CFD 入门练习 5

1.对带源项的扩散方程 $u_t = u_{xx} + \pi^2 \sin(\pi x), x \in [0,1], t \geq 0$ **,满足以下初始条件** $u(x,0) = x^2 - x$ **,及边界条件** $u(0,t) = u(1,t) = 0$ **。**

(1)求该方程的解析稳态解。

(2)使用 FOHS 引入辅助变量，将上述方程改写为双曲方程组，考虑均匀网格(8,16,32,64,128…)，用不同的数值方法求解稳态解，并与(1)中的解析解进行对比，测试原始变量 u **和它在** x **方向的导数的空间精度。**

解：(1)方程的解析稳态解为： $u(x,t) = \sin(\pi x)$ 。

(2)本题使用 DG(P0)+DG(P0)，DG(P0P1)+DG(P0)，DG(P0P2)+rDG(P0P1)三种显式欧拉方法求解变形后双曲方程组的稳态解。这里仅展示网格数为 8,16,32,64，CFL=0.01 的稳态数值解与解析解的比较图，并给出 3 种方法的空间精度比较图。

比较 U **稳态数值解与解析解：**

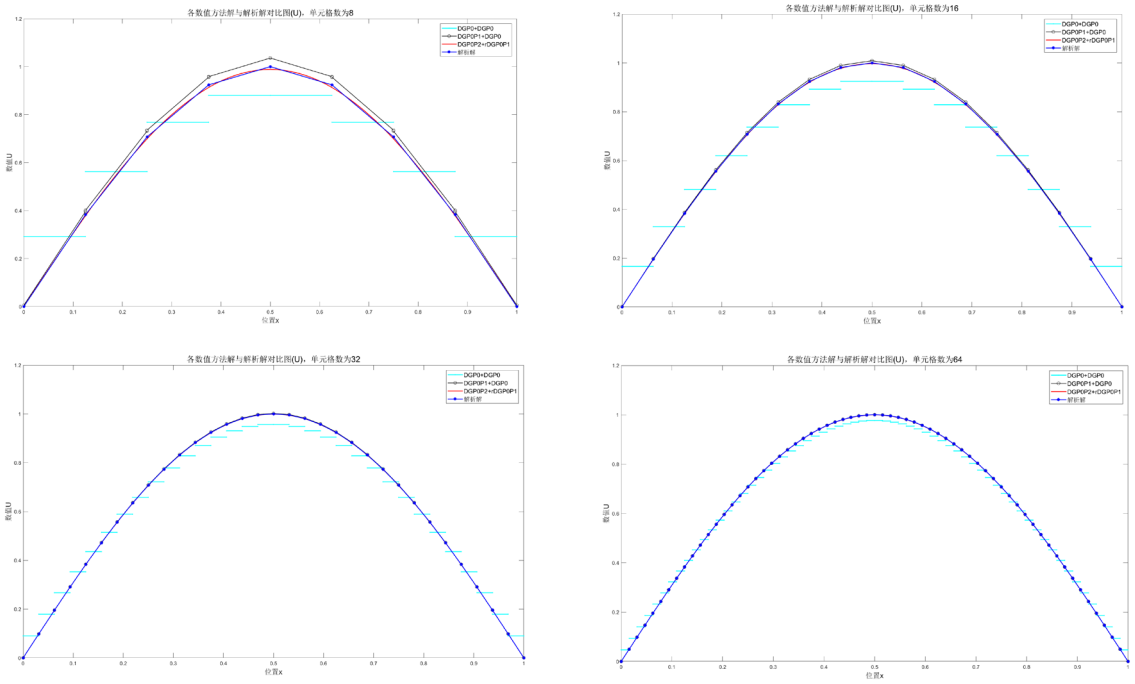

图 1：3 种数值方法求得的 U 稳态解与解析解比较图

此外，这里给出单元格数为 32 与 64 的U稳态解与解析解局部（放大）比较图。

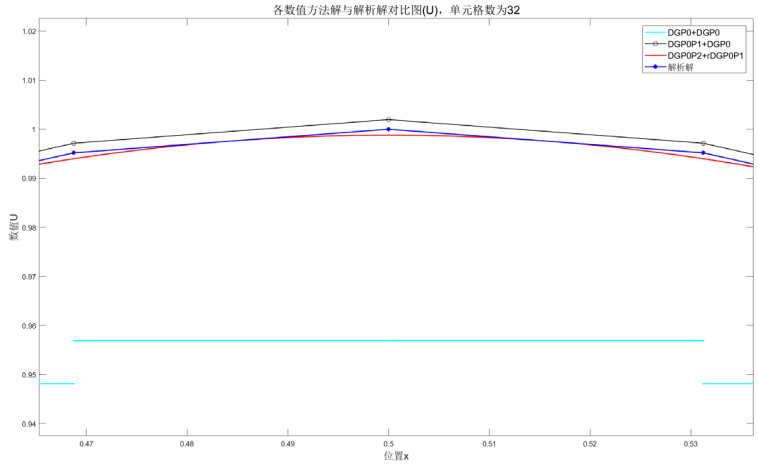

图 2：U稳态解与解析解局部（放大）比较图（单元格数为 32）

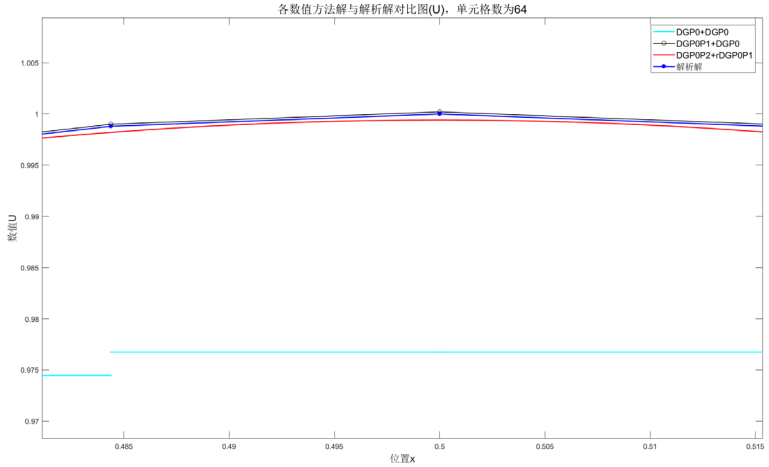

图 3：U稳态解与解析解局部（放大）比较图（单元格数为 64）

比较 3 种方法U稳态数值解的空间精度：

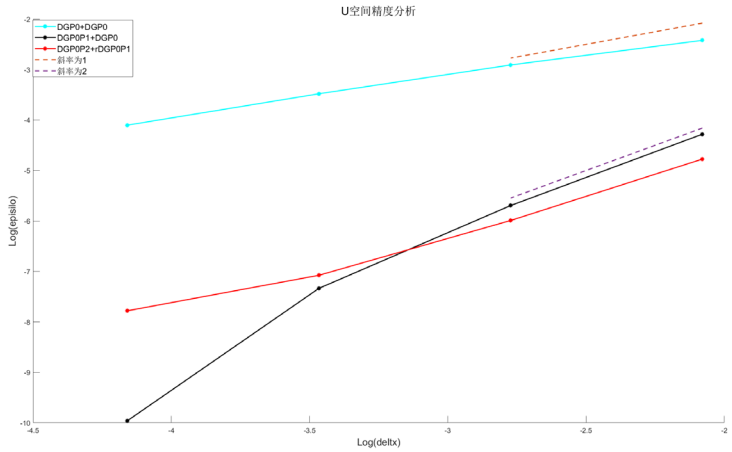

图 4：3 种方法U稳态数值解的空间精度

比较U_x稳态数值解与解析解：

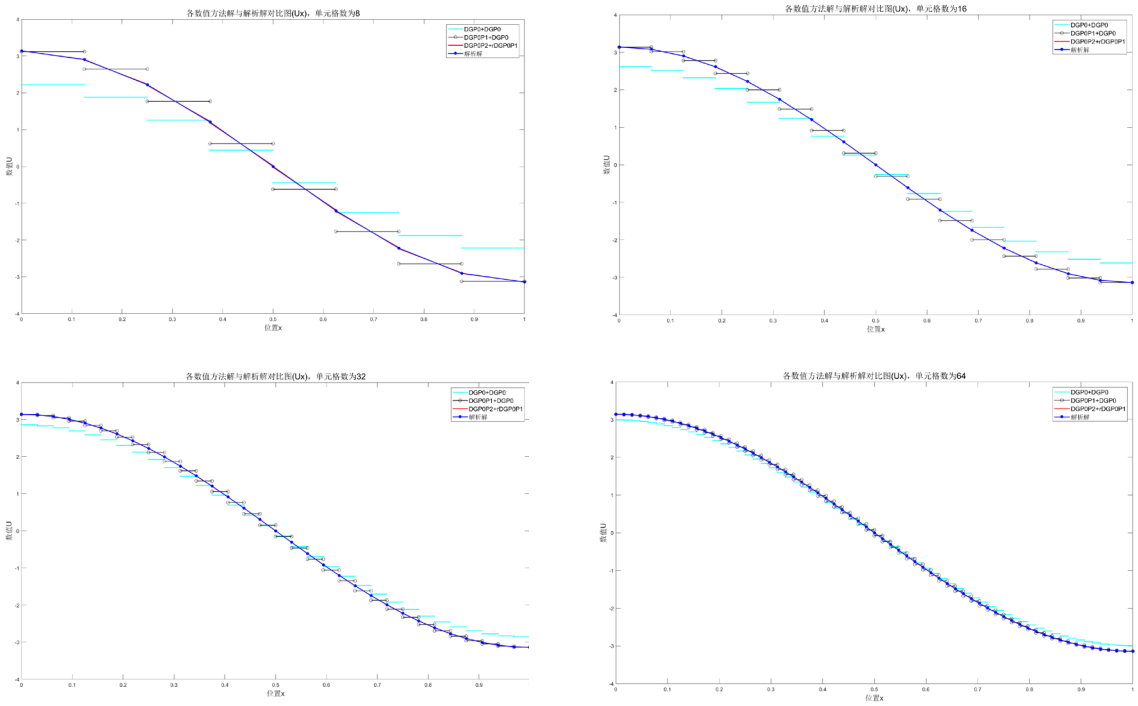

图 5：3 种数值方法求得的U_x稳态解与解析解比较图

比较 3 种方法U_x稳态数值解的空间精度：

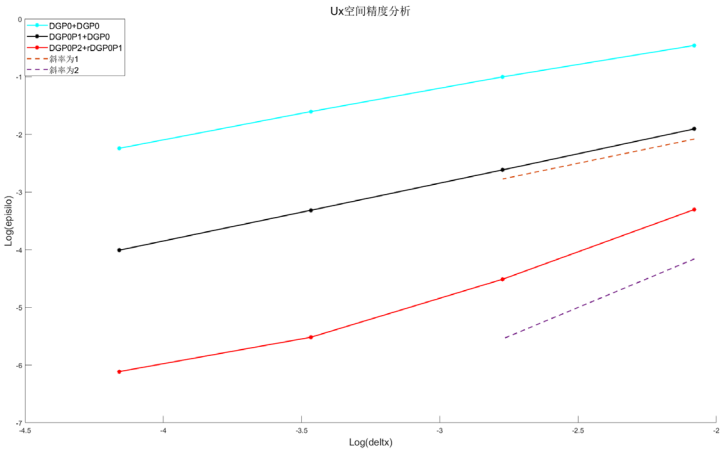

图 6：3 种方法U_x稳态数值解的空间精度

附录

本题修改后采用主程序调用子程序的模式

主程序

```
clc
clear all
close all
%% Pre-processing
Unit=64;CFL=0.01;endto=0.5;endx=1;deltx=1/Unit;numberx=endx/deltx+1;
Uexasolution=zeros(2,numberx);
UDGP0plusDGP0=zeros(2,numberx-1);
UDGP0P1plusDGP0=zeros(2,numberx-1);
UDGP0P2plusrDGP0P1=zeros(2,numberx-1);
Unumsolution1=zeros(1,2);
Unumsolution2=zeros(2,numberx-1);
Acc=zeros(3,4);a1=[1/8,1/16,1/32,1/64];a2=[1/8,1/16];
%% solve the question
  %solve the exasolution
x=0;
for k=1:numberx
      Uexasolution(1,k)=sin(pi*x);
      Uexasolution(2,k)=pi*cos(pi*x);
      x=x+deltx;
end

UDGP0plusDGP0=subDGP0plusDGP0(Unit,CFL,endto);%acquire u and ux
UDGP0P1plusDGP0=subDGP0P1plusDGP0(Unit,CFL,endto);%acquire u and ux
UDGP0P2plusrDGP0P1=subDGP0P2plusrDGP0P1(Unit,CFL,endto);%acquire u and
ux*deltx and uxx*deltx^2

%% post-processing
%plot the u
%plot the DGP0
figure
  x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=UDGP0plusDGP0(1,1);Unumsolution1(1,2)=UDGP0plusDGP0
(1,1);
  plot(x,Unumsolution1,'-c.','linewidth',1.5);hold on
  H1=plot(x,Unumsolution1,'-c.','linewidth',1.5);hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;
```

```
