

# 课题组组会-练习 4

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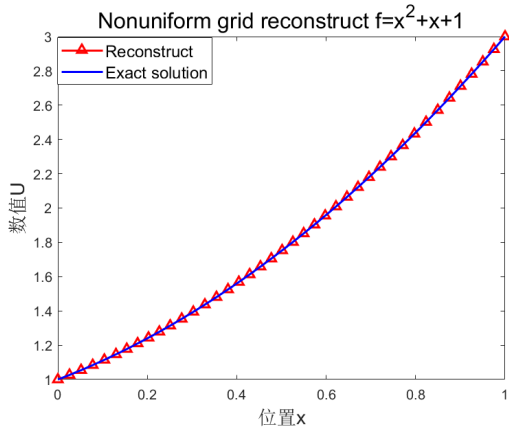
## 一 练习及结果

1. 在  $x \in [0, 1]$  的均匀网格上尝试使用 Least Squares Recovery(LSR), Least Squares Reconstruction(LSr) 和 Hybrid Least Squares Reconstruction(HLSr) 对  $f(x)$  及  $g(x)$  进行 P1P2 重构, 其中  $f(x) = 1 + x + x^2, g(x) = \sin(\pi x)$ , 测试重构精度。如果网格为不均匀网格呢?

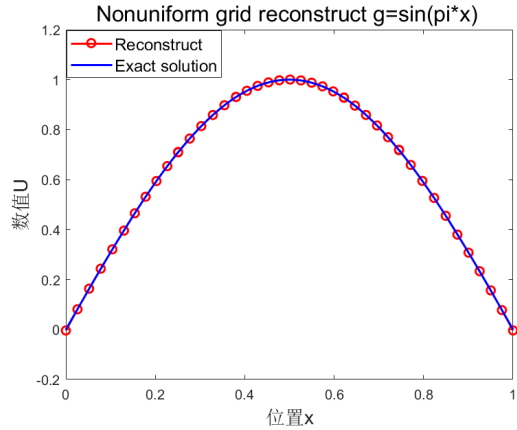
2. 在  $x \in [0, 1]$  的均匀网格上尝试使用 Hybrid Least Squares Reconstruction(HLSr) 对  $f(x)$  及  $g(x)$  进行 Hyperbolic rDG 的 DG(P0P2)+rDG(P0P1) 重构, 其中  $f(x) = 1 + x + x^2 + x^3, g(x) = 1 + x + x^2 + x^3 + x^4, h(x) = \sin(\pi x)$ , 测试重构精度。如果网格为不均匀网格呢?

解:

(1) 分别用三种重构方法对  $f(x)$  与  $g(x)$  进行 P1P2 重构, 下面展示 LSR 方法的重构图 (非均匀网格):



(a) f 重构



(b) g 重构

图 1: LSR

LSr 与 HLSr 的重构图与图 1 类似, 这里不重复展示。

下面展示不同网格下  $f(x)$  与  $g(x)$  的精度比较图:

