课题组组会-练习 3

王程

2023 年 10 月 24 日

一　练习及结果

1. 对带源项的扩散方程 $u_t = u_{xx} + \pi^2 sin\left(\pi x\right), x \in [0,1], t \geq 0$, 满足以下初始条件 $u\left(x,0\right) = x^2 - x$, 及边界条件 $u\left(0,t\right) = u\left(1,t\right) = 0$.

(1) 将空间离散格式改为 DG(P0P1)+DG(P0)，时间离散方式使用 a) 显式欧拉格式,b)TVD-RK3,c)BDF1 在均匀网格下进行求解。

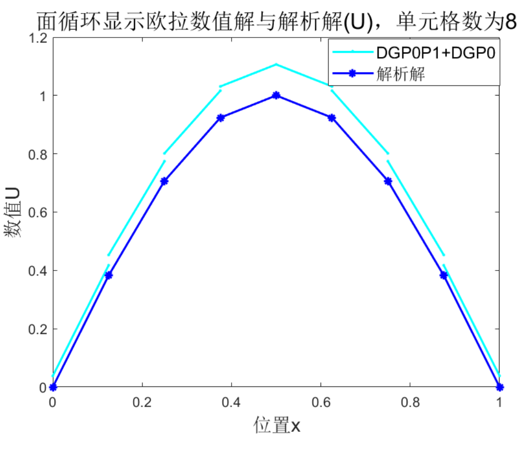
(2) 在网格生成时，对内部点坐标进行随机扰动，扰动范围为 $\pm 5\% \Delta x_{uniform}$，重新对问题进行求解，并测试精度。

2 在 $x \in [0,1]$ 的均匀网格上尝试使用 Green-Gauss Reconstruction 对 $f\left(x\right)$ 及 $g\left(x\right)$ 进行 P1P2 重构，其中 $f\left(x\right) = 1 + x + x^2$,$g\left(x\right) = sin\left(\pi x\right)$，测试重构精度。如果网格为不均匀网格呢？