Unumsolution1(1,1)=UDGP0plusDGP0(1,i);Unumsolution1(1,2)=UDGP0plusDGP0(
1,i);
      plot(x,Unumsolution1,'-c.','linewidth',1.5)
end

%plot the DGP0P1
for i=1:numberx-1
Unumsolution2(1,i)=UDGP0P1plusDGP0(1,i)-0.5*UDGP0P1plusDGP0(2,i)*deltx;
Unumsolution2(2,i)=UDGP0P1plusDGP0(1,i)+0.5*UDGP0P1plusDGP0(2,i)*deltx;
end
  x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=Unumsolution2(1,1);Unumsolution1(1,2)=Unumsolution2(2,1);
  plot(x,Unumsolution1,'-ko','linewidth',1);hold on
  H2=plot(x,Unumsolution1,'-ko','linewidth',1);hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution2(1,i);Unumsolution1(1,2)=Unumsolution2(2,i);
      plot(x,Unumsolution1,'-ko','linewidth',1)
end
%plot the DGP0P2
  hold on
  k=1;
  x=0*deltx:0.1*deltx:1*deltx;
  p=[0.5*UDGP0P2plusrDGP0P1(3,k)/deltx^2,UDGP0P2plusrDGP0P1(2,k)/deltx-
UDGP0P2plusrDGP0P1(3,k)*(k-
0.5)/deltx,0.5*(UDGP0P2plusrDGP0P1(3,k)/deltx^2)*((k-
0.5)*deltx)^2+UDGP0P2plusrDGP0P1(1,k)-UDGP0P2plusrDGP0P1(2,k)/deltx*((k-
0.5)*deltx)-UDGP0P2plusrDGP0P1(3,k)/24];
y=polyval(p,x);
  plot(x,y,'-r','linewidth',1.5);hold on
  H3=plot(x,y,'-r','linewidth',1.5);hold on
for k=2:numberx-1
      x=(k-1)*deltx:0.1*deltx:k*deltx;
  p=[0.5*UDGP0P2plusrDGP0P1(3,k)/deltx^2,UDGP0P2plusrDGP0P1(2,k)/deltx-
UDGP0P2plusrDGP0P1(3,k)*(k-
0.5)/deltx,0.5*(UDGP0P2plusrDGP0P1(3,k)/deltx^2)*((k-
0.5)*deltx)^2+UDGP0P2plusrDGP0P1(1,k)-UDGP0P2plusrDGP0P1(2,k)/deltx*((k-
0.5)*deltx)-UDGP0P2plusrDGP0P1(3,k)/24];
      y=polyval(p,x);
      plot(x,y,'-r','linewidth',1.5)
end
```

```
%plot the exact
y=0:deltx:endx;
plot(y,Uexasolution(1,:),'-b*','linewidth',1.5)
H4=plot(y,Uexasolution(1,:),'-b*','linewidth',1.5);hold on
lgd=legend([H1,H2,H3,H4],'DGP0+DGP0','DGP0P1+DGP0','DGP0P2+rDGP0P1','
解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('各数值方法解与解析解对比图(U)，单元格数为 64','fontsize',16)
hold off

%plot the ux
%DGP0
 figure
 x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=UDGP0plusDGP0(2,1);Unumsolution1(1,2)=UDGP0plusDGP0
(2,1);
 plot(x,Unumsolution1,'-c.','linewidth',1.5);hold on
 H1=plot(x,Unumsolution1,'-c.','linewidth',1.5);hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=UDGP0plusDGP0(2,i);Unumsolution1(1,2)=UDGP0plusDGP0(
