CFD 入门练习 5

1.对带源项的扩散方程 $u_t = u_{xx} + \pi^2 \sin(\pi x), x \in [0,1], t \geq 0$ **,满足以下初始条件** $u(x,0) = x^2 - x$ **,及边界条件** $u(0,t) = u(1,t) = 0$ **。**

(1)求该方程的解析稳态解。

(2)使用 FOHS 引入辅助变量，将上述方程改写为双曲方程组，考虑均匀网格(8,16,32,64,128…)，用不同的数值方法求解稳态解，并与(1)中的解析解进行对比，测试原始变量 u **和它在** x **方向的导数的空间精度。**

解：(1)方程的解析稳态解为： $u(x,t) = \sin(\pi x)$ 。

(2)本题使用 DG(P0)+DG(P0)，DG(P0P1)+DG(P0)，DG(P0P2)+rDG(P0P1)三种显式欧拉方法求解变形后双曲方程组的稳态解。这里仅展示网格数为 8,16,32,64,128，CFL=0.01 的稳态数值解与解析解的比较图，并给出误差与空间步长的关系图(处理后)以及最终的空间精度。

1. DG(P0)+DG(P0)

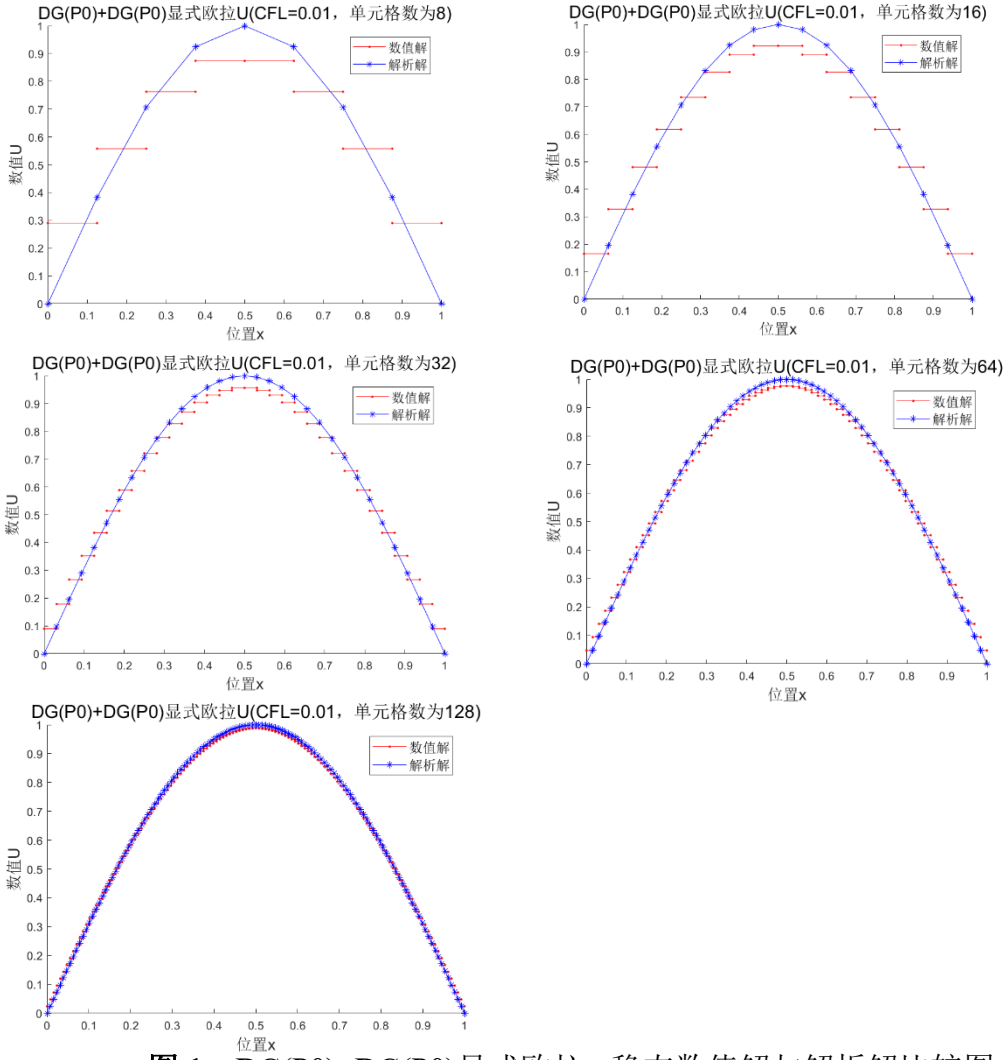

图 1：DG(P0)+DG(P0)显式欧拉 u 稳态数值解与解析解比较图

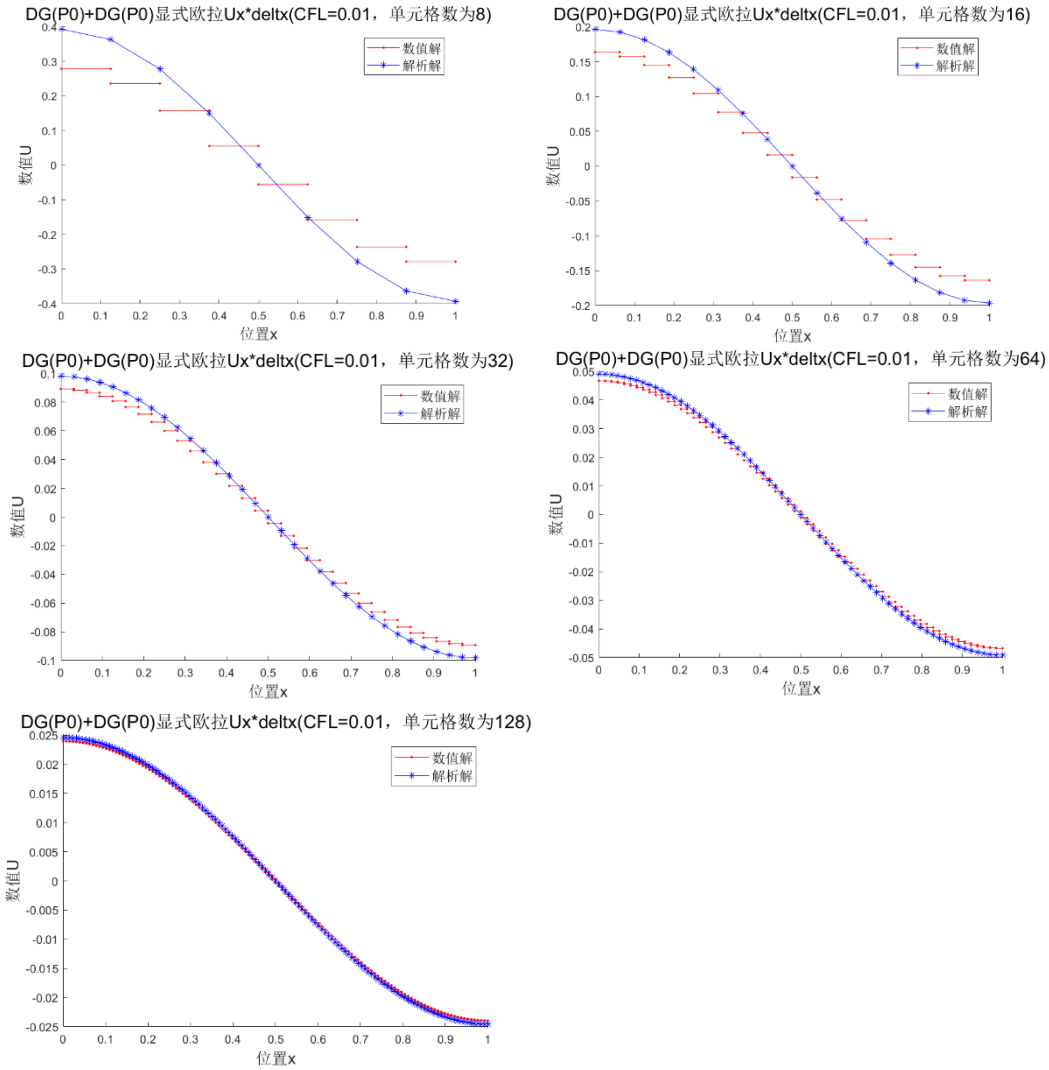

图 2：DG(P0)+DG(P0)显式欧拉 u_x 稳态数值解与解析解比较图

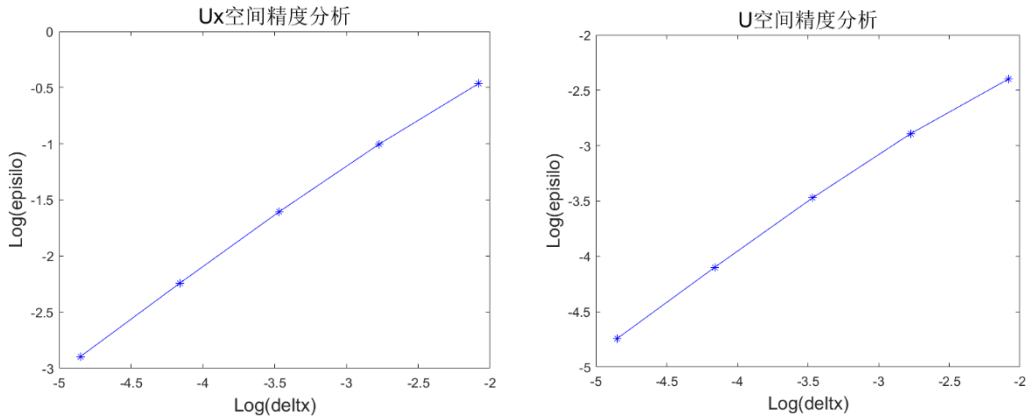

图 3：DG(P0)+DG(P0)显式欧拉 u 与 u_x 空间精度分析图

最终 u 空间精度为 1， u_x 空间精度为 1。

2. DG(P0P1)+DG(P0)

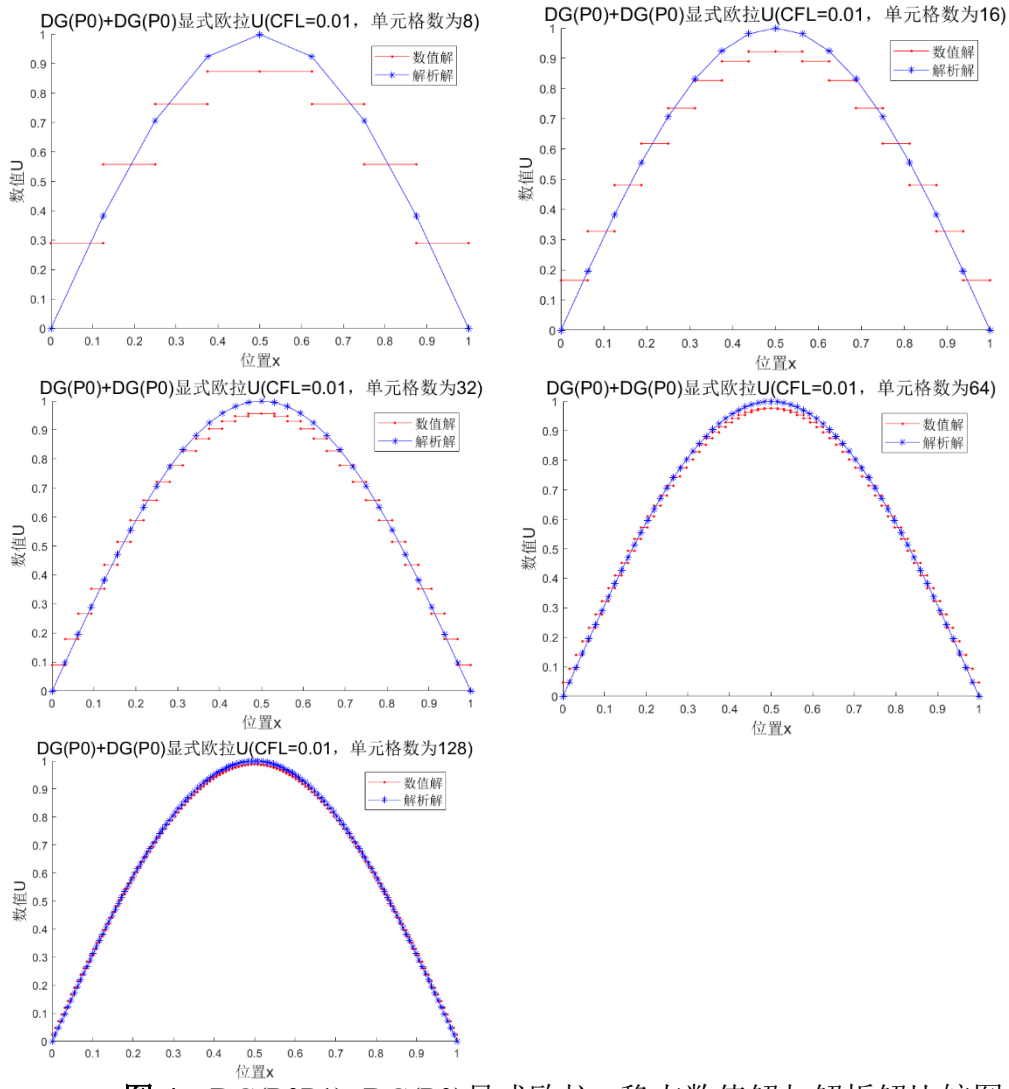

图 4：DG(P0P1)+DG(P0)显式欧拉 u 稳态数值解与解析解比较图

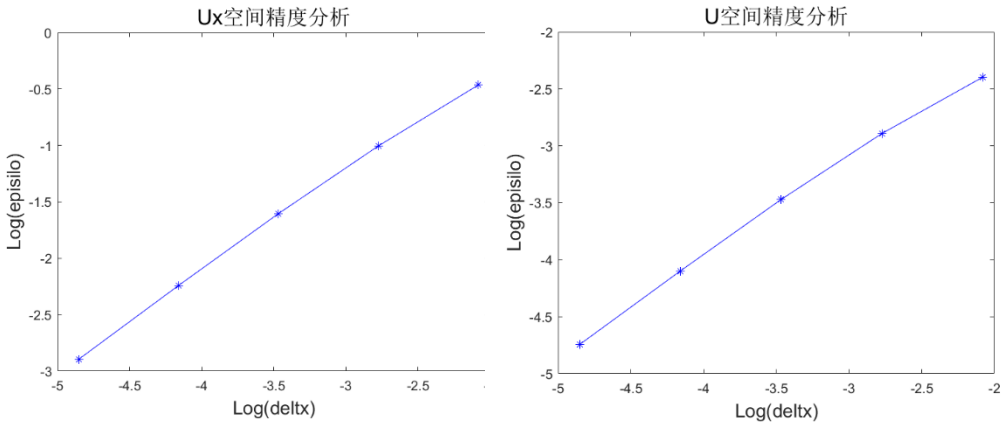

图 5：DG(P0P1)+DG(P0)显式欧拉 u 与 u_x 空间精度分析图

最终 u 空间精度为 0.4， u_x 空间精度为 1。

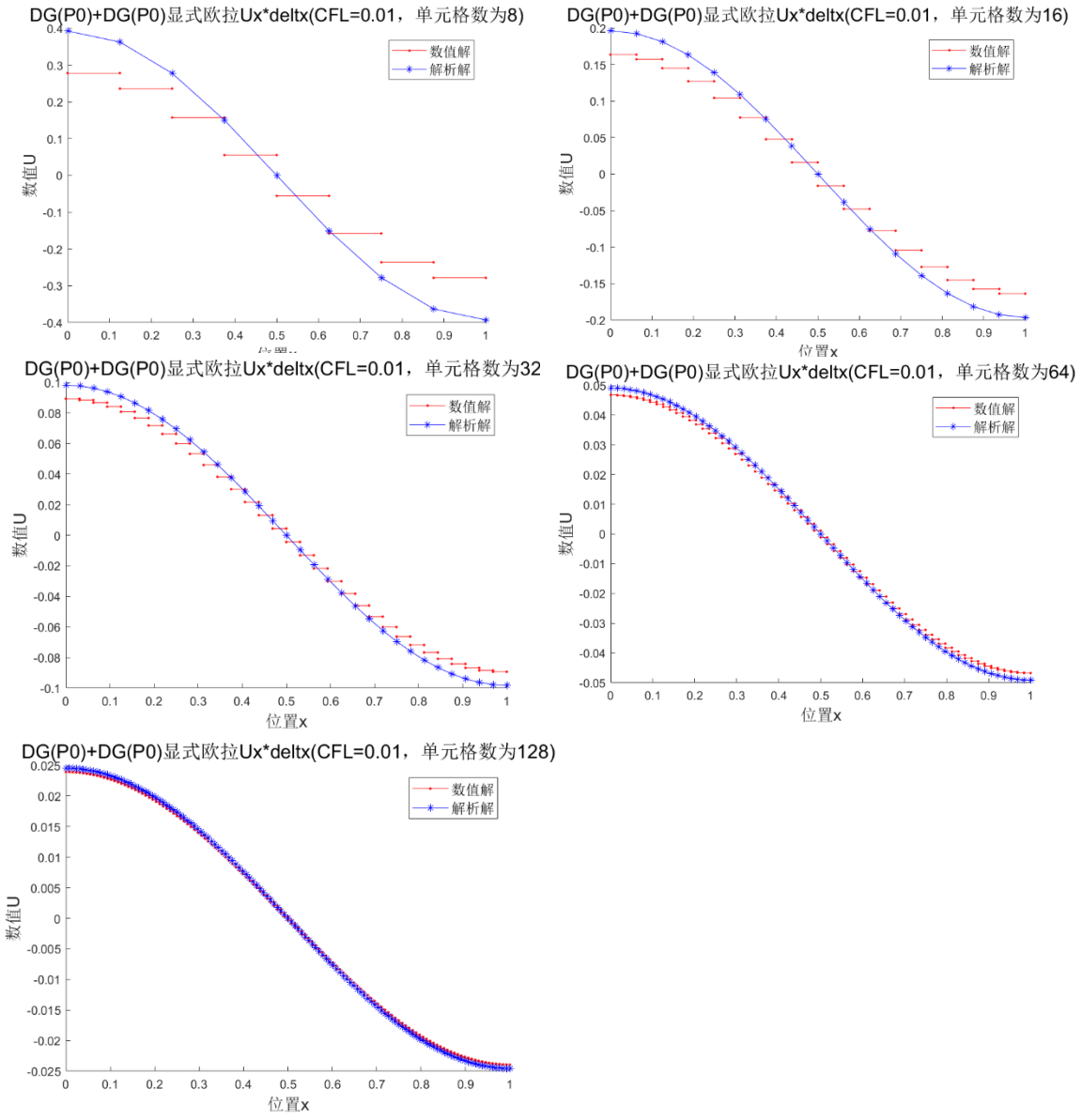

图 6：DG(P0P1)+DG(P0)显式欧拉 u_x 稳态数值解与解析解比较图

