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Symbolic and Evolutionary Artificial Intelligence

Sultan Mahmud

Supervised by:

Prof. Marco Cococcioni

Federico Rossi

Lorenzo Fiaschi

Contents

1. Introduction

In the evolving landscape of machine learning, efficiency in computations and optimal resource utilisation are crucial, especially as models and datasets grow in complexity. Posit number formats, known for their variable precision and efficiency, offer a promising alternative to traditional floating-point formats. These advantages make posits particularly appealing for use during the inference phase of machine learning algorithms. This project specifically investigates the use of posit number formats in the inference phase of non-linear Support Vector Machines (SVMs) after training these models with floating-point numbers. The primary objective is to examine the impact of precision loss when converting learned parameters from floats to posits and to explore strategies to mitigate this degradation.

Background Information on SVMs and Number Formats (Floats and Posits)

Support Vector Machines (SVMs) are robust supervised learning algorithms widely used for classification and regression tasks. SVMs are particularly effective in high-dimensional spaces and can be adapted for various data distributions using different kernel functions, enabling them to model non-linear relationships effectively.

Floating-point arithmetic, commonly used in training SVMs, provides uniform precision across a broad range of values but can introduce inefficiencies and rounding errors under certain conditions. Posits, an alternative number format, offer tapered precision—greater accuracy over critical ranges and less elsewhere—making them potentially more suitable for specific computational needs within machine learning.

Statement of the Problem Regarding Precision Loss During Format Conversion from Floats to Posits

Transitioning SVM models trained with floating-point numbers to use posits for inference involves converting model parameters, such as weights and biases, from floats to posits. This conversion can lead to precision loss, particularly pronounced in regions where posits exhibit lower density compared to floats. Such precision loss can alter the model's performance, potentially degrading the accuracy and reliability of its predictions in practical applications.

Objectives and Scope of Your Project

- Assess the extent of performance degradation in non-linear SVMs caused by precision loss during the conversion of model parameters from float to posit formats.
- Investigate the relationship between various float and posit format configurations and their impact on the inference accuracy of non-linear SVMs.
- Develop and evaluate training strategies that incorporate posit constraints aimed at reducing performance losses during inference with posits.

The scope of this project will focus exclusively on non-linear SVMs, utilising different kernel functions and a range of datasets to analyse the effects comprehensively. By concentrating on non-linear models, this investigation aims to highlight and address the specific challenges posed by using posits in scenarios where the model complexity and the non-linearity of data distributions are significant. This focused approach intends to yield insights into tailored training modifications that can effectively counteract the impacts of precision loss encountered during posit-based inference.

2. Previous Studies

Impact of Bounding Dual Solutions in Non-linear SVMs with Posit Constraints
Introduction to Non-linear SVMs with Posit Constraints

Support Vector Machines (SVMs), particularly in their non-linear form, are widely recognized for their ability to model complex relationships within datasets using kernel functions. In the adaptation of SVMs to use posit number formats, especially during inference, the conversion of model parameters from traditional floating-point representations introduces unique challenges. Precision loss during this conversion can significantly impact the model's accuracy due to the misrepresentation of support vectors—key elements that define the SVM's decision boundary.

Bounding Dual Solutions: A Strategic Approach

To address the challenges posed by precision loss in posits, bounding the dual variables within the SVM's optimization formulation has emerged as a promising strategy. This approach constrains the values of the dual variables (μ, η) to ensure that the support vectors maintain their integrity even after the conversion from floats to posits.

Mathematical Formulation and Implementation

$$\underset{\mu,\eta}{\text{maximize}} \quad -\frac{1}{2}\sum_{i=1}^{L}\sum_{i=1}^{L} y^i y^j (x^i)^T x^j (\mu_i - \eta_i)(\mu_j - \eta_j) + \sum_{i=1}^{L}(\mu_i - \eta_i)$$

$$\text{subject to} \quad \sum_{i=1}^{L}(\mu_i - \eta_i)y^i = 0,$$

$$0 \leq \mu_i - \eta_i \leq C, \quad i = 1,\ldots,L,$$

$$l \leq \mu_i \leq h, \quad i = 1,\ldots,L,$$

$$l \leq \eta_i \leq h, \quad i = 1,\ldots,L$$

$$w^* = \sum_{i=1}^{L} \lambda_i^* y_i x_i$$

$$b^* = \frac{1}{y_i} - (w^*)^T x_i$$

The optimization problem focuses on maximizing the margin between the separating hyperplane and the support vectors, subject to constraints that bound the dual variables. The objective and constraints are defined as:

