CFD 入门练习 3

1.对一维线性波动方程 $u_t + u_x = 0, x \in [0,1], t \geq 0$ **,满足以下初始条件** $u(x,0) = u_0(x)$ **,及边界条件** $u(0,t) = u(1,t) = 0$ **。其中**

$$u_0(x) = \begin{cases} 0 & ,x < 0.2 \\ 1 & ,0.2 \leq x \leq 0.3 \\ 2(x-0.3)^3 - 3(x-0.3)^2 + 1 & ,0.3 < x \leq 0.4 \\ 0 & ,x > 0.4 \end{cases}$$

考虑均匀网格 $\Delta x = 0.01$ **,并定义 CFL=** $\Delta t / \Delta x$ **,编写程序用以下数值方法使用显式格式计算不同 CFL 值时候，** $t = 0.35$ **时刻的数值解，并与解析解进行对比。**

(1) 有限差分法

这里仅展示最终的数值解与解析解比较图。(CFL 分别为 0.001，0.01，0.1，1)，详细代码见附录。

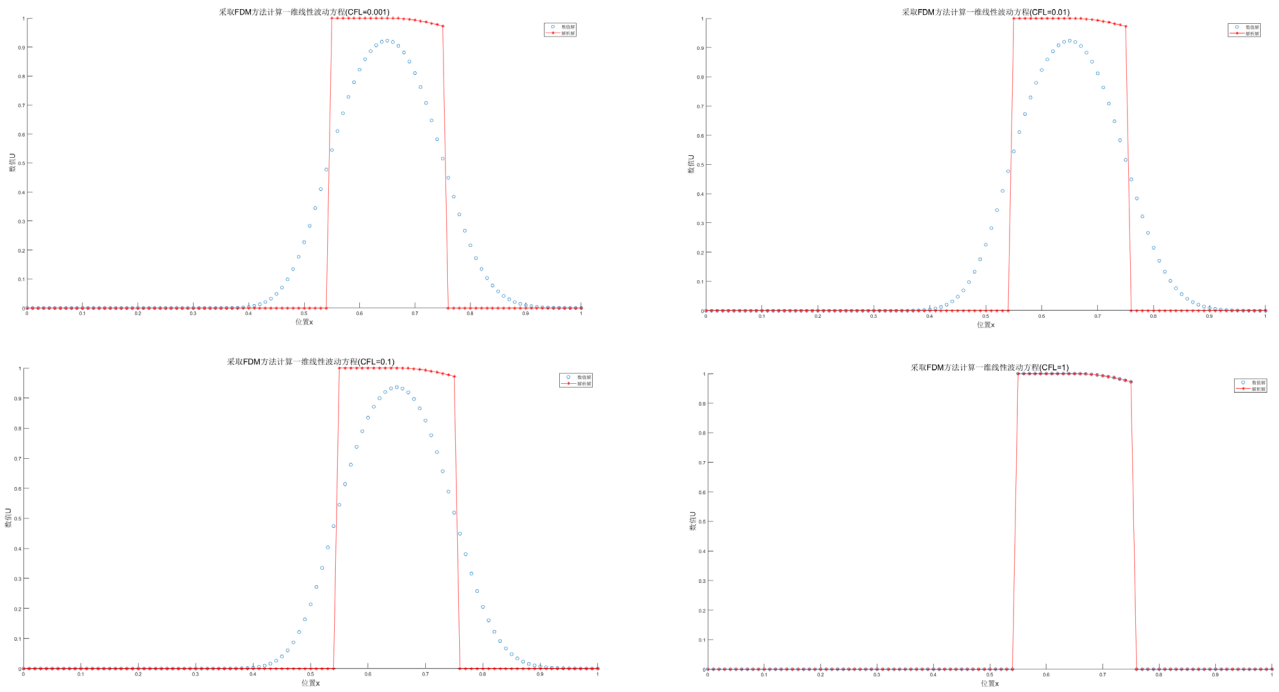

图 1：FDM 方法下不同 CFL 对应的数值解与解析解比较图

(2)有限体积法

这里仅展示最终的数值解与解析解比较图。(CFL 取值同上)