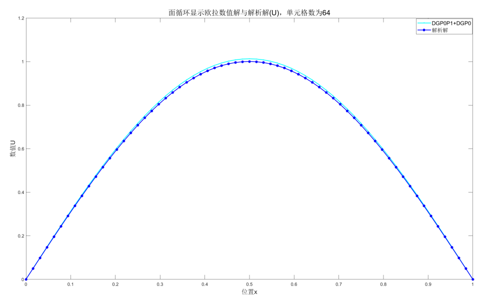
解：

(1) 更改空间离散格式，下面给出三种时间离散格式下 (单元格数为 8，64) 的方程数值解与解析解对比图以及空间精度。

A)**Explicit Euler**

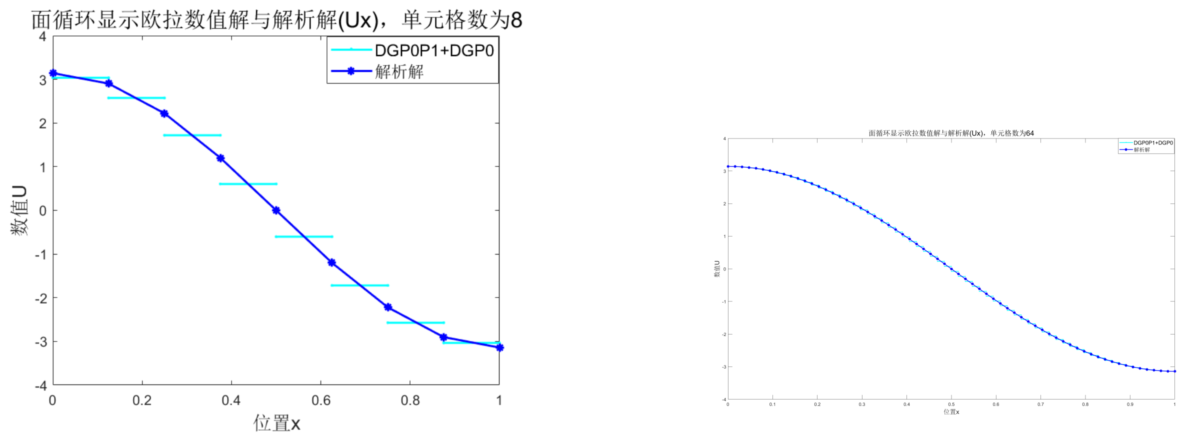


(a) 8 elements



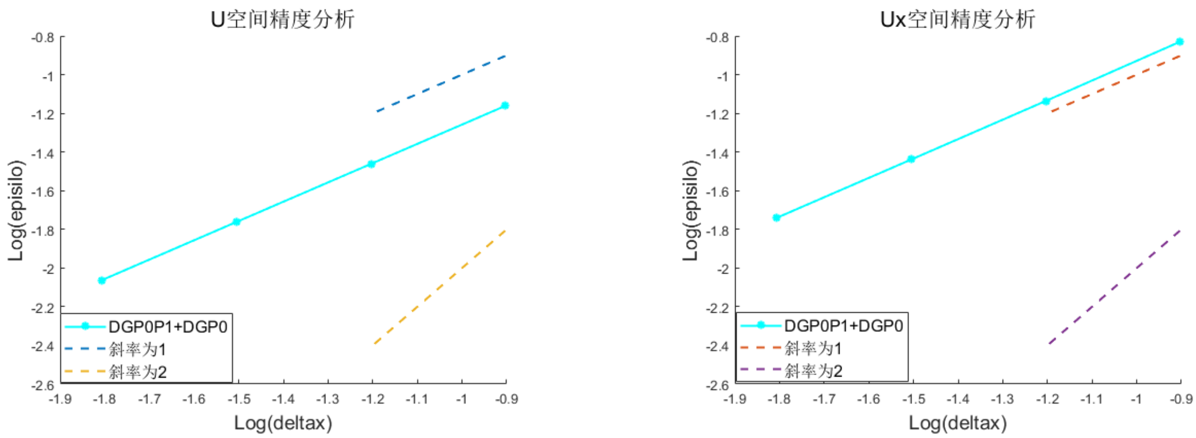
(b) 64 elements

图 1: Explicit Euler U



(a) 8 elements　　**(b)** 64 elements

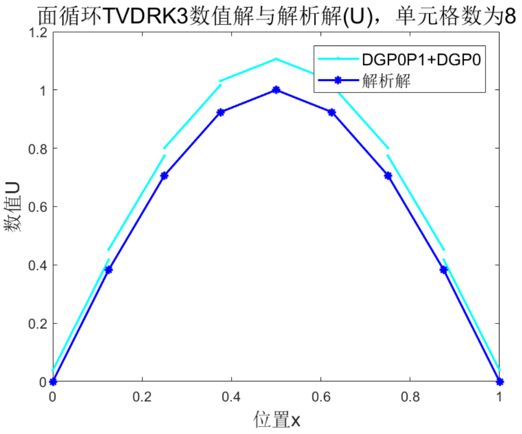
图 2: Explicit Euler Ux



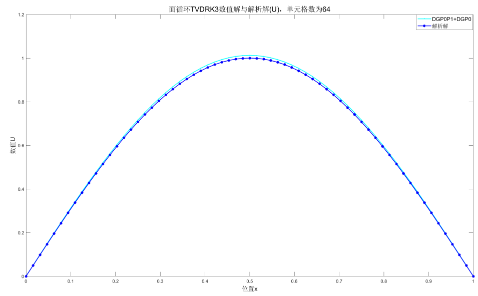
(a) U　　**(b)** Ux

图 3: 空间精度

B)**TVD-RK3**

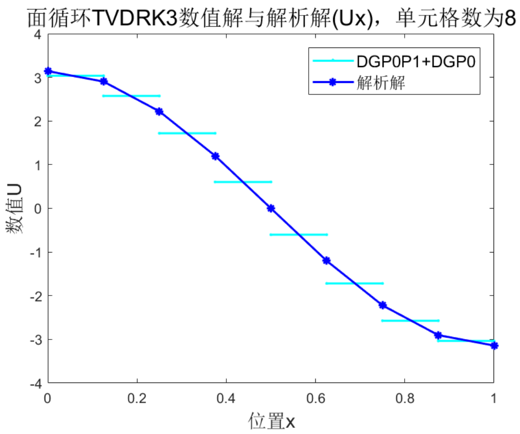


(a) 8 elements

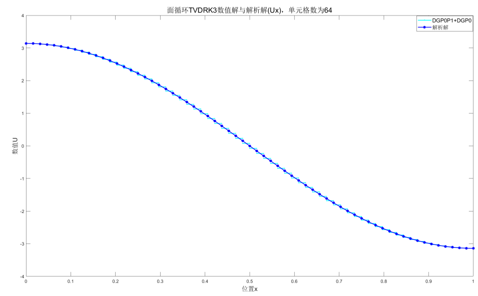


(b) 64 elements

图 4: TVD-RK3 U

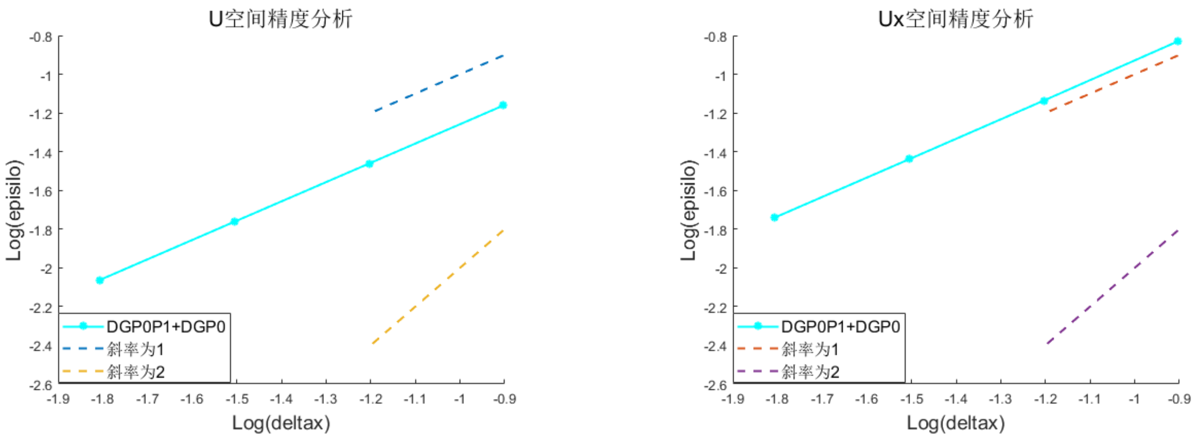


(a) 8 elements



(b) 64 elements

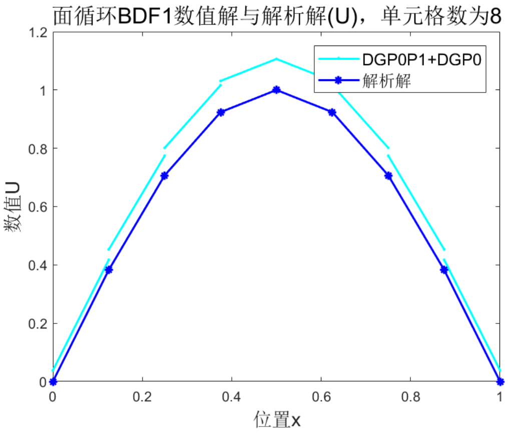
图 5: TVD-RK3 Ux