2,i);
      plot(x,Unumsolution1,'-c.','linewidth',1.5)
end

%DGP0P1
 x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=UDGP0P1plusDGP0(2,1);Unumsolution1(1,2)=UDGP0P1plus
DGP0(2,1);
 plot(x,Unumsolution1,'-ko','linewidth',1);hold on
 H2=plot(x,Unumsolution1,'-ko','linewidth',1);hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=UDGP0P1plusDGP0(2,i);Unumsolution1(1,2)=UDGP0P1plusD
GP0(2,i);
      plot(x,Unumsolution1,'-ko','linewidth',1)
end
```

```
%DGP0P2
  for i=1:numberx-1
Unumsolution2(1,i)=UDGP0P2plusrDGP0P1(2,i)/deltx-
0.5*UDGP0P2plusrDGP0P1(3,i)/deltx;
Unumsolution2(2,i)=UDGP0P2plusrDGP0P1(2,i)/deltx+0.5*UDGP0P2plusrDGP0P1
(3,i)/deltx;
end
  x=0*deltx:deltx:1*deltx;

Unumsolution1(1,1)=Unumsolution2(1,1);Unumsolution1(1,2)=Unumsolution2(2,1);
  plot(x,Unumsolution1,'-r','linewidth',1.5);hold on
  H3=plot(x,Unumsolution1,'-r','linewidth',1.5);hold on
for i=2:numberx-1
      x=(i-1)*deltx:deltx:i*deltx;

Unumsolution1(1,1)=Unumsolution2(1,i);Unumsolution1(1,2)=Unumsolution2(2,i);
      plot(x,Unumsolution1,'-r','linewidth',1.5)
end

%exact
y=0:deltx:endx;
plot(y,Uexasolution(2,:),'-b*','linewidth',1.5)
H4=plot(y,Uexasolution(2,:),'-b*','linewidth',1.5);hold on
lgd=legend([H1,H2,H3,H4],'DGP0+DGP0','DGP0P1+DGP0','DGP0P2+rDGP0P1','
解析解');
lgd.FontSize=12;
xlabel('位置 x','fontsize',14)
ylabel('数值 U','fontsize',14)
title('各数值方法解与解析解对比图(Ux)，单元格数为 64','fontsize',16)
hold off

%determine the accuracy of space U
%DGP0
Acc(1,1)=accuracy(8,subDGP0plusDGP0(8,CFL,endto));
Acc(1,2)=accuracy(16,subDGP0plusDGP0(16,CFL,endto));
Acc(1,3)=accuracy(32,subDGP0plusDGP0(32,CFL,endto));
Acc(1,4)=accuracy(64,subDGP0plusDGP0(64,CFL,endto));


%DGP0P1
Acc(2,1)=accuracy1(8,subDGP0P1plusDGP0(8,CFL,endto));
Acc(2,2)=accuracy1(16,subDGP0P1plusDGP0(16,CFL,endto));
Acc(2,3)=accuracy1(32,subDGP0P1plusDGP0(32,CFL,endto));
```

```
Acc(2,4)=accuracy1(64,subDGP0P1plusDGP0(64,CFL,endto));


%DGP0P2
Acc(3,1)=accuracy2(8,subDGP0P2plusrDGP0P1(8,CFL,endto));
Acc(3,2)=accuracy2(16,subDGP0P2plusrDGP0P1(16,CFL,endto));
Acc(3,3)=accuracy2(32,subDGP0P2plusrDGP0P1(32,CFL,endto));
Acc(3,4)=accuracy2(64,subDGP0P2plusrDGP0P1(64,CFL,endto));


figure
hold on