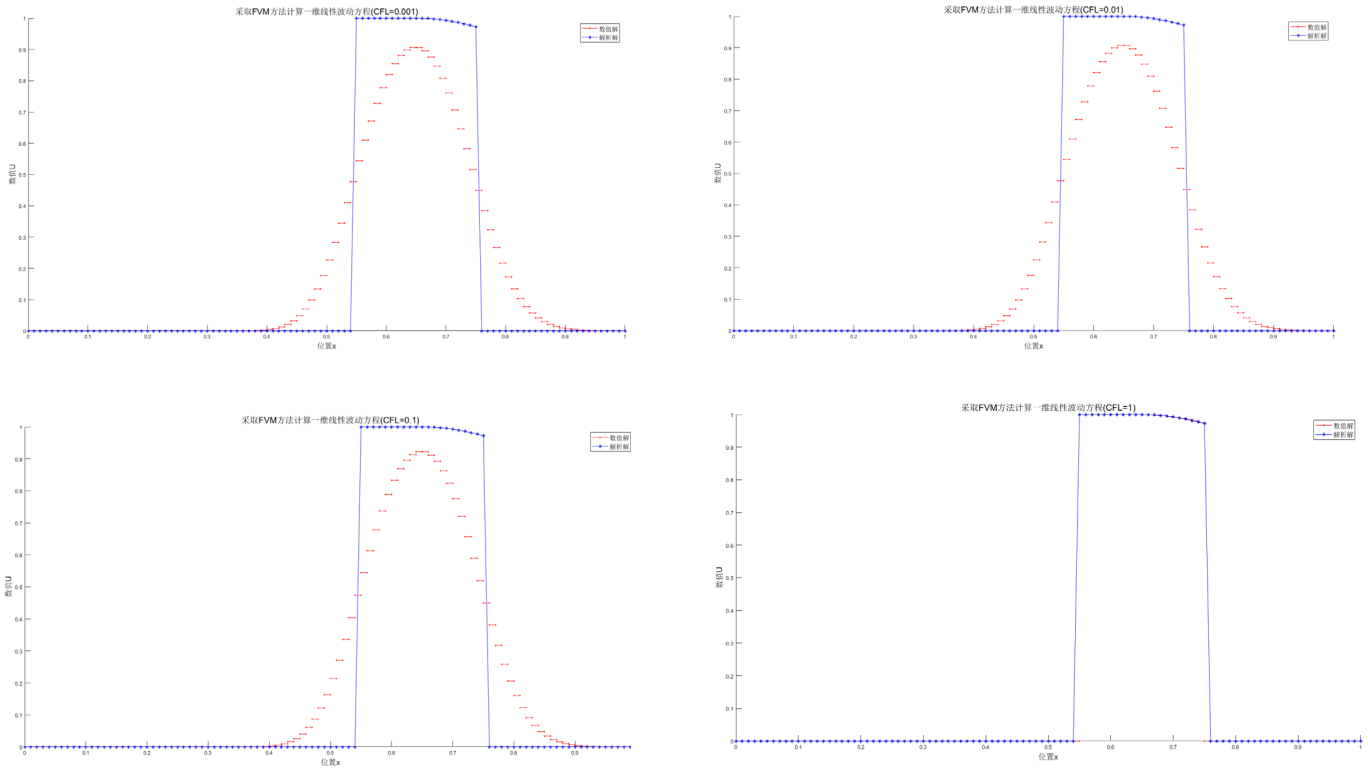

图 2：FVM 方法下不同 CFL 对应的数值解与解析解比较图

(3)间断伽辽金法

这里仅展示最终的数值解与解析解比较图。(CFL 取值同上)

