课题组组会-练习 4

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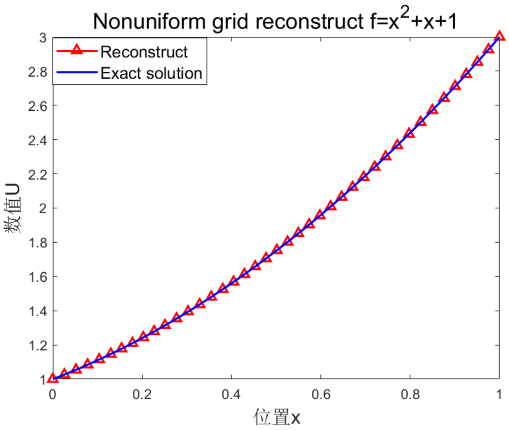
一 练习及结果

1. 在 $x \in [0,1]$ 的均匀网格上尝试使用 Least Squares Recovery(LSR), Least Squares Reconstruction(LSr) 和 Hybrid Least Squares Reconstruction(HLSr) 对 $f(x)$ 及 $g(x)$ 进行 P1P2 重构, 其中 $f(x) = 1 + x + x^2, g(x) = sin(\pi x)$，测试重构精度。如果网格为不均匀网格呢？

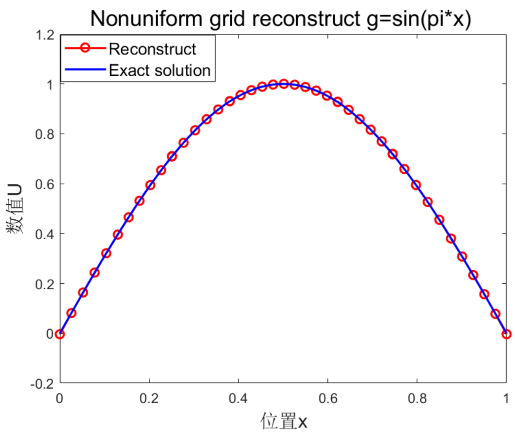
2. 在 $x \in [0,1]$ 的均匀网格上尝试使用 Hybrid Least Squares Reconstruction(HLSr) 对 $f(x)$ 及 $g(x)$ 进行 Hyperbolic rDG 的 DG(P0P2)+rDG(P0P1) 重构，其中 $f(x) = 1 + x + x^2 + x^3, g(x) = 1 + x + x^2 + x^3 + x^4, h(x) = sin(\pi x)$，测试重构精度。如果网格为不均匀网格呢？