3. DG(P0P2)+rDG(P0P1)

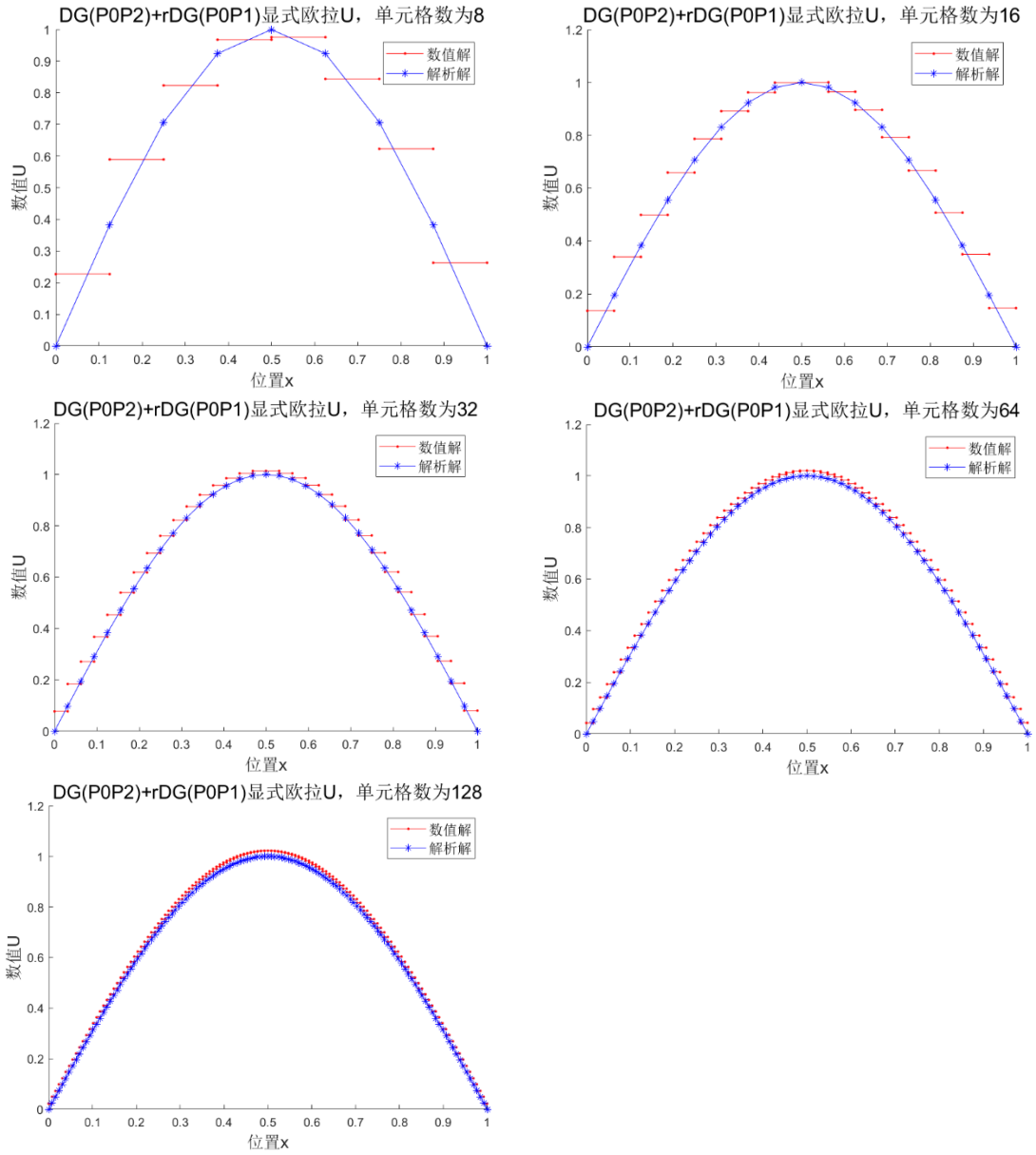

图 7：DG(P0P2)+rDG(P0P1)显式欧拉 u 稳态数值解与解析解比较图

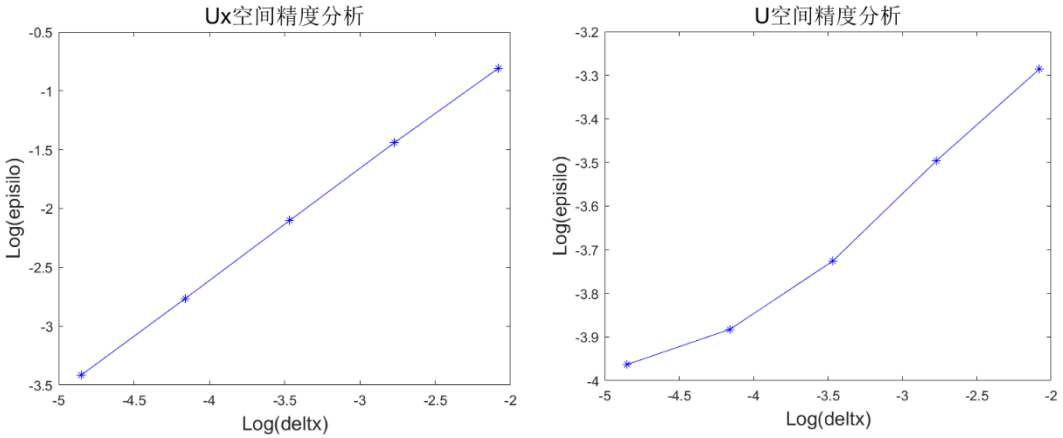

图 8：DG(P0P2)+rDG(P0P1)显式欧拉 u 与 u_x 空间精度分析图

最终 u 空间精度为 1， u_x 空间精度为 1。

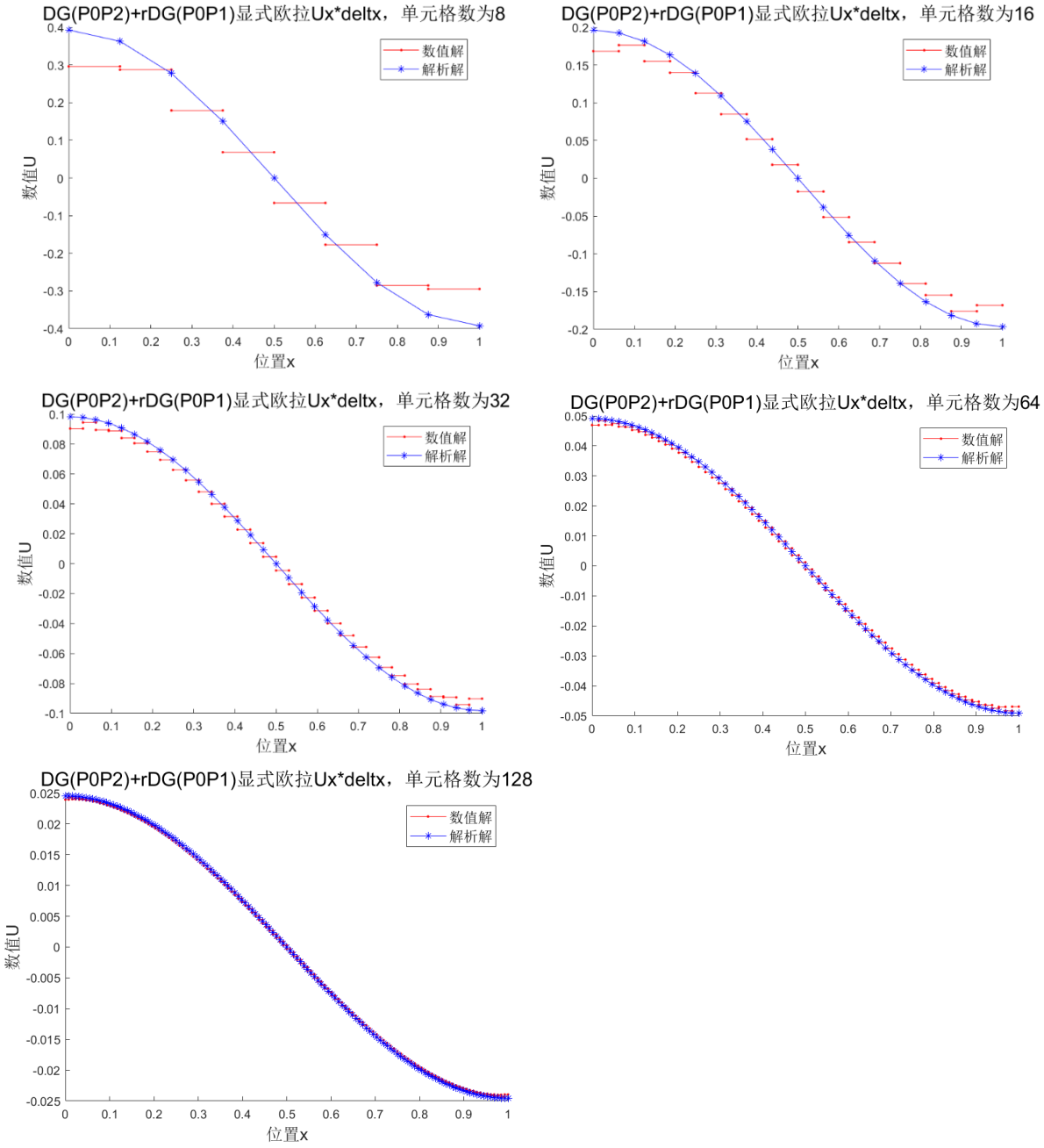

图 9： DG(P0P2)+rDG(P0P1)显式欧拉 u_x 稳态数值解与解析解比较图

附录

DGP0+DGP0

```
clc
clear all
close all
%% Pre-processing
Unit=128;deltx=1/Unit;CFL=0.01;tol=0.01;
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);deltto=CFL*deltx/abslambda;%伪时间变量
endx=1;endto=0.5;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(2,numberx);
Unumsolution1=zeros(1,2);
B1=1;
C=[B1,0;0,B1/deltx];Mto=[deltx,0;0,1/deltx];
A=[abslambda,0;0,abslambda];
R=zeros(2,numberx-1);
F1=zeros(2,numberx-1);
F2=zeros(2,numberx-1);
V0=zeros(1,numberx-1);
%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(1,k)=(x+deltx/2)^2-(x+deltx/2);
    k=k+1;
end
V0=Ucurrent(1,:);
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=(2*(x+deltx)-1)*deltx;
    k=k+1;
end
k=1;
for x=0:deltx:endx-deltx
    R(1,k)=pi*(cos(pi*x)-cos(pi*(x+deltx)));
