

# 课题组组会-练习 6

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## 一 练习及结果

1. 在  $x \in [0, 1]$  的均匀网格上尝试使用 Variational Reconstruction (VR) 对  $f(x), g(x)$  分别进行 P0P2, P1P2 重构, 其中  $f(x) = 1 + x + x^2, g(x) = \sin(\pi x)$ .

a) 尝试调整不同阶次的权重系数及边界面权重系数, 测试重构精度。

b) 如果网格为不均匀网格呢?

c) 如果考虑 Hyperbolic rDG, 进行 DG(P0P2)+rDG(P0P1), 使用 VR 重构算法, 结果如何?

### a) 解: P0P2

本题均考虑非均匀网格, 均匀网格视为非均匀网格的特例。

下面给出 Interfacial jump integration 的定义以及 P0P2 的限制方程组。

$$I = \sum_{f=1}^{N_f} I_f, I_f = \frac{1}{d_{LR}} \left( \omega_0^2 [u]^2 + \omega_1^2 [u_x]^2 d_{LR}^2 + \omega_2^2 [u_{xx}]^2 d_{LR}^4 \right), [u] = u_L - u_R$$

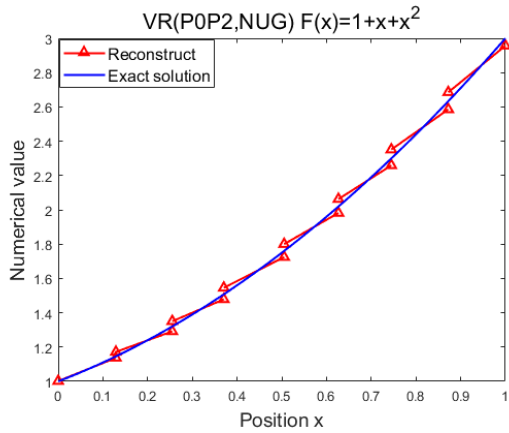
限制方程组:

$$\begin{cases} \frac{\partial I_f}{\partial u_{2L}} = 0 \\ \frac{\partial I_f}{\partial u_{3L}} = 0 \\ \frac{\partial I_f}{\partial u_{2R}} = 0 \\ \frac{\partial I_f}{\partial u_{3R}} = 0 \end{cases}$$

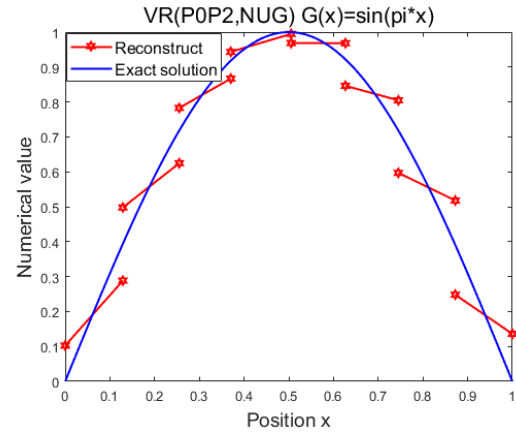
即

$$\left\{ \begin{array}{l} \frac{2}{d_{LR}} \left( \omega_0^2 \left( \sum_{i=1}^3 u_{iL} B_{iL} - \sum_{i=1}^3 u_{iR} B_{iR} \right) B_{2L} + \omega_1^2 \left( \frac{1}{\Delta x_{ieL}} \sum_{i=2}^3 u_{iL} B_{i-1L} - \frac{1}{\Delta x_{ieR}} \sum_{i=2}^3 u_{iR} B_{i-1R} \right) \frac{1}{\Delta x_{ieL}} d_{LR}^2 \right) = 0 \\ \frac{2}{d_{LR}} \left( \omega_0^2 \left( \sum_{i=1}^3 u_{iL} B_{iL} - \sum_{i=1}^3 u_{iR} B_{iR} \right) B_{3L} + \omega_1^2 \left( \frac{1}{\Delta x_{ieL}} \sum_{i=2}^3 u_{iL} B_{i-1L} - \frac{1}{\Delta x_{ieR}} \sum_{i=2}^3 u_{iR} B_{i-1R} \right) B_{2L} \frac{1}{\Delta x_{ieL}} d_{LR}^2 + \omega_2^2 (\Delta x_{ieL}^{-2} u_{3L} - \Delta x_{ieR}^{-2} u_{3R}) \Delta x_{ieL}^{-2} d_{LR}^4 \right) = 0 \\ \frac{-2}{d_{LR}} \left( \omega_0^2 \left( \sum_{i=1}^3 u_{iL} B_{iL} - \sum_{i=1}^3 u_{iR} B_{iR} \right) B_{2R} + \omega_1^2 \left( \frac{1}{\Delta x_{ieL}} \sum_{i=2}^3 u_{iL} B_{i-1L} - \frac{1}{\Delta x_{ieR}} \sum_{i=2}^3 u_{iR} B_{i-1R} \right) \frac{1}{\Delta x_{ieR}} d_{LR}^2 \right) = 0 \\ \frac{-2}{d_{LR}} \left( \omega_0^2 \left( \sum_{i=1}^3 u_{iL} B_{iL} - \sum_{i=1}^3 u_{iR} B_{iR} \right) B_{3R} + \omega_1^2 \left( \frac{1}{\Delta x_{ieL}} \sum_{i=2}^3 u_{iL} B_{i-1L} - \frac{1}{\Delta x_{ieR}} \sum_{i=2}^3 u_{iR} B_{i-1R} \right) B_{2R} \frac{1}{\Delta x_{ieR}} d_{LR}^2 + \omega_2^2 (\Delta x_{ieL}^{-2} u_{3L} - \Delta x_{ieR}^{-2} u_{3R}) \Delta x_{ieR}^{-2} d_{LR}^4 \right) = 0 \end{array} \right.$$