Using MATLAB's Optimization Toolbox, this bounded dual problem was implemented using quadratic programming. The constraints were tailored to accommodate the specific representational characteristics of posits, focusing on minimizing the loss of critical support vector information during format conversion.

Dataset and Kernel Choice

The Wisconsin Breast Cancer Dataset (WDBC), consisting of detailed features of breast mass cell nuclei, was selected for the experimental evaluation. A Gaussian kernel was utilized due to its universal kernel properties, appropriate for handling the non-linear separations typical in medical diagnostic data.

Detailed Findings from Scenario-Based Evaluations

The evaluation involved a series of scenarios designed to rigorously test the effectiveness of bounding dual variables under non-linear SVM configurations with posit constraints:

Baseline : The model operated without any constraints using traditional floating-point formats, establishing a performance benchmark.
Constrained Model : This scenario introduced the posit constraints at inference while maintaining higher precision types during training. This hybrid approach was necessary to manage the demanding computations of the Gaussian kernel without sacrificing the precision needed for effective learning.
The results indicated a marked improvement in the model's performance when dual variable constraints were applied. Specifically, the constrained model demonstrated

enhanced stability and reliability in its predictions compared to the baseline. These improvements were particularly pronounced in tests where the Gaussian kernel's sensitivity to data representation played a significant role in the decision-making process.

WDBC Summary (C = 1, γ = 0.001)						
Constraints	Normalization	UB	Posit Variable	Smart Computation	Inference	Accuracy
Without	[-1, 1]	X	X	No	Double	0.89
Without	[-useed, useed]	X	X	No	Double	0.78
Without	[-1, 1]	X	Dual	Mixed Precision (Float)	Posit	0.83
Without	[-useed, useed]	X	Dual	Mixed Precision (Float)	Posit	0.41
With	[-1, 1]	1.2	Dual	Mixed Precision (Float)	Posit	0.84
With	[-useed, useed]	1.2	Dual	Mixed Precision (Float)	Posit	0.60

The study underscores the potential of bounding dual variables in non-linear SVMs to mitigate adverse effects caused by precision loss during the transition from floating-point to posit number formats. This approach not only preserves the integrity of support vectors but also enhances the overall robustness of the model against variations in data representability. Future research should focus on refining these bounding techniques, exploring dynamic adjustment strategies that cater to varying dataset characteristics and kernel functions, thereby broadening the applicability and effectiveness of non-linear SVMs in real-world applications.

3. Training and Testing Strategy

The training and testing strategies implemented for this study are designed to investigate the efficacy and efficiency of using Posit number systems in machine learning, specifically within the context of Support Vector Machines (SVMs) applied to non-linear problems. A distinctive feature of this study is the use of mixed precision

levels during inference phases to optimize computational resources while maintaining high accuracy levels.

With γ = 0.01, the influence of each training point is more localized, creating a more complex and potentially more accurate decision boundary. This higher precision in capturing the relationship between data points improves accuracy.

Training Strategy

The SVM models were trained using MATLAB using a data normalization range of [−1,+1] to ensure uniform scaling across features, a platform renowned for its robust numerical computing capabilities. The training process incorporated Posit constraints to adapt the SVM algorithm to the characteristics of Posit arithmetic. This training approach was aimed at:

Customizing the SVM Model: The SVM was tailored to leverage the dynamic range and precision offered by different Posit configurations, thereby ensuring that the model is inherently compatible with Posit-based computations.

Optimizing Model Parameters: Through MATLAB's optimization toolbox, the hyperparameters of the SVM, including the regularization parameter C and kernel parameters, were adjusted to find the optimal balance between model complexity and generalization ability.

