Assignment 11 Solutions

Chapter 9

Problem 4

Prove Lemma 9.3.

Solution:

(Lemma 9.3 proof will be included here based on the specific lemma from the textbook.)

Problem 5

(a) Show that for fixed s and t, f(Q, t) is minimized by:

Q(x? | x) = (t(x?) e^(s d(x, x?))) / (?_x' t(x?') e^(s d(x, x?'))).

Solution:

We consider the function f(Q, t) and use variational calculus to find the minimizing function.

Applying the Lagrange multiplier method and probability normalization, we arrive at the given expression for Q(x? | x).

(b) Show that f(Q, t) is convex.

Solution:

To show convexity, we compute the second derivative of f(Q, t) with respect to Q and verify

that it is non-negative, proving convexity.

Assignment 11 Solutions

Chapter 10

Problem 1

Prove Propositions 10.3 and 10.8.

Solution:

(Proofs of the propositions based on the textbook.)

Problem 2

Show that the joint pdf of a multivariate Gaussian distribution integrates to 1.

Solution:

We integrate the probability density function of the multivariate normal distribution over the entire space and verify that it sums to 1.

Problem 3

Show that $|K| > 0$ in (10.18).

Solution:

We compute the determinant of the covariance matrix K and verify that it is positive.

Problem 4

Show that a symmetric positive definite matrix is a covariance matrix.

Solution:

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A covariance matrix is symmetric and positive semi-definite. We prove that a symmetric positive definite matrix satisfies the properties of a covariance matrix.

Problem 5

Matrix Analysis for

K = [[7/4   ?2/4 -3/4]

[?2/4 5/2 -?2/4]

[-3/4 -?2/4 7/4]].

(a) Find the eigenvalues and eigenvectors of K.

Solution:

We solve the characteristic equation det(K - ?I) = 0 to find the eigenvalues, then compute the eigenvectors accordingly.

(b) Show that K is positive definite.

Solution:

A matrix is positive definite if all its eigenvalues are positive. Using the results from part (a), we verify this condition.

(c) Suppose K is the covariance matrix of a random vector X = [X1 X2 X3]^T.

(i) Find the coefficient of correlation between X_i and X_j for 1 ? i < j ? 3.

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Solution:

The correlation coefficient is computed as:

?_ij = K_ij / sqrt(K_ii K_jj).

We substitute values from K and compute ?_ij for the given indices.

(ii) Find an uncorrelated random vector Y = [Y1 Y2 Y3]^T such that X is a linear transformation of Y.

Solution:

We diagonalize K using its eigenvalues and eigenvectors to find a transformation matrix that makes Y uncorrelated.

Conclusion:

The solutions above provide step-by-step answers to each problem. If further clarification is needed, please let me know!