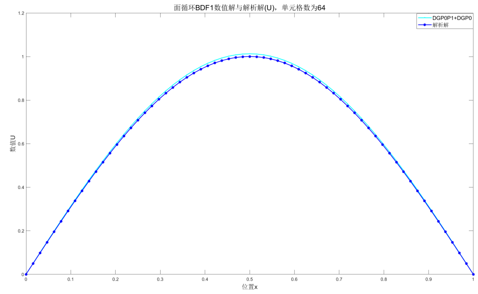
(a) U　　**(b)** Ux

图 6: 空间精度

C)**BDF1**

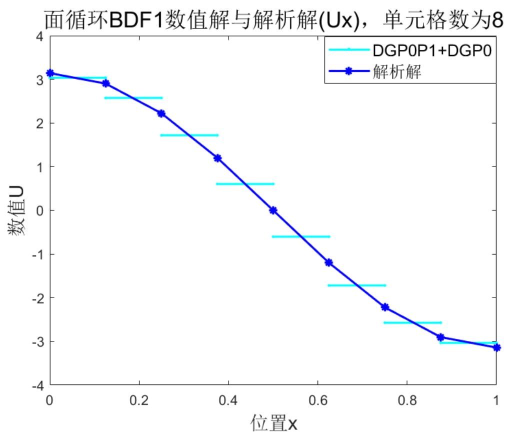


(a) 8 elements

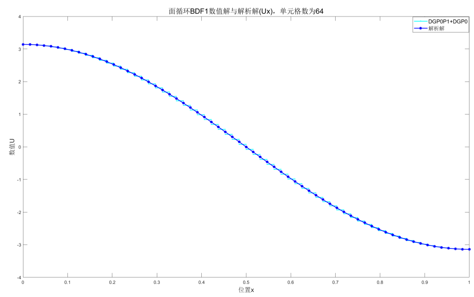


(b) 64 elements

图 **7:** BDF1 U

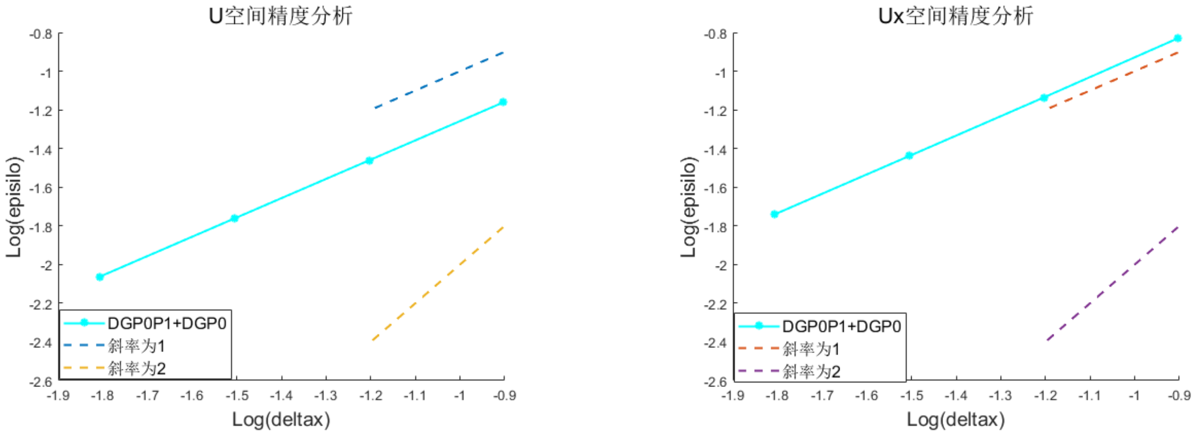


(a) 8 elements



(b) 64 elements

图 **8:** BDF1 Ux



(a) U

(b) Ux

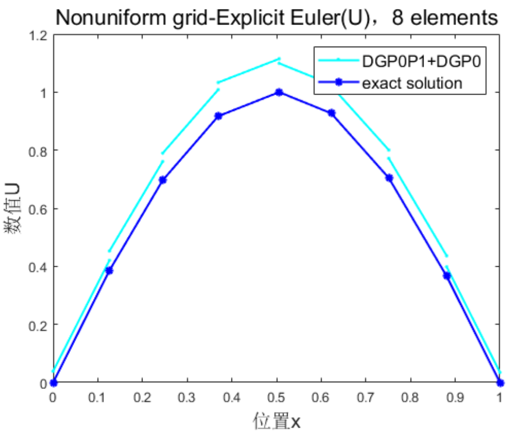
图 9: 空间精度

解：

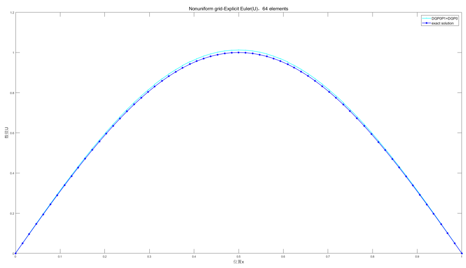
(2) 生成网格时对网格点进行扰动，这会影响 Δx，进而会影响到每个单元上的 $\Delta\tau$，这里采用了两种向前推进方法：a）取所有单元的最小 $\Delta\tau$ 进行推进 (如下面的 TVD-RK3)，b）每个单元取自己对应的 $\Delta\tau$，最后终止的 $End\tau$ 取最大的那个 (如下面的 Obvious Euler 和 BDF1)。

以下展示非均匀网格下三种时间离散格式下 (单元格数为 8，64) 的方程数值解与解析解对比图以及空间精度（这里仍然采取的均匀网格的 Δx），结果显示两种推进方式得到的 U 的精度相似，但方法 b) 得到的 Ux 精度较好。