通过求解该线性方程组进行  $f(x)$  与  $g(x)$  的重构, 需要注意的是, LU-SGS 得到的解误差较大, 需要 SGS(k) 进行求解, 且  $k=21$ 。下面展示权重  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$ , Nelem=8 时的重构图和精度分析图:



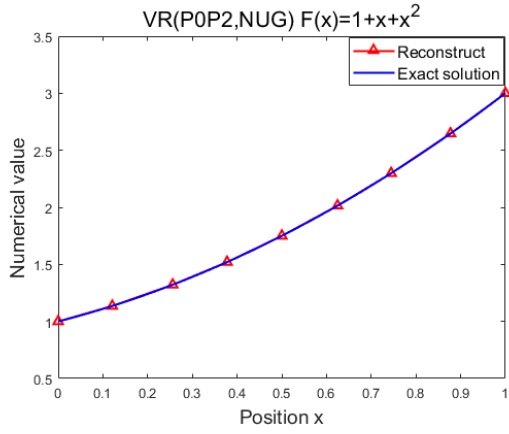
(a) f 重构



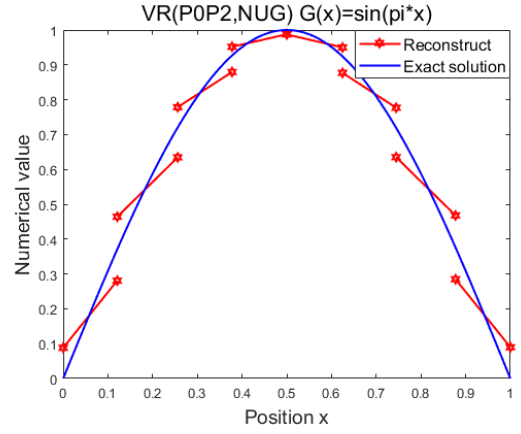
(b) g 重构

图 1: VR P0P2 重构 (LU-SGS)

调整权重系数以及边界面权重系数, 测试重构精度, 得到以下结论:

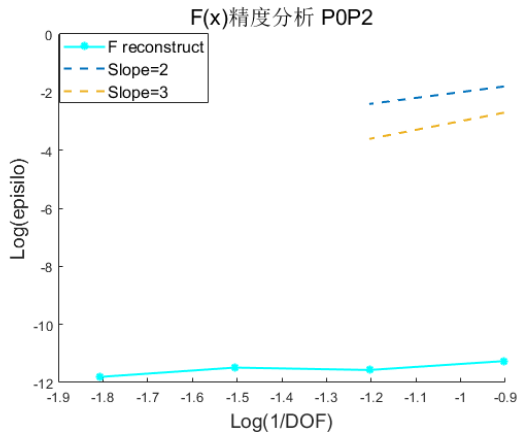


(a) f 重构

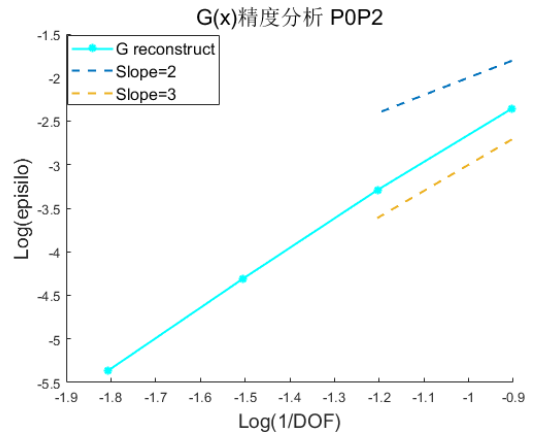


(b) g 重构

图 2: VR P0P2 重构 (SGS(21))



(a) f 精度分析



(b) g 精度分析

图 3: VR P0P2 精度分析

表 1:  $F(x)$  精度与  $\omega_0, \omega_1, \omega_2, \omega_b$  的关系

$\omega_0$	$\omega_1$	$\omega_2$	$\omega_b$	精度表现
0	—	—	—	差
—	0	—	—	差
—	—	0	—	差
—	—	—	↓ 1	↑
—	—	↑ 1	—	↑
—	↑ 1	—	—	↑
↑ 1	—	—	—	↑

表 2:  $G(x)$  精度与  $\omega_0, \omega_1, \omega_2, \omega_b$  的关系

$\omega_0$	$\omega_1$	$\omega_2$	$\omega_b$	精度表现
0	—	—	—	差
—	—	—	↓ 1	↑
—	—	↑ 0.5	—	↑
—	↑ 0.5	—	—	↑
↑ 1	—	—	—	↑

### 解: P1P2

与 P0P2 类似, IJI 的定义不变, 但是限制方程组改变:

$$\begin{cases} \frac{\partial I_f}{\partial u_{3L}} = 0 \\ \frac{\partial I_f}{\partial u_{3R}} = 0 \end{cases}$$

该重构可用托马斯精确求解, 下面展示权重  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$ , Nelem=8 时的重构图和精度分析图:

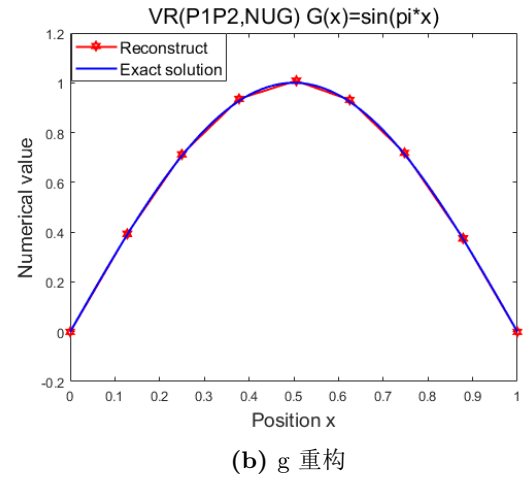
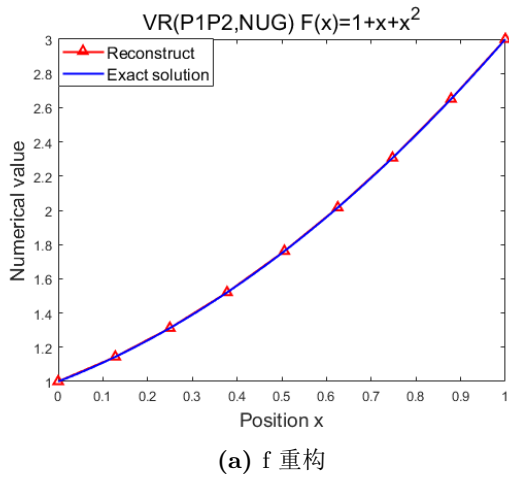


图 4: VR P1P2 重构

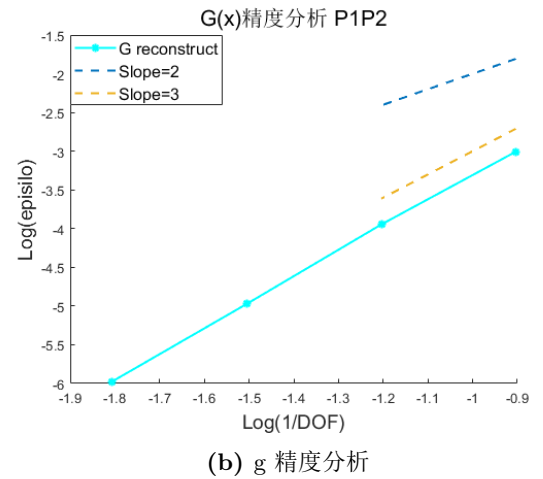
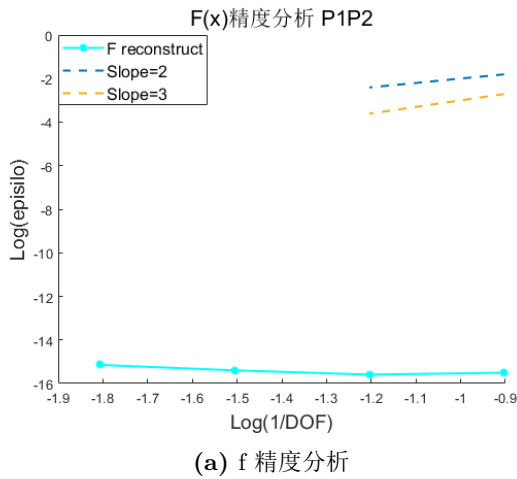


图 5: VR P1P2 精度分析

调整权重系数以及边界面权重系数，测试重构精度，得到以下结论：

表 3:  $F(x)$  精度与  $\omega_0, \omega_1, \omega_2, \omega_b$  的关系 (P1P2)

$\omega_0$	$\omega_1$	$\omega_2$	$\omega_b$	精度表现
—	—	—	$\uparrow \frac{\omega_0}{\sqrt{2}}$	$\uparrow$
—	—	$\uparrow 1$	—	$\uparrow$
—	$\uparrow 0.5$	—	—	$\uparrow$
改变	—	—	—	几乎无影响

表 4:  $G(x)$  精度与  $\omega_0, \omega_1, \omega_2, \omega_b$  的关系 (P1P2)

$\omega_0$	$\omega_1$	$\omega_2$	$\omega_b$	精度表现
—	—	—	$\uparrow \frac{\omega_0}{\sqrt{2}}$	$\uparrow$
—	—	$\uparrow 1$	—	$\uparrow$
—	$\uparrow 1$	—	—	$\uparrow$
改变	—	—	—	几乎无影响

b) 解:

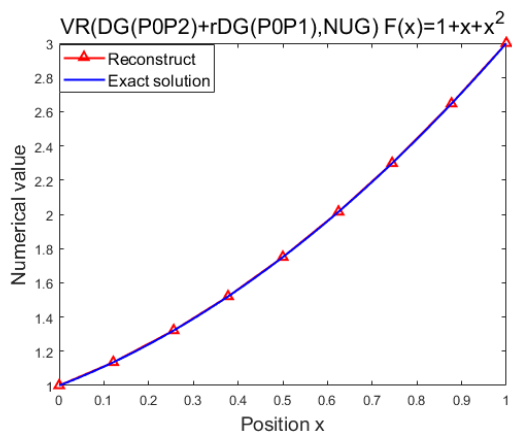
考虑 Hyperbolic rDG, 注意到

$$u = \begin{pmatrix} \varphi \\ v \end{pmatrix} = \begin{pmatrix} B_1 & B_2 & B_3 \\ 0 & B_1 \Delta x^{-1} & B_2 \Delta x^{-1} \end{pmatrix} \begin{pmatrix} \bar{\varphi} \\ \bar{\varphi}_x \Delta x \\ \varphi_{xx}^{c,R} \Delta x^2 \end{pmatrix}$$