plot(log(a1),log(Acc(1,:)),'-c*','linewidth',1.5)
H1=plot(log(a1),log(Acc(1,:)),'-c*','linewidth',1.5);
plot(log(a1),log(Acc(2,:)),'-k*','linewidth',1.5)
H2=plot(log(a1),log(Acc(2,:)),'-k*','linewidth',1.5);
plot(log(a1),log(Acc(3,:)),'-r*','linewidth',1.5)
H3=plot(log(a1),log(Acc(3,:)),'-r*','linewidth',1.5);
plot(log(a2),1*log(a2),'--','linewidth',1.5)
H4=plot(log(a2),1*log(a2),'--','linewidth',1.5);
plot(log(a2),2*log(a2),'--','linewidth',1.5)
H5=plot(log(a2),2*log(a2),'--','linewidth',1.5);
lgd=legend([H1,H2,H3,H4,H5],'DGP0+DGP0','DGP0P1+DGP0','DGP0P2+rDGP0P1
','斜率为 1','斜率为 2');
lgd.FontSize=12;
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('U 空间精度分析','fontsize',16)

%determine the accuracy of space Ux
%DGP0
Acc(1,1)=accuracyUx(8,subDGP0plusDGP0(8,CFL,endto));
Acc(1,2)=accuracyUx(16,subDGP0plusDGP0(16,CFL,endto));
Acc(1,3)=accuracyUx(32,subDGP0plusDGP0(32,CFL,endto));
Acc(1,4)=accuracyUx(64,subDGP0plusDGP0(64,CFL,endto));


%DGP0P1
Acc(2,1)=accuracyUx(8,subDGP0P1plusDGP0(8,CFL,endto));
Acc(2,2)=accuracyUx(16,subDGP0P1plusDGP0(16,CFL,endto));
Acc(2,3)=accuracyUx(32,subDGP0P1plusDGP0(32,CFL,endto));
Acc(2,4)=accuracyUx(64,subDGP0P1plusDGP0(64,CFL,endto));
```

```
%DGP0P2
Acc(3,1)=accuracyUx1(8,subDGP0P2plusrDGP0P1(8,CFL,endto));
Acc(3,2)=accuracyUx1(16,subDGP0P2plusrDGP0P1(16,CFL,endto));
Acc(3,3)=accuracyUx1(32,subDGP0P2plusrDGP0P1(32,CFL,endto));
Acc(3,4)=accuracyUx1(64,subDGP0P2plusrDGP0P1(64,CFL,endto));

figure
hold on
plot(log(a1),log(Acc(1,:)),'-c*','linewidth',1.5)
H1=plot(log(a1),log(Acc(1,:)),'-c*','linewidth',1.5);
plot(log(a1),log(Acc(2,:)),'-k*','linewidth',1.5)
H2=plot(log(a1),log(Acc(2,:)),'-k*','linewidth',1.5);
plot(log(a1),log(Acc(3,:)),'-r*','linewidth',1.5)
H3=plot(log(a1),log(Acc(3,:)),'-r*','linewidth',1.5);
plot(log(a2),1*log(a2),'--','linewidth',1.5)
H4=plot(log(a2),1*log(a2),'--','linewidth',1.5);
plot(log(a2),2*log(a2),'--','linewidth',1.5)
H5=plot(log(a2),2*log(a2),'--','linewidth',1.5);
lgd=legend([H1,H2,H3,H4,H5],'DGP0+DGP0','DGP0P1+DGP0','DGP0P2+rDGP0P1
','斜率为 1','斜率为 2');
lgd.FontSize=12;
xlabel('Log(deltx)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('Ux 空间精度分析','fontsize',16)
```

