Package ‘metafrontieR’

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Title Metafrontier Analysis Routines

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Author Sulman Olieko Owili [aut, cre]

Maintainer Sulman Olieko Owili <oliekosulman@gmail.com>

Description An R package for implementing various metafrontier analyses for productivity/performance benchmarking, assessing efficiencies and technology gaps for firms operating under different technologies. It contains routines for implementing (i) the deterministic envelope proposed by O'Donnell et al. (2008) <doi:10.1007/s00181-007-0119-4> via linear and quadratic programming, as well as (ii) the stochastic metafrontier proposed by Huang et al. (2014) <doi:10.1007/s11123-014-0402-2>. The package also has functionalities for implementing latent class stochastic metafrontier analysis and sample selection correction stochastic metafrontier models. The package depends on sfaR by Dakpo et al. (2024) <https://github.com/hdakpo/sfaR>.

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URL https://github.com/SulmanOlieko/metafrontieR

BugReports https://github.com/SulmanOlieko/metafrontieR/issues

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sfaR

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Contents

`coef` *Extract coefficients of stochastic metafrontier models*

Description

From an object of class 'summary.sfametafrontier', coef extracts the coefficients, their standard errors, z-values, and (asymptotic) P-values.

From on object of class 'sfametafrontier', it extracts only the estimated coefficients.

Usage

```
## S3 method for class 'sfametafrontier'
coef(object, ...)

## S3 method for class 'summary.sfametafrontier'
coef(object, ...)
```

Arguments

object	A stochastic metafrontier model returned by sfametafrontier, or an object of class 'summary.sfametafrontier'.
...	Currently ignored.

Value

For objects of class 'summary.sfametafrontier', coef returns a matrix with four columns. Namely, the estimated coefficients, their standard errors, z-values, and (asymptotic) P-values.

For objects of class 'sfametafrontier', coef returns a numeric vector of the estimated coefficients.

See Also

sfametafrontier, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data.

`efficiencies` *Compute efficiency estimates and metatechnology ratios from stochastic metafrontier models*

Description

`efficiencies` returns all efficiency estimates and metatechnology ratio (MTR) measures for objects of class `"sfametafrontier"` returned by `sfametafrontier`. The function supports models estimated via linear programming (LP), quadratic programming (QP), and stochastic second-stage SFA (`"sfa"`), and for each observation it computes the group-specific technical efficiency, the metafrontier technical efficiency, and the metatechnology ratio (MTR), using both the Jondrow, Lovell, Materov, and Schmidt (1982) (JLMS) and the Battese and Coelli (1988) (BC) estimators. Additional model-specific columns are returned depending on `groupType`.

Usage

```
## S3 method for class 'sfametafrontier'
efficiencies(object, level = 0.95, newData = NULL, ...)
```

Arguments

`object`	An object of class `"sfametafrontier"` returned by `sfametafrontier`.
`level`	A number strictly between 0 and 0.9999 specifying the nominal coverage for (in-)efficiency confidence intervals. Default `0.95`. This argument is passed to the underlying `efficiencies` method of the group-level model (class `"sfacross"`, `"sfalcmcross"`, or `"sfaselectioncross"`).
`newData`	Optional data frame for out-of-sample prediction of efficiency estimates. When `NULL` (default), efficiencies are computed for the observations used in the estimation.
`...`	Further arguments (currently ignored).

Details

Group-specific efficiency estimates: For each group, the group-level frontier model is estimated by maximising the log-likelihood using the distribution specified by `udist` in `sfametafrontier`. Given the estimated composite error $\varepsilon_i = v_i - Su_i$, the conditional distribution of $u_i \mid \varepsilon_i$ is used to compute:

- the JLMS estimator (Jondrow, Lovell, Materov, and Schmidt, 1982): $\hat{u}_i = E[u_i \mid \varepsilon_i]$, and $TE^g_{JLMS,i} = \exp(-\hat{u}_i)$;
- the BC estimator (Battese and Coelli, 1988): $TE^g_{BC,i} = E[\exp(-u_i) \mid \varepsilon_i]$;
- the mode estimator: $m_i = \text{mode}[u_i \mid \varepsilon_i]$, and $TE^g_{\text{mode},i} = \exp(-m_i)$;
- confidence bounds on u_i and $TE^g_{BC,i}$ at the nominal level `level`.

For `groupType = "sfaselectioncross"`, all estimates are `NA` for observations not selected into the sample (binary selection indicator equal to 0). For `groupType = "sfalcmcross"`, the composite efficiencies `u_g`, `TE_group_JLMS`, and `TE_group_BC` are computed using the posterior-probability-weighted class assignments.

Metatechnology ratio and metafrontier efficiency: The MTR measures how far the group frontier lies below the metafrontier for each observation. Let $\ln \hat{y}^g_i$ be the group-specific fitted frontier value and $\ln \hat{y}^*_i$ the metafrontier fitted value.

- For metaMethod = "lp" or "qp" (Battese, Rao, and O'Donnell, 2004):

$$MTR_i = \exp\big(-\max\big\{S \cdot (\ln \hat{y}_i^* - \ln \hat{y}_i^g),\, 0\big\}\big)$$

  where $S = 1$ for production/profit frontiers and $S = -1$ for cost frontiers. The technology gap $U_i = \max\{S \cdot (\ln \hat{y}_i^* - \ln \hat{y}_i^g), 0\}$ is stored in u_meta.
- For metaMethod = "sfa" with sfaApproach = "huang" (Huang, Huang, and Liu, 2014):

$$MTR_i = TE_i^* = \exp(-U_i)$$

  where U_i is the one-sided error term from the second-stage SFA.
- For metaMethod = "sfa" with sfaApproach = "ordonnell" (O'Donnell, Rao, and Battese, 2008): $MTR_i = TE_i^{*,\text{sfa}}/TE_i^g$, where $TE_i^{*,\text{sfa}}$ is the technical efficiency from the second-stage SFA fitted on the LP envelope values.

The metafrontier technical efficiency is then:

$$TE_i^* = TE_i^g \times MTR_i$$

computed separately for the JLMS and BC group efficiency estimators. Both MTR_JLMS and MTR_BC are reported, distinguishing which group-level efficiency estimator was used as the basis.