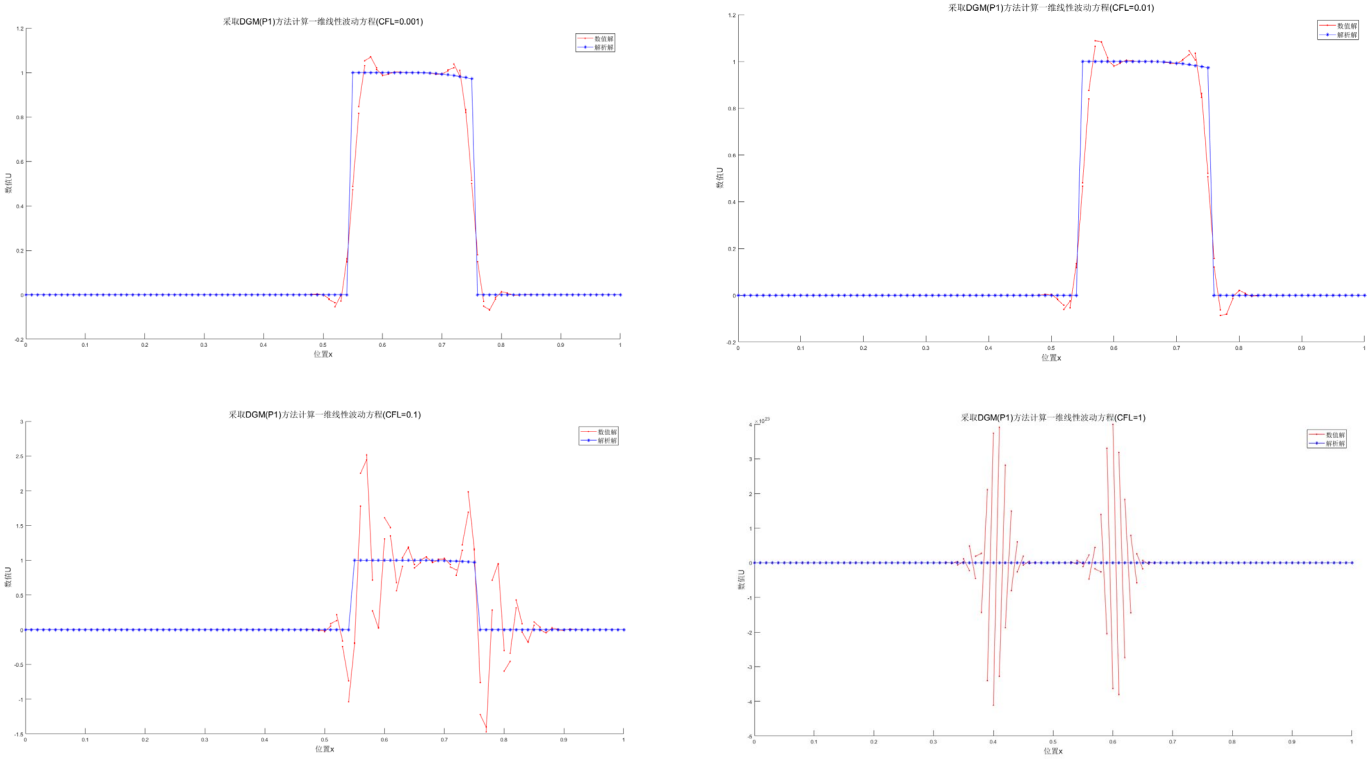

图 3：DGM(P1)方法下不同 CFL 对应的数值解与解析解比较图

附录

FDM 方法

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(1,numberx);
Unext=zeros(1,numberx);
Unumsolution=zeros(1,numberx);
Uexasolution=zeros(1,numberx);
UL=0;UR=0;
%% solve the question
%initial    condition set up
Ucurrent(1,1)=UL;Ucurrent(1,numberx)=UR;k=2;
for x=deltx:deltx:endx-deltx
    if x<0.2
        U=0;Ucurrent(1,k)=U;
    elseif x>=0.2&&x<=0.3
        U=1;Ucurrent(1,k)=U;
    elseif x>0.3&&x<=0.4
        U=2*(x-0.3)^3-3*(x-0.3)^2+1;Ucurrent(1,k)=U;
    elseif x>0.4
        U=0;Ucurrent(1,k)=U;
    end
    k=k+1;
end

%solve the numsolution
for n=deltt:deltt:endt
    for k=2:numberx-1
        Unext(1,k)=CFL*(Ucurrent(1,k-1)-Ucurrent(1,k))+Ucurrent(1,k);
    end
    Unext(1,1)=UL;Unext(1,numberx)=UR;Ucurrent=Unext;
end
Unumsolution=Ucurrent;

%solve the exasolution
Uexasolution(1,1)=UL;Uexasolution(1,numberx)=UR;k=2;
for x=deltx:deltx:endx-deltx
```