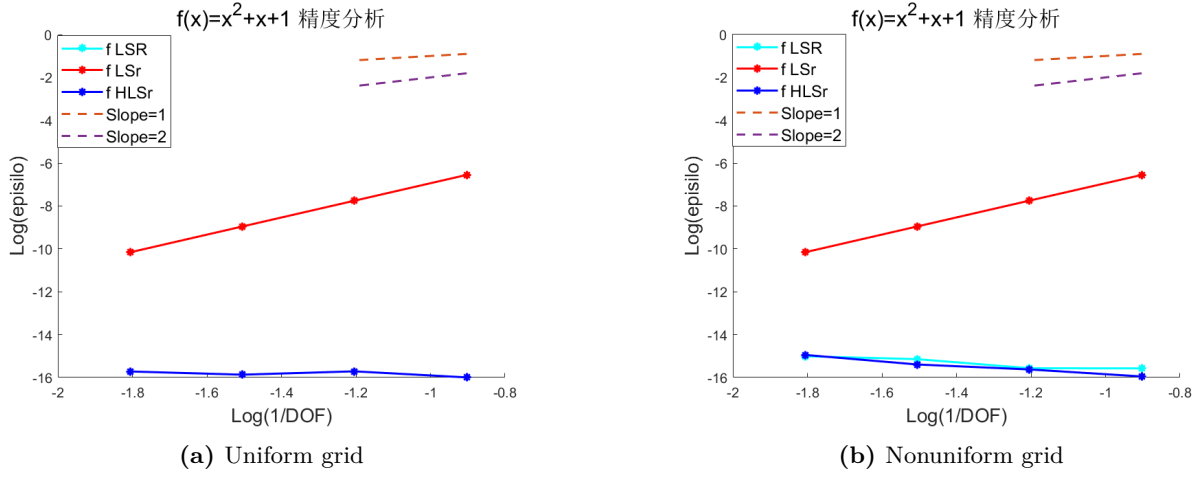


图 2:  $f$  精度分析

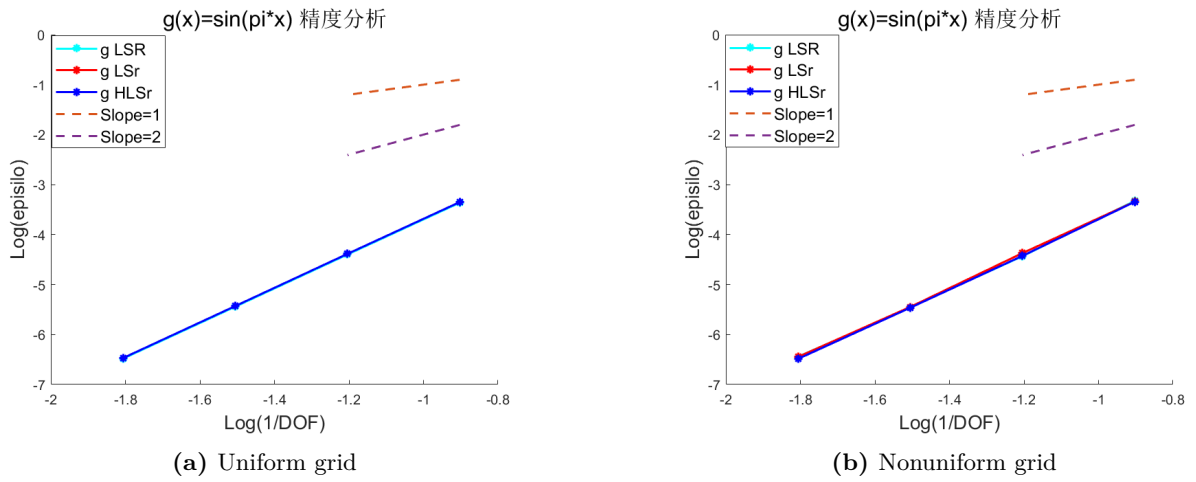


图 3:  $g$  精度分析

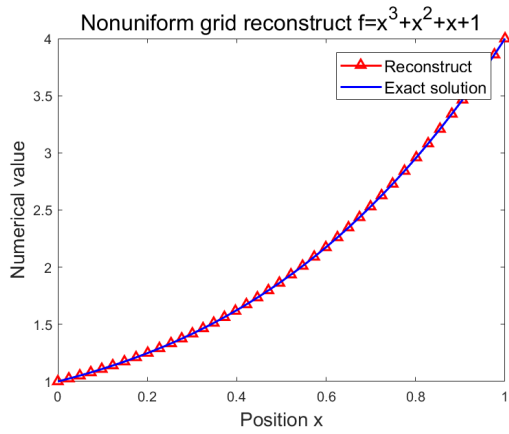
注意: 虽然 LSR 与 HLSr 方法下  $f(x)$  没有 Order, 但其误差值达到了机器误差。下述表格能更直观地看出各个方法之间的差别。

表 1: 非均匀网格下各方法比较

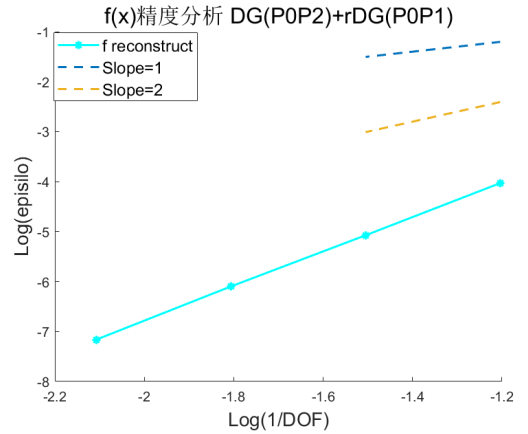
Num of cells	P1P2(LSR)		P1P2(LSr)		P1P2(HLSr)	
	L2-errors	order	L2-errors	order	L2-errors	order
case 1: $f(x) = 1 + x + x^2$						
8	1.70E-16		2.74E-8		1.56E-16	
16	2.95E-16	—	1.77E-8	3.95	2.17E-16	—
32	4.10E-16	—	1.11E-9	3.40	6.05E-16	—
64	1.00E-15	—	6.94E-11	4.00	9.37E-16	—
case 2: $g(x) = \sin(\pi x)$						
8	4.51E-4		4.13E-4		4.53E-4	
16	4.28E-5	3.40	4.06E-5	3.35	3.96E-5	3.52
32	3.81E-6	3.49	3.68E-6	3.46	3.48E-6	3.50
64	3.57E-7	3.41	3.37E-7	3.45	3.42E-7	3.35