A)**Nonuniform grid explicit euler**

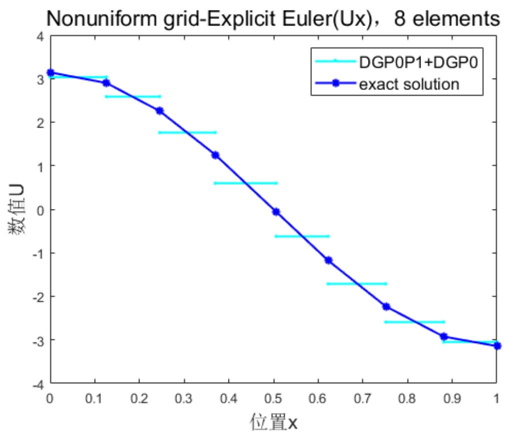


(a) 8 elements

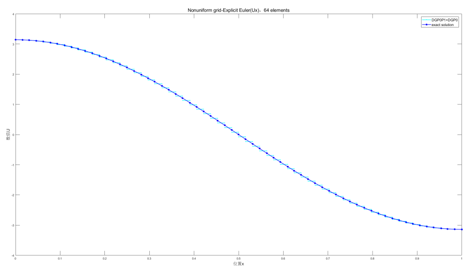


(b) 64 elements

图 **10:** Nonuniform grid Explicit euler U

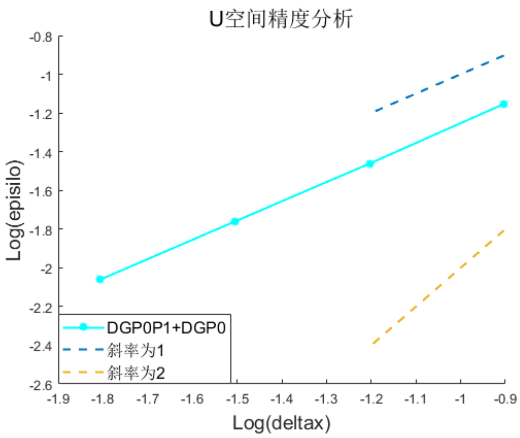


(a) 8 elements

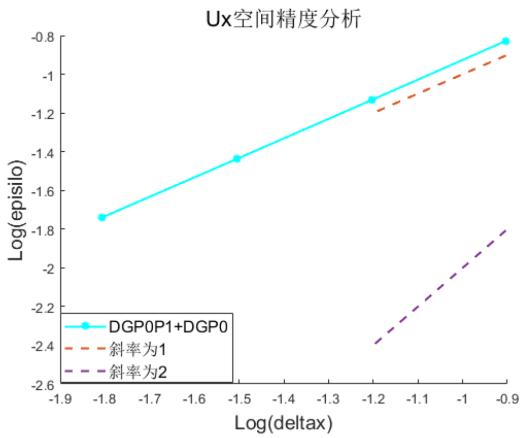


(b) 64 elements

图 **11:** Nonuniform grid Explicit euler Ux



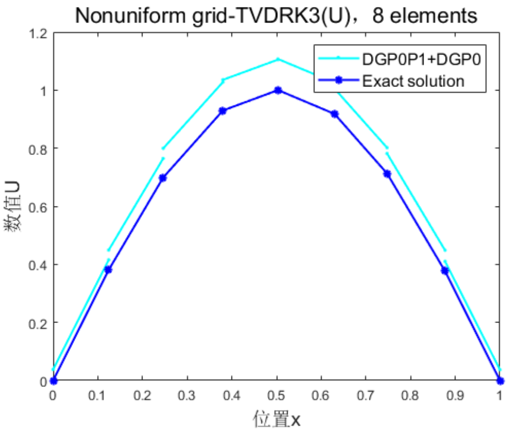
(a) U



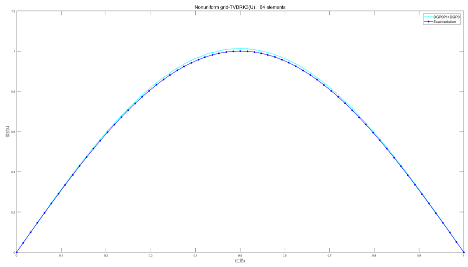
(b) Ux

图 12: 空间精度

B)**Nonuniform grid TVDRK3**

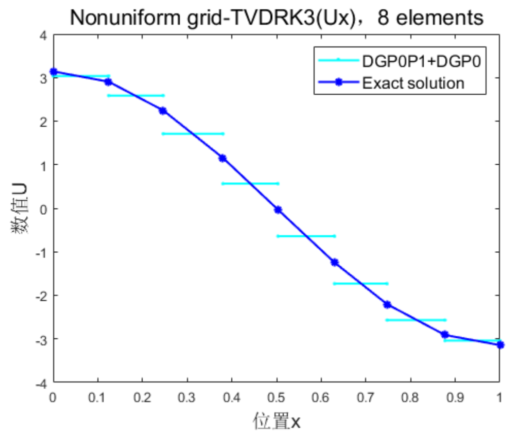


(a) 8 elements

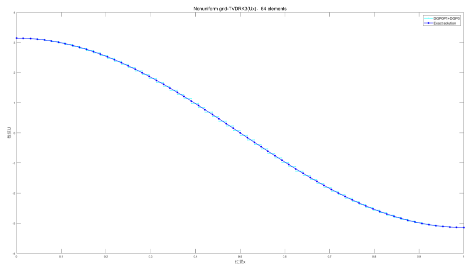


(b) 64 elements

图 **13:** Nonuniform grid TVDRK3 U

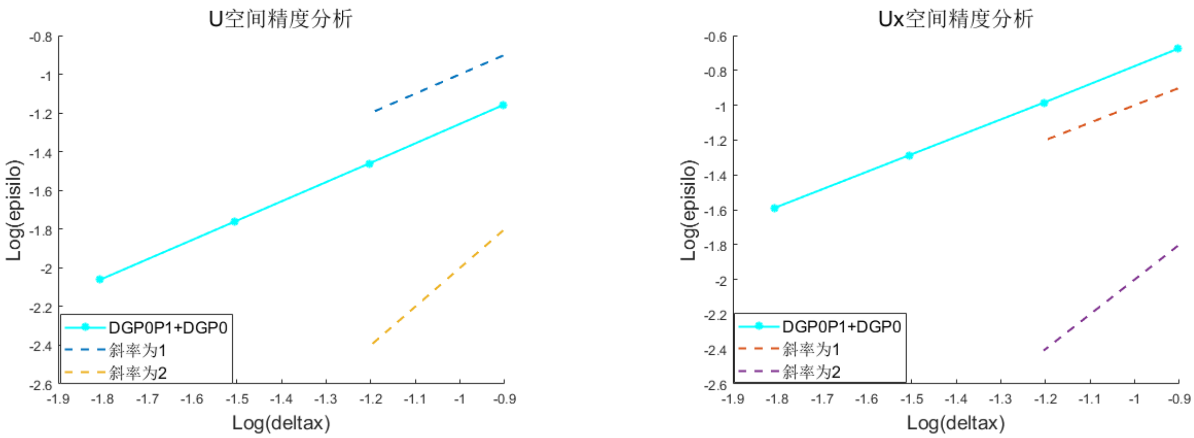


(a) 8 elements