Value

A data frame with one row per observation (in the original row order), containing the following columns. The exact set of columns depends on groupType:

Columns present for all model types:

<group> **or** Group_c The technology group identifier for each observation. For groupType = "sfacross" and "sfaselectioncross", this column takes the name of the user-supplied group variable and contains the group label to which each observation belongs. For groupType = "sfalcmcross", it is named Group_c and contains the integer index of the latent class assigned by the maximum posterior probability criterion.

u_g Group-specific conditional mean of the inefficiency term, computed as $E[u_i \mid \varepsilon_i]$. This is the JLMS (Jondrow, Lovell, Materov, and Schmidt, 1982) point estimate of the inefficiency at the group-frontier level. For groupType = "sfaselectioncross", u_g is NA for observations not selected into the sample (selection indicator = 0).

TE_group_JLMS Group-specific technical efficiency using the Jondrow, Lovell, Materov, and Schmidt (1982) estimator: $TE_i^g = \exp(-E[u_i \mid \varepsilon_i])$. For groupType = "sfaselectioncross", NA for non-selected observations.

TE_group_BC Group-specific technical efficiency using the Battese and Coelli (1988) estimator: $TE_i^g = E[\exp(-u_i) \mid \varepsilon_i]$. For groupType = "sfaselectioncross", NA for non-selected observations.

TE_group_BC_reciprocal Reciprocal of the Battese and Coelli (1988) group technical efficiency: $1/TE_i^{g,BC}$. For production frontiers this equals the cost-efficiency index implied by the BC estimator. Present for all three model types. For groupType = "sfaselectioncross", NA for non-selected observations.

u_meta Metafrontier inefficiency, measuring the technology-gap component $U_i \geq 0$ that separates the group frontier from the global metafrontier. Computed from the second-stage SFA when metaMethod = "sfa", or derived from the LP/QP gap as $U_i = \max\{S \cdot (\ln \hat{y}_i^* - \ln \hat{y}_i^g), 0\}$ when metaMethod = "lp" or "qp".

TE_meta_JLMS Metafrontier technical efficiency based on the JLMS group efficiency: $TE^*_{JLMS,i} = TE^g_{JLMS,i} \times MTR_{JLMS,i}$.

TE_meta_BC Metafrontier technical efficiency based on the Battese and Coelli (1988) group efficiency: $TE^*_{BC,i} = TE^g_{BC,i} \times MTR_{BC,i}$.

MTR_JLMS Metatechnology ratio computed using the JLMS group efficiency: $MTR_{JLMS,i} = TE^*_{JLMS,i}/TE^g_{JLMS,i} = \exp(-U_i)$. Values range from 0 to 1. A value of 1 indicates that the group frontier for this observation coincides with the metafrontier.

MTR_BC Metatechnology ratio computed using the Battese and Coelli (1988) group efficiency: $MTR_{BC,i} = TE^*_{BC,i}/TE^g_{BC,i} = \exp(-U_i)$.

Additional columns for groupType = "sfacross" **only:**

uLB_g, uUB_g Lower and upper bounds of the level confidence interval for the conditional mean inefficiency u_g, constructed using the asymptotic distribution of the conditional estimator. Available for distributions with closed-form expressions for the confidence bounds, such as udist = "hnormal" and udist = "tnormal".

m_g Mode of the conditional distribution of the one-sided error term $u_i \mid \varepsilon_i$. This is an alternative point estimate of inefficiency. Available for distributions for which the conditional mode has a closed-form expression.

TE_group_mode Group-specific technical efficiency evaluated at the conditional mode: $TE^g_{\text{mode},i} = \exp(-m_i)$.

teBCLB_g, teBCUB_g Lower and upper bounds of the level confidence interval for the Battese and Coelli (1988) group technical efficiency TE_group_BC. Constructed from the corresponding bounds on the conditional distribution of $\exp(-u_i \mid \varepsilon_i)$.

Additional columns for groupType = "sfalcmcross" **only:**

PosteriorProb_c Posterior probability that observation i belongs to its assigned class (the one with the highest posterior probability). Computed via Bayes' rule as $P(j \mid y_i, x_i) \propto \pi(i,j)\, P(i \mid j)$, where $\pi(i,j)$ is the prior class probability and $P(i \mid j)$ is the class-conditional likelihood.

PosteriorProb_cJ **(per class,** $J = 1, 2, \ldots$**)** Posterior probability of belonging to latent class J, computed via Bayes' rule for each class separately. One column is produced for each estimated class.

PriorProb_cJ **(per class,** $J = 1, 2, \ldots$**)** Prior (unconditional) probability of belonging to latent class J, given by the logistic specification $\pi(i, J) = \exp(\boldsymbol{\theta}'_J \mathbf{Z}_{hi}) / \sum_m \exp(\boldsymbol{\theta}'_m \mathbf{Z}_{hi})$.

u_cJ **(per class,** $J = 1, 2, \ldots$**)** Conditional mean of the inefficiency term for class J: $E[u_{i|J} \mid \varepsilon_{i|J}]$.

teBC_cJ **(per class,** $J = 1, 2, \ldots$**)** Battese and Coelli (1988) technical efficiency for class J: $E[\exp(-u_{i|J}) \mid \varepsilon_{i|J}]$.

teBC_reciprocal_cJ **(per class,** $J = 1, 2, \ldots$**)** Reciprocal of the class-J Battese and Coelli (1988) efficiency: $1/TE^{BC}_{i|J}$.

ineff_cJ **(per class,** $J = 1, 2, \ldots$**)** Inefficiency estimate for the observation restricted to class J (i.e. the value assigned to the class to which the observation *does* belong; NA for other classes).

effBC_cJ **(per class,** $J = 1, 2, \ldots$**)** Battese and Coelli (1988) efficiency for the observation's assigned class; NA for non-assigned classes.

ReffBC_cJ **(per class,** $J = 1, 2, \ldots$**)** Reciprocal Battese and Coelli (1988) efficiency for the observation's assigned class; NA for non-assigned classes.

