,1)/2+1/12*Uxx(1))+Ucurrent(1,1);
    Unext(:,1)=Ucurrent(:,1)+M\R1*deltt;
    Ucurrent=Unext;
    for k=2:numberx-2
    b=[Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1)-Ucurrent(1,k)-0.5*Ucurrent(2,k);
        Ucurrent(2,k+1)-Ucurrent(2,k);
        Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1)-Ucurrent(1,k)+0.5*Ucurrent(2,k);
        Ucurrent(2,k-1)-Ucurrent(2,k)];
    Uxx(k)=A\b;
    end
    b=[Ucurrent(1,2)-0.5*Ucurrent(2,2)-Ucurrent(1,1)-0.5*Ucurrent(2,1);
        Ucurrent(2,2)-Ucurrent(2,1);
        Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-1)-Ucurrent(1,1)+0.5*Ucurrent(2,1);
        Ucurrent(2,numberx-1)-Ucurrent(2,1)];
    Uxx(1)=A\b;
    b=[Ucurrent(1,1)-0.5*Ucurrent(2,1)-Ucurrent(1,numberx-1)-0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,1)-Ucurrent(2,numberx-1);
        Ucurrent(1,numberx-2)+0.5*Ucurrent(2,numberx-2)-Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,numberx-2)-Ucurrent(2,numberx-1)];
    Uxx(numberx-1)=A\b;
end

for i=1:numberx-1
```

```
Unumsolution(1,i)=Ucurrent(1,i)+Ucurrent(2,i)*(-1/2);
Unumsolution(2,i)=Ucurrent(1,i)+Ucurrent(2,i)*(1/2);
end

%solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(2*pi*(x-endt));
    k=k+1;
end


%% post-processing
figure
hold on
x=0*deltx:deltx:1*deltx;
Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(2,1);
plot(x,Unumsolution1,'-ro');hold on
H1=plot(x,Unumsolution1,'-ro');hold on
for i=2:numberx-1
    x=(i-1)*deltx:deltx:i*deltx;
    Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(2,i);
    plot(x,Unumsolution1,'-ro')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('rDG(P1P2)显式欧拉(CFL=1，ENDT=0.35)','fontsize',16)
hold off


%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
for x=0:deltx:endx-deltx
   for i=1:3
       xi=deltx/2*t(i)+0.5*(2*x+deltx);
       for m=1:numberx-1
           if xi>(m-1)*deltx&&xi<m*deltx
```