(b) 64 elements

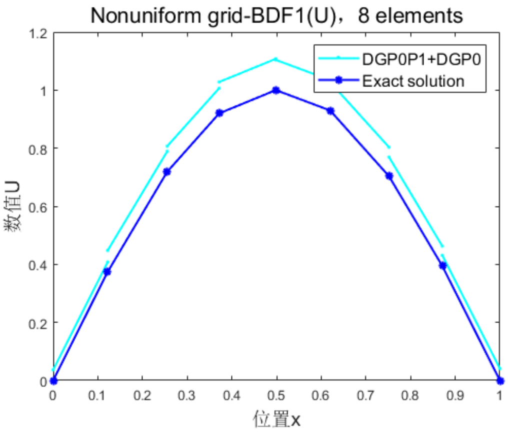
图 **14:** Nonuniform grid TVDRK3 Ux



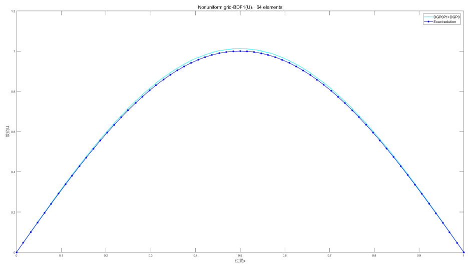
(a) U　　**(b)** Ux

图 15: 空间精度

C)**Nonuniform grid BDF1**

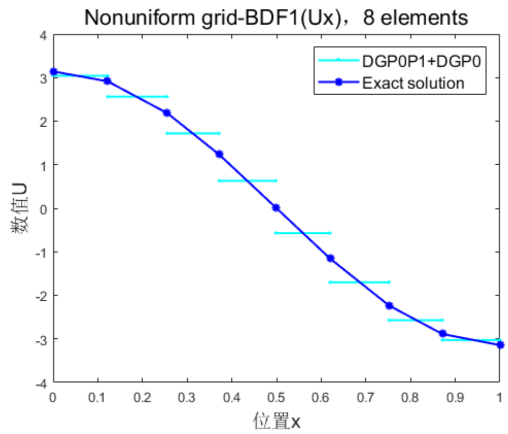


(a) 8 elements

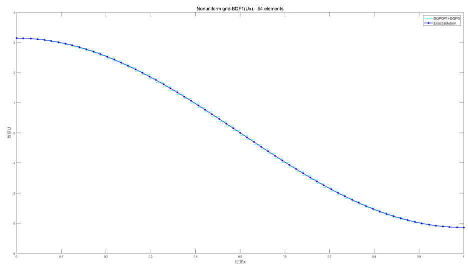


(b) 64 elements

图 16: Nonuniform grid BDF1 U

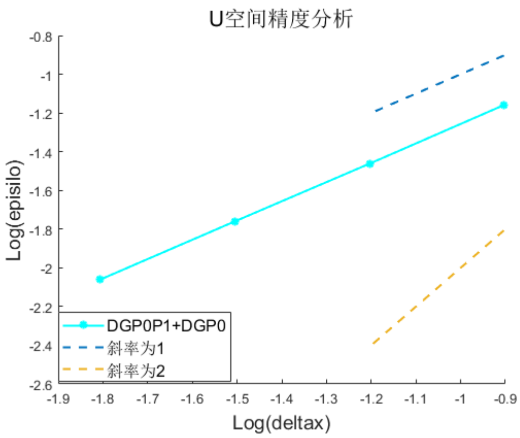


(a) 8 elements

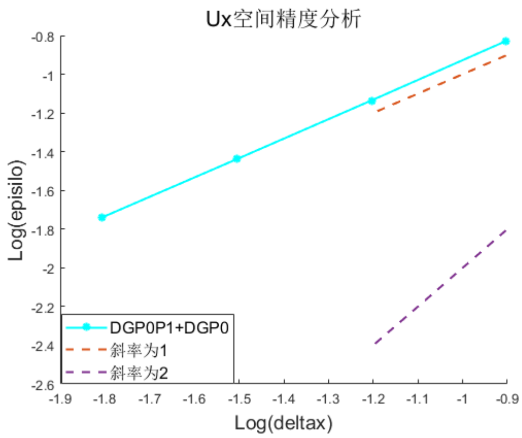


(b) 64 elements

图 17: Nonuniform grid BDF1 Ux



(a) U



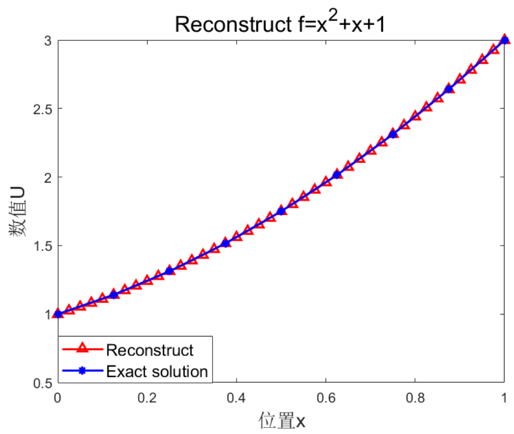
(b) Ux

图 18: 空间精度

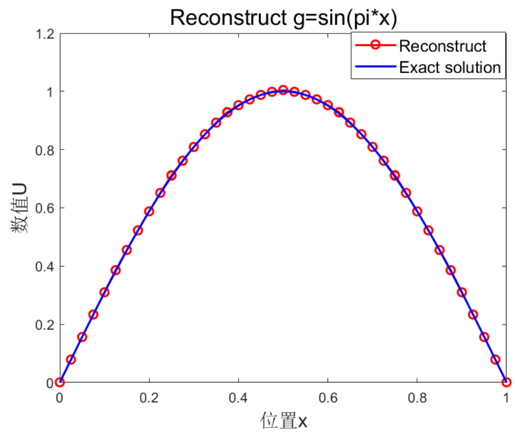
解：

2. 以下对 $f(x), g(x)$ 进行均匀网格与非均匀网格下的 P1P2 Green-Gauss Reconstruction，并给出非均匀网格下的重构精度图 (仍然采取均匀网格的 Δx).