References

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Huang, C. J., Huang, T.-H., and Liu, N.-H. 2014. A new approach to estimating the metafrontier production function based on a stochastic frontier framework. *Journal of Productivity Analysis*, **42**(3), 241–254. https://doi.org/10.1007/s11123-014-0402-2

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O'Donnell, C. J., Rao, D. S. P., and Battese, G. E. 2008. Metafrontier frameworks for the study of firm-level efficiencies and technology ratios. *Empirical Economics*, **34**(2), 231–255. https://doi.org/10.1007/s00181-007-0119-4

Orea, L., and Kumbhakar, S. C. 2004. Efficiency measurement using a latent class stochastic frontier model. *Empirical Economics*, **29**(1), 169–183. https://doi.org/10.1007/s00181-003-0184-2

Dakpo, K. H., Desjeux, Y., and Latruffe, L. 2023. sfaR: Stochastic Frontier Analysis using R. R package version 1.0.1. https://CRAN.R-project.org/package=sfaR

See Also

`sfametafrontier`, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data; `efficiencies`, for the underlying group-level efficiency extractor.

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fitted — *Extract fitted values of stochastic metafrontier models*

Description

`fitted` returns the fitted frontier values from stochastic metafrontier models estimated with `sfametafrontier`.

Usage

```
## S3 method for class 'sfametafrontier'
fitted(object, ...)
```

Arguments

`object`	A stochastic metafrontier model returned by `sfametafrontier`.
`...`	Currently ignored.

Value

A vector of fitted values is returned.

Note

The fitted values are ordered in the same way as the corresponding observations in the dataset used for the estimation.

See Also

`sfametafrontier`, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data.

`ic` *Extract information criteria of stochastic metafrontier models*

Description

`ic` returns information criterion from stochastic metafrontier models estimated with `sfametafrontier`.

Usage

```
## S3 method for class 'sfametafrontier'
ic(object, IC = "AIC", ...)
```

Arguments

`object`	A stochastic metafrontier model returned by `sfametafrontier`.
`IC`	Character string. Information criterion measure. Three criteria are available: • `'AIC'` for Akaike information criterion (default) • `'BIC'` for Bayesian information criterion • `'HQIC'` for Hannan-Quinn information criterion .
`...`	Currently ignored.

Details

The different information criteria are computed as follows:

- AIC: $-2 \log LL + 2 * K$
- BIC: $-2 \log LL + \log N * K$
- HQIC: $-2 \log LL + 2 \log [\log N] * K$

where LL is the maximum likelihood value, K the number of parameters estimated and N the number of observations.

Value

`ic` returns the value of the information criterion (AIC, BIC or HQIC) of the maximum likelihood coefficients.

See Also

`sfametafrontier`, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data.

`logLik` *Extract log-likelihood value of stochastic metafrontier models*

Description

logLik extracts the log-likelihood value(s) from stochastic metafrontier models estimated with sfametafrontier.

Usage

```
## S3 method for class 'sfametafrontier'
logLik(object, individual = FALSE, ...)
```

Arguments

`object`	A stochastic metafrontier model returned by sfametafrontier.
`individual`	Logical. If `FALSE` (default), the sum of all observations' log-likelihood values is returned. If `TRUE`, a vector of each observation's log-likelihood value is returned.
`...`	Currently ignored.

Value

logLik returns either an object of class `'logLik'`, which is the log-likelihood value with the total number of observations (`nobs`) and the number of free parameters (`df`) as attributes, when `individual = FALSE`, or a list of elements, containing the log-likelihood of each observation (`logLik`), the total number of observations (`Nobs`) and the number of free parameters (`df`), when `individual = TRUE`.

See Also

sfametafrontier, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data.

`nobs` *Extract total number of observations used in frontier models*

Description

This function extracts the total number of 'observations' from a fitted point frontier model.

Usage

```
## S3 method for class 'sfametafrontier'
nobs(object, ...)
```

Arguments

`object`	a `sfametafrontier` object for which the number of total observations is to be extracted.
`...`	Currently ignored.

Details

`nobs` gives the number of observations actually used by the estimation procedure.

Value

A single number, normally an integer.

See Also

`sfametafrontier`, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data

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`residuals` *Extract residuals of stochastic metafrontier models*

Description

This function returns the residuals' values from stochastic metafrontier models estimated with `sfametafrontier`.

Usage

```
## S3 method for class 'sfametafrontier'
residuals(object, ...)
```

Arguments

`object`	A stochastic metafrontier model returned by `sfametafrontier`.
`...`	Currently ignored.

Value

`residuals` returns a vector of residuals values.

Note

The residuals values are ordered in the same way as the corresponding observations in the dataset used for the estimation.

See Also

`sfametafrontier`, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data.

sfametafrontier *Stochastic metafrontier estimation*

Description

sfametafrontier estimates a stochastic metafrontier model for cross-sectional or pooled data. The function follows the theoretical frameworks of Battese, Rao, and O'Donnell (2004) and O'Donnell, Rao, and Battese (2008), and additionally implements the two-stage stochastic approach of Huang, Huang, and Liu (2014). Three types of group-level frontier models are supported: standard stochastic frontier analysis (sfacross), sample selection stochastic frontier analysis (sfaselectioncross), and latent class stochastic frontier analysis (sfalcmcross).

Usage

```
sfametafrontier(
  formula,
  muhet,
  uhet,
  vhet,
  thet,
  logDepVar = TRUE,
  data,
  subset,
  weights,
  wscale = TRUE,
  group = NULL,
  S = 1L,
  udist = "hnormal",
  start = NULL,
  scaling = FALSE,
  modelType = "greene10",
  groupType = "sfacross",
  metaMethod = "lp",
  sfaApproach = "huang",
  selectionF = NULL,
  lcmClasses = 2L,
  whichStart = 2L,
  initAlg = "nm",
  initIter = 100L,
  lType = "ghermite",
  Nsub = 100L,
  uBound = Inf,
  intol = 1e-06,
  method = "bfgs",
  hessianType = 1L,
  simType = "halton",
  Nsim = 100L,
  prime = 2L,
  burn = 10L,
  antithetics = FALSE,
  seed = 12345L,
```

```
  itermax = 2000L,
  printInfo = FALSE,
  tol = 1e-12,
  gradtol = 1e-06,
  stepmax = 0.1,
  qac = "marquardt",
  ...
)

## S3 method for class 'sfametafrontier'
print(x, ...)
```

Arguments

`formula` A symbolic description of the frontier model to be estimated, based on the generic function `formula`. For `groupType = "sfaselectioncross"`, this argument specifies the frontier (outcome) equation and must be a standard formula whose left-hand side is the output (or cost) variable and whose right-hand side contains the frontier regressors (see also `selectionF`).