(2) 在  $x \in [0, 1]$  上使用 Hybrid Least Squares Reconstruction(HLSr) 对  $f(x)$  及  $g(x)$  与进行  $h(x)$  Hyperbolic rDG 的 DG(P0P2)+rDG(P0P1) 重构, 分别对  $\varphi$  和  $v$  选取方程进行重构求解, 最终得到如下结果 (仅考虑非均匀网格, 均匀网格视为非均匀网格的一个特例):

A)  $f(x) = x^3 + x^2 + x + 1$



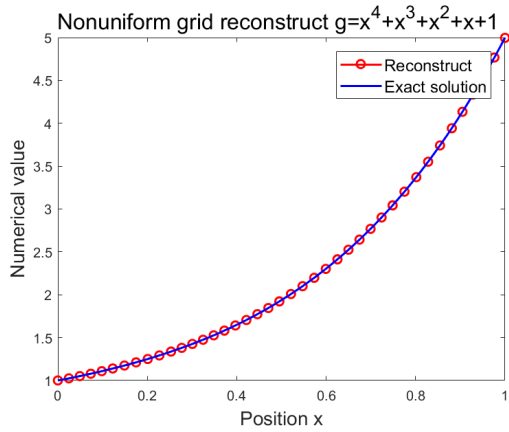
(a) 重构图



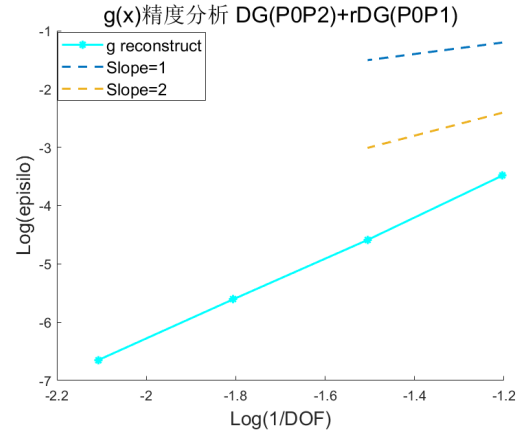
(b) 精度分析

图 4:  $f(x)$

**B)**  $g(x) = x^4 + x^3 + x^2 + x + 1$



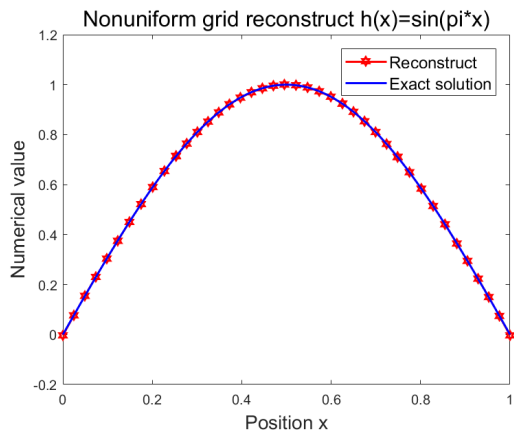
(a) 重构图



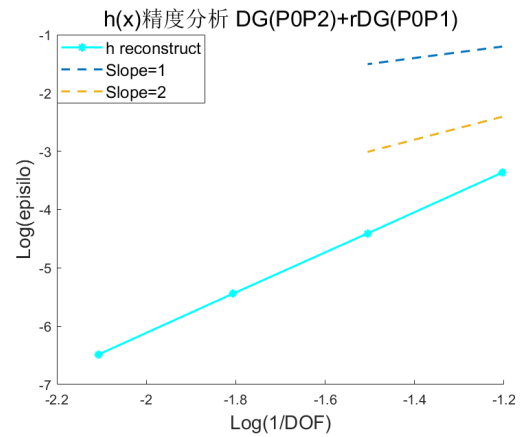
(b) 精度分析

图 5:  $g(x)$

**C)**  $h(x) = \sin(\pi x)$



(a) 重构图



(b) 精度分析

图 6:  $h(x)$

## 二 附录 (代码, 仅展示部分)

### LSR

```
1  clc
2  clear all
3  close all
4  %Unit=8;endx=1;deltax=endx/Unit;numberx=Unit+1;
5  %记录内点位置, 上下浮动不超过百分之 5
6  Grid=zeros(1, numberx);
7  for i=2:numberx-1
8  Grid(1, i)=(i-1)*deltax+(0.1*rand(1)-0.05)*deltax;
9  end
10 Grid(1, numberx)=endx;
11 f=@(x)x.^2+x+1;F=@(x)2*x+1;
12 g=@(y)sin(pi*y);G=@(y)pi*cos(pi*y);
13 Unumsolution=zeros(2, Unit);
14 Ureconstruct=zeros(1, Unit);
15 Unumsolution1=zeros(1, 2);
16 Unumsolution2=zeros(2, numberx-1);
17 Acc=zeros(3, 4); a1=[1/8, 1/16, 1/32, 1/64]; a2=[1/8, 1/16];
18 %for k=1:Unit
19 Unumsolution(1, k)=(Grid(k+1)-Grid(k)+0.5*(Grid(k+1)^2-Grid(k)^2)+(Grid(k
    +1)^3-Grid(k)^3)/3)/(Grid(k+1)-Grid(k));
20 Unumsolution(2, k)=F(0.5*(Grid(k)+Grid(k+1)))*(Grid(k+1)-Grid(k)); %store
    Uxc*deltax
21 end
22
23 %Reconstruct Uxx
24 for k=2:numberx-2
25 xci=0.5*(Grid(k)+Grid(k+1)); %xci
26 xci1=0.5*(Grid(k+1)+Grid(k+2)); %xci+1
27 xci2=0.5*(Grid(k-1)+Grid(k)); %xci-1
28 A=[(xci1-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k+2)-Grid(k+1))
    ^2/(Grid(k+1)-Grid(k))^2-1);
29 (Grid(k+2)-Grid(k+1))*(xci1-xci)/(Grid(k+1)-Grid(k))^2;
30 (xci2-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k)-Grid(k-1))^2/(Grid
    (k+1)-Grid(k))^2-1);
31 (Grid(k)-Grid(k-1))*(xci2-xci)/(Grid(k+1)-Grid(k))^2];
32 b=[Unumsolution(1, k+1)-Unumsolution(1, k)-Unumsolution(2, k)*(xci1-xci)/(
    Grid(k+1)-Grid(k));
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33 Unumsolution(2,k+1)-Unumsolution(2,k)*(Grid(k+2)-Grid(k+1))/(Grid(k+1)-
    Grid(k));
34 Unumsolution(1,k-1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci2-xci)/(Grid
    (k+1)-Grid(k));
35 Unumsolution(2,k-1)-Unumsolution(2,k)*(Grid(k)-Grid(k-1))/(Grid(k+1)-
    Grid(k));
36 Ureconstruct(k)=A\b;
37 end
38 %boundary
39 %Left
40 xci=0.5*(Grid(1)+Grid(2));%xci
41 xci1=0.5*(Grid(2)+Grid(3));%xci+1
42 A=[(xci1-xci)^2/(2*(Grid(2)-Grid(1))^2)+1/24*((Grid(3)-Grid(2))^2/(Grid
    (2)-Grid(1))^2-1);