A)**Uniform grid**



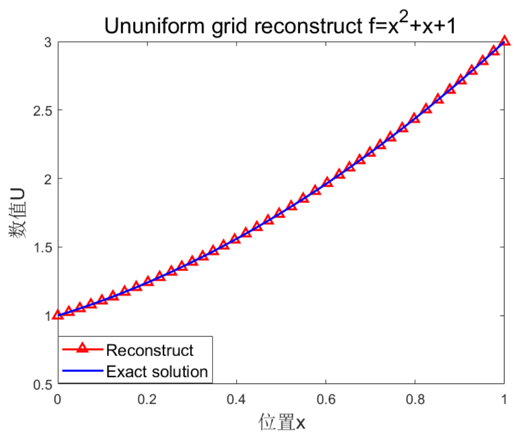
(a) f 重构



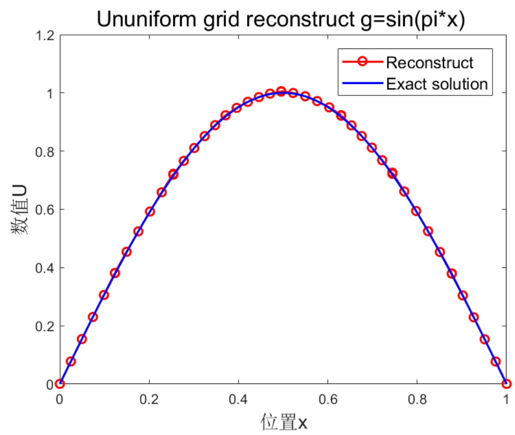
(b) g 重构

图 19: Uniform grid Green gauss Reconstruction

B)**Nonuniform grid**

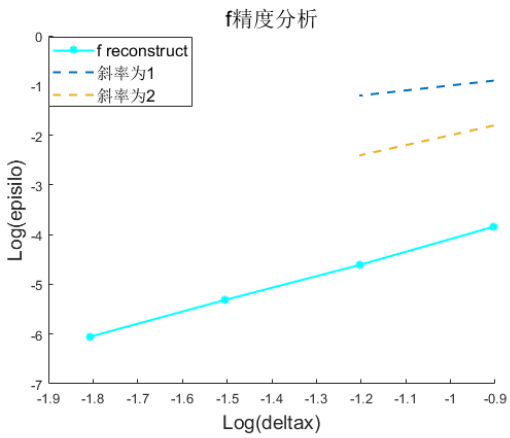


(a) f 重构

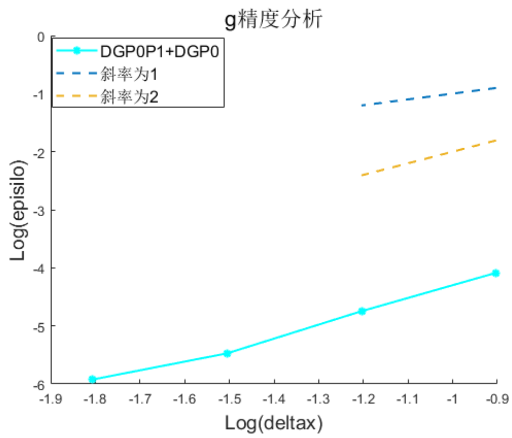


(b) g 重构

图 20: Nonuniform grid Green gauss Reconstruction



(a) f 重构



(b) g 重构

图 21: 精度分析

二　附录 (代码, 仅展示部分)

Nonuniform grid BDF1

```
clc
clear all
close all
% Pre-processing
Unit=64;CFL=0.01;endtau=2;endx=1;deltax=1/Unit;numberx=endx/deltax+1;
%记录内点位置
Uexasolution=zeros(2,numberx);
UDGP0P1plusDGP0=zeros(2,numberx-1);
Unumsolution1=zeros(1,2);
Unumsolution2=zeros(2,numberx-1);
Acc=zeros(3,4);a1=[1/8,1/16,1/32,1/64];a2=[1/8,1/16];
% solve the question
%solve the numsolution
[UDGP0P1plusDGP0,Grid,endtau01plus0]=BDF1subDGP0P1plusDGP0(Unit,CFL,
    endtau);%acquire u and
    ux

%solve the exasolution
x=0;
for k=1:numberx
x=Grid(1,k);
Uexasolution(1,k)=sin(pi*x);
Uexasolution(2,k)=pi*cos(pi*x);
end
% post-processing
%plot the u
%plot the DGP0P1plusDGP0
figure
Unumsolution1(1,1)=UDGP0P1plusDGP0(1,1)-UDGP0P1plusDGP0(2,1)*(Grid(2)-
    Grid(1))/2;Unumsolution1(1,2)=UDGP0P1plusDGP0(1,1)+UDGP0P1plusDGP0
    (2,1)*(Grid(2)-Grid(1))/2;
plot([Grid(1,1),Grid(1,2)],Unumsolution1,'-c.','linewidth',1.5);hold on
H1=plot([Grid(1,1),Grid(1,2)],Unumsolution1,'-c.','linewidth',1.5);hold
    on
for i=2:numberx-1
Unumsolution1(1,1)=UDGP0P1plusDGP0(1,i)-UDGP0P1plusDGP0(2,i)*(Grid(i+1)-
    Grid(i))/2;Unumsolution1(1,2)=UDGP0P1plusDGP0(1,i)+UDGP0P1plusDGP0(2,
```

```
i)*(Grid(i+1)-Grid(i))/2;
plot([Grid(1,i),Grid(1,i+1)],Unumsolution1,'-c.','linewidth',1.5)
end

%plot the exact
y=Grid;
plot(y,Uexasolution(1,:),'-b*','linewidth',1.5)
H2=plot(y,Uexasolution(1,:),'-b*','linewidth',1.5);hold on
lgd=legend([H1,H2],'DGP0P1+DGP0','Exact solution');
lgd.FontSize=12;
xlabel('位置x','fontsize',14)
ylabel('数值U','fontsize',14)
title('Ununiform grid-BDF1(U), 64 element','fontsize',16)
hold off


%plot the ux
%DGP0P1plusDGP0
figure
Unumsolution1(1,1)=UDGP0P1plusDGP0(2,1);Unumsolution1(1,2)=UDGP0P1plusDGP0(2,1);
plot([Grid(1,1),Grid(1,2)],Unumsolution1,'-c.','linewidth',1.5);hold on
H1=plot([Grid(1,1),Grid(1,2)],Unumsolution1,'-c.','linewidth',1.5);hold on
for i=2:numberx-1
Unumsolution1(1,1)=UDGP0P1plusDGP0(2,i);Unumsolution1(1,2)=UDGP0P1plusDGP0(2,i);
plot([Grid(1,i),Grid(1,i+1)],Unumsolution1,'-c.','linewidth',1.5)
end

%exact
y=Grid;
plot(y,Uexasolution(2,:),'-b*','linewidth',1.5)
H2=plot(y,Uexasolution(2,:),'-b*','linewidth',1.5);hold on
lgd=legend([H1,H2],'DGP0P1+DGP0','Exact solution');
lgd.FontSize=12;
xlabel('位置x','fontsize',14)
ylabel('数值U','fontsize',14)
title('Ununiform grid-BDF1(Ux), 64 element','fontsize',16)
hold off
```

```
%determine the accuracy of space U
[U,G,~]=BDF1subDGP0P1plusDGP0(8,CFL,endtau);
Acc(1,1)=AccuracyU(8,U,G);
[U,G,~]=BDF1subDGP0P1plusDGP0(16,CFL,endtau);
Acc(1,2)=AccuracyU(16,U,G);
[U,G,~]=BDF1subDGP0P1plusDGP0(32,CFL,endtau);
Acc(1,3)=AccuracyU(32,U,G);
[U,G,~]=BDF1subDGP0P1plusDGP0(64,CFL,endtau);
Acc(1,4)=AccuracyU(64,U,G);

figure
hold on
plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);

H2=plot(log10(a2),1*log10(a2),'--','linewidth',1.5);
plot(log10(a2),2*log10(a2),'--','linewidth',1.5)
H3=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
lgd=legend([H1,H2,H3],'DGP0P1+DGP0','斜率为1','斜率为2');
lgd.FontSize=12;
xlabel('Log(deltax)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('U空间精度分析','fontsize',16)

%determine the accuracy of space Ux
%DGP0P1
[U,G,~]=BDF1subDGP0P1plusDGP0(8,CFL,endtau);
Acc(1,1)=AccuracyUx(8,U,G);
[U,G,~]=BDF1subDGP0P1plusDGP0(16,CFL,endtau);
Acc(1,2)=AccuracyUx(16,U,G);
[U,G,~]=BDF1subDGP0P1plusDGP0(32,CFL,endtau);
Acc(1,3)=AccuracyUx(32,U,G);
[U,G,~]=BDF1subDGP0P1plusDGP0(64,CFL,endtau);
Acc(1,4)=AccuracyUx(64,U,G);

figure
hold on
plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);
```

```
plot(log10(a2),1*log10(a2),'--','linewidth',1.5)
H2=plot(log10(a2),1*log10(a2),'--','linewidth',1.5);
plot(log10(a2),2*log10(a2),'--','linewidth',1.5)
H3=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
lgd=legend([H1,H2,H3],'DGP0P1+DGP0','斜率为1','斜率为2');
lgd.FontSize=12;
xlabel('Log(deltax)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('Ux空间精度分析','fontsize',16)
```