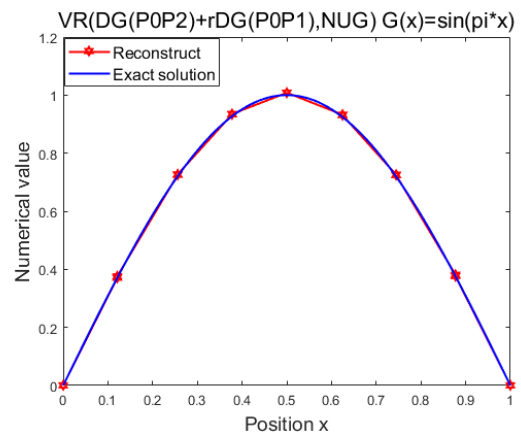
$$\Rightarrow \begin{cases} \varphi_x = v \\ \varphi_{xx} = v_x \end{cases} \text{ 所以, 若 } u_1 = \bar{\varphi}_i, u_2 = \bar{\varphi}_x \Delta x_i, u_3 = \varphi_{xx}^{c,R} \Delta x_i^2, \text{ 则 } \varphi = \sum_{i=1}^3 u_i B_i$$

$$I_f = \frac{1}{d_{LR}} \left( \omega_0^2 [\varphi]^2 + \omega_1^2 [v]^2 d_{LR}^2 + \omega_2^2 [v_x]^2 d_{LR}^4 \right) = \frac{1}{d_{LR}} \left( \omega_0^2 [\varphi]^2 + \omega_1^2 [\varphi_x]^2 d_{LR}^2 + \omega_2^2 [\varphi_{xx}]^2 d_{LR}^4 \right)$$

这与 P1P2 的 VR 是等价的，只不过  $u_2$  存储的量发生了改变。权重系数与边界系数对于重构精度的影响与 P1P2 的 VR 类似，这里不予展示。下面展示这种情况下权重  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$ , Nelem=8 时的重构图和精度分析图：

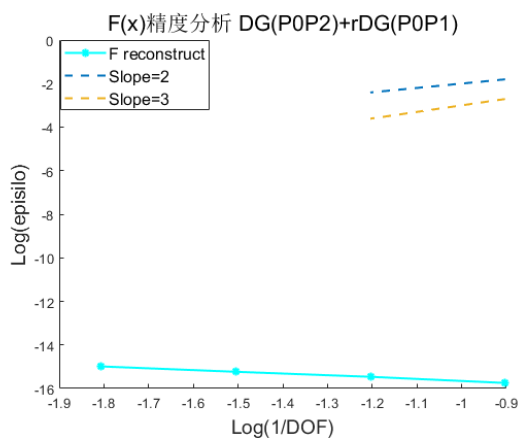


(a) f 重构

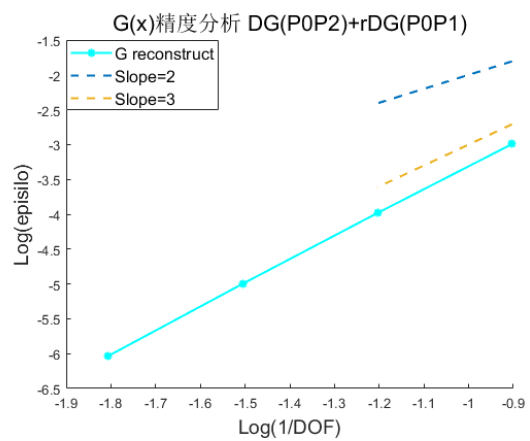


(b) g 重构

图 6: VR DG(P0P2)+rDG(P0P1)