解：

(1) 分别用三种重构方法对 $f(x)$ 与 $g(x)$ 进行 P1P2 重构，下面展示 LSR 方法的重构图 (非均匀网格)：



(a) f 重构

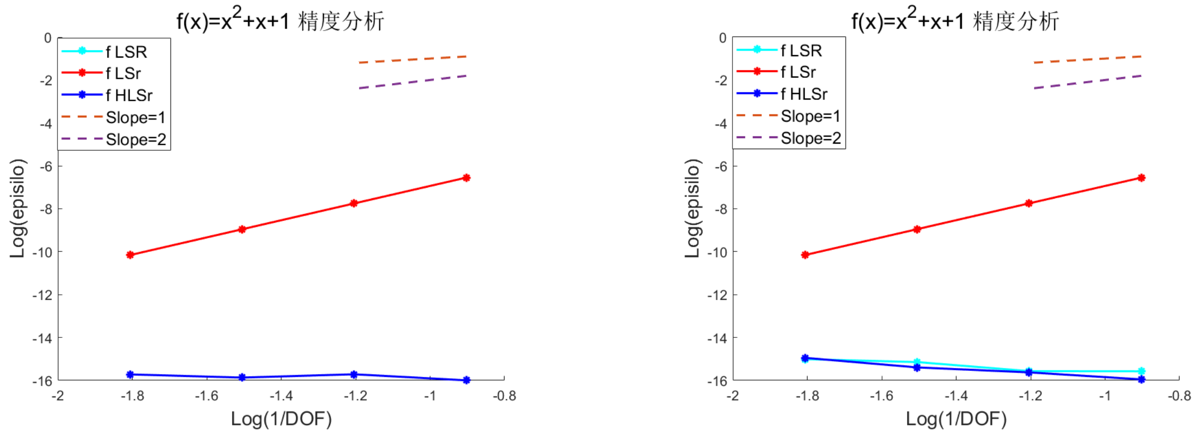


(b) g 重构

图 1: LSR

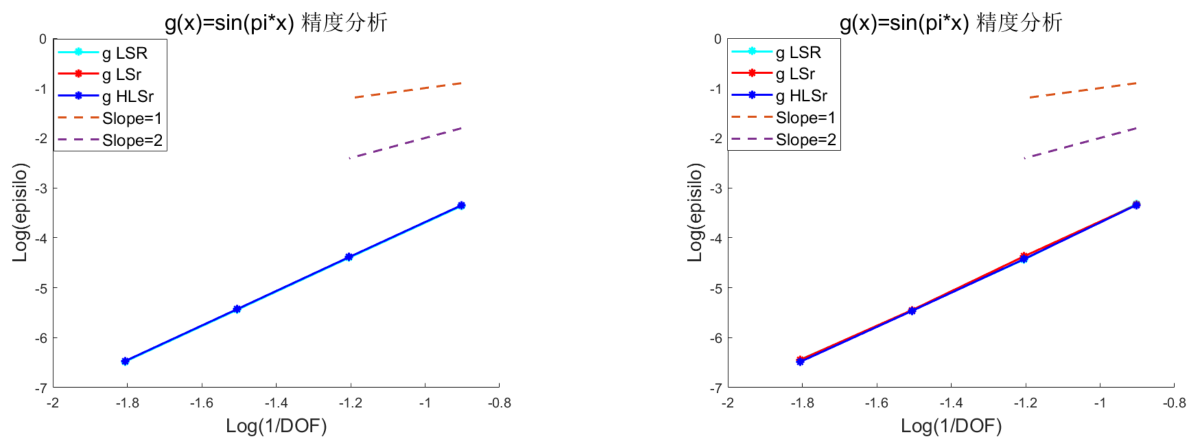
LSr 与 HLSr 的重构图与图 1 类似，这里不重复展示。

下面展示不同网格下 $f\,(x)$ 与 $g\,(x)$ 的精度比较图:



(a) Uniform grid　　**(b)** Nonuniform grid

图 2: f 精度分析



(a) Uniform grid　　**(b)** Nonuniform grid

图 3: g 精度分析

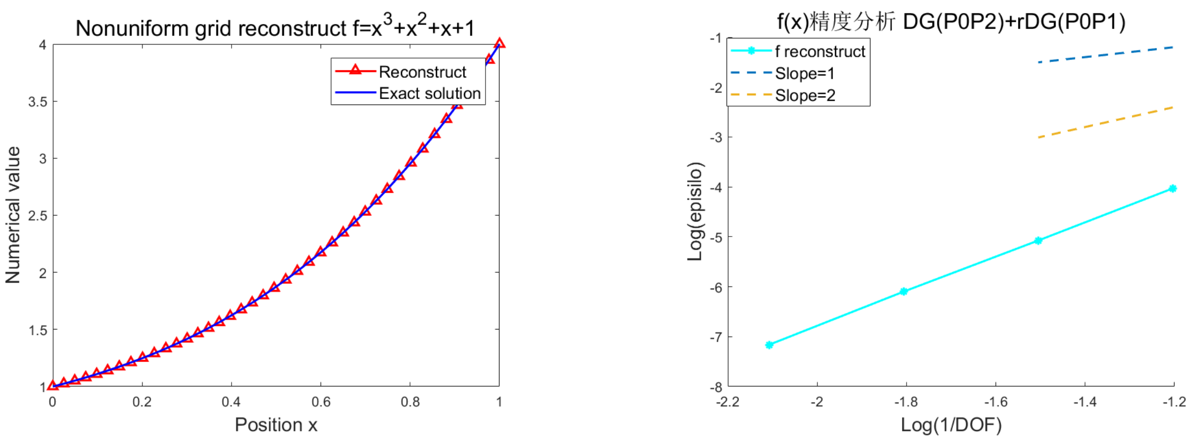
注意：虽然 LSR 与 HLSr 方法下 $f\,(x)$ 没有 Order，但其误差值达到了机器误差。下述表格能更直观地看出各个方法之间的差别。