子程序

subDGP0plusDGP0(用来计算此方法的数值解)

```
function Unumsolution=subDGP0plusDGP0(Unit,CFL,endto)
deltx=1/Unit;tol=0.01;
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);deltto=CFL*deltx/abslambda;%伪时间变量
endx=1;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
B1=1;
C=[B1,0;0,B1/deltx];Mto=[deltx,0;0,1/deltx];
A=[abslambda,0;0,abslambda];
R=zeros(2,numberx-1);
F1=zeros(2,numberx-1);
F2=zeros(2,numberx-1);
V0=zeros(1,numberx-1);
%% solve the question
```

```
%initial    condition set up
x=0;
for k=1:numberx-1
      Ucurrent(1,k)=(x+deltx/2)^2-(x+deltx/2);
      x=x+deltx;
end
V0=Ucurrent(1,:);

x=0;
for k=1:numberx-1
      Ucurrent(2,k)=(2*(x+deltx/2)-1)*deltx;
      x=x+deltx;
end

x=0;
for k=1:numberx-1
      R(1,k)=pi*(cos(pi*x)-cos(pi*(x+deltx)));
      R(2,k)=-Ucurrent(2,k)/(Tr*deltx);
      x=x+deltx;
end

for k=2:numberx-1
      F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-Ucurrent(1,k-1)/(Tr*deltx)]+[-
nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k);Ucurrent(2,k)/deltx]-[Ucurrent(1,k-1);Ucurrent(2,k-1)/deltx]);
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-
Ucurrent(1,1)/(Tr*deltx)])-0.5*A*([Ucurrent(1,1);Ucurrent(2,1)/deltx]-
[0;Ucurrent(2,1)/deltx]);

for k=1:numberx-2
      F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)]+[-
nu*Ucurrent(2,k+1)/deltx;-Ucurrent(1,k+1)/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k+1);Ucurrent(2,k+1)/deltx]-
[Ucurrent(1,k);Ucurrent(2,k)/deltx]);
end
 F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;-Ucurrent(1,numberx-
1)/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;-0/(Tr*deltx)])-
0.5*A*([0;Ucurrent(2,numberx-1)/deltx]-[Ucurrent(1,numberx-
1);Ucurrent(2,numberx-1)/deltx]);
for k=1:numberx-1
      R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
 end
```

```
 %solve the exasolution
x=0;
for k=1:numberx
      Uexasolution(1,k)=sin(pi*x);
      Uexasolution(2,k)=pi*cos(pi*x)*deltx;
      x=x+deltx;
end

%solve the numsolution
for n=deltto:deltto:endto
      for k=1:numberx-1
          Unext(:,k)=Ucurrent(:,k)+Mto\R(:,k)*deltto;
      end
       if var(Ucurrent(1,:)-Unext(1,:))<tol*V0
            break
       end
      Ucurrent=Unext;

x=0;
for k=1:numberx-1
      R(1,k)=pi*(cos(pi*x)-cos(pi*(x+deltx)));
      R(2,k)=-Ucurrent(2,k)/(Tr*deltx);
      x=x+deltx;
end

for k=2:numberx-1
      F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-Ucurrent(1,k-1)/(Tr*deltx)]+[-
nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k);Ucurrent(2,k)/deltx]-[Ucurrent(1,k-1);Ucurrent(2,k-1)/deltx]);
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-
Ucurrent(1,1)/(Tr*deltx)])-0.5*A*([Ucurrent(1,1);Ucurrent(2,1)/deltx]-
[0;Ucurrent(2,1)/deltx]);

for k=1:numberx-2
      F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;-Ucurrent(1,k)/(Tr*deltx)]+[-
nu*Ucurrent(2,k+1)/deltx;-Ucurrent(1,k+1)/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k+1);Ucurrent(2,k+1)/deltx]-
[Ucurrent(1,k);Ucurrent(2,k)/deltx]);
end
 F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;-Ucurrent(1,numberx-
1)/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;-0/(Tr*deltx)])-
0.5*A*([0;Ucurrent(2,numberx-1)/deltx]-[Ucurrent(1,numberx-
1);Ucurrent(2,numberx-1)/deltx]);
```

```
 for k=1:numberx-1
     R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
 end

end
Unumsolution(1,:)=Ucurrent(1,:);Unumsolution(2,:)=Ucurrent(2,:)/deltx;
end
```