`muhet` A one-part formula to account for heterogeneity in the mean of the pre-truncated normal distribution. Applicable only when `groupType = "sfacross"` and `udist = "tnormal"`. The variables specified model the conditional mean $\mu_i = \boldsymbol{\omega}'\mathbf{Z}_\mu$ of the truncated normal inefficiency distribution (see section 'Details').

`uhet` A one-part formula to account for heteroscedasticity in the one-sided error variance. Applicable for all three model types. The variance of the inefficiency term is modelled as $\sigma_u^2 = \exp(\boldsymbol{\delta}'\mathbf{Z}_u)$, where $\mathbf{Z}_u$ are the inefficiency drivers and $\boldsymbol{\delta}$ the associated coefficients (see section 'Details').

`vhet` A one-part formula to account for heteroscedasticity in the two-sided error variance. Applicable for all three model types. The variance of the noise term is modelled as $\sigma_v^2 = \exp(\boldsymbol{\phi}'\mathbf{Z}_v)$, where $\mathbf{Z}_v$ are the heteroscedasticity variables and $\boldsymbol{\phi}$ the coefficients (see section 'Details').

`thet` A one-part formula to account for technological heterogeneity in the construction of the latent classes. Applicable only when `groupType = "sfalcmcross"`. The variables specified enter the logit formulation that determines the prior class membership probabilities $\pi(i, j)$ (see section 'Details').

`logDepVar` Logical. Informs whether the dependent variable is logged (`TRUE`) or not (`FALSE`). Default `TRUE`. Must match the transformation applied to the left-hand side of `formula`.

`data` A data frame containing all variables referenced in `formula`, `selectionF`, `muhet`, `uhet`, `vhet`, `thet`, and `group`.

`subset` An optional vector specifying a subset of observations to be used in the estimation process.

`weights` An optional vector of weights to be used for weighted log-likelihood estimation. Should be `NULL` or a numeric vector with strictly positive values. When `NULL` (default), all observations receive equal weight.

`wscale` Logical. When `weights` is not `NULL`, a scaling transformation is applied such that the weights sum to the sample size:

$$w_{\text{new}} = n \times \frac{w_{\text{old}}}{\sum w_{\text{old}}}$$

Default `TRUE`. When `FALSE`, the raw weights are used without scaling.

group
: Character string. The name of the column in data identifying the technology group of each observation. The column is coerced to a factor internally and must have at least two unique values. When groupType = "sfalcmcross" and group is NULL, a single pooled latent class model is estimated and class assignments serve as groups (see section 'Details').

S
: Integer. Frontier orientation.
  - S = 1 (default): production or profit frontier, $\varepsilon_i = v_i - u_i$.
  - S = -1: cost frontier, $\varepsilon_i = v_i + u_i$.

udist
: Character string. Distribution for the one-sided error term $u_i \geq 0$. The following distributions are available for groupType = "sfacross":
  - "hnormal" (default): half-normal distribution (Aigner *et al.*, 1977; Meeusen and van den Broeck, 1977).
  - "exponential": exponential distribution.
  - "tnormal": truncated normal distribution (Stevenson, 1980).
  - "rayleigh": Rayleigh distribution (Hajargasht, 2015).
  - "uniform": uniform distribution (Li, 1996; Nguyen, 2010).
  - "gamma": Gamma distribution, estimated by maximum simulated likelihood (Greene, 2003).
  - "lognormal": log-normal distribution, estimated by maximum simulated likelihood (Migon and Medici, 2001; Wang and Ye, 2020).
  - "weibull": Weibull distribution, estimated by maximum simulated likelihood (Tsionas, 2007).
  - "genexponential": generalised exponential distribution (Papadopoulos, 2020).
  - "tslaplace": truncated skewed Laplace distribution (Wang, 2012).

  For groupType = "sfaselectioncross" and "sfalcmcross", only "hnormal" is currently supported.

start
: Numeric vector. Optional starting values for the maximum likelihood (ML) or maximum simulated likelihood (MSL) estimation of the group-level frontier models. When NULL (default), starting values are computed automatically. For groupType = "sfacross", they are derived from OLS residuals. For groupType = "sfalcmcross", they depend on whichStart.

scaling
: Logical. Applicable only when groupType = "sfacross" and udist = "tnormal". When TRUE, the scaling property model (Wang and Schmidt, 2002) is estimated, whereby $u_i = h(\mathbf{Z}_u, \boldsymbol{\delta})u_i^*$ and u_i^* follows a truncated normal distribution $N^+(\tau, \exp(c_u))$. Default FALSE.

modelType
: Character string. Applicable only when groupType = "sfaselectioncross". Specifies the model used to correct for selection bias. Currently, only "greene10" (default) is supported, corresponding to the two-step approach of Greene (2010): a probit model is estimated for the selection equation, and its inverse Mills ratio is included as a correction term in the stochastic frontier second step.

groupType
: Character string. Type of frontier model estimated for each technology group. Three options are available:
  - "sfacross" (default): standard cross-sectional stochastic frontier analysis (sfacross). Groups are defined by the group variable. All 10 distributions for udist are supported, along with heteroscedasticity in both error components (uhet, vhet), heterogeneity in the truncated mean (muhet), and the scaling property.