BDF1subDGP0P1plusDGP0

```
function [Unumsolution,Grid,N]=subDGP0P1plusDGP0(Unit,CFL,endtau)
endx=1;deltax=endx/Unit;tol=10^(-10);
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);
numberx=Unit+1;
%记录内点位置
Grid=zeros(1,numberx);
for i=2:numberx-1
Grid(1,i)=(i-1)*deltax+(0.1*rand(1)-0.05)*deltax;
end
Grid(1,numberx)=endx;

deltatau=zeros(1,numberx-1);
for i=1:numberx-1
deltatau(i)=CFL*(Grid(1,i+1)-Grid(1,i))/abslambda;%伪时间变量
end

% Ucurrent=zeros(2,numberx-1);
% Unext=zeros(2,numberx-1);

B1=1;
%C=[B1,0;0,B1/deltax];
%Mtau=[deltax,0;0,deltax/12+1/deltax];A=[abslambda,0;0,abslambda];
R=zeros(2*Unit,1);
Rd=zeros(2,numberx-1);
Rb=zeros(2,numberx-1);
Fn=zeros(2,numberx);

```

```

% solve the question
%构建 LHS
%Mtau/deltatau
LHS1=zeros(2*Unit,2*Unit);
for k=1:numberx-1
LHS1(2*k-1,2*k-1)=(Grid(k+1)-Grid(k))/deltatau(k);
LHS1(2*k,2*k)=((Grid(k+1)-Grid(k))/12+1/(Grid(k+1)-Grid(k)))/deltatau(k);
end
%Rdomain
for k=1:numberx-1
LHS2(2*k,2*k)=(nu+1/Tr)/(Grid(k+1)-Grid(k));
end

%Rboundary
LHS3=zeros(2*Unit,2*Unit);
for iface=2:numberx-1
ieL=iface-1;
ieR=iface;
CL=[B1,1/2;0,B1/(Grid(ieL+1)-Grid(ieL))];
CR=[B1,-1/2;0,B1/(Grid(ieR+1)-Grid(ieR))];
%diag
LHS3(2*ieL-1:2*ieL,2*ieL-1:2*ieL)=LHS3(2*ieL-1:2*ieL,2*ieL-1:2*ieL)+CL'*[abslambda/2,-nu/2;-1/(2*Tr),abslambda/2]*CL;
LHS3(2*ieR-1:2*ieR,2*ieR-1:2*ieR)=LHS3(2*ieR-1:2*ieR,2*ieR-1:2*ieR)-CR'*[-abslambda/2,-nu/2;-1/(2*Tr),-abslambda/2]*CR;
%upper
LHS3(2*ieL-1:2*ieL,2*ieR-1:2*ieR)=LHS3(2*ieL-1:2*ieL,2*ieR-1:2*ieR)+CL'*[-abslambda/2,-nu/2;-1/(2*Tr),-abslambda/2]*CR;
%lower
LHS3(2*ieR-1:2*ieR,2*ieL-1:2*ieL)=LHS3(2*ieR-1:2*ieR,2*ieL-1:2*ieL)-CR'*[abslambda/2,-nu/2;-1/(2*Tr),abslambda/2]*CL;
end
CL=[B1,1/2;0,B1/(Grid(numberx)-Grid(numberx-1))];
CR=[B1,-1/2;0,B1/(Grid(2)-Grid(1))];
LHS3(2*1-1:2*1,2*1-1:2*1)=LHS3(2*1-1:2*1,2*1-1:2*1)-CR'*([abslambda/2,-nu/2;-1/(2*Tr),abslambda/2]*[0,0;0,1/deltax]+[-abslambda/2,-nu/2;-1/(2*Tr),-abslambda/2]*CR);
LHS3(2*(numberx-1)-1:2*(numberx-1),2*(numberx-1)-1:2*(numberx-1))=LHS3
```

```
    (2*(numberx-1)-1:2*(numberx-1),2*(numberx-1)-1:2*(numberx-1))+CL'*([
    abslambda/2,-nu/2;-1/(2*Tr),abslambda/2]+[-abslambda/2,-nu/2;-1/(2*Tr
    ),-abslambda/2]*[0,0;0,1])*CL;
LHS=LHS1+LHS2+LHS3;

%取出我们所需要的 D
D=zeros(2*Unit,2*Unit);
for iface=2:numberx
ieL=iface-1;
D(2*ieL-1:2*ieL,2*ieL-1:2*ieL)=LHS(2*ieL-1:2*ieL,2*ieL-1:2*ieL);
end
%为循环所预设的一些量
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2*Unit,1);
%initial condition set up

for k=1:numberx-1
x=Grid(k);
Ucurrent(1,k)=(x+(Grid(k+1)-Grid(k))/2)^2-(x+(Grid(k+1)-Grid(k))/2);
Ucurrent(2,k)=(2*(x+(Grid(k+1)-Grid(k))/2)-1)*(Grid(k+1)-Grid(k));%
    Ucurrent(2,:) 存储的是
    Ux*deltx
end


%Rdomain
for k=1:numberx-1
Rd(1,k)=gauss1(Grid(k),Grid(k+1));
%Rd(1,k)=pi*(cos(pi*x)-cos(pi*(x+deltax)));
%Rd(2,k)=-nu*Ucurrent(2,k)/deltax+(-pi/deltax*(deltax/2*cos(pi*(x+deltax))-(-
    deltax/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltax))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltax);
Rd(2,k)=-nu*Ucurrent(2,k)/(Grid(k+1)-Grid(k))-Ucurrent(2,k)/(Tr*(Grid(k
    +1)-Grid(k)))+gauss2(Grid(k),Grid(k+1));
end


%Rboundary
for iface=2:numberx-1
ieL=iface-1;
ieR=iface;
B2L=1/2;
B2R=-1/2;
```

```
Fn(:,iface)=0.5*([-nu*Ucurrent(2,ieL)/(Grid(ieL+1)-Grid(ieL));-(Ucurrent(1,ieL)+B2L*Ucurrent(2,ieL))/Tr]+[-nu*Ucurrent(2,ieR)/(Grid(ieR+1)-Grid(ieR));-(Ucurrent(1,ieR)+B2R*Ucurrent(2,ieR))/Tr])-0.5*A*([Ucurrent(1,ieR)+B2R*Ucurrent(2,ieR);Ucurrent(2,ieR)/(Grid(ieR+1)-Grid(ieR))]-[Ucurrent(1,ieL)+B2L*Ucurrent(2,ieL);Ucurrent(2,ieL)/(Grid(ieL+1)-Grid(ieL))]);