    R(2,k)=-Ucurrent(2,k)/(Tr*deltx);
    k=k+1;
end
```

```
for k=2:numberx-1
    F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-Ucurrent(1,k-1)/(Tr*deltx)]+[-
nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)]-0.5*A*(Ucurrent(:,k)-
Ucurrent(:,k-1)));
    k=k+1;
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-
Ucurrent(1,1)/(Tr*deltx)])-0.5*A*(Ucurrent(:,1)-[0;Ucurrent(2,1)]);

for k=1:numberx-2
    F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)]+[-
nu*Ucurrent(2,k+1)/deltx;-Ucurrent(1,k+1)/(Tr*deltx)]-0.5*A*(Ucurrent(:,k+1)-
Ucurrent(:,k)));
    k=k+1;
end
 F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;-Ucurrent(1,numberx-
1)/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;-0/(Tr*deltx)])-
0.5*A*([0;Ucurrent(2,numberx-1)]-Ucurrent(:,numberx-1));

 for k=1:numberx-1
    R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
 end

 %solve the exasolution
k=1;
for x=0:deltx:endx
    Uexasolution(1,k)=sin(pi*x);
    Uexasolution(2,k)=pi*cos(pi*x)*deltx;
    k=k+1;
end

%solve the numsolution
for n=deltto:deltto:endto
    for k=1:numberx-1
        Unext(:,k)=Ucurrent(:,k)+Mto\R(:,k)*deltto;
    end
     if var(Ucurrent(1,:)-Unext(1,:))<tol*V0
          break
     end
    Ucurrent=Unext;

    k=1;
for x=0:deltx:endx-deltx
    R(1,k)=pi*(cos(pi*x)-cos(pi*(x+deltx)));
```

```
        R(2,k)=-Ucurrent(2,k)/(deltx*Tr);
        k=k+1;
end

for k=2:numberx-1
        F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-Ucurrent(1,k-1)/(Tr*deltx)]+[-
nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)])-0.5*A*(Ucurrent(:,k)-
Ucurrent(:,k-1));
        k=k+1;
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-
Ucurrent(1,1)/(Tr*deltx)])-0.5*A*(Ucurrent(:,1)-[0;Ucurrent(2,1)]);

for k=1:numberx-2
        F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)]+[-
nu*Ucurrent(2,k+1)/deltx;-Ucurrent(1,k+1)/(Tr*deltx)])-0.5*A*(Ucurrent(:,k+1)-
Ucurrent(:,k));
        k=k+1;
end
  F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;-Ucurrent(1,numberx-
1)/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;-0/(Tr*deltx)])-
0.5*A*([0;Ucurrent(2,numberx-1)]-Ucurrent(:,numberx-1));