43 (Grid(3)-Grid(2))*(xci1-xci)/(Grid(2)-Grid(1))^2];
44 b=[Unumsolution(1,2)-Unumsolution(1,1)-Unumsolution(2,1)*(xci1-xci)/(
    Grid(2)-Grid(1));
45 Unumsolution(2,2)-Unumsolution(2,1)*(Grid(3)-Grid(2))/(Grid(2)-Grid(1))
    ];
46 Ureconstruct(1)=A\b;
47 %Right
48 xci=0.5*(Grid(numberx-1)+Grid(numberx-1+1));%xci
49 xci2=0.5*(Grid(numberx-1-1)+Grid(numberx-1));%xci-1
50 A=[(xci2-xci)^2/(2*(Grid(numberx-1+1)-Grid(numberx-1))^2)+1/24*((Grid(
    numberx-1)-Grid(numberx-1-1))^2/(Grid(numberx-1+1)-Grid(numberx-1))
    ^2-1);
51 (Grid(numberx-1)-Grid(numberx-1-1))*(xci2-xci)/(Grid(numberx-1+1)-Grid(
    numberx-1))^2];
52 b=[Unumsolution(1,numberx-1-1)-Unumsolution(1,numberx-1)-Unumsolution(2,
    numberx-1)*(xci2-xci)/(Grid(numberx-1+1)-Grid(numberx-1));
53 Unumsolution(2,numberx-1-1)-Unumsolution(2,numberx-1)*(Grid(numberx-1)-
    Grid(numberx-1-1))/(Grid(numberx-1+1)-Grid(numberx-1))];
54 Ureconstruct(numberx-1)=A\b;
55
56 %Post-proceeding
57 figure
58 k=1;
59 x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
60 p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2),Unumsolution(2,k)/(Grid(k
    +1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(k)

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        k))^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))
        ^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k)
        )/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
61 y=polyval(p,x);
62 plot(x,y,'-r^','linewidth',1.5);hold on
63 H1=plot(x,y,'-r^','linewidth',1.5);hold on
64
65 for k=2:numberx-1
66 x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
67 p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2),Unumsolution(2,k)/(Grid(k
        +1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(k)
        )^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))
        ^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k)
        )/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
68 y=polyval(p,x);plot(x,y,'-r^','linewidth',1.5);
69 end
70
71 hold on
72 x=Grid(1):0.01*(Grid(numberx)-Grid(1)):Grid(numberx);
73 plot(x,f(x),'-b','linewidth',1.5);
74 H2=plot(x,f(x),'-b','linewidth',1.5);
75 lgd=legend([H1,H2],'Reconstruct','Exact solution');
76 lgd.FontSize=12;
77 xlabel('位置x','fontsize',14)
78 ylabel('数值U','fontsize',14)
79 title('Nonuniform grid reconstruct f=x^2+x+1','fontsize',16)
80 hold off
81
82 %计算精度
83 Acc(1,1)=Accuracy(8);Acc(1,2)=Accuracy(16);
84 Acc(1,3)=Accuracy(32);
85 Acc(1,4)=Accuracy(64);
86 for k=1:3
87 accuracyf(k)=(log10(Acc(1,k+1))-log10(Acc(1,k)))/(log10(a1(1,k+1))-