- "sfaselectioncross": sample selection stochastic frontier analysis (sfaselectioncross). Corrects for sample selection bias via the generalised Heckman approach (Greene, 2010). Requires selectionF. Only observations for which the selection indicator equals one enter the frontier and metafrontier; efficiency estimates for non-selected observations are NA. Only udist = "hnormal" is supported.
- "sfalcmcross": latent class stochastic frontier analysis (sfalcmcross). Estimates a finite mixture of frontier models with the number of classes determined by lcmClasses. When group is supplied, a separate latent class model is estimated per group-stratum and combined for the metafrontier. When group is omitted, a single pooled model is estimated and class assignments serve as technology groups. Supports thet for class-membership covariates and uhet, vhet for within-class heteroscedasticity. Only udist = "hnormal" is supported.

metaMethod
: Character string. Method for estimating the global metafrontier that envelopes all group frontiers. Three options are available:
- "lp" (default): deterministic linear programming envelope. Finds the parameter vector $\boldsymbol{\beta}^*$ minimising $\sum_i |\ln \hat{f}(x_i, \boldsymbol{\beta}^*) - \ln \hat{f}(x_i, \hat{\boldsymbol{\beta}}_{(g)})|$ subject to $\ln \hat{f}(x_i, \boldsymbol{\beta}^*) \geq \ln \hat{f}(x_i, \hat{\boldsymbol{\beta}}_{(g)})$ for all observations and all groups (Battese *et al.*, 2004).
- "qp": deterministic quadratic programming envelope. Minimises the sum of squared deviations under the same envelope constraint.
- "sfa": stochastic metafrontier estimated by a second-stage pooled SFA. The specific construction of the dependent variable is determined by sfaApproach.

sfaApproach
: Character string. Applicable only when metaMethod = "sfa". Determines how the second-stage SFA is constructed:
- "huang" (default): the group-specific fitted frontier value $\ln \hat{y}_i^g$ for each observation is used as the dependent variable in a pooled cross-sectional SFA (Huang, Huang, and Liu, 2014). The technology gap $U_i \geq 0$ and second-stage noise V_i are estimated directly by the SFA procedure.
- "ordonnell": the column-wise maximum of all group-fitted frontier values (the deterministic LP envelope) is used as the dependent variable in the second-stage SFA (O'Donnell, Rao, and Battese, 2008).

selectionF
: A two-sided formula specifying the sample selection equation, e.g., selected ~ z1 + z2. The left-hand side must be a binary (0/1) indicator already present in data: 1 means the observation participates in the frontier and metafrontier; 0 means it is excluded (efficiency estimates will be NA). Alternatively, a named list of formulas, one per group level, may be supplied to allow group-specific selection equations. Required when groupType = "sfaselectioncross"; ignored otherwise.

lcmClasses
: Integer. Number of latent classes to be estimated per group when groupType = "sfalcmcross". Must be between 2 and 5 (default 2). The optimal number of classes can be selected based on information criteria (see ic).

whichStart
: Integer. Strategy for obtaining starting values in the latent class model (groupType = "sfalcmcross"):
- 1: starting values are obtained from the method of moments.
- 2 (default): the model is initialised by first solving a homoscedastic pooled cross-sectional SFA using the algorithm specified by initAlg for at most initIter iterations.

`initAlg` Character string. Optimisation algorithm used during the initialisation of the latent class model when `whichStart = 2`. Only algorithms from the `maxLik` package are supported:

- `"nm"` (default): Nelder-Mead (see `maxNM`).
- `"bfgs"`: Broyden-Fletcher-Goldfarb-Shanno (see `maxBFGS`).
- `"bhhh"`: Berndt-Hall-Hall-Hausman (see `maxBHHH`).
- `"nr"`: Newton-Raphson (see `maxNR`).
- `"cg"`: Conjugate Gradient (see `maxCG`).
- `"sann"`: Simulated Annealing (see `maxSANN`).

`initIter` Integer. Maximum number of iterations for the initialisation algorithm when `whichStart = 2` and `groupType = "sfalcmcross"`. Default `100`.

`lType` Character string. Specifies how the likelihood is evaluated for the selection model (`groupType = "sfaselectioncross"`). Five options are available:

- `"ghermite"` (default): Gauss-Hermite quadrature (see `gaussHermiteData`).
- `"kronrod"`: Gauss-Kronrod quadrature (see `integrate`).
- `"hcubature"`: adaptive integration over hypercubes (see `hcubature`).
- `"pcubature"`: p-adaptive cubature (see `pcubature`).
- `"msl"`: maximum simulated likelihood (controlled by `simType`, `Nsim`, `prime`, `burn`, `antithetics`, and `seed`).

`Nsub` Integer. Number of quadrature nodes or integration subdivisions when `lType` is `"ghermite"`, `"kronrod"`, `"hcubature"`, or `"pcubature"`. Applicable only when `groupType = "sfaselectioncross"`. Default `100`.

`uBound` Numeric. Upper bound for the numerical integration of the inefficiency component when `lType` is `"kronrod"`, `"hcubature"`, or `"pcubature"`. For Gauss-Hermite the bound is automatically infinite. Applicable only when `groupType = "sfaselectioncross"`. Default `Inf`.

`intol` Numeric. Integration tolerance for the quadrature approaches `"kronrod"`, `"hcubature"`, and `"pcubature"`. Applicable only when `groupType = "sfaselectioncross"`. Default `1e-6`.

`method` Character string. Optimisation algorithm for the main ML/MSL estimation of each group-level frontier model. Default `"bfgs"`. Eleven algorithms are available:

- `"bfgs"`: Broyden-Fletcher-Goldfarb-Shanno (see `maxBFGS`).
- `"bhhh"`: Berndt-Hall-Hall-Hausman (see `maxBHHH`).
- `"nr"`: Newton-Raphson (see `maxNR`).
- `"nm"`: Nelder-Mead (see `maxNM`).
- `"cg"`: Conjugate Gradient (see `maxCG`).
- `"sann"`: Simulated Annealing (see `maxSANN`).
- `"ucminf"`: quasi-Newton optimisation with BFGS updating of the inverse Hessian and soft line search (see `ucminf`).
- `"mla"`: Marquardt-Levenberg algorithm (see `mla`).
- `"sr1"`: Symmetric Rank 1 trust-region method (see `trust.optim`).
- `"sparse"`: trust-region method with sparse Hessian (see `trust.optim`).
- `"nlminb"`: PORT routines optimisation (see `nlminb`).

`hessianType` Integer. Determines how the Hessian is computed for the group-level models.

- `1` (default for `"sfacross"` and `"sfalcmcross"`): analytic Hessian.

- 2 (default for "sfaselectioncross"): BHHH approximation $\mathbf{G}'\mathbf{G}$, where $\mathbf{G}$ is the matrix of individual score contributions. This default applies to the selection model because estimation is conducted in two steps.

simType
: Character string. Simulation method for maximum simulated likelihood (MSL). Applicable to groupType = "sfacross" when udist is "gamma", "lognormal", or "weibull", and to groupType = "sfaselectioncross" when lType = "msl":
  - "halton" (default): Halton quasi-random sequences.
  - "ghalton": Generalised-Halton sequences.
  - "sobol": Sobol low-discrepancy sequences.
  - "uniform": pseudo-random uniform draws.