  for k=1:numberx-1
        R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
  end

end
Unumsolution=Ucurrent;



%% post-processing
%calculate the exact value
  figure
  hold on
  x=0*deltx:deltx:1*deltx;
  Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(1,1);
  plot(x,Unumsolution1,'-r.');hold on
  H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
        x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(1,i);
```

```
        plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0)+DG(P0)显式欧拉 U(CFL=0.01，单元格数为 128)','fontsize',16)
hold off

  figure
  hold on
  x=0*deltx:deltx:1*deltx;
  Unumsolution1(1,1)=Unumsolution(2,1);Unumsolution1(1,2)=Unumsolution(2,1);
  plot(x,Unumsolution1,'-r.');hold on
  H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
        x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution(2,i);Unumsolution1(1,2)=Unumsolution(2,i);
        plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(2,:),'-b*')
H2=plot(y,Uexasolution(2,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0)+DG(P0)显式欧拉 Ux*deltx(CFL=0.01，单元格数为
128)','fontsize',16)
hold off

%calculate the accuracy of space
I1=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
            xi=deltx/2*t(i)+0.5*(2*x+deltx);
            for m=1:numberx-1
```

```
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(pi*xi)-Unumsolution(1,m))^2;k=k+1;
            end
        end
        I1=I1+W(i)*fi;
    end
end
I1=I1*0.5*deltx;
I1=sqrt(I1)

I2=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(pi*cos(pi*xi)-Unumsolution(2,m)/deltx)^2;k=k+1;
            end
        end
        I2=I2+W(i)*fi;
    end
end
I2=I2*0.5*deltx;
I2=sqrt(I2)

figure
a1=[0.0910,0.0555,0.0311,0.0166,0.0087];
a2=[1/8,1/16,1/32,1/64,1/128];
plot(log(a2),log(a1),'-b*')
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('U 空间精度分析','fontsize',16)
afa1=round(log(a1)/log(a2))

figure
a1=[0.6287,0.3665,0.2012,0.1064,0.0553];
a2=[1/8,1/16,1/32,1/64,1/128];
plot(log(a2),log(a1),'-b*')
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('Ux 空间精度分析','fontsize',16)
afa2=round(log(a1)/log(a2))
```