Rb(:,ieL)=Rb(:,ieL)-[1,B2L;0,1/(Grid(ieL+1)-Grid(ieL))]'*Fn(:,iface);
Rb(:,ieR)=Rb(:,ieR)+[1,B2R;0,1/(Grid(ieR+1)-Grid(ieR))]'*Fn(:,iface);
end
Fn(:,1)=0.5*([-nu*Ucurrent(2,1)/(Grid(1+1)-Grid(1));0]+[-nu*Ucurrent(2,1)/(Grid(1+1)-Grid(1));-(Ucurrent(1,1)+B2R*Ucurrent(2,1))/Tr])-0.5*A*([Ucurrent(1,1)+B2R*Ucurrent(2,1);Ucurrent(2,1)/(Grid(1+1)-Grid(1))]-[0;Ucurrent(2,1)/(Grid(1+1)-Grid(1))]);
Fn(:,numberx)=0.5*([-nu*Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(numberx-1));-(Ucurrent(1,numberx-1)+B2L*Ucurrent(2,numberx-1))/Tr]+[-nu*Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(numberx-1));0])-0.5*A*([0;Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(numberx-1))]-[Ucurrent(1,numberx-1)+B2L*Ucurrent(2,numberx-1);Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(numberx-1))]);
Rb(:,1)=Rb(:,1)+[1,B2R;0,1/(Grid(2)-Grid(1))]'*Fn(:,1);
Rb(:,numberx-1)=Rb(:,numberx-1)-[1,B2L;0,1/(Grid(numberx)-Grid(numberx-1))]'*Fn(:,numberx);

%R 组装
for k=1:numberx-1
R(2*k-1:2*k,1)=Rd(:,k)+Rb(:,k);
end

%进行必要的向量等价转变
for k=1:numberx-1
Unext(2*k-1:2*k,1)=Ucurrent(:,k);
end

%循环迭代
for n=1:floor(endtau/max(deltatau))
X=D\R;
if max(X)<tol
N=max(n*deltatau);
break
end
```

```
Unext=Unext+X;
Rd=zeros(2,numberx-1);
Rb=zeros(2,numberx-1);
for k=1:numberx-1
Ucurrent(:,k)=Unext(2*k-1:2*k,1);
end
%Rdomain
for k=1:numberx-1
Rd(1,k)=gauss1(Grid(k),Grid(k+1));
%Rd(1,k)=pi*(cos(pi*x)-cos(pi*(x+deltax)));
%Rd(2,k)=-nu*Ucurrent(2,k)/deltax+(-pi/deltax*(deltax/2*cos(pi*(x+deltax))-(-
    deltax/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltax))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltax);
Rd(2,k)=-nu*Ucurrent(2,k)/(Grid(k+1)-Grid(k))-Ucurrent(2,k)/(Tr*(Grid(k
    +1)-Grid(k)))+gauss2(Grid(k),Grid(k+1));
end

%Rboundary
for iface=2:numberx-1
ieL=iface-1;
ieR=iface;
B2L=1/2;
B2R=-1/2;
Fn(:,iface)=0.5*([-nu*Ucurrent(2,ieL)/(Grid(ieL+1)-Grid(ieL));-(Ucurrent
    (1,ieL)+B2L*Ucurrent(2,ieL))/Tr]+[-nu*Ucurrent(2,ieR)/(Grid(ieR+1)-
    Grid(ieR));-(Ucurrent(1,ieR)+B2R*Ucurrent(2,ieR))/Tr])-0.5*A*([
    Ucurrent(1,ieR)+B2R*Ucurrent(2,ieR);Ucurrent(2,ieR)/(Grid(ieR+1)-Grid
    (ieR))]-[Ucurrent(1,ieL)+B2L*Ucurrent(2,ieL);Ucurrent(2,ieL)/(Grid(
    ieL+1)-Grid(ieL))]);
Rb(:,ieL)=Rb(:,ieL)-[1,B2L;0,1/(Grid(ieL+1)-Grid(ieL))]'*Fn(:,iface);
Rb(:,ieR)=Rb(:,ieR)+[1,B2R;0,1/(Grid(ieR+1)-Grid(ieR))]'*Fn(:,iface);
end
Fn(:,1)=0.5*([-nu*Ucurrent(2,1)/(Grid(1+1)-Grid(1));0]+[-nu*Ucurrent
    (2,1)/(Grid(1+1)-Grid(1));-(Ucurrent(1,1)+B2R*Ucurrent(2,1))/Tr])
    -0.5*A*([Ucurrent(1,1)+B2R*Ucurrent(2,1);Ucurrent(2,1)/(Grid(1+1)-
    Grid(1))]-[0;Ucurrent(2,1)/(Grid(1+1)-Grid(1))]);
Fn(:,numberx)=0.5*([-nu*Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(
    numberx-1));-(Ucurrent(1,numberx-1)+B2L*Ucurrent(2,numberx-1))/Tr]+[-
    nu*Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(numberx-1));0])-0.5*A
    *([0;Ucurrent(2,numberx-1)/(Grid(numberx)-Grid(numberx-1))]-[Ucurrent
    (1,numberx-1)+B2L*Ucurrent(2,numberx-1);Ucurrent(2,numberx-1)/(Grid(
```

```
numberx)-Grid(numberx-1))]);
Rb(:,1)=Rb(:,1)+[1,B2R;0,1/(Grid(2)-Grid(1))]'*Fn(:,1);
Rb(:,numberx-1)=Rb(:,numberx-1)-[1,B2L;0,1/(Grid(numberx)-Grid(numberx-1))]'*Fn(:,numberx);

%R 组装
for k=1:numberx-1
R(2*k-1:2*k,1)=Rd(:,k)+Rb(:,k);
end
end
if n==floor(endtau/max(deltatau))
N=max(n*deltatau);
end
Unumsolution(1,:)=Ucurrent(1,:);
for k=1:numberx-1
Unumsolution(2,k)=Ucurrent(2,k)/(Grid(k+1)-Grid(k));
end
end
```