        log10(a1(1,k)));
88 end
89 figure
90 hold on
91 plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
92 H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);

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93
94 H2=plot(log10(a2),1*log10(a2),'--','linewidth',1.5);
95 plot(log10(a2),2*log10(a2),'--','linewidth',1.5)
96 H3=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
97 lgd=legend([H1,H2,H3],'f reconstruct','斜率为1','斜率为2');
98 lgd.FontSize=12;
99 xlabel('Log(deltax)','fontsize',14)
100 ylabel('Log(episilo)','fontsize',14)
101 title('f精度分析','fontsize',16)
102
103 %对 g
104 for k=1:Unit
105 Unumsolution(1,k)=(cos(pi*Grid(k))-cos(pi*Grid(k+1)))/(pi*(Grid(k+1)-
    Grid(k)));
106 Unumsolution(2,k)=G(0.5*(Grid(k)+Grid(k+1)))*(Grid(k+1)-Grid(k));%store
    Uxc*deltax
107 end
108
109 %Reconstruct Uxx
110 for k=2:numberx-2
111 xci=0.5*(Grid(k)+Grid(k+1));%xci
112 xci1=0.5*(Grid(k+1)+Grid(k+2));%xci+1
113 xci2=0.5*(Grid(k-1)+Grid(k));%xci-1
114 A=[(xci1-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k+2)-Grid(k+1))
    ^2/(Grid(k+1)-Grid(k))^2-1);
115 (Grid(k+2)-Grid(k+1))*(xci1-xci)/(Grid(k+1)-Grid(k))^2;
116 (xci2-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k)-Grid(k-1))^2/(Grid
    (k+1)-Grid(k))^2-1);
117 (Grid(k)-Grid(k-1))*(xci2-xci)/(Grid(k+1)-Grid(k))^2];
118 b=[Unumsolution(1,k+1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci1-xci)/(
    Grid(k+1)-Grid(k));
119 Unumsolution(2,k+1)-Unumsolution(2,k)*(Grid(k+2)-Grid(k+1))/(Grid(k+1)-
    Grid(k));
120 Unumsolution(1,k-1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci2-xci)/(Grid
    (k+1)-Grid(k));
121 Unumsolution(2,k-1)-Unumsolution(2,k)*(Grid(k)-Grid(k-1))/(Grid(k+1)-
    Grid(k))];
122 Ureconstruct(k)=A\b;
123 end
124 %boundary

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125 %Left
126 xci=0.5*(Grid(1)+Grid(2)); %xci
127 xci1=0.5*(Grid(2)+Grid(3)); %xci+1
128 A=[(xci1-xci)^2/(2*(Grid(2)-Grid(1))^2)+1/24*((Grid(3)-Grid(2))^2/(Grid
    (2)-Grid(1))^2-1);
129 (Grid(3)-Grid(2))*(xci1-xci)/(Grid(2)-Grid(1))^2];
130 b=[Unumsolution(1,2)-Unumsolution(1,1)-Unumsolution(2,1)*(xci1-xci)/(
    Grid(2)-Grid(1));
131 Unumsolution(2,2)-Unumsolution(2,1)*(Grid(3)-Grid(2))/(Grid(2)-Grid(1))
    ];
132 Ureconstruct(1)=A\b;
133 %Right
134 xci=0.5*(Grid(numberx-1)+Grid(numberx-1+1)); %xci
135 xci2=0.5*(Grid(numberx-1-1)+Grid(numberx-1)); %xci-1
136 A=[(xci2-xci)^2/(2*(Grid(numberx-1+1)-Grid(numberx-1))^2)+1/24*((Grid(
    numberx-1)-Grid(numberx-1-1))^2/(Grid(numberx-1+1)-Grid(numberx-1))