DGP0P1+DGP0

```
clc
clear all
close all
%% Pre-processing
Unit=128;deltx=1/Unit;CFL=0.01;tol=0.01;
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);deltto=CFL*deltx/abslambda;%伪时间变量
endx=1;endto=0.5;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(2,numberx);
Unumsolution1=zeros(1,2);
Unumsolution2=zeros(2,numberx-1);
Mto=[deltx,0;0,deltx/12+1/deltx];
A=[abslambda,0;0,abslambda];
R=zeros(2,numberx-1);
F1=zeros(2,numberx-1);
F2=zeros(2,numberx-1);
V0=zeros(1,numberx-1);
%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(1,k)=(x+deltx/2)^2-(x+deltx/2);
    k=k+1;
end
V0=Ucurrent(1,:);
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=(2*(x+deltx/2)-1)*deltx;
    k=k+1;
end

k=1;
for x=0:deltx:endx-deltx
    R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
    R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
    k=k+1;
end
```

```
for k=2:numberx-1
      F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-0.5*(-nu*Ucurrent(2,k-1)/deltx)-
(Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1))/(Tr*deltx)]+[-nu*Ucurrent(2,k)/deltx;-0.5*(-
nu*Ucurrent(2,k)/deltx)-(Ucurrent(1,k)+0.5*Ucurrent(2,k))/(Tr*deltx)])-
0.5*A*(Ucurrent(:,k)-Ucurrent(:,k-1));
      k=k+1;
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
(Ucurrent(1,1)+0.5*Ucurrent(2,1))/(Tr*deltx)])-0.5*A*(Ucurrent(:,1)-
[0;Ucurrent(2,1)]);

for k=1:numberx-2
      F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;0.5*(-nu*Ucurrent(2,k)/deltx)-
(Ucurrent(1,k)+0.5*Ucurrent(2,k))/(Tr*deltx)]+[-nu*Ucurrent(2,k+1)/deltx;0.5*(-
nu*Ucurrent(2,k+1)/deltx)-(Ucurrent(1,k+1)+0.5*Ucurrent(2,k+1))/(Tr*deltx)])-
0.5*A*(Ucurrent(:,k+1)-Ucurrent(:,k));
      k=k+1;
end
  F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;0.5*(-
nu*Ucurrent(2,numberx-1)/deltx)-(Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-
1))/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;0.5*(-nu*Ucurrent(2,numberx-
1)/deltx)-0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)]-Ucurrent(:,numberx-1));

  for k=1:numberx-1
      R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
  end

  %solve the exasolution
k=1;
for x=0:deltx:endx
      Uexasolution(1,k)=sin(pi*x);
      Uexasolution(2,k)=pi*cos(pi*x)*deltx;
      k=k+1;
end

%solve the numsolution
for n=deltto:deltto:endto
      for k=1:numberx-1
          Unext(:,k)=Ucurrent(:,k)+Mto\R(:,k)*deltto;
      end
       if var(Ucurrent(1,:)-Unext(1,:))<tol*V0
            break
       end
```

```
    Ucurrent=Unext;

k=1;
for x=0:deltx:endx-deltx
    R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
    R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
    k=k+1;
end

for k=2:numberx-1
    F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-0.5*(-nu*Ucurrent(2,k-1)/deltx)-
Ucurrent(1,k-1)/(Tr*deltx)]+[-nu*Ucurrent(2,k)/deltx;-0.5*(-nu*Ucurrent(2,k)/deltx)-
Ucurrent(1,k)/(Tr*deltx)])-0.5*A*(Ucurrent(:,k)-Ucurrent(:,k-1));
    k=k+1;
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
Ucurrent(1,1)/(Tr*deltx)])-0.5*A*(Ucurrent(:,1)-[0;Ucurrent(2,1)]);

for k=1:numberx-2
    F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;-0.5*(-nu*Ucurrent(2,k)/deltx)-
Ucurrent(1,k)/(Tr*deltx)]+[-nu*Ucurrent(2,k+1)/deltx;-0.5*(-
nu*Ucurrent(2,k+1)/deltx)-Ucurrent(1,k+1)/(Tr*deltx)])-0.5*A*(Ucurrent(:,k+1)-
Ucurrent(:,k));
    k=k+1;
end
  F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;-0.5*(-
nu*Ucurrent(2,numberx-1)/deltx)-Ucurrent(1,numberx-1)/(Tr*deltx)]+[-
nu*Ucurrent(2,numberx-1)/deltx;-0.5*(-nu*Ucurrent(2,numberx-1)/deltx)-
0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)]-Ucurrent(:,numberx-1));

  for k=1:numberx-1
    R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
  end
end
Unumsolution=Ucurrent;

for i=1:numberx-1
Unumsolution2(1,i)=Unumsolution(1,i)+Unumsolution(2,i)*(-deltx/2);
Unumsolution2(2,i)=Unumsolution(1,i)+Unumsolution(2,i)*(deltx/2);
end
```

```
%% post-processing
%calculate the exact value
 figure
 hold on
 x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=Unumsolution2(1,1);Unumsolution1(1,2)=Unumsolution2(1,1);
 plot(x,Unumsolution1,'-r.');hold on
 H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution2(1,i);Unumsolution1(1,2)=Unumsolution2(1,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0P1)+DG(P0)显式欧拉 U(CFL=0.01，单元格数为 128)','fontsize',16)
hold off

 figure
 hold on
 x=0*deltx:deltx:1*deltx;
 Unumsolution1(1,1)=Unumsolution(2,1);Unumsolution1(1,2)=Unumsolution(2,1);
 plot(x,Unumsolution1,'-r.');hold on
 H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution(2,i);Unumsolution1(1,2)=Unumsolution(2,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(2,:),'-b*')
H2=plot(y,Uexasolution(2,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
```

```
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0P1)+DG(P0)显式欧拉 Ux*deltx(CFL=0.01， 单元格数为
128)','fontsize',16)
hold off

%calculate the accuracy of space
I1=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(pi*xi)-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-
((m-1)*deltx+deltx/2))))^2;k=k+1;
            end
        end
        I1=I1+W(i)*fi;
    end
end
I1=I1*0.5*deltx;
I1=sqrt(I1)

I2=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(pi*cos(pi*xi)-Unumsolution(2,m)/deltx)^2;k=k+1;
            end
        end
        I2=I2+W(i)*fi;
    end
end
I2=I2*0.5*deltx;
I2=sqrt(I2)

figure
a1=[0.0740,0.0427,0.0208,0.0114,0.0120];
a2=[1/8,1/16,1/32,1/64,1/128];
plot(log(a2),log(a1),'-b*')
```

```
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('U 空间精度分析','fontsize',16)
afa1=round(log(a1)/log(a2))

figure
a1=[0.6407,0.3750,0.2067,0.1100,0.0578];
a2=[1/8,1/16,1/32,1/64,1/128];
plot(log(a2),log(a1),'-b*')
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('Ux 空间精度分析','fontsize',16)
afa2=round(log(a1)/log(a2))
```

