

Soal tambahan

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$1) A \cdot A^T = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 17 & 8 \\ 8 & 17 \end{bmatrix}$$

2.) mencari λ dan \bar{u}

$$\det \begin{bmatrix} 17-\lambda & 8 \\ 8 & 17-\lambda \end{bmatrix} = 0$$

$$\Leftrightarrow (17-\lambda)(17-\lambda) - 64 = 0$$

$$\Leftrightarrow \lambda^2 - 34\lambda + 225 = 0$$

$$\Leftrightarrow (\lambda - 9)(\lambda - 25) = 0$$

$$\therefore \lambda = 9 \quad ; \quad \lambda = 25$$

$$\left\{ \begin{array}{l} \text{Untuk } \lambda = 25 \\ ((A \cdot A^T) - \lambda I) \bar{u} = 0 \end{array} \right.$$

$$\begin{bmatrix} -8 & 8 \\ 8 & -8 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

$$-8u_1 + 8u_2 = 0$$

$$u_2 = \frac{8u_1}{8}$$

$$u_2 = u_1 \Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$\text{Untuk } \lambda = 9$$

$$\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 0$$

$$8u_1 + 8u_2 = 0$$

$$u_2 = -u_1 \Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

3.) Matriks Σ

$$\lambda_1 = 25 \quad ; \quad \lambda_2 = 9$$

$$\Sigma = \begin{bmatrix} \sqrt{25} & 0 & 0 \\ 0 & \sqrt{9} & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

4.) Matriks U

$$U = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

5.) Matriks V

$$A^T \cdot A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 12 & 2 \\ 12 & 13 & -2 \\ 2 & -2 & 8 \end{bmatrix}$$

$$\lambda_1 = 9 ; \lambda_2 = 25 ;$$

Untuk $\lambda_1 = 9$

$$\begin{bmatrix} 4 & 12 & 2 \\ 12 & 4 & -2 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = 0$$

$$4u_1 - 12u_2 + 2u_3 = 0$$

$$\vec{u} = \begin{bmatrix} -0,17 \\ -0,16 \\ 0,23 \end{bmatrix}$$

Untuk $\lambda_2 = 25$

$$\begin{bmatrix} -12 & 12 & 2 \\ 12 & -12 & -2 \\ 2 & -2 & -17 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = 0$$

$$\vec{u} = \begin{bmatrix} 0 \\ 0,5 \\ 0,9 \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} & -\frac{2}{3} \\ \frac{1}{\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{2}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{6} & -\frac{2}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{6} & \frac{2}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix}$$

$$\therefore Z = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix} ; V = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{6} & -\frac{2}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{6} & \frac{2}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{bmatrix} ; U = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$