    ^2-1);
137 (Grid(numberx-1)-Grid(numberx-1-1))*(xci2-xci)/(Grid(numberx-1+1)-Grid(
    numberx-1))^2];
138 b=[Unumsolution(1,numberx-1-1)-Unumsolution(1,numberx-1)-Unumsolution(2,
    numberx-1)*(xci2-xci)/(Grid(numberx-1+1)-Grid(numberx-1));
139 Unumsolution(2,numberx-1-1)-Unumsolution(2,numberx-1)*(Grid(numberx-1)-
    Grid(numberx-1-1))/(Grid(numberx-1+1)-Grid(numberx-1))];
140 Ureconstruct(numberx-1)=A\b;
141 %Post-proceeding
142 figure
143 k=1;
144 x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
145 p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2), Unumsolution(2,k)/(Grid(k
    +1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(
    k))^2, Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))
    ^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k)
    )/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
146 y=polyval(p,x);
147 plot(x,y,'-ro','linewidth',1.5); hold on
148 H1=plot(x,y,'-ro','linewidth',1.5); hold on
149
150 for k=2:numberx-1
151 x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
152 p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2), Unumsolution(2,k)/(Grid(k

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+1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(k))^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
153 y=polyval(p,x);
154 plot(x,y,'-ro','linewidth',1.5);
155 end
156 hold on
157 x=Grid(1):0.01*(Grid(numberx)-Grid(1)):Grid(numberx);
158 plot(x,g(x),'-b','linewidth',1.5);
159 H2=plot(x,g(x),'-b','linewidth',1.5);
160 lgd=legend([H1,H2],'Reconstruct','Exact solution');
161 lgd.FontSize=12;
162 xlabel('位置x','fontsize',14)
163 ylabel('数值U','fontsize',14)
164 title('Nonuniform grid reconstruct g=sin(pi*x)','fontsize',16)
165 hold off
166
167 %计算精度
168 Acc(1,1)=Accuracyg(8);
169 Acc(1,2)=Accuracyg(16);
170 Acc(1,3)=Accuracyg(32);
171 Acc(1,4)=Accuracyg(64);
172 for k=1:3
173 accuracyg(k)=(log10(Acc(1,k+1))-log10(Acc(1,k)))/(log10(a1(1,k+1))-log10(a1(1,k)));
174 end
175 figure
176 hold on
177 plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
178 H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);
179 H2=plot(log10(a2),1*log10(a2),'--','linewidth',1.5);
180 plot(log10(a2),2*log10(a2),'--','linewidth',1.5)
181 H3=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
182 lgd=legend([H1,H2,H3],'g reconstruct','斜率为1','斜率为2');
183 lgd.FontSize=12;
184 xlabel('Log(deltax)','fontsize',14)
185 ylabel('Log(episilo)','fontsize',14)
186 title('g精度分析','fontsize',16)

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