Nsim
: Integer. Number of simulation draws for MSL. Default 100.

prime
: Integer. Prime number used to construct Halton or Generalised-Halton sequences. Default 2.

burn
: Integer. Number of leading draws discarded from the Halton sequence to reduce serial correlation. Default 10.

antithetics
: Logical. If TRUE, antithetic draws are added: the first Nsim/2 draws are taken, and the remaining Nsim/2 are $1 -$ draw. Default FALSE.

seed
: Integer. Random seed for simulation draws, ensuring reproducibility of MSL estimates. Default 12345.

itermax
: Integer. Maximum number of iterations for the main optimisation. Default 2000. For method = "sann", it is recommended to increase this substantially (e.g., itermax = 20000).

printInfo
: Logical. If TRUE, optimisation progress is printed during estimation of each group-level model. Default FALSE.

tol
: Numeric. Convergence tolerance. The algorithm is considered converged when the change in the log-likelihood between successive iterations is smaller than tol in absolute value. Default 1e-12.

gradtol
: Numeric. Gradient convergence tolerance. The algorithm is considered converged when the Euclidean norm of the gradient is smaller than gradtol. Default 1e-6.

stepmax
: Numeric. Maximum step length used by the "ucminf" algorithm. Default 0.1.

qac
: Character string. Quadratic Approximation Correction for the "bhhh" and "nr" algorithms when the Hessian is not negative definite:
  - "marquardt" (default): step length is decreased while also shifting closer to the gradient direction.
  - "stephalving": step length is halved, preserving the current direction.

  See maxBHHH and maxNR for details.

...
: Additional arguments passed through to the second-stage SFA call when metaMethod = "sfa".

x
: An object of class "sfametafrontier", as returned by sfametafrontier, for use with the print method.

Details

Standard stochastic frontier (groupType = "sfacross")**:** The stochastic frontier model is defined as:

$$y_i = \alpha + \mathbf{x}_i'\boldsymbol{\beta} + v_i - Su_i$$

where y is the output (cost, revenue, or profit), $\mathbf{x}$ is the vector of frontier regressors, $u_i \geq 0$ is the one-sided inefficiency term with variance σ_u^2, and v_i is the symmetric noise term with variance σ_v^2.

Estimation is by ML for all distributions except "gamma", "lognormal", and "weibull", for which MSL is used with Halton, Generalised-Halton, Sobol, or uniform draws. Antithetic draws are available for the uniform case.

To account for heteroscedasticity, the variances are modelled as $\sigma_u^2 = \exp(\boldsymbol{\delta}'\mathbf{Z}_u)$ and $\sigma_v^2 = \exp(\boldsymbol{\phi}'\mathbf{Z}_v)$. For the truncated normal distribution, heterogeneity in the pre-truncation mean is modelled as $\mu_i = \boldsymbol{\omega}'\mathbf{Z}_\mu$. The scaling property (Wang and Schmidt, 2002) can also be imposed for the truncated normal.

Sample selection frontier (`groupType = "sfaselectioncross"`): This model extends the Heckman (1979) selection framework to the stochastic frontier setting (Greene, 2010; Dakpo *et al.*, 2021). The selection and frontier equations are:

$$y_{1i}^* = \mathbf{Z}_{si}'\boldsymbol{\gamma} + w_i, \quad w_i \sim \mathcal{N}(0,1)$$

$$y_{2i}^* = \mathbf{x}_i'\boldsymbol{\beta} + v_i - Su_i$$

where $y_{1i} = \mathbf{1}(y_{1i}^* > 0)$ is the binary selection indicator and $y_{2i} = y_{2i}^*$ is observed only when $y_{1i} = 1$. Selection bias arises from $\rho = \mathrm{Corr}(w_i, v_i) \neq 0$. Only selected observations enter the frontier and metafrontier estimation; efficiency estimates for non-selected observations are `NA`.

Latent class frontier (`groupType = "sfalcmcross"`): The latent class model (Orea and Kumbhakar, 2004) estimates a finite mixture of J frontier models:

$$y_i = \alpha_j + \mathbf{x}_i'\boldsymbol{\beta}_j + v_{i|j} - Su_{i|j}$$

The prior class probability follows a logit specification:

$$\pi(i,j) = \frac{\exp(\boldsymbol{\theta}_j'\mathbf{Z}_{hi})}{\sum_{m=1}^{J}\exp(\boldsymbol{\theta}_m'\mathbf{Z}_{hi})}$$

Class assignment is based on the maximum posterior probability computed via Bayes' rule. When `group` is omitted, a single pooled model is estimated and class assignments serve as technology groups.

Metafrontier estimation: The global metafrontier $f(x_i, \boldsymbol{\beta}^*)$ envelopes all group frontiers. With LP (Battese *et al.*, 2004), $\boldsymbol{\beta}^*$ minimises $\sum_i |\ln \hat{f}(x_i, \boldsymbol{\beta}^*) - \ln \hat{f}(x_i, \hat{\boldsymbol{\beta}}_{(g)})|$ subject to $\ln \hat{f}(x_i, \boldsymbol{\beta}^*) \geq \ln \hat{f}(x_i, \hat{\boldsymbol{\beta}}_{(g)})$. QP minimises the squared analogue. The stochastic approaches (Huang *et al.*, 2014; O'Donnell *et al.*, 2008) treat the technology gap U_i as a one-sided error in a second-stage SFA. Group and metafrontier efficiencies are:

$$TE_i^g = \exp(-u_i^g), \quad MTR_i = \exp(-U_i), \quad TE_i^* = TE_i^g \times MTR_i$$

Both Jondrow *et al.* (1982) and Battese and Coelli (1988) estimators are provided for each measure. See `efficiencies` for details.

Value

`sfametafrontier` returns an object of class `"sfametafrontier"`, which is a list containing:

`call` The matched call.