subDGP0P1plusDGP0

```
function Unumsolution=subDGP0P1plusDGP0(Unit,CFL,endto)
%% Pre-processing
deltx=1/Unit;tol=0.01;
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);deltto=CFL*deltx/abslambda;%伪时间变量
endx=1;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(2,numberx-1);
Mto=[deltx,0;0,deltx/12+1/deltx];
A=[abslambda,0;0,abslambda];
R=zeros(2,numberx-1);
F1=zeros(2,numberx-1);
F2=zeros(2,numberx-1);
V0=zeros(1,numberx-1);

%% solve the question
%initial    condition set up
x=0;
for k=1:numberx-1
      Ucurrent(1,k)=(x+deltx/2)^2-(x+deltx/2);
      x=x+deltx;
end
V0=Ucurrent(1,:);

x=0;
for k=1:numberx-1
      Ucurrent(2,k)=(2*(x+deltx/2)-1)*deltx;
      x=x+deltx;
end

x=0;
for k=1:numberx-1
```

```
        R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
        R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
        x=x+deltx;
end

for k=2:numberx-1
        F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-0.5*(-nu*Ucurrent(2,k-1)/deltx)-
(Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1))/(Tr*deltx)]+[-nu*Ucurrent(2,k)/deltx;-0.5*(-
nu*Ucurrent(2,k)/deltx)-(Ucurrent(1,k)-0.5*Ucurrent(2,k))/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k)-0.5*Ucurrent(2,k);Ucurrent(2,k)/deltx]-[Ucurrent(1,k-
1)+0.5*Ucurrent(2,k-1);Ucurrent(2,k-1)/deltx]);
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
(Ucurrent(1,1)-0.5*Ucurrent(2,1))/(Tr*deltx)])-0.5*A*([Ucurrent(1,1)-
0.5*Ucurrent(2,1);Ucurrent(2,1)/deltx]-[0;Ucurrent(2,1)/deltx]);

for k=1:numberx-2
        F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;0.5*(-nu*Ucurrent(2,k)/deltx)-
(Ucurrent(1,k)+0.5*Ucurrent(2,k))/(Tr*deltx)]+[-nu*Ucurrent(2,k+1)/deltx;0.5*(-
nu*Ucurrent(2,k+1)/deltx)-(Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1))/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1);Ucurrent(2,k+1)/deltx]-
[Ucurrent(1,k)+0.5*Ucurrent(2,k);Ucurrent(2,k)/deltx]);
end
  F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;0.5*(-
nu*Ucurrent(2,numberx-1)/deltx)-(Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-
1))/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;0.5*(-nu*Ucurrent(2,numberx-
1)/deltx)-0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)/deltx]-
[Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-1);Ucurrent(2,numberx-1)/deltx]);

  for k=1:numberx-1
        R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
  end


%solve the numsolution
for n=deltto:deltto:endto
        for k=1:numberx-1
            Unext(:,k)=Ucurrent(:,k)+Mto\R(:,k)*deltto;
        end
         if var(Ucurrent(1,:)-Unext(1,:))<tol*V0
               break
         end
```

```
        Ucurrent=Unext;
x=0;
for k=1:numberx-1
        R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
        R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
        x=x+deltx;
end

for k=2:numberx-1
        F1(:,k)=0.5*([-nu*Ucurrent(2,k-1)/deltx;-0.5*(-nu*Ucurrent(2,k-1)/deltx)-
(Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1))/(Tr*deltx)]+[-nu*Ucurrent(2,k)/deltx;-0.5*(-
nu*Ucurrent(2,k)/deltx)-(Ucurrent(1,k)-0.5*Ucurrent(2,k))/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k)-0.5*Ucurrent(2,k);Ucurrent(2,k)/deltx]-[Ucurrent(1,k-
1)+0.5*Ucurrent(2,k-1);Ucurrent(2,k-1)/deltx]);
end
F1(:,1)=0.5*([-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
0/(Tr*deltx)]+[-nu*Ucurrent(2,1)/deltx;-0.5*(-nu*Ucurrent(2,1)/deltx)-
(Ucurrent(1,1)-0.5*Ucurrent(2,1))/(Tr*deltx)])-0.5*A*([Ucurrent(1,1)-
0.5*Ucurrent(2,1);Ucurrent(2,1)/deltx]-[0;Ucurrent(2,1)/deltx]);

for k=1:numberx-2
        F2(:,k)=0.5*([-nu*Ucurrent(2,k)/deltx;0.5*(-nu*Ucurrent(2,k)/deltx)-
(Ucurrent(1,k)+0.5*Ucurrent(2,k))/(Tr*deltx)]+[-nu*Ucurrent(2,k+1)/deltx;0.5*(-
nu*Ucurrent(2,k+1)/deltx)-(Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1))/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k+1)-0.5*Ucurrent(2,k+1);Ucurrent(2,k+1)/deltx]-
[Ucurrent(1,k)+0.5*Ucurrent(2,k);Ucurrent(2,k)/deltx]);
end
  F2(:,numberx-1)=0.5*([-nu*Ucurrent(2,numberx-1)/deltx;0.5*(-
nu*Ucurrent(2,numberx-1)/deltx)-(Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-
1))/(Tr*deltx)]+[-nu*Ucurrent(2,numberx-1)/deltx;0.5*(-nu*Ucurrent(2,numberx-
1)/deltx)-0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)/deltx]-
[Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-1);Ucurrent(2,numberx-1)/deltx]);

  for k=1:numberx-1
        R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
  end
end
Unumsolution(1,:)=Ucurrent(1,:);Unumsolution(2,:)=Ucurrent(2,:)/deltx;

end
```

