Solutions to Assignment 10

Chapter 8

Problem 4: Rate-Distortion Function for a Product Source

Given two independent random variables X and Y with rate-distortion functions $R_X(D_X)$ and $R_Y(D_Y)$, respectively, and a combined distortion measure:

$$d((x,y), (\hat{x}, \hat{y})) = d_X(x, \hat{x}) + d_Y(y, \hat{y}),$$

we prove that the rate-distortion function for (X, Y) satisfies:

$$R(D) = \min_{D_X + D_Y = D} (R_X(D_X) + R_Y(D_Y)).$$

Solution:

1. Using Shannon's rate-distortion theorem, we analyze mutual information expressions and confirm the separability of independent sources under the given distortion metric.
2. Applying convexity properties of R(D), we derive the minimization structure.

Problem 6: Asymptotic Optimality in Rate-Distortion and Channel Coding

Solution:

- We show that for an i.i.d. information source, rate-distortion coding followed by channel coding remains asymptotically optimal.
- We utilize the single-letter distortion measure and memoryless channel assumption to derive optimality conditions.

Chapter 9

Problem 3: Initial Input Distribution in the BA Algorithm

Solution:

- The BA Algorithm iteratively refines input distributions for computing channel capacity.
- We explain why an initial distribution with zero probability masses can cause convergence issues, leading to incorrect capacity computations.