`groupModels` A named list of fitted group-level frontier objects, one per technology group. Each element is of class `"sfacross"`, `"sfaselectioncross"`, or `"sfalcmcross"`, depending on `groupType`.

metaSfaObj	The fitted metafrontier object. For metaMethod = "sfa", an object of class "sfacross" from the second-stage SFA. For metaMethod = "lp" or "qp", a list containing the optimisation result and the estimated envelope coefficients.
metaRes	Estimated metafrontier coefficients (with standard errors, z-values, and p-values for metaMethod = "sfa", or the plain coefficient vector for deterministic envelopes).
formula	The formula supplied to the call.
metaMethod	The metafrontier estimation method used.
sfaApproach	The second-stage SFA approach; NA when metaMethod is not "sfa".
groupType	The type of group-level frontier model estimated.
group	The name of the grouping variable.
groups	Character vector of unique group labels.
S	The frontier orientation (1 or -1).
dataTable	The data used in estimation, augmented with .mf_yhat_group (group-specific fitted frontier values) and .mf_yhat_meta (metafrontier fitted values).
lcmNoGroup	Logical. TRUE when groupType = "sfalcmcross" and group was not supplied.
lcmObj	When lcmNoGroup = TRUE, the pooled sfalcmcross object.

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See Also

sfacross, sfaselectioncross, sfalcmcross, efficiencies, summary.sfametafrontier, ic

Examples

```
## Not run:
############################################################################
## -------- SECTION 1: Standard SFA Group Frontier ----------------------##
## Using the rice production dataset (ricephil) from Battese et al.      ##
## Groups are formed based on farm area terciles (small/medium/large).   ##
############################################################################

data("ricephil", package = "sfaR")
ricephil$group <- cut(ricephil$AREA,
  breaks = quantile(ricephil$AREA, probs = c(0, 1/3, 2/3, 1), na.rm = TRUE),
  labels = c("small", "medium", "large"),
  include.lowest = TRUE
)
```

```
## 1a. sfacross groups + LP metafrontier
##     Deterministic envelope via linear programming (Battese et al., 2004).
meta_sfacross_lp <- sfametafrontier(
  formula    = log(PROD) ~ log(AREA) + log(LABOR) + log(NPK),
  data       = ricephil,
  group      = "group",
  S          = 1,
  udist      = "hnormal",
  groupType  = "sfacross",
  metaMethod = "lp"
)
summary(meta_sfacross_lp)
# Retrieve individual efficiency and metatechnology ratio estimates:
ef_lp <- efficiencies(meta_sfacross_lp)
head(ef_lp)

## 1b. sfacross groups + QP metafrontier
##     Deterministic envelope via quadratic programming.
meta_sfacross_qp <- sfametafrontier(
  formula    = log(PROD) ~ log(AREA) + log(LABOR) + log(NPK),
  data       = ricephil,
  group      = "group",
  S          = 1,
  udist      = "hnormal",
  groupType  = "sfacross",
  metaMethod = "qp"
)
summary(meta_sfacross_qp)

## 1c. sfacross groups + Two-stage SFA metafrontier (Huang et al., 2014)
##     The group-specific fitted frontier values serve as the dependent
##     variable in the second-stage SFA, yielding a stochastic technology gap.
meta_sfacross_huang <- sfametafrontier(
  formula     = log(PROD) ~ log(AREA) + log(LABOR) + log(NPK),
  data        = ricephil,
  group       = "group",
  S           = 1,
  udist       = "hnormal",
  groupType   = "sfacross",
  metaMethod  = "sfa",
  sfaApproach = "huang"
)
summary(meta_sfacross_huang)
ef_huang <- efficiencies(meta_sfacross_huang)

## 1d. sfacross groups + O'Donnell et al. (2008) stochastic metafrontier
##     The LP deterministic envelope is used as the second-stage dependent
##     variable: the metafrontier is estimated stochastically around the
##     envelope.
meta_sfacross_odonnell <- sfametafrontier(
  formula     = log(PROD) ~ log(AREA) + log(LABOR) + log(NPK),
  data        = ricephil,
  group       = "group",
  S           = 1,
  udist       = "hnormal",
  groupType   = "sfacross",
```

```
  metaMethod  = "sfa",
  sfaApproach = "ordonnell"
)
summary(meta_sfacross_odonnell)

###########################################################################
## -------- SECTION 2: Latent Class (LCM) Group Frontier ---------------##
## No observed group variable: a pooled sfalcmcross model assigns       ##
## observations to 2 latent technology classes; these classes become the ##
## technology groups for the metafrontier.                               ##
###########################################################################

data("utility", package = "sfaR")

## 2a. sfalcmcross (pooled, 2 classes) + LP metafrontier
meta_lcm_lp <- sfametafrontier(
  formula    = log(tc/wf) ~ log(y) + log(wl/wf) + log(wk/wf),
  data       = utility,
  S          = -1,
  groupType  = "sfalcmcross",
  lcmClasses = 2,
  metaMethod = "lp"
)
summary(meta_lcm_lp)
ef_lcm_lp <- efficiencies(meta_lcm_lp)
# Per-class posterior probabilities and class-specific efficiencies are
# included alongside group and metafrontier efficiencies.

## 2b. sfalcmcross (pooled, 2 classes) + QP metafrontier
meta_lcm_qp <- sfametafrontier(
  formula    = log(tc/wf) ~ log(y) + log(wl/wf) + log(wk/wf),
  data       = utility,
  S          = -1,
  groupType  = "sfalcmcross",
  lcmClasses = 2,
  metaMethod = "qp"
)
summary(meta_lcm_qp)

## 2c. sfalcmcross (pooled, 2 classes) + Two-stage SFA metafrontier
##     (Huang et al., 2014)
meta_lcm_huang <- sfametafrontier(
  formula     = log(tc/wf) ~ log(y) + log(wl/wf) + log(wk/wf),
  data        = utility,
  S           = -1,
  groupType   = "sfalcmcross",
  lcmClasses  = 2,
  metaMethod  = "sfa",
  sfaApproach = "huang"
)
summary(meta_lcm_huang)
ef_lcm_huang <- efficiencies(meta_lcm_huang)

## 2d. sfalcmcross (pooled, 2 classes) + O'Donnell et al. (2008)
meta_lcm_odonnell <- sfametafrontier(
  formula     = log(tc/wf) ~ log(y) + log(wl/wf) + log(wk/wf),
  data        = utility,
```

```
  S           = -1,
  groupType   = "sfalcmcross",
  lcmClasses  = 2,
  metaMethod  = "sfa",
  sfaApproach = "ordonnell"
)
summary(meta_lcm_odonnell)

###########################################################################
## -------- SECTION 3: Sample Selection SFA Group Frontier -------------##
## Simulated dataset with a Heckman selection mechanism. Only selected  ##
## observations (d == 1) participate in the frontier and metafrontier.  ##
## Efficiency estimates for non-selected observations are NA.           ##
###########################################################################

N <- 2000; set.seed(12345)
z1 <- rnorm(N); z2 <- rnorm(N)
v1 <- rnorm(N); v2 <- rnorm(N)
g  <- rnorm(N)
e1 <- v1
e2 <- 0.7071 * (v1 + v2)
ds <- z1 + z2 + e1
d  <- ifelse(ds > 0, 1, 0)        # binary selection indicator
group <- ifelse(g > 0, 1, 0)      # two technology groups (0 and 1)
u  <- abs(rnorm(N))
x1 <- rnorm(N); x2 <- rnorm(N)
y  <- x1 + x2 + e2 - u
dat <- as.data.frame(cbind(y=y, x1=x1, x2=x2, z1=z1, z2=z2, d=d, group=group))

## 3a. sfaselectioncross + LP metafrontier
##     Selection bias is corrected via the Greene (2010) two-step probit
##     approach. The LP envelope envelopes both groups' selected-sample
##     frontier fitted values.
meta_sel_lp <- sfametafrontier(
  formula    = y ~ x1 + x2,
  selectionF = d ~ z1 + z2,
  data       = dat,
  group      = "group",
  S          = 1L,
  udist      = "hnormal",
  groupType  = "sfaselectioncross",
  modelType  = "greene10",
  lType      = "kronrod",
  Nsub       = 100,
  uBound     = Inf,
  method     = "bfgs",
  itermax    = 2000,
  metaMethod = "lp"
)
summary(meta_sel_lp)
ef_sel_lp <- efficiencies(meta_sel_lp)

## 3b. sfaselectioncross + QP metafrontier
meta_sel_qp <- sfametafrontier(
  formula    = y ~ x1 + x2,
  selectionF = d ~ z1 + z2,
  data       = dat,
```

```
  group      = "group",
  S          = 1L,
  udist      = "hnormal",
  groupType  = "sfaselectioncross",
  modelType  = "greene10",
  lType      = "kronrod",
  Nsub       = 100,
  uBound     = Inf,
  method     = "bfgs",
  itermax    = 2000,
  metaMethod = "qp"
)
summary(meta_sel_qp)

## 3c. sfaselectioncross + Two-stage SFA metafrontier (Huang et al., 2014)
meta_sel_huang <- sfametafrontier(
  formula     = y ~ x1 + x2,
  selectionF  = d ~ z1 + z2,
  data        = dat,
  group       = "group",
  S           = 1L,
  udist       = "hnormal",
  groupType   = "sfaselectioncross",
  modelType   = "greene10",
  lType       = "kronrod",
  Nsub        = 100,
  uBound      = Inf,
  simType     = "halton",
  Nsim        = 300,
  prime       = 2L,
  burn        = 10,
  antithetics = FALSE,
  seed        = 12345,
  method      = "bfgs",
  itermax     = 2000,
  metaMethod  = "sfa",
  sfaApproach = "huang"
)
summary(meta_sel_huang)
ef_sel_huang <- efficiencies(meta_sel_huang)

## 3d. sfaselectioncross + O'Donnell et al. (2008) stochastic metafrontier
meta_sel_odonnell <- sfametafrontier(
  formula     = y ~ x1 + x2,
  selectionF  = d ~ z1 + z2,
  data        = dat,
  group       = "group",
  S           = 1L,
  udist       = "hnormal",
  groupType   = "sfaselectioncross",
  modelType   = "greene10",
  lType       = "kronrod",
  Nsub        = 100,
  uBound      = Inf,
  method      = "bfgs",
  itermax     = 2000,
  metaMethod  = "sfa",
```

```
  sfaApproach = "ordonnell"
)
summary(meta_sel_odonnell)

## End(Not run)
```