(a) f 精度分析



(b) g 精度分析

图 7: VR DG(P0P2)+rDG(P0P1) 精度分析

## 二 附录

### 2 代码, 仅展示部分

#### VRP0P2

```
1  clc
2  clear all
3  close all
4  %Pre-proceeding
5  Unit=8;endx=1;deltax=endx/Unit;numberx=Unit+1;omega0=1;omega1=1;omega2
    =0.5;omegab=omega0/sqrt(2);
6  %记录内点位置, 上下浮动不超过百分之 5
7  Grid=zeros(1,numberx);
8  Deltax=zeros(1,Unit);
9  for i=2:numberx-1
10 Grid(1,i)=(i-1)*deltax+(0.1*rand(1)-0.05)*deltax;
11 end
12 Grid(1,numberx)=endx;
13 for i=2:numberx
14 Deltax(i-1)=Grid(1,i)-Grid(1,i-1);%记录每个单元的区间长度
15 end
16 f=@(x)1+x+x.^2;F=@(x)2.*x+1;
17 h=@(x)sin(pi*x);H=@(x)pi*cos(pi*x);
18 Unumsolution=zeros(1,Unit);
19 Ureconstruct=zeros(2*Unit,1);
20 A=sparse(1:2*Unit,1:2*Unit,0,2*Unit,2*Unit);
21 R=zeros(2*Unit,1);
22 Unumsolution1=zeros(1,2);
23 Unumsolution2=zeros(2,numberx-1);
24 Acc=zeros(3,4);a1=[1/8,1/16,1/32,1/64];a2=[1/8,1/16];
25
26 %Proceeding
27 %对 f
28 %initial
29 for i=1:numberx-1
30 Unumsolution(1,i)=(Grid(i+1)-Grid(i)+(Grid(i+1)^2-Grid(i)^2)/2+(Grid(i
    +1)^3-Grid(i)^3)/3)/(Grid(i+1)-Grid(i));
31 end
32
33 %构建大型分块稀疏矩阵
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```

34 for iface=2:numberx-1
35 ieL=iface-1; xciL=0.5*(Grid(ieL)+Grid(ieL+1));
36 ieR=iface; xciR=0.5*(Grid(ieR)+Grid(ieR+1));
37 dLR=0.5*(Deltax(ieR)+Deltax(ieL));
38 B2L=(Grid(iface)-xciL)/Deltax(ieL); B2R=(Grid(iface)-xciR)/Deltax(ieR);
39 B3L=0.5*B2L^2-1/24; B3R=0.5*B2R^2-1/24;
40 %diag
41 A(2*ieL-1,2*ieL-1)=A(2*ieL-1,2*ieL-1)+2*(omega0^2*B2L^2+omega1^2*dLR^2/
    Deltax(ieL)^2)/dLR;
42 A(2*ieL-1,2*ieL)=A(2*ieL-1,2*ieL)+2*(omega0^2*B3L*B2L+omega1^2*dLR^2*B2L
    /Deltax(ieL)^2)/dLR;
43 A(2*ieL,2*ieL-1)=A(2*ieL,2*ieL-1)+2*(omega0^2*B2L*B3L+omega1^2*dLR^2*B2L
    /Deltax(ieL)^2)/dLR;
44 A(2*ieL,2*ieL)=A(2*ieL,2*ieL)+2*(omega0^2*B3L^2+omega1^2*dLR^2*B2L^2/
    Deltax(ieL)^2+omega2^2*dLR^4/Deltax(ieL)^4)/dLR;
45 A(2*ieR-1,2*ieR-1)=A(2*ieR-1,2*ieR-1)+2*(omega0^2*B2R^2+omega1^2*dLR^2/
    Deltax(ieR)^2)/dLR;
46 A(2*ieR-1,2*ieR)=A(2*ieR-1,2*ieR)+2*(omega0^2*B3R*B2R+omega1^2*dLR^2*B2R
    /Deltax(ieR)^2)/dLR;
47 A(2*ieR,2*ieR-1)=A(2*ieR,2*ieR-1)+2*(omega0^2*B2R*B3R+omega1^2*dLR^2*B2R
    /Deltax(ieR)^2)/dLR;
48 A(2*ieR,2*ieR)=A(2*ieR,2*ieR)+2*(omega0^2*B3R^2+omega1^2*dLR^2*B2R^2/
    Deltax(ieR)^2+omega2^2*dLR^4/Deltax(ieR)^4)/dLR;
49
50 %upper
51 A(2*ieL-1,2*ieR-1)=A(2*ieL-1,2*ieR-1)-2*(omega0^2*B2R*B2L+omega1^2*dLR
    ^2/(Deltax(ieL)*Deltax(ieR)))/dLR;
52 A(2*ieL-1,2*ieR)=A(2*ieL-1,2*ieR)-2*(omega0^2*B3R*B2L+omega1^2*dLR^2*B2R
    /(Deltax(ieL)*Deltax(ieR)))/dLR;
53 A(2*ieL,2*ieR-1)=A(2*ieL,2*ieR-1)-2*(omega0^2*B2R*B3L+omega1^2*dLR^2*B2L
    /(Deltax(ieL)*Deltax(ieR)))/dLR;
54 A(2*ieL,2*ieR)=A(2*ieL,2*ieR)-2*(omega0^2*B3R*B3L+omega1^2*dLR^2*B2R*B2L
    /(Deltax(ieL)*Deltax(ieR))+omega2^2*dLR^4/(Deltax(ieL)^2*Deltax(ieR)
    ^2))/dLR;
55 %lower
56 A(2*ieR-1,2*ieL-1)=A(2*ieR-1,2*ieL-1)-2*(omega0^2*B2L*B2R+omega1^2*dLR
    ^2/(Deltax(ieL)*Deltax(ieR)))/dLR;
57 A(2*ieR-1,2*ieL)=A(2*ieR-1,2*ieL)-2*(omega0^2*B3L*B2R+omega1^2*dLR^2*B2L
    /(Deltax(ieR)*Deltax(ieL)))/dLR;
58 A(2*ieR,2*ieL-1)=A(2*ieR,2*ieL-1)-2*(omega0^2*B2L*B3R+omega1^2*dLR^2*B2R

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        /(Deltax(ieR)*Deltax(ieL))/dLR;
59 A(2*ieR,2*ieL)=A(2*ieR,2*ieL)-2*(omega0^2*B3L*B3R+omega1^2*dLR^2*B2L*B2R
        /(Deltax(ieR)*Deltax(ieL))+omega2^2*dLR^4/(Deltax(ieR)^2*Deltax(ieL)
        ^2))/dLR;
60
61 %RHS
62 R(2*ieL-1)=R(2*ieL-1)-2*omega0^2*(Unumsolution(ieL)-Unumsolution(ieR))*
        B2L/dLR;
63 R(2*ieL)=R(2*ieL)-2*omega0^2*(Unumsolution(ieL)-Unumsolution(ieR))*B3L/
        dLR;
64 R(2*ieR-1)=R(2*ieR-1)+2*omega0^2*(Unumsolution(ieL)-Unumsolution(ieR))*
        B2R/dLR;
65 R(2*ieR)=R(2*ieR)+2*omega0^2*(Unumsolution(ieL)-Unumsolution(ieR))*B3R/
        dLR;
66 end
67
68 %B.C
69 %left
70 iface=1;
71 ieR=iface;
72 xciR=0.5*(Grid(ieR)+Grid(ieR+1));
73 B2R=(Grid(iface)-xciR)/Deltax(ieR);
74 B3R=0.5*B2R^2-1/24;
75 A(2*ieR-1,2*ieR-1)=A(2*ieR-1,2*ieR-1)+4*omegab^2*B2R^2/Deltax(ieR);
76 A(2*ieR-1,2*ieR)=A(2*ieR-1,2*ieR)+4*omegab^2*B2R*B3R/Deltax(ieR);
77 A(2*ieR,2*ieR-1)=A(2*ieR,2*ieR-1)+4*omegab^2*B2R*B3R/Deltax(ieR);
78 A(2*ieR,2*ieR)=A(2*ieR,2*ieR)+4*omegab^2*B3R^2/Deltax(ieR);
79
80 R(2*ieR-1)=R(2*ieR-1)+4*omegab^2*(1-Unumsolution(ieR))*B2R/Deltax(ieR);
81 R(2*ieR)=R(2*ieR)+4*omegab^2*(1-Unumsolution(ieR))*B3R/Deltax(ieR);