```
                    fi=(sin(2*pi*(xi-endt))-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-((m-
1)*deltx+deltx/2))))^2;
                end
            end
            I=I+W(i)*fi;
        end
    end
I=I*0.5*deltx;
I=sqrt(I)
```

rDGP1P2- TVDRK3

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=0.001;deltt=CFL*deltx;
endx=1;endt=10;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Uxx=zeros(1,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(1,numberx);
M=[deltx,0;0,deltx/12];
R=zeros(2,1);R1=zeros(2,1);
Unumsolution1=zeros(1,2);
Unk=zeros(4,numberx-1);%store the intermediate quantity of TVDRK3
Uhold=zeros(2,numberx-1);
A=[1/12;1/2;1/12;-1/2];
afa=[0,1/4,2/3];beta=[1,1/4,2/3];gama=[1,3/4,1/3];
%% solve the question
%initial    condition set up(dimensionless)
%Uc
k=1;
for x=0:deltx:endx-deltx
   Ucurrent(1,k)=(cos(2*pi*x)-cos(2*pi*(x+deltx)))/(2*pi*deltx);
      k=k+1;
end

%Uxc
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=2*pi*cos(2*pi*(x+deltx/2))*deltx;
      k=k+1;
```

```
end

%Uxxc
for k=2:numberx-2
    b=[Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1)-Ucurrent(1,k)-0.5*Ucurrent(2,k);
        Ucurrent(2,k+1)-Ucurrent(2,k);
        Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1)-Ucurrent(1,k)+0.5*Ucurrent(2,k);
        Ucurrent(2,k-1)-Ucurrent(2,k)];
    Uxx(k)=A\b;
end
b=[Ucurrent(1,2)-0.5*Ucurrent(2,2)-Ucurrent(1,1)-0.5*Ucurrent(2,1);
        Ucurrent(2,2)-Ucurrent(2,1);
        Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-1)-Ucurrent(1,1)+0.5*Ucurrent(2,1);
        Ucurrent(2,numberx-1)-Ucurrent(2,1)];
Uxx(1)=A\b;
b=[Ucurrent(1,1)-0.5*Ucurrent(2,1)-Ucurrent(1,numberx-1)-0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,1)-Ucurrent(2,numberx-1);
        Ucurrent(1,numberx-2)+0.5*Ucurrent(2,numberx-2)-Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,numberx-2)-Ucurrent(2,numberx-1)];
Uxx(numberx-1)=A\b;

Unk([1,2],:)=Ucurrent;Uhold=Ucurrent;%store

%solve the numsolution TVDRK3
for n=deltt:deltt:endt
    for istage=1:3
        R1(1,1)=Ucurrent(1,numberx-1)+Ucurrent(2,numberx-1)/2+1/12*Uxx(numberx-1)-
Ucurrent(1,1)-Ucurrent(2,1)/2-1/12*Uxx(1);
        R1(2,1)=-0.5*(Ucurrent(1,numberx-1)+Ucurrent(2,numberx-1)/2+1/12*Uxx(numberx-
1)+Ucurrent(1,1)+Ucurrent(2,1)/2+1/12*Uxx(1))+Ucurrent(1,1);

Unk([3,4],1)=gama(istage)*Unk([1,2],1)+afa(istage)*Uhold([1,2],1)+M\R1*beta(istage)*deltt;
    for k=2:numberx-1
        f2=Ucurrent(1,k)+Ucurrent(2,k)/2+1/12*Uxx(k);
        f1=Ucurrent(1,k-1)+Ucurrent(2,k-1)/2+1/12*Uxx(k-1);
        R(1,1)=f1-f2;
        R(2,1)=-0.5*(f1+f2)+Ucurrent(1,k);
       Unk([3,4],k)=gama(istage)*Unk([1,2],k)+afa(istage)*Uhold([1,2],k)+M\R*beta(istage)*deltt;
    end
    Uhold([1,2],:)=Unk([3,4],:);
    end
```

```
Unext([1,2],:)=Uhold([1,2],:);Ucurrent=Unext;Unk([1,2],:)=Unext([1,2],:);Uhold([1,2],:)=Unext([1,2]
,:);
    %update Uxxc
    for k=2:numberx-2
    b=[Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1)-Ucurrent(1,k)-0.5*Ucurrent(2,k);
        Ucurrent(2,k+1)-Ucurrent(2,k);
        Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1)-Ucurrent(1,k)+0.5*Ucurrent(2,k);
        Ucurrent(2,k-1)-Ucurrent(2,k)];
    Uxx(k)=A\b;
    end
    b=[Ucurrent(1,2)-0.5*Ucurrent(2,2)-Ucurrent(1,1)-0.5*Ucurrent(2,1);
        Ucurrent(2,2)-Ucurrent(2,1);
        Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-1)-Ucurrent(1,1)+0.5*Ucurrent(2,1);
        Ucurrent(2,numberx-1)-Ucurrent(2,1)];
    Uxx(1)=A\b;
    b=[Ucurrent(1,1)-0.5*Ucurrent(2,1)-Ucurrent(1,numberx-1)-0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,1)-Ucurrent(2,numberx-1);
        Ucurrent(1,numberx-2)+0.5*Ucurrent(2,numberx-2)-Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1);
        Ucurrent(2,numberx-2)-Ucurrent(2,numberx-1)];
    Uxx(numberx-1)=A\b;
end


for i=1:numberx-1
Unumsolution(1,i)=Ucurrent(1,i)+Ucurrent(2,i)*(-1/2);
Unumsolution(2,i)=Ucurrent(1,i)+Ucurrent(2,i)*(1/2);
end

%solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(2*pi*(x-endt));
    k=k+1;
end


%% post-processing
figure
hold on
x=0*deltx:deltx:1*deltx;
Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(2,1);
plot(x,Unumsolution1,'-ro');hold on
```

```
H1=plot(x,Unumsolution1,'-ro');hold on
for i=2:numberx-1
    x=(i-1)*deltx:deltx:i*deltx;
    Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(2,i);
    plot(x,Unumsolution1,'-ro')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('rDG(P1P2)TVDRK3(CFL=0.001，ENDT=10)','fontsize',16)
hold off


%calculate the accuracy of space
I=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(2*pi*(xi-endt))-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-((m-
1)*deltx+deltx/2))))^2;
            end
        end
        I=I+W(i)*fi;
    end
end
I=I*0.5*deltx;
I=sqrt(I)
```