表 1: 非均匀网格下各方法比较

Num of cells	P1P2(LSR)		P1P2(LSr)		P1P2(HLSr)	
	L2-errors	order	L2-errors	order	L2-errors	order
case 1: $f(x) = 1 + x + x^2$						
8	1.70E-16		2.74E-8		1.56E-16	
16	2.95E-16	—	1.77E-8	3.95	2.17E-16	—
32	4.10E-16	—	1.11E-9	3.40	6.05E-16	—
64	1.00E-15	—	6.94E-11	4.00	9.37E-16	—
case 2: $g(x) = sin(\pi x)$						
8	4.51E-4		4.13E-4		4.53E-4	
16	4.28E-5	3.40	4.06E-5	3.35	3.96E-5	3.52
32	3.81E-6	3.49	3.68E-6	3.46	3.48E-6	3.50
64	3.57E-7	3.41	3.37E-7	3.45	3.42E-7	3.35

(2) 在 $x \in [0,1]$ 上使用 Hybrid Least Squares Reconstruction(HLSr) 对 $f(x)$ 及 $g(x)$ 与进行 $h(x)$ Hyperbolic rDG 的 DG(P0P2)+rDG(P0P1) 重构, 分别对 φ 和 v 选取方程进行重构求解，最终得到如下结果 (仅考虑非均匀网格，均匀网格视为非均匀网格的一个特例)：

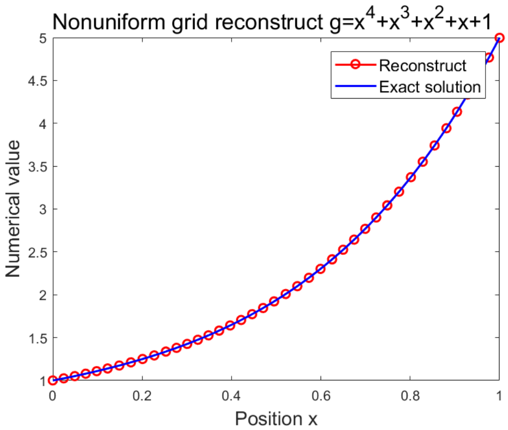
A) $f(x) = x^3 + x^2 + x + 1$



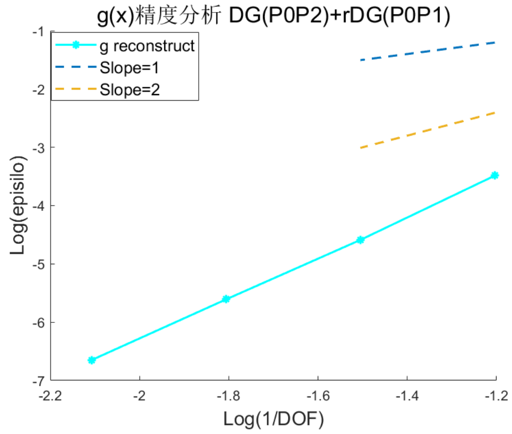
(a) 重构图 (b) 精度分析

图 4: f(x)

B)$g\left(x\right)=x^4+x^3+x^2+x+1$



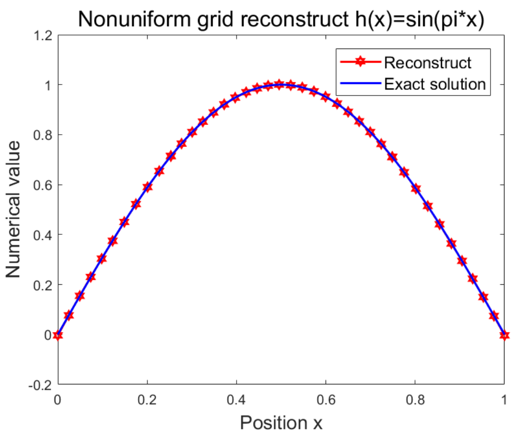
(a) 重构图



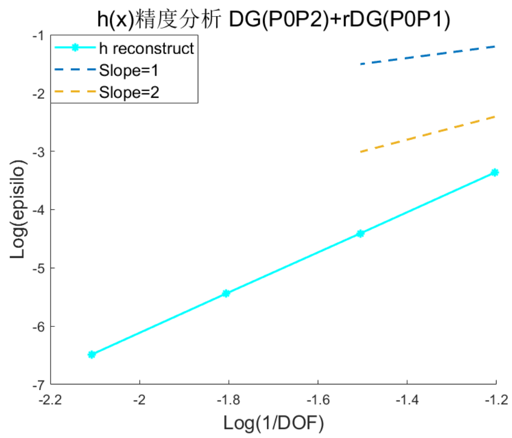
(b) 精度分析

图 5: g(x)