82
83 %Right
84 iface=numberx;
85 ieL=iface-1;
86 xciL=0.5*(Grid(ieL)+Grid(ieL+1));
87 B2L=(Grid(iface)-xciL)/Deltax(ieL);
88 B3L=0.5*B2L^2-1/24;
89
90 A(2*ieL-1,2*ieL-1)=A(2*ieL-1,2*ieL-1)+4*omegab^2*B2L^2/Deltax(ieL);
91 A(2*ieL-1,2*ieL)=A(2*ieL-1,2*ieL)+4*omegab^2*B2L*B3L/Deltax(ieL);

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92 A(2*ieL,2*ieL-1)=A(2*ieL,2*ieL-1)+4*omegab^2*B2L*B3L/Deltax(ieL);
93 A(2*ieL,2*ieL)=A(2*ieL,2*ieL)+4*omegab^2*B3L^2/Deltax(ieL);
94
95 R(2*ieL-1)=R(2*ieL-1)-4*omegab^2*(Unumsolution(ieL)-3)*B2L/Deltax(ieL);
96 R(2*ieL)=R(2*ieL)-4*omegab^2*(Unumsolution(ieL)-3)*B3L/Deltax(ieL);
97
98
99
100 %LU-SGS 解三对角矩阵
101 %取出我们所需要的 D
102 D=zeros(2*Unit,2*Unit);
103 for iface=2:numberx
104     ieL=iface-1;
105     D(2*ieL-1:2*ieL,2*ieL-1:2*ieL)=A(2*ieL-1:2*ieL,2*ieL-1:2*ieL);
106 end
107 %取出我们所需要的 L
108 L=zeros(2*Unit,2*Unit);
109 for iface=2:numberx-1
110     ieR=iface;
111     ieL=iface-1;
112     L(2*ieR-1:2*ieR,2*ieL-1:2*ieL)=A(2*ieR-1:2*ieR,2*ieL-1:2*ieL);
113 end
114
115 %取出我们所需要的 U
116 U=zeros(2*Unit,2*Unit);
117 for iface=2:numberx-1
118     ieR=iface;
119     ieL=iface-1;
120     U(2*ieL-1:2*ieL,2*ieR-1:2*ieR)=A(2*ieL-1:2*ieL,2*ieR-1:2*ieR);
121 end
122 b=R;%用来存储最初的 rhs
123 Ureconstruct0=zeros(2*Unit,1);
124 for k=1:30%k 表示 SGS(k)
125     %Forward sweep
126     ie=1;
127     Ureconstruct(ie:ie+1,1)=D(ie:ie+1,ie:ie+1)\R(ie:ie+1,1);
128     for ie=2:numberx-1
129         R(2*ie-1:2*ie,1)=R(2*ie-1:2*ie,1)-L(2*ie-1:2*ie,2*(ie-1)-1:2*(ie-1))*
            Ureconstruct(2*(ie-1)-1:2*(ie-1),1);
130     Ureconstruct(2*ie-1:2*ie,1)=D(2*ie-1:2*ie,2*ie-1:2*ie)\R(2*ie-1:2*ie,1);

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```

131 end
132 %Backward sweep
133 for ie=1:numberx-1
134 R(2*ie-1:2*ie,1)=D(2*ie-1:2*ie,2*ie-1:2*ie)*Ureconstruct(2*ie-1:2*ie,1)
135 ;
136 end
137 ie=numberx-1;
138 Ureconstruct(2*ie-1:2*ie)=D(2*ie-1:2*ie,2*ie-1:2*ie)\R(2*ie-1:2*ie,1);
139 for ie=numberx-2:-1:1
140 R(2*ie-1:2*ie,1)=R(2*ie-1:2*ie,1)-U(2*ie-1:2*ie,2*(ie+1)-1:2*(ie+1))*
    Ureconstruct(2*(ie+1)-1:2*(ie+1),1);
141 Ureconstruct(2*ie-1:2*ie,1)=D(2*ie-1:2*ie,2*ie-1:2*ie)\R(2*ie-1:2*ie,1);
142 end
143 deltaUreconstruct=Ureconstruct;
144 Ureconstruct=Ureconstruct0+Ureconstruct;
145 if max(deltaUreconstruct)<10^(-10)
146 break;
147 end
148 %重新整理 R
149 R=b-A*Ureconstruct;
150 Ureconstruct0=Ureconstruct;
151 end
152 %Post-proceeding
153 figure
154 k=1;
155 x=Grid(k):1*(Grid(k+1)-Grid(k)):Grid(k+1);
156 xci=(Grid(k+1)+Grid(k))/2;
157 p=@(x)Unumsolution(1,k)+Ureconstruct(2*k-1,1)*(x-xci)/Deltax(k)+
    Ureconstruct(2*k,1)*(0.5*((x-xci)/Deltax(k)).^2-1/24);
158 y=p(x);
159 plot(x,y,'-r^','linewidth',1.5);hold on
160 H1=plot(x,y,'-r^','linewidth',1.5);hold on
161
162 for k=2:numberx-1
163 x=Grid(k):1*(Grid(k+1)-Grid(k)):Grid(k+1);
164 xci=(Grid(k+1)+Grid(k))/2;
165 p=@(x)Unumsolution(1,k)+Ureconstruct(2*k-1,1)*(x-xci)/Deltax(k)+
    Ureconstruct(2*k,1)*(0.5*((x-xci)/Deltax(k)).^2-1/24);
166 y=p(x);

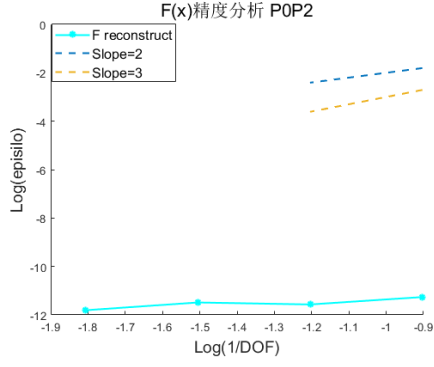
```

```

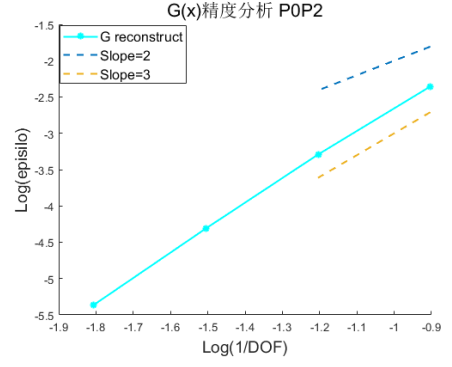
167 plot(x,y,'-r^','linewidth',1.5);
168 end
169
170 hold on
171 x=Grid(1):0.01*(Grid(numberx)-Grid(1)):Grid(numberx);
172 plot(x,f(x),'-b','linewidth',1.5);
173 H2=plot(x,f(x),'-b','linewidth',1.5);
174 lgd=legend([H1,H2],'Reconstruct','Exact solution');
175 lgd.FontSize=12;
176 xlabel('Position x','fontsize',14)
177 ylabel('Numerical value','fontsize',14)
178 title('VR(POP2,NUG) F(x)=1+x+x^2','fontsize',16)
179 hold off
180
181 %计算精度
182 Acc(1,1)=Accuracy(8);
183 Acc(1,2)=Accuracy(16);
184 Acc(1,3)=Accuracy(32);
185 Acc(1,4)=Accuracy(64);
186 for k=1:3
187 accuracyf(k)=(log10(Acc(1,k+1))-log10(Acc(1,k)))/(log10(a1(1,k+1))-
    log10(a1(1,k)));
188 end
189
190 figure
191 hold on
192 plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
193 H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);
194
195 H2=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
196 plot(log10(a2),3*log10(a2),'--','linewidth',1.5)
197 H3=plot(log10(a2),3*log10(a2),'--','linewidth',1.5);
198 lgd=legend([H1,H2,H3],'F reconstruct','Slope=2','Slope=3');
199 lgd.FontSize=12;
200 xlabel('Log(1/D0F)','fontsize',14)
201 ylabel('Log(episilo)','fontsize',14)
202 title('F(x)精度分析 POP2','fontsize',16)