Testing Strategy

After training, the optimal dual variables were quantized into Posit formats, specifically using 8, 16, and 32-bit Posits with varying exponent bits. This step was crucial for evaluating the performance of Posits in representing model parameters that directly influence the decision boundary. For inference, a mixed precision approach was adopted, where the optimal dual variables were represented using Posits, and the decision function was computed using higher precision types:

Mixed Precision Inference: Inferences were performed using Posits for representing the dataset and the optimal dual variables derived from the training phase. However, to enhance precision in critical computational tasks, higher precision types (such as Double and Float) were used for calculating the decision function.

Utilization of Gaussian Kernel: The decision to use higher precision types for the Gaussian kernel computation was driven by the need to handle exponential functions accurately. Posits, while efficient for general arithmetic, can introduce significant errors in exponential calculations due to their limited precision and range in smaller bit configurations.

4. Experiments

In this section of the report, we delve into the practical application of the training and testing strategies previously outlined, by conducting a series of experiments designed to evaluate the effectiveness and robustness of the non-linear SVM model when operating under mixed precision settings. The experiments are structured to test various configurations of Posit arithmetic across different scenarios, highlighting how these configurations influence the model's accuracy and computational efficiency.

Experiment 1:

Dynamic Range Adjustment in SVM with Posit Constraints

This experiment aims to assess the performance of a non-linear Support Vector Machine (SVM) when employing dynamic range adjustments for Posit constraints. By adapting the bounds based on the maximum kernel values, the experiment investigates whether tailored bounds enhance the SVM's ability to accurately model complex decision boundaries.

Kernel matrix K was computed for all pairs in the training set, using the Gaussian function to measure similarity. The maximum value of each row in the kernel matrix (K_max) determines the lower (lb) and upper (ub) bounds for the dual variables of the corresponding data point, aiming to customize the SVM's slackness adaptively based on the data's characteristics.

The dynamic adjustment of bounds based on the Gaussian kernel values aimed to ensure that each support vector's influence is appropriately scaled, potentially improving the model's sensitivity to nuances in the data.

- For each row in K, find the maximum value Kmax[i].
- Set the bounds for the optimization variables λ based on Kmax:lb[i] = low × Kmax:[i] and ub[i] = Kmax[i]

WDBC Summary (C = 1, γ = 0.01)					
Constraints	Normalization	Posit Variable	Smart Computation	Inference	Accuracy
With	[-1, 1]	Dual	-	Double	73.10
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit8	95.322

With	[-1, 1]	Dual	Mixed Precision (Float)	Posit16	95.322
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit32	95.322

A higher gamma value creates a more localized decision boundary, making the model sensitive to individual training points. This can lead to better capture of complex patterns in the data during testing.

By adjusting the bounds based on K_max, the model dynamically scales the influence of support vectors. This scaling can help in more accurately modeling the decision boundary, especially when the kernel values vary significantly. The dynamic range adjustment helped mitigate the effects of quantization errors by scaling the bounds adaptively.

The adaptive bounds ensure that the support vectors' contributions are appropriately scaled, enhancing the model's sensitivity to nuances in the data.

Experiment 2:

Worst-case Bounds Adjustment in SVM with Posit Constraints

Experiment Two explores the performance impact of applying a worst-case bounds adjustment strategy on a non-linear SVM. This method involves setting the regularization constraints based on the maximum value of the Gaussian kernel across all entries. The aim is to evaluate whether such a conservative approach can stabilize the model's behavior in extreme cases, potentially enhancing its robustness.

In worst-case adjustment, the maximum kernel value is computed across all entries in the kernel matrix, considering the maximum value globally. The initial lower and upper bounds are multiplied by this global maximum kernel value. This approach assumes a worst-case scenario where the maximum kernel value across all entries represents the upper bound for all training samples, potentially leading to more conservative bounds.