`summary` *Summary of results for stochastic metafrontier models*

Description

Create and print summary results for stochastic metafrontier models returned by `sfametafrontier`.

Usage

```
## S3 method for class 'sfametafrontier'
summary(object, ...)

## S3 method for class 'summary.sfametafrontier'
print(x, digits = max(3, getOption("digits") - 2), ...)
```

Arguments

`object`	An object of class `'sfametafrontier'` returned by the function `sfametafrontier`.
`...`	Currently ignored.
`x`	An object of class `'summary.sfametafrontier'`.
`digits`	Numeric. Number of digits displayed in values.

Value

The `summary` method returns a list of class `'summary.sfametafrontier'` that contains the same elements as an object returned by `sfametafrontier` with the following additional elements:

`AIC`	Akaike information criterion.
`BIC`	Bayesian information criterion.
`HQIC`	Hannan-Quinn information criterion.
`metaRes`	Matrix of metafrontier estimates, their standard errors, z-values, and asymptotic P-values.
`effStats`	A list of efficiency statistics including group means and class membership probabilities.
`grpSummaries`	A list of summary objects for each group model.

See Also

`sfametafrontier`, for the stochastic metafrontier analysis model fitting function for cross-sectional or pooled data.

vcov *Compute variance-covariance matrix of stochastic metafrontier models*

Description

vcov computes the variance-covariance matrix of the maximum likelihood (ML) coefficients from stochastic metafrontier models estimated with sfametafrontier.

Usage

```
## S3 method for class 'sfametafrontier'
vcov(object, ...)
```

Arguments

object	A stochastic metafrontier model returned by sfametafrontier.
...	Currently ignored

Details

The variance-covariance matrix is obtained by the inversion of the negative Hessian matrix. Depending on the distribution and the 'hessianType' option, the analytical/numeric Hessian or the bhhh Hessian is evaluated.

Value

The variance-covariance matrix of the maximum likelihood coefficients is returned.

See Also

sfametafrontier, for the stochastic metafrontier analysis model fitting function using cross-sectional or pooled data.

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