subDGP0P2plusrDGP0P1

```
function Unumsolution=subDGP0P2plusrDGP0P1(Unit,CFL,endto)
%% Pre-processing
deltx=1/Unit;tol=0.01;
nu=1;Lr=1/(2*pi);Tr=Lr^2/nu;
abslambda=sqrt(nu/Tr);deltto=CFL*deltx/abslambda;%伪时间变量
endx=1;
numberx=endx/deltx+1;
Ucurrent=zeros(2,numberx-1);
Ucurrent1=zeros(1,numberx-1);
Unext=zeros(2,numberx-1);
Unumsolution=zeros(3,numberx-1);
Mto=[deltx,0;0,deltx/12+1/deltx];
A=[abslambda,0;0,abslambda];
R=zeros(2,numberx-1);
F1=zeros(2,numberx-1);
F2=zeros(2,numberx-1);
V0=zeros(1,numberx-1);

%initial    condition set up
x=0;
for k=1:numberx-1
      Ucurrent(1,k)=(x+deltx/2)^2-(x+deltx/2);
      x=x+deltx;
end
V0=Ucurrent(1,:);

x=0;
for k=1:numberx-1
      Ucurrent(2,k)=(2*(x+deltx/2)-1)*deltx;
      x=x+deltx;
end


for k=1:numberx-1
      Ucurrent1(1,k)=2*deltx^2;
end

x=0;
for k=1:numberx-1
      R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
      R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
      x=x+deltx;
```

```
end

for k=2:numberx-1
      F1(:,k)=0.5*([-nu*(Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx);-0.5*(-
nu*(Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx))-(Ucurrent(1,k-
1)+0.5*Ucurrent(2,k-1)+Ucurrent1(1,k-1)/12)/(Tr*deltx)]+[-nu*(Ucurrent(2,k)/deltx-
0.5*Ucurrent1(1,k)/deltx);-0.5*(-nu*(Ucurrent(2,k)/deltx-0.5*Ucurrent1(1,k)/deltx))-
(Ucurrent(1,k)-0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12)/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k)-0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12;Ucurrent(2,k)/deltx-
0.5*Ucurrent1(1,k)/deltx]-[Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1)+Ucurrent1(1,k-
1)/12;Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx]);
end
F1(:,1)=0.5*([-nu*(Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx);-0.5*(-
nu*(Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx))-0/(Tr*deltx)]+[-
nu*(Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx);-0.5*(-nu*(Ucurrent(2,1)/deltx-
0.5*Ucurrent1(1,1)/deltx))-(Ucurrent(1,1)-
0.5*Ucurrent(2,1)+Ucurrent1(1,1)/12)/(Tr*deltx)])-0.5*A*([Ucurrent(1,1)-
0.5*Ucurrent(2,1)+Ucurrent1(1,1)/12;Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx]-
[0;Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx]);

for k=1:numberx-2
      F2(:,k)=0.5*([-nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx);0.5*(-
nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx))-
(Ucurrent(1,k)+0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12)/(Tr*deltx)]+[-
nu*(Ucurrent(2,k+1)/deltx-0.5*Ucurrent1(1,k+1)/deltx);0.5*(-
nu*(Ucurrent(2,k+1)/deltx-0.5*Ucurrent1(1,k+1)/deltx))-(Ucurrent(1,k+1)-
0.5*Ucurrent(2,k+1)+Ucurrent1(1,k+1)/12)/(Tr*deltx)])-0.5*A*([Ucurrent(1,k+1)-
0.5*Ucurrent(2,k+1)+Ucurrent1(1,k+1)/12;Ucurrent(2,k+1)/deltx-
0.5*Ucurrent1(1,k+1)/deltx]-
[Ucurrent(1,k)+0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12;Ucurrent(2,k)/deltx+0.5*Ucurre
nt1(1,k)/deltx]);
end
  F2(:,numberx-1)=0.5*([-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx);0.5*(-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx))-(Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1)+Ucurrent1(1,numberx-1)/12)/(Tr*deltx)]+[-
nu*(Ucurrent(2,numberx-1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx);0.5*(-
nu*(Ucurrent(2,numberx-1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx))-
0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)/deltx+0.5*Ucurrent1(1,numberx-
1)/deltx]-[Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-
1)+Ucurrent1(1,numberx-1)/12;Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx]);

  for k=1:numberx-1
```

```
        R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
    end

    %solve the exasolution
k=1;
for x=0:deltx:endx
        Uexasolution(1,k)=sin(pi*x);
        Uexasolution(2,k)=pi*cos(pi*x);
        k=k+1;
end

%solve the numsolution
for n=deltto:deltto:endto
        for k=1:numberx-1
            Unext(:,k)=Ucurrent(:,k)+Mto\R(:,k)*deltto;
        end
         if var(Ucurrent(1,:)-Unext(1,:))<tol*V0
              break
         end
        Ucurrent=Unext;

     for k=2:numberx-2
              Ucurrent1(1,k)=[deltx;-deltx]\[Ucurrent(2,k+1)*deltx-
Ucurrent(2,k)*deltx;Ucurrent(2,k-1)*deltx-Ucurrent(2,k)*deltx];
     end
        Ucurrent1(1,1)=[deltx;-deltx]\[Ucurrent(2,2)*deltx-
Ucurrent(2,1)*deltx;Ucurrent(2,1)*deltx-Ucurrent(2,1)*deltx];
        Ucurrent1(1,numberx-1)=[deltx;-deltx]\[Ucurrent(2,numberx-1)*deltx-
Ucurrent(2,numberx-1)*deltx;Ucurrent(2,numberx-2)*deltx-Ucurrent(2,numberx-
1)*deltx];

x=0;
for k=1:numberx-1
        R(1,k)=0+pi*(cos(pi*x)-cos(pi*(x+deltx)));
        R(2,k)=-nu*Ucurrent(2,k)/deltx+(-pi/deltx*(deltx/2*cos(pi*(x+deltx))-(-
deltx/2)*cos(pi*x)-1/pi*(sin(pi*(x+deltx))-sin(pi*x))))-Ucurrent(2,k)/(Tr*deltx);
        x=x+deltx;
end

for k=2:numberx-1
        F1(:,k)=0.5*([-nu*(Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx);-0.5*(-
nu*(Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx))-(Ucurrent(1,k-
1)+0.5*Ucurrent(2,k-1)+Ucurrent1(1,k-1)/12)/(Tr*deltx)]+[-nu*(Ucurrent(2,k)/deltx-
0.5*Ucurrent1(1,k)/deltx);-0.5*(-nu*(Ucurrent(2,k)/deltx-0.5*Ucurrent1(1,k)/deltx))-
```

```
(Ucurrent(1,k)-0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12)/(Tr*deltx)])-
0.5*A*([Ucurrent(1,k)-0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12;Ucurrent(2,k)/deltx-
0.5*Ucurrent1(1,k)/deltx]-[Ucurrent(1,k-1)+0.5*Ucurrent(2,k-1)+Ucurrent1(1,k-
1)/12;Ucurrent(2,k-1)/deltx+0.5*Ucurrent1(1,k-1)/deltx]);
end
F1(:,1)=0.5*([-nu*(Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx);-0.5*(-
nu*(Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx))-0/(Tr*deltx)]+[-
nu*(Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx);-0.5*(-nu*(Ucurrent(2,1)/deltx-
0.5*Ucurrent1(1,1)/deltx))-(Ucurrent(1,1)-
0.5*Ucurrent(2,1)+Ucurrent1(1,1)/12)/(Tr*deltx)])-0.5*A*([Ucurrent(1,1)-
0.5*Ucurrent(2,1)+Ucurrent1(1,1)/12;Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx]-
[0;Ucurrent(2,1)/deltx-0.5*Ucurrent1(1,1)/deltx]);

for k=1:numberx-2
    F2(:,k)=0.5*([-nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx);0.5*(-
nu*(Ucurrent(2,k)/deltx+0.5*Ucurrent1(1,k)/deltx))-
(Ucurrent(1,k)+0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12)/(Tr*deltx)]+[-
nu*(Ucurrent(2,k+1)/deltx-0.5*Ucurrent1(1,k+1)/deltx);0.5*(-
nu*(Ucurrent(2,k+1)/deltx-0.5*Ucurrent1(1,k+1)/deltx))-(Ucurrent(1,k+1)-
0.5*Ucurrent(2,k+1)+Ucurrent1(1,k+1)/12)/(Tr*deltx)])-0.5*A*([Ucurrent(1,k+1)-
0.5*Ucurrent(2,k+1)+Ucurrent1(1,k+1)/12;Ucurrent(2,k+1)/deltx-
0.5*Ucurrent1(1,k+1)/deltx]-
[Ucurrent(1,k)+0.5*Ucurrent(2,k)+Ucurrent1(1,k)/12;Ucurrent(2,k)/deltx+0.5*Ucurre
nt1(1,k)/deltx]);
end
  F2(:,numberx-1)=0.5*([-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx);0.5*(-nu*(Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx))-(Ucurrent(1,numberx-
1)+0.5*Ucurrent(2,numberx-1)+Ucurrent1(1,numberx-1)/12)/(Tr*deltx)]+[-
nu*(Ucurrent(2,numberx-1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx);0.5*(-
nu*(Ucurrent(2,numberx-1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx))-
0/(Tr*deltx)])-0.5*A*([0;Ucurrent(2,numberx-1)/deltx+0.5*Ucurrent1(1,numberx-
1)/deltx]-[Ucurrent(1,numberx-1)+0.5*Ucurrent(2,numberx-
1)+Ucurrent1(1,numberx-1)/12;Ucurrent(2,numberx-
1)/deltx+0.5*Ucurrent1(1,numberx-1)/deltx]);

  for k=1:numberx-1
      R(:,k)=R(:,k)+F1(:,k)-F2(:,k);
  end

end
Unumsolution([1,2],:)=Ucurrent;Unumsolution(3,:)=Ucurrent1;
end
```