C)$h\left(x\right)=sin\left(\pi x\right)$



(a) 重构图



(b) 精度分析

图 6: h(x)

二 附录 (代码, 仅展示部分)

LSR

```
clc
clear all
close all
%Unit=8;endx=1;deltax=endx/Unit;numberx=Unit+1;
%记录内点位置, 上下浮动不超过百分之 5
Grid=zeros(1,numberx);
for i=2:numberx-1
Grid(1,i)=(i-1)*deltax+(0.1*rand(1)-0.05)*deltax;
end
Grid(1,numberx)=endx;
f=@(x)x.^2+x+1;F=@(x)2*x+1;
g=@(y)sin(pi*y);G=@(y)pi*cos(pi*y);
Unumsolution=zeros(2,Unit);
Ureconstruct=zeros(1,Unit);
Unumsolution1=zeros(1,2);
Unumsolution2=zeros(2,numberx-1);
Acc=zeros(3,4);a1=[1/8,1/16,1/32,1/64];a2=[1/8,1/16];
%for k=1:Unit
Unumsolution(1,k)=(Grid(k+1)-Grid(k)+0.5*(Grid(k+1)^2-Grid(k)^2)+(Grid(k
    +1)^3-Grid(k)^3)/3)/(Grid(k+1)-Grid(k));
Unumsolution(2,k)=F(0.5*(Grid(k)+Grid(k+1)))*(Grid(k+1)-Grid(k));%store
    Uxc*deltax
end

%Reconstruct Uxx
for k=2:numberx-2
xci=0.5*(Grid(k)+Grid(k+1));%xci
xci1=0.5*(Grid(k+1)+Grid(k+2));%xci+1
xci2=0.5*(Grid(k-1)+Grid(k));%xci-1
A=[(xci1-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k+2)-Grid(k+1))
    ^2/(Grid(k+1)-Grid(k))^2-1);
(Grid(k+2)-Grid(k+1))*(xci1-xci)/(Grid(k+1)-Grid(k))^2;
(xci2-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k)-Grid(k-1))^2/(Grid
    (k+1)-Grid(k))^2-1);
(Grid(k)-Grid(k-1))*(xci2-xci)/(Grid(k+1)-Grid(k))^2];
b=[Unumsolution(1,k+1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci1-xci)/(
    Grid(k+1)-Grid(k));
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```
Unumsolution(2,k+1)-Unumsolution(2,k)*(Grid(k+2)-Grid(k+1))/(Grid(k+1)-
    Grid(k));
Unumsolution(1,k-1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci2-xci)/(Grid
    (k+1)-Grid(k));
Unumsolution(2,k-1)-Unumsolution(2,k)*(Grid(k)-Grid(k-1))/(Grid(k+1)-
    Grid(k))];
Ureconstruct(k)=A\b;
end
%boundary
%Left
xci=0.5*(Grid(1)+Grid(2));%xci
xci1=0.5*(Grid(2)+Grid(3));%xci+1
A=[(xci1-xci)^2/(2*(Grid(2)-Grid(1))^2)+1/24*((Grid(3)-Grid(2))^2/(Grid
    (2)-Grid(1))^2-1);
(Grid(3)-Grid(2))*(xci1-xci)/(Grid(2)-Grid(1))^2];
b=[Unumsolution(1,2)-Unumsolution(1,1)-Unumsolution(2,1)*(xci1-xci)/(
    Grid(2)-Grid(1));
Unumsolution(2,2)-Unumsolution(2,1)*(Grid(3)-Grid(2))/(Grid(2)-Grid(1))
    ];
Ureconstruct(1)=A\b;
%Right
xci=0.5*(Grid(numberx-1)+Grid(numberx-1+1));%xci
xci2=0.5*(Grid(numberx-1-1)+Grid(numberx-1));%xci-1
A=[(xci2-xci)^2/(2*(Grid(numberx-1+1)-Grid(numberx-1))^2)+1/24*((Grid(
    numberx-1)-Grid(numberx-1-1))^2/(Grid(numberx-1+1)-Grid(numberx-1))
    ^2-1);
(Grid(numberx-1)-Grid(numberx-1-1))*(xci2-xci)/(Grid(numberx-1+1)-Grid(
    numberx-1))^2];
b=[Unumsolution(1,numberx-1-1)-Unumsolution(1,numberx-1)-Unumsolution(2,
    numberx-1)*(xci2-xci)/(Grid(numberx-1+1)-Grid(numberx-1));
Unumsolution(2,numberx-1-1)-Unumsolution(2,numberx-1)*(Grid(numberx-1)-
    Grid(numberx-1-1))/(Grid(numberx-1+1)-Grid(numberx-1))];
Ureconstruct(numberx-1)=A\b;

%Post-proceeding
figure
k=1;
x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2),Unumsolution(2,k)/(Grid(k