```
    if x-endt<0.2
        U=0;Uexasolution(1,k)=U;
    elseif x-endt>=0.2&&x-endt<=0.3
        U=1;Uexasolution(1,k)=U;
    elseif x-endt>0.3&&x-endt<=0.4
        U=2*(x-endt-0.3)^3-3*(x-endt-0.3)^2+1;Uexasolution(1,k)=U;
    elseif   x-endt>0.4
         U=0;Uexasolution(1,k)=U;
    end
    k=k+1;
end

%% post-processing
%calculate the exact value
x=0:deltx:endx;
figure
scatter(x,Unumsolution)
hold on
plot(x,Uexasolution,'-r*')
legend('数值解','解析解')
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('采取 FDM 方法计算一维线性波动方程(CFL=1)','fontsize',16)

hold off
%calculate the variance
B=Uexasolution-Unumsolution;
Var=var(B);
```

FVM 方法

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(1,numberx-1);
Unext=zeros(1,numberx-1);
Unumsolution=zeros(1,numberx-1);
Uexasolution=zeros(1,numberx);
```

```
UL=0;UR=0;
Unumsolution1=zeros(1,2);
B=zeros(2,numberx-1);
%% solve the question
%initial    condition set up
k=1;
for x=0:deltx:endx-deltx
      if x<0.2&&x+deltx<0.2
            U=0;Ucurrent(1,k)=U;
      elseif x<0.2&&x+deltx>0.2&&x+deltx<=0.3
             U=0+(x+deltx-0.2)/deltx;Ucurrent(1,k)=U;
      elseif x>=0.2&&x<=0.3&&x+deltx>=0.2&&x+deltx<=0.3
            U=1;Ucurrent(1,k)=U;
      elseif x>=0.2&&x<=0.3&&x+deltx>0.3&&x+deltx<=0.4
            U=(0.3-x)/deltx+(0.5*(x+deltx-0.3)^4-(x+deltx-0.3)^3+x+deltx-
0.3)/deltx;Ucurrent(1,k)=U;
      elseif x>0.3&&x<=0.4&&x+deltx>0.3&&x+deltx<=0.4
            U=(0.5*((x+deltx-0.3)^4-(x-0.3)^4)-((x+deltx-0.3)^3-(x-
0.3)^3)+deltx)/deltx;Ucurrent(1,k)=U;
      elseif x>0.3&&x<=0.4&&x+deltx>0.4
             U=(0.5*((0.4-0.3)^4-(x-0.3)^4)-((0.4-0.3)^3-(x-0.3)^3)+0.4-
x)/deltx;Ucurrent(1,k)=U;
      elseif x>0.4
            U=0;Ucurrent(1,k)=U;
      end
      k=k+1;
end

%solve the numsolution
for n=deltt:deltt:endt
      for k=2:numberx-1
          Unext(1,k)=Ucurrent(1,k)+CFL*(Ucurrent(1,k-1)-Ucurrent(1,k));
      end
      Unext(1,1)=UL;Ucurrent=Unext;
end
Unumsolution=Ucurrent;


k=1;
for x=0:deltx:endx
      if x-endt<0.2
            U=0;Uexasolution(1,k)=U;
      elseif x-endt>0.2&&x-endt<=0.3
             U=1;Uexasolution(1,k)=U;
```

```
        elseif x-endt>0.3&&x-endt<=0.4
            U=2*(x-endt-0.3)^3-3*(x-endt-0.3)^2+1;Uexasolution(1,k)=U;
        elseif x-endt>0.4
            U=0;Uexasolution(1,k)=U;
        end
        k=k+1;
end
%% post-processing
%calculate the exact value
 figure
 hold on
 x=0*deltx:deltx:1*deltx;
 Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(1,1);
 plot(x,Unumsolution1,'-r.');hold on
 H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
        x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(1,i);
        plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('采取 FVM 方法计算一维线性波动方程(CFL=1)','fontsize',16)
hold off
%calculate the variance
for i=1:numberx-1
        B(1,i)=Unumsolution(1,i)-Uexasolution(1,i);
        B(2,i)=Unumsolution(1,i)-Uexasolution(1,i+1);
end

Var=var(B(1,:))+var(B(2,:))
```