- Identify the maximum kernel value across all kernel entries K_max = max(K(:));
- Use Kmax to set uniform lower and upper bounds for all dual variables λ,
      lb = low × Kmax × ones(2 × I, 1)
      ub = high × Kmax × ones(2 × I, 1)

WDBC Summary (C = 1, γ = 0.01)					
Constraints	Normalization	Posit Variable	Smart Computation	Inference	Accuracy
With	[-1, 1]	Dual	-	Double	95.322
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit8	95.321
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit16	95.321
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit32	95.321

The focus is on mitigating the effects of high-magnitude kernel values that could potentially distort the decision function of the SVM. The worst-case bounds adjustment strategy sets the bounds for the dual variables in the SVM optimization process based on the global maximum value of the Gaussian kernel matrix.

The worst-case bounds adjustment ensures that the SVM remains robust even when faced with extreme kernel values. By setting bounds based on the maximum kernel value, the model is less likely to be adversely affected by outliers or high-magnitude values in the kernel matrix. Preventing any single data point from disproportionately influencing the optimization process.

The consistency in accuracy across different posit precisions (Posit8, Posit16, Posit32) indicates that the worst-case bounds adjustment effectively mitigates the precision issues typically associated with posit formats.

The experiment shows minimal performance degradation due to precision loss when using worst-case bounds, fulfilling the objective of assessing the impact of float-to-posit conversion. The consistent performance across Posit8, Posit16, and Posit32 suggests that the chosen configuration effectively mitigates precision issues.

Experiment 3:

Density-Sensitive Regularization in SVM with Posit Constraints

The primary goal of Experiment three is to investigate the impact of adjusting the regularization parameter C based on the median value of the kernel matrix in a non-linear SVM. This experiment tests whether such density-sensitive regularization can enhance the model's performance by better adapting to variations in data density within the feature space.

- C is set as 1/K median, intending to tailor the SVM's sensitivity to the central density of the kernel values.
- Scaling the lower and upper bounds for the dual variables based on the median value,
  lb = low × K_median × ones(2 × l, 1)
  ub = high × K_median × ones(2 × l, 1),

WDBC Summary (C = 1/K median, γ = 0.01)					
Constraints	Normalization	Posit Variable	Smart Computation	Inference	Accuracy
With	[-1, 1]	Dual	-	Double	95.322
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit8	93.567
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit16	95.321
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit32	95.321

By setting the bounds based on the median kernel value, the SVM model becomes more sensitive to the central density of the data. This allows the model to better adapt to regions where the data is more densely packed, potentially improving its ability to capture subtle patterns in the data.

The slight drop in accuracy for Posit8 highlights the impact of precision loss when converting from floats to posits. Posit8 has a smaller dynamic range and fewer bits to represent values, which can lead to greater quantization errors, particularly in regions where the kernel values vary significantly.
Posit16 and Posit32, with their higher precision, are better able to represent the kernel values accurately, resulting in minimal performance degradation compared to the float-based model.

The experiment confirms that precision loss in lower-bit posit formats (Posit8) can lead to noticeable performance degradation, fulfilling the objective of assessing the impact of float-to-posit conversion. The results demonstrate that higher precision posit formats (Posit16, Posit32) can maintain model accuracy, while lower precision formats may suffer from accuracy loss.

Experiment 4:

Quantile-based Thresholding

In this approach, the 95th percentile of the kernel matrix K is computed using the quantile function. The 95th percentile represents a threshold value below which 95% of the kernel values fall. It indicates a high similarity threshold in the feature space. The regularization parameter C is then set to the reciprocal of this quantile value. This essentially means that a smaller value of C will be used when the kernel values are higher, implying a high degree of similarity between data points.

Quantile-based Thresholding adjusts the regularization parameter C based on the distribution of kernel values, specifically targeting the high-similarity regions of the feature space.

- quantile(K(:), 0.95) reshapes the kernel matrix into a vector and finds the value below which 95% of the kernel data falls.

- C = 1 / K_quantile, Setting C inversely to K_quantile, targeting the upper range of typical interactions.