DGP0P2+rDGP0P1

```
clc
clear all
close all
%% Pre-processing
Unit=128;deltx=1/Unit;CFL=0.01;tol=0.01;
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);deltto=CFL*deltx/abslambda;%伪时间变量
endx=1;endto=0.5;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Ucurrent1=zeros(1,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(2,numberx);
Unumsolution1=zeros(1,2);
Unumsolution2=zeros(2,numberx-1);
Mto=[deltx,0;0,deltx/12+1/deltx];
A=[abslambda,0;0,abslambda];
R=zeros(2,numberx-1);
F1=zeros(2,numberx-1);
F2=zeros(2,numberx-1);
V0=zeros(1,numberx-1);
%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
      Ucurrent(1,k)=(x+deltx/2)^2-(x+deltx/2);
      k=k+1;
end
```

```
V0=Ucurrent(1,:);
k=1;
for x=0:deltx:endx-deltx
    Ucurrent(2,k)=(2*(x+deltx/2)-1)*deltx;
    k=k+1;
end
k=1;
for x=0:deltx:endx-deltx
    Ucurrent1(1,k)=2*deltx^2;
    k=k+1;
end

k=1;
for x=0:deltx:endx-deltx
    R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
    R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
    k=k+1;
end

for k=2:numberx-1
    F1(:,k)=0.5*([-nu*(Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx);-0.5*(-
nu*(Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx))-(Ucurrent(1,k-
1)+0.5*Ucurrent(2,k-1))/(Tr*deltx)]+[-
nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx);-0.5*(-
nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx))-
(Ucurrent(1,k)+0.5*Ucurrent(2,k))/(Tr*deltx)])-0.5*A*(Ucurrent(:,k)-Ucurrent(:,k-
1));
    k=k+1;
end
F1(:,1)=0.5*([-nu*(Ucurrent(2,1)/deltx+0.5*Ucurrent1(1,1)/deltx);-0.5*(-
nu*(Ucurrent(2,1)/deltx+0.5*Ucurrent1(1,1)/deltx))-0/(Tr*deltx)]+[-
nu*(Ucurrent(2,1)/deltx+0.5*Ucurrent1(1,1)/deltx);-0.5*(-
nu*(Ucurrent(2,1)/deltx+0.5*Ucurrent1(1,1)/deltx))-
(Ucurrent(1,1)+0.5*Ucurrent(2,1))/(Tr*deltx)])-0.5*A*(Ucurrent(:,1)-
[0;Ucurrent(2,1)]);

for k=1:numberx-2
    F2(:,k)=0.5*([-nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx);0.5*(-
nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx))-
(Ucurrent(1,k)+0.5*Ucurrent(2,k))/(Tr*deltx)]+[-
nu*(Ucurrent(2,k+1)/deltx+0.5*Ucurrent1(1,k+1)/deltx);0.5*(-
nu*(Ucurrent(2,k+1)/deltx+0.5*Ucurrent1(1,k+1)/deltx))-
```

```
(Ucurrent(1,k+1)+0.5*Ucurrent(2,k+1))/(Tr*deltx)])-0.5*A*(Ucurrent(:,k+1)-
Ucurrent(:,k));
      k=k+1;
end
 F2(:,numberx-1)=0.5*([-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx);0.5*(-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx))-(Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1))/(Tr*deltx)]+[-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx);0.5*(-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx))-0/(Tr*deltx)])-
0.5*A*([0;Ucurrent(2,numberx-1)]-Ucurrent(:,numberx-1));

 for k=1:numberx-1
      R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
 end

 %solve the exasolution
k=1;
for x=0:deltx:endx
      Uexasolution(1,k)=sin(pi*x);
      Uexasolution(2,k)=pi*cos(pi*x)*deltx;
      k=k+1;
end

%solve the numsolution
for n=deltto:deltto:endto
      for k=1:numberx-1
          Unext(:,k)=Ucurrent(:,k)+Mto\R(:,k)*deltto;
      end
       if var(Ucurrent(1,:)-Unext(1,:))<tol*V0
            break
       end
      Ucurrent=Unext;

k=1;
for x=0:deltx:endx-deltx
      R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
      R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
      k=k+1;
end

for k=2:numberx-1
```

```
        F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-0.5*(-nu*Ucurrent(2,k-1)/deltx)-
Ucurrent(1,k-1)/(Tr*deltx)]+[-nu*Ucurrent(2,k)/deltx;-0.5*(-nu*Ucurrent(2,k)/deltx)-
Ucurrent(1,k)/(Tr*deltx)]-0.5*A*(Ucurrent(:,k)-Ucurrent(:,k-1)));
        k=k+1;
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
Ucurrent(1,1)/(Tr*deltx)])-0.5*A*(Ucurrent(:,1)-[0;Ucurrent(2,1)]);

for k=1:numberx-2
        F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;-0.5*(-nu*Ucurrent(2,k)/deltx)-
Ucurrent(1,k)/(Tr*deltx)]+[-nu*Ucurrent(2,k+1)/deltx;-0.5*(-
nu*Ucurrent(2,k+1)/deltx)-Ucurrent(1,k+1)/(Tr*deltx)]-0.5*A*(Ucurrent(:,k+1)-
Ucurrent(:,k)));
        k=k+1;
end
  F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;-0.5*(-
nu*Ucurrent(2,numberx-1)/deltx)-Ucurrent(1,numberx-1)/(Tr*deltx)]+[-
nu*Ucurrent(2,numberx-1)/deltx;-0.5*(-nu*Ucurrent(2,numberx-1)/deltx)-
0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)]-Ucurrent(:,numberx-1));

  for k=1:numberx-1
        R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
        for k=2:numberx-2
              Ucurrent1(1,k)=[deltx;-deltx]\[Ucurrent(2,k+1)-
Ucurrent(2,k);Ucurrent(2,k-1)-Ucurrent(2,k)];
        end
        Ucurrent1(1,1)=[deltx;-deltx]\[Ucurrent(2,2)-Ucurrent(2,1);Ucurrent(2,1)-
Ucurrent(2,1)];
        Ucurrent1(1,numberx-1)=[deltx;-deltx]\[Ucurrent(2,numberx-1)-
Ucurrent(2,numberx-1);Ucurrent(2,numberx-2)-Ucurrent(2,numberx-1)];
  end
end
Unumsolution=Ucurrent;

for i=1:numberx-1
Unumsolution2(1,i)=Unumsolution(1,i)+Unumsolution(2,i)*(-deltx/2);
Unumsolution2(2,i)=Unumsolution(1,i)+Unumsolution(2,i)*(deltx/2);
end


%% post-processing
%calculate the exact value
  figure
```

```
  hold on
  x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=Unumsolution2(1,1);Unumsolution1(1,2)=Unumsolution2(1,1);
  plot(x,Unumsolution1,'-r.');hold on
  H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution2(1,i);Unumsolution1(1,2)=Unumsolution2(1,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0P2)+rDG(P0P1)显式欧拉 U，单元格数为 128','fontsize',16)
hold off

  figure
  hold on
  x=0*deltx:deltx:1*deltx;
  Unumsolution1(1,1)=Unumsolution(2,1);Unumsolution1(1,2)=Unumsolution(2,1);
  plot(x,Unumsolution1,'-r.');hold on
  H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution(2,i);Unumsolution1(1,2)=Unumsolution(2,i);
      plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(2,:),'-b*')
H2=plot(y,Uexasolution(2,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('DG(P0P2)+rDG(P0P1)显式欧拉 Ux*deltx，单元格数为 128','fontsize',16)
```

```
hold off

%calculate the accuracy of space
I1=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(pi*xi)-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-
((m-1)*deltx+deltx/2))))^2;k=k+1;
            end
        end
        I1=I1+W(i)*fi;
    end
end
I1=I1*0.5*deltx;
I1=sqrt(I1)

I2=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(pi*cos(pi*xi)-Unumsolution(2,m)/deltx)^2;k=k+1;
            end
        end
        I2=I2+W(i)*fi;
    end
end
I2=I2*0.5*deltx;
I2=sqrt(I2)

figure
a1=[0.0374,0.0303,0.0241,0.0206,0.0190];
a2=[1/8,1/16,1/32,1/64,1/128];
plot(log(a2),log(a1),'-b*')
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('U 空间精度分析','fontsize',16)
afa1=round(log(a1)/log(a2))
```

```
figure
a1=[0.4460,0.2363,0.1223,0.0630,0.0328];
a2=[1/8,1/16,1/32,1/64,1/128];
plot(log(a2),log(a1),'-b*')
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('Ux 空间精度分析','fontsize',16)
afa2=round(log(a1)/log(a2))
```