DGM(*P*1)方法

```
clc
clear all
close all
%% Pre-processing
deltx=0.01;CFL=1;deltt=CFL*deltx;
endx=1;endt=0.35;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Uexasolution=zeros(1,numberx);
UL=0;UR=0;
M=[deltx,0;0,deltx^3/12];
R=zeros(2,1);
B=zeros(2,numberx-1);
Unumsolution1=zeros(1,2);
%% solve the question
%initial    condition set up
%Uc
k=1;
for x=0:deltx:endx-deltx
    if x+deltx/2<0.2
        U=0;Ucurrent(1,k)=U;
    elseif x+deltx/2>0.2&&x+deltx/2<=0.3
        U=1;Ucurrent(1,k)=U;
    elseif x+deltx/2>0.3&&x+deltx/2<=0.4
        U=2*(x+deltx/2-0.3)^3-3*(x+deltx/2-0.3)^2+1;Ucurrent(1,k)=U;
    elseif x+deltx/2>0.4
        U=0;Ucurrent(1,k)=U;
    end
    k=k+1;
end
%Uxc
k=1;
for x=0:deltx:endx-deltx
    if x+deltx/2<0.2
        U=0;Ucurrent(2,k)=U;
    elseif x+deltx/2>0.2&&x+deltx/2<=0.3
        U=0;Ucurrent(2,k)=U;
    elseif x+deltx/2>0.3&&x+deltx/2<=0.4
        U=6*(x+deltx/2-0.3)^2-6*(x+deltx/2-0.3);Ucurrent(2,k)=U;
    elseif x+deltx/2>0.4
```

```
            U=0;Ucurrent(2,k)=U;
        end
        k=k+1;
end
%solve the numsolution
for n=deltt:deltt:endt
        for k=2:numberx-1
            f1=Ucurrent(1,k-1)+Ucurrent(2,k-1)*deltx/2;
            f2=Ucurrent(1,k)+Ucurrent(2,k)*deltx/2;
            R(1,1)=f1-f2;
            R(2,1)=-deltx/2*(f1+f2)+deltx*Ucurrent(1,k);
            Unext(:,k)=Ucurrent(:,k)+M\R*deltt;
        end
        Unext(1,1)=UL;Unext(2,1)=UL;Ucurrent=Unext;
end
for i=1:numberx-1
Unumsolution(1,i)=Ucurrent(1,i)+Ucurrent(2,i)*(-deltx/2);
Unumsolution(2,i)=Ucurrent(1,i)+Ucurrent(2,i)*(deltx/2);
end

%solve the exasolution
k=1;
for x=0:deltx:endx
        if x-endt<0.2
            U=0;Uexasolution(1,k)=U;
        elseif x-endt>0.2&&x-endt<=0.3
             U=1;Uexasolution(1,k)=U;
        elseif x-endt>0.3&&x-endt<=0.4
            U=2*(x-endt-0.3)^3-3*(x-endt-0.3)^2+1;Uexasolution(1,k)=U;
        elseif x-endt>0.4
             U=0;Uexasolution(1,k)=U;
        end
        k=k+1;
end


%% post-processing
figure
hold on
x=0*deltx:deltx:1*deltx;
Unumsolution1(1,1)=Unumsolution(1,1);Unumsolution1(1,2)=Unumsolution(2,1);
plot(x,Unumsolution1,'-r.');hold on
H1=plot(x,Unumsolution1,'-r.');hold on
for i=2:numberx-1
```

```
        x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution(1,i);Unumsolution1(1,2)=Unumsolution(2,i);
        plot(x,Unumsolution1,'-r.')
end
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*')
H2=plot(y,Uexasolution(1,:),'-b*');hold on
legend('数值解');hold on
lgd=legend([H1,H2],'数值解','解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('采取 DGM(P1)方法计算一维线性波动方程(CFL=0.1)','fontsize',16)
hold off

for i=1:numberx-1
        B(1,i)=Uexasolution(1,i)-Unumsolution(1,i);
end
for i=1:numberx-1
        B(2,i)=Uexasolution(1,i+1)-Unumsolution(2,i);
end
Var=var(B(1,:))+var(B(2,:))
```