```

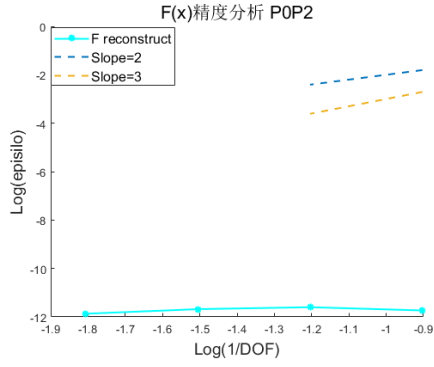
## 2 部分精度比较图



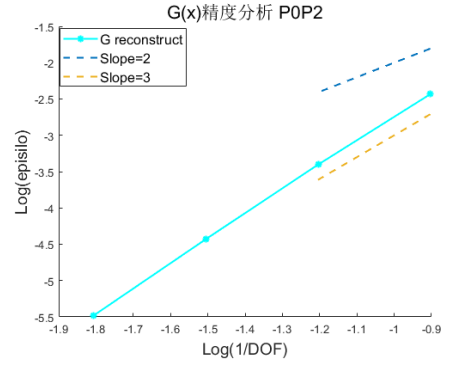
(a)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$



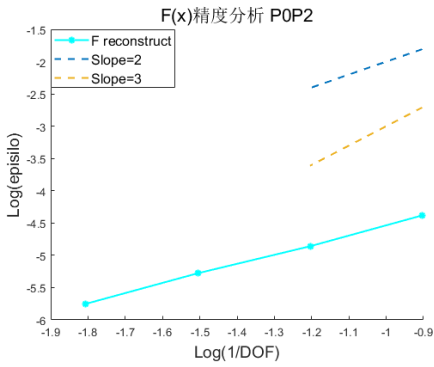
(b)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$



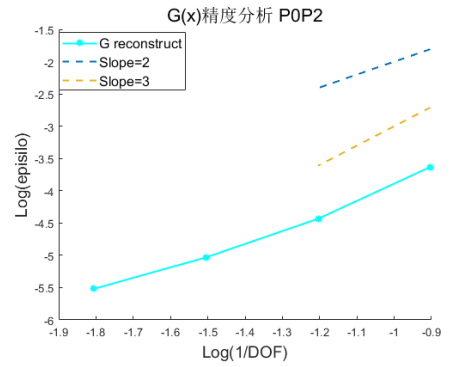
(c)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = 1$



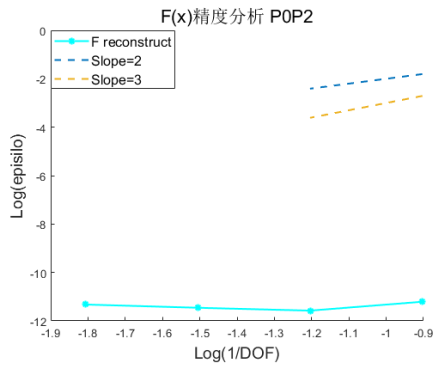
(d)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = 1$



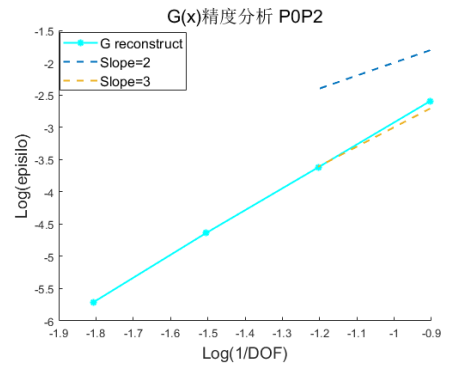
(e)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0, \omega_b = \frac{\omega_0}{\sqrt{2}}$