    +1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(
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    k))^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))
    ^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k)
    )/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
y=polyval(p,x);
plot(x,y,'-r^','linewidth',1.5);hold on
H1=plot(x,y,'-r^','linewidth',1.5);hold on

for k=2:numberx-1
x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2),Unumsolution(2,k)/(Grid(k
    +1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(
    k))^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))
    ^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k)
    )/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
y=polyval(p,x);plot(x,y,'-r^','linewidth',1.5);
end

hold on
x=Grid(1):0.01*(Grid(numberx)-Grid(1)):Grid(numberx);
plot(x,f(x),'-b','linewidth',1.5);
H2=plot(x,f(x),'-b','linewidth',1.5);
lgd=legend([H1,H2],'Reconstruct','Exact solution');
lgd.FontSize=12;
xlabel('位置x','fontsize',14)
ylabel('数值U','fontsize',14)
title('Nonuniform grid reconstruct f=x^2+x+1','fontsize',16)
hold off

%计算精度
Acc(1,1)=Accuracy(8);Acc(1,2)=Accuracy(16);
Acc(1,3)=Accuracy(32);
Acc(1,4)=Accuracy(64);
for k=1:3
accuracyf(k)=(log10(Acc(1,k+1))-log10(Acc(1,k)))./(log10(a1(1,k+1))-
    log10(a1(1,k)));
end
figure
hold on
plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);
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```

H2=plot(log10(a2),1*log10(a2),'--','linewidth',1.5);
plot(log10(a2),2*log10(a2),'--','linewidth',1.5)
H3=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
lgd=legend([H1,H2,H3],'f reconstruct','斜率为1','斜率为2');
lgd.FontSize=12;
xlabel('Log(deltax)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('f精度分析','fontsize',16)

%对 g
for k=1:Unit
Unumsolution(1,k)=(cos(pi*Grid(k))-cos(pi*Grid(k+1)))/(pi*(Grid(k+1)-
    Grid(k)));
Unumsolution(2,k)=G(0.5*(Grid(k)+Grid(k+1)))*(Grid(k+1)-Grid(k));%store
    Uxc*deltax
end

%Reconstruct Uxx
for k=2:numberx-2
xci=0.5*(Grid(k)+Grid(k+1));%xci
xci1=0.5*(Grid(k+1)+Grid(k+2));%xci+1
xci2=0.5*(Grid(k-1)+Grid(k));%xci-1
A=[(xci1-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k+2)-Grid(k+1))
    ^2/(Grid(k+1)-Grid(k))^2-1);
(Grid(k+2)-Grid(k+1))*(xci1-xci)/(Grid(k+1)-Grid(k))^2;
(xci2-xci)^2/(2*(Grid(k+1)-Grid(k))^2)+1/24*((Grid(k)-Grid(k-1))^2/(Grid
    (k+1)-Grid(k))^2-1);
(Grid(k)-Grid(k-1))*(xci2-xci)/(Grid(k+1)-Grid(k))^2];
b=[Unumsolution(1,k+1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci1-xci)/(
    Grid(k+1)-Grid(k));
Unumsolution(2,k+1)-Unumsolution(2,k)*(Grid(k+2)-Grid(k+1))/(Grid(k+1)-
    Grid(k));
Unumsolution(1,k-1)-Unumsolution(1,k)-Unumsolution(2,k)*(xci2-xci)/(Grid
    (k+1)-Grid(k));
Unumsolution(2,k-1)-Unumsolution(2,k)*(Grid(k)-Grid(k-1))/(Grid(k+1)-
    Grid(k))];
Ureconstruct(k)=A\b;
end
%boundary
```

```matlab
%Left
xci=0.5*(Grid(1)+Grid(2));%xci
xci1=0.5*(Grid(2)+Grid(3));%xci+1
