

**Hajee Mohammad Danesh Science and Technology University, Dinajpur**

B. Sc. in CSE, Level 2, Semester I, Online Examination 2020

Course Title: Mathematics III, Course code: MAT 201

Full Marks: 90

Time: 1.30 hours

*The figures in the right margin indicate the full marks for respective question.*

**Section A**

**Answer any 3 (three) questions.**

1. (a) Define unit vector. If  $\vec{a}=3\hat{i}-\hat{j}-4\hat{k}, \vec{b}=-2\hat{i}+4\hat{j}-3\hat{k}, \vec{c}=\hat{i}+2\hat{j}+\hat{k}$ , Find the unit vector parallel to  $3\vec{a}-2\vec{b}+4\vec{c}$ . 7
- (b) A rigid body is rotating at the rate of 2.5 radians per second about an axis  $AB$  where  $A$  and  $B$  are the points  $(1, -2, 1)$  and  $(3, -4, 2)$ . Find the velocity of the point  $p$  at  $(5, -1, -1)$  of the body. 8
2. (a) Find the unit tangent and unit normal vector at  $t = 2$  on the curve  $x = t^2 - 1, y = 4t - 3, z = 2t^2 - 6t$  where  $t$  is any variable. 7
- (b) The coordinates of a moving particle are given by  $x = 4t - \frac{t^2}{2}$  and  $y = 3 + 6t - \frac{t^3}{6}$ . Find the velocity and acceleration of the particle when  $t = 2 \text{ sec}$ . 8
3. (a) Find the scalar  $m$  such that the scalar product of  $\vec{i} + \vec{j} + \vec{k}$  with the unit vector parallel to the sum of  $2\vec{i} + 4\vec{j} - 5\vec{k}$  and  $m\vec{i} + 2\vec{j} + 3\vec{k}$  is equal to one. 7
- (b) Show that  $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$  is irrotational and hence find its scalar potential. 8
4. (a) If  $\vec{F} = 3xy\vec{i} - y^2\vec{j}$ , evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the curve in the  $xy$  plane,  $y = 2x^2$ , from  $(0, 0)$  and  $(1, 2)$ . 7
- (b) Find the work done in moving a particle once round the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$  under the field of force given by  $\vec{F} = (2x - y + z)\vec{i} + (x + y - z^2)\vec{j} + (3x - 2y + 4z)\vec{k}$  is the field of force conservative. 8

**Section B**

**Answer any 3 (three) questions.**

5. (a) Define complex number. Find the complex number  $z$  if  $\arg(z + 1) = \frac{\pi}{6}$  and  $\arg(z - 1) = \frac{2\pi}{3}$ . 7
- (b) Separate into real and imaginary parts  $\sqrt{i}^{\sqrt{i}}$ . 8
6. (a) Define complex variable. Test the analyticity of the function  $w = \sin z$  and hence derive that:  $\frac{d}{dz}(\sin z) = \cos z$ . 7
- (b) An electrostatic field in the  $xy$  plane is given by the potential function  $\phi = 3x^2y - y^3$ , find the stream function. 8
7. (a) Define Laplace Transformation. Find the Laplace transform of  $\cos^2 t$ . 7
- (b) Obtain the Laplace transform of  $t^2 e^t \sin 4t$ . 8
8. (a) Find the Laplace transform of the waveform  $f(t) = \frac{2t}{3}; 0 \leq t \leq 3$ . 7
- (b) Show that the Laplace transform of the function  $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ t-1, & 2 < t < 3 \\ 7, & t > 3 \end{cases}$  is  $\frac{2}{s^3} - \frac{e^{-2s}}{s^3} (2 + 3s + 3s^2) + \frac{e^{-3s}}{s^2} (5s - 1)$ . 8