(f)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0, \omega_b = \frac{\omega_0}{\sqrt{2}}$

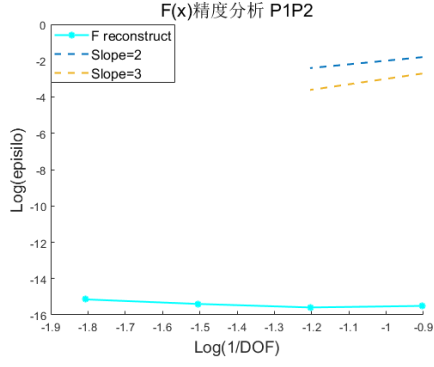


(g)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0.5, \omega_b = \frac{\omega_0}{\sqrt{2}}$

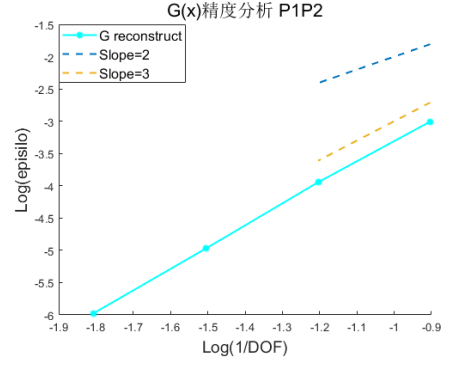


(h)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0.5, \omega_b = \frac{\omega_0}{\sqrt{2}}$

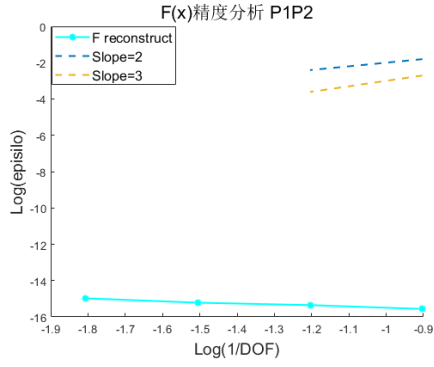
图 8: VR 精度分析 (P0P2 调整权重系数)



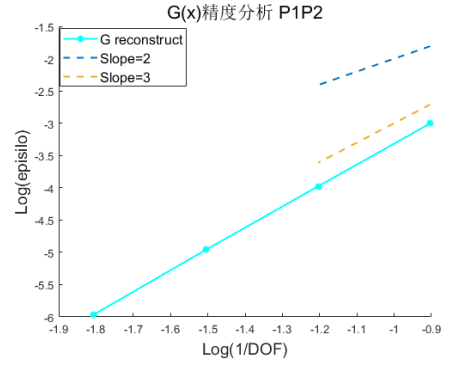
(a)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$



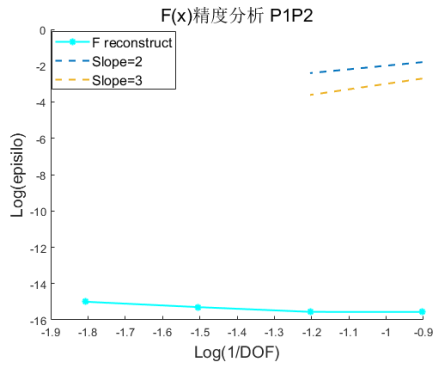
(b)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = \frac{\omega_0}{\sqrt{2}}$



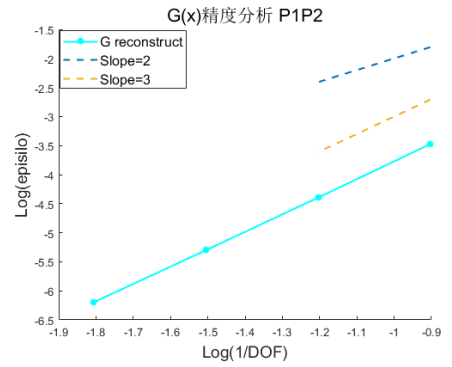
(c)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = 1$



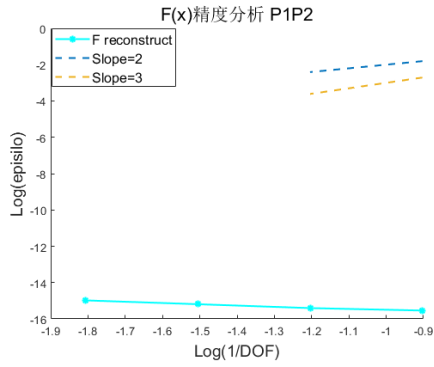
(d)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 1, \omega_b = 1$



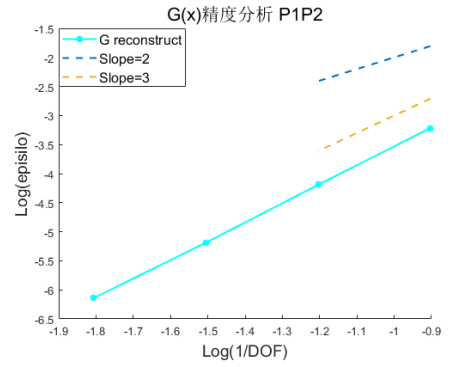
(e)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0, \omega_b = \frac{\omega_0}{\sqrt{2}}$



(f)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0, \omega_b = \frac{\omega_0}{\sqrt{2}}$



(g)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0.5, \omega_b = \frac{\omega_0}{\sqrt{2}}$



(h)  $\omega_0 = 1, \omega_1 = 1, \omega_2 = 0.5, \omega_b = \frac{\omega_0}{\sqrt{2}}$

图 9: VR 精度分析 (P1P2 调整权重系数)