- Scaling the lower and upper bounds for the dual variables based on the quantile value,
  lb = low × K_quantile × ones(2 × l, 1)
  ub = high × K_quantile × ones(2 × l, 1),

WDBC Summary (C = 1/ K quantile, γ = 0.01)					
Constraints	Normalization	Posit Variable	Smart Computation	Inference	Accuracy
With	[-1, 1]	Dual	-	Double	95.322
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit8	93.567
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit16	95.321
With	[-1, 1]	Dual	Mixed Precision (Float)	Posit32	95.321

By setting C based on the 95th percentile of the kernel values, the SVM model places greater emphasis on the high-similarity regions of the feature space. This can

potentially enhance the model's sensitivity to data points that are closely related, improving its ability to capture important patterns.

The quantile-based thresholding focuses on high-similarity regions (top 5% of kernel values). Representing these high-precision areas accurately is crucial for maintaining model performance. Posit8 may not capture the nuances in these regions as effectively as higher precision formats.

5. Key Findings

The exploration of using posit number formats for inference in non-linear Support Vector Machines (SVMs) has yielded valuable insights into the potential benefits and challenges associated with this approach. Through a series of carefully designed experiments, this study has highlighted the nuanced effects of precision loss when converting model parameters from floating-point representations to posits. Here, we summarize the key findings and propose strategies to mitigate precision loss, enhancing the reliability and accuracy of SVMs using posits during inference.

- Impact of Precision Loss: The experiments consistently showed that lower precision posit formats, such as Posit8, resulted in noticeable performance degradation compared to higher precision formats (Posit16 and Posit32) and traditional floating-point representations. This was particularly evident in scenarios involving complex kernel functions and higher gamma values, where precision loss could significantly distort the decision boundary of the SVM.

- Effectiveness of Bounding Techniques: Bounding dual variables based on dynamic adjustments (e.g., maximum, median, and quantile values of the kernel matrix) proved to be effective strategies. These approaches helped maintain model stability and accuracy by scaling the influence of support vectors according to data characteristics, thereby mitigating the adverse effects of precision loss.

- Mixed Precision Inference: Utilizing a mixed precision approach, where posit formats were used for representing dual variables and higher precision types (float or double) for critical computations (e.g., Gaussian kernel), demonstrated a balanced trade-off. This strategy preserved computational efficiency while minimizing the negative impact on model accuracy.

- Adaptive Regularization: Adjusting the regularization parameter C based on kernel matrix statistics (median and quantile values) allowed the model to adapt to varying data densities, enhancing its ability to capture subtle

patterns. This adaptive approach helped maintain performance even when using lower precision posits.

Lessons Learned

- Precision Matters: The choice of posit precision significantly influences the performance of SVMs during inference. Higher precision posits (Posit16 and Posit32) offer a more reliable alternative to floats, especially in data-sensitive applications.

- Importance of Dynamic Scaling: Dynamic adjustment of bounds based on kernel matrix values ensures that the model remains robust against precision loss. This method effectively counters the limitations of posit formats by tailoring the influence of support vectors.

- Mixed Precision as a Viable Strategy: Combining posits with higher precision computations in critical areas strikes a balance between efficiency and accuracy. This approach leverages the strengths of both number formats, providing a practical solution for real-world applications.

Mitigation Strategies

To mitigate precision loss during inference with posit formats, the following strategies are recommended:

- Dynamic Bound Adjustment: Implement dynamic scaling of dual variable bounds based on kernel matrix statistics (maximum, median, quantile) to ensure robust model performance across varying data characteristics.

- Mixed Precision Inference: Adopt a mixed precision approach, using posits for storage and representation of dual variables while employing higher precision types for key computations, particularly in non-linear kernels.

- Adaptive Regularization: Adjust the regularization parameter C dynamically based on kernel matrix distributions to enhance the model's sensitivity to data density, improving its ability to capture important patterns without being overly influenced by precision loss.

6. Conclusion

The transition from floating-point to posit number formats for SVM inference presents both opportunities and challenges. While precision loss is a significant concern, especially with lower precision posits, strategic adjustments to dual variable bounds and the adoption of mixed precision techniques can effectively mitigate these issues. By dynamically scaling bounds and leveraging higher precision for critical

calculations, it is possible to harness the efficiency of posits without compromising the accuracy and reliability of SVM models. This study provides a foundation for further research into optimizing machine learning algorithms using alternative number formats, paving the way for more efficient and accurate computational methods in the evolving landscape of machine learning.