A=[(xci1-xci)^2/(2*(Grid(2)-Grid(1))^2)+1/24*((Grid(3)-Grid(2))^2/(Grid(2)-Grid(1))^2-1);
(Grid(3)-Grid(2))*(xci1-xci)/(Grid(2)-Grid(1))^2];
b=[Unumsolution(1,2)-Unumsolution(1,1)-Unumsolution(2,1)*(xci1-xci)/(Grid(2)-Grid(1));
Unumsolution(2,2)-Unumsolution(2,1)*(Grid(3)-Grid(2))/(Grid(2)-Grid(1))];
Ureconstruct(1)=A\b;
%Right
xci=0.5*(Grid(numberx-1)+Grid(numberx-1+1));%xci
xci2=0.5*(Grid(numberx-1-1)+Grid(numberx-1));%xci-1
A=[(xci2-xci)^2/(2*(Grid(numberx-1+1)-Grid(numberx-1))^2)+1/24*((Grid(numberx-1)-Grid(numberx-1-1))^2/(Grid(numberx-1+1)-Grid(numberx-1))^2-1);
(Grid(numberx-1)-Grid(numberx-1-1))*(xci2-xci)/(Grid(numberx-1+1)-Grid(numberx-1))^2];
b=[Unumsolution(1,numberx-1-1)-Unumsolution(1,numberx-1)-Unumsolution(2,numberx-1)*(xci2-xci)/(Grid(numberx-1+1)-Grid(numberx-1));
Unumsolution(2,numberx-1-1)-Unumsolution(2,numberx-1)*(Grid(numberx-1)-Grid(numberx-1-1))/(Grid(numberx-1+1)-Grid(numberx-1))];
Ureconstruct(numberx-1)=A\b;
%Post-proceeding
figure
k=1;
x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2),Unumsolution(2,k)/(Grid(k+1)-Grid(k))-Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(k))^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))^2/(2*(Grid(k+1)-Grid(k))^2)-Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)-Grid(k))-Ureconstruct(k)/24];
y=polyval(p,x);
plot(x,y,'-ro','linewidth',1.5);hold on
H1=plot(x,y,'-ro','linewidth',1.5);hold on

for k=2:numberx-1
x=Grid(k):0.2*(Grid(k+1)-Grid(k)):Grid(k+1);
p=[Ureconstruct(k)/(2*(Grid(k+1)-Grid(k))^2),Unumsolution(2,k)/(Grid(k
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+1)−Grid(k))−Ureconstruct(k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)−Grid(k))^2,Unumsolution(1,k)+Ureconstruct(k)*(0.5*(Grid(k+1)+Grid(k)))^2/(2*(Grid(k+1)−Grid(k))^2)−Unumsolution(2,k)*0.5*(Grid(k+1)+Grid(k))/(Grid(k+1)−Grid(k))−Ureconstruct(k)/24];
y=polyval(p,x);
plot(x,y,'-ro','linewidth',1.5);
end
hold on
x=Grid(1):0.01*(Grid(numberx)−Grid(1)):Grid(numberx);
plot(x,g(x),'-b','linewidth',1.5);
H2=plot(x,g(x),'-b','linewidth',1.5);
lgd=legend([H1,H2],'Reconstruct','Exact solution');
lgd.FontSize=12;
xlabel('位置x','fontsize',14)
ylabel('数值U','fontsize',14)
title('Nonuniform grid reconstruct g=sin(pi*x) ','fontsize',16)
hold off

%计算精度
Acc(1,1)=Accuracyg(8);
Acc(1,2)=Accuracyg(16);
Acc(1,3)=Accuracyg(32);
Acc(1,4)=Accuracyg(64);
for k=1:3
accuracyg(k)=(log10(Acc(1,k+1))−log10(Acc(1,k)))./(log10(a1(1,k+1))−log10(a1(1,k)));
end
figure
hold on
plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5)
H1=plot(log10(a1),log10(Acc(1,:)),'-c*','linewidth',1.5);
H2=plot(log10(a2),1*log10(a2),'--','linewidth',1.5);
plot(log10(a2),2*log10(a2),'--','linewidth',1.5)
H3=plot(log10(a2),2*log10(a2),'--','linewidth',1.5);
lgd=legend([H1,H2,H3],'g reconstruct','斜率为1','斜率为2');
lgd.FontSize=12;
xlabel('Log(deltax)','fontsize',14)
ylabel('Log(episilo)','fontsize',14)
title('g精度分析','fontsize',16)
```