accuracy(计算 P0U 的空间精度)

```
function A=accuracy(Unit,Unumsolution)
%% Pre-processing
deltx=1/Unit;endx=1;
numberx=endx/deltx+1;
%calculate the accuracy of space DGp0+DGP0
I1=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(pi*xi)-Unumsolution(1,m))^2;k=k+1;
            end
        end
        I1=I1+W(i)*fi;
    end
end
I1=I1*0.5*deltx;
A=sqrt(I1);
end
```

accuracy1(计算 P0P1U 的空间精度)

```
function A1=accuracy1(Unit,Unumsolution)
%% Pre-processing
deltx=1/Unit;endx=1;
numberx=endx/deltx+1;
%calculate the accuracy of space
I1=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(pi*xi)-(Unumsolution(1,m)+Unumsolution(2,m)*(xi-((m-
1)*deltx+deltx/2))))^2;k=k+1;
            end
        end
        I1=I1+W(i)*fi;
    end
end
I1=I1*0.5*deltx;
```

```
A1=sqrt(I1);
end
```

accuracy2(计算 P0P2U 的空间精度)

```
function A2=accuracy2(Unit,Unumsolution)
%% Pre-processing
deltx=1/Unit;endx=1;
numberx=endx/deltx+1;
%calculate the accuracy of space
I1=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(sin(pi*xi)-(Unumsolution(1,m)+Unumsolution(2,m)/deltx*(xi-
((m-1)*deltx+deltx/2))+Unumsolution(3,m)*(-1/24+(xi-((m-
1)*deltx+deltx/2))^2/(2*deltx^2))))^2;k=k+1;
            end
        end
        I1=I1+W(i)*fi;
    end
end
I1=I1*0.5*deltx;
A2=sqrt(I1);
end
```

accuracyUx(计算 P0Ux 的空间精度)

```
function A1=accuracyUx(Unit,Unumsolution)
%% Pre-processing
deltx=1/Unit;endx=1;
numberx=endx/deltx+1;
%calculate the accuracy of space
I2=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(pi*cos(pi*xi)-Unumsolution(2,m))^2;k=k+1;
            end
        end
```

```
            I2=I2+W(i)*fi;
        end
    end
    I2=I2*0.5*deltx;
    A1=sqrt(I2);
    end
```

accuracyUx1(计算 rDGP0P1Ux 的空间精度)

```
function A1=accuracyUx1(Unit,Unumsolution)
%% Pre-processing
deltx=1/Unit;endx=1;
numberx=endx/deltx+1;
%calculate the accuracy of space
I2=0;t=[-1/sqrt(5),0,1/sqrt(5)];W=[5/9,8/9,5/9];
k=1;%determine the correctness of the program
for x=0:deltx:endx-deltx
    for i=1:3
        xi=deltx/2*t(i)+0.5*(2*x+deltx);
        for m=1:numberx-1
            if xi>(m-1)*deltx&&xi<m*deltx
                fi=(pi*cos(pi*xi)-(Unumsolution(2,m)/deltx+(xi-((m-
1)*deltx+deltx/2))*Unumsolution(3,m)/deltx^2))^2;k=k+1;
            end
        end
        I2=I2+W(i)*fi;
    end
end
I2=I2*0.5*deltx;
A1=sqrt(I2);
end
```