LIMC
April 20, 2024

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Details

QUESTIONS
This is a twenty-question contest.

ANSWERS
The choices are A, B, C, D , and E, only one option is corrrect.

The answers are circled out of the possible solutions. Scratch papers would be provided.

SCORING
Correct answers are awarded 2, 3 or 4 points. No extra points awarded for incorrect or unanswered answers.

MATERIALS
No aids are allowed except for: scratch papers, pen and pencils, rulers, protractors, erasers and compasses.

No calculators are allowed.

TIME
You have a total of 65 minutes on this contest. 5 minutes reading time and after that the supervisor would indicate a signal to start.

1. What is the largest digit sum of one integer (for example 109 has a digit sum of 1+0+9=10) among all integers from 1 to 2024?
(2 points)

A. 18
B. 8
C. 29
D. 28
E. 31

2. Find x if: $\sqrt{4 + \sqrt{2 + \sqrt{x}}\sqrt{2 + \sqrt{x}}} + 6 = 3$: (2 points)

A. 9
B. 4
C. 16.
D. 25
E. There is no real solution for x.

3. The points (3,2) and (11,4) are both within the function of
y=$log_2 (ax + b)$, find a+b. (2 points)

A. 0.5
B. 1
C. 1.5
D. 2
E. 2.5

4. Prof. Collatz created a machine which receives a single positive integer x as input and continuously performs operations on it. Every second, the machine does the following:

If $x = 1$, **SHUT DOWN.**

If x is even, replace x with $\frac{x}{2}$.
Otherwise, replace x with $3x + 4$.

Professor noticed that in some cases the machine will never shut down. For how many initial values of x, from 1 to 100 inclusive, will the machine eventually shut down? (3 points)

    A. 5
    B. 7
    C. 33
    D. 50
    E. 67

5. A triangle has one angle that is 12° larger than another, and the third angle is two times larger in measure than one of the first two. The smallest angle measures: (2 points)

    A. 42°
    B. 54°
    C. 36°
    D. 88°
    E. Not enough information to say more precisely.

6. Given that f(x) is $(x + 3)^{2^{(x^2-9)}} log|x^2 + x - 1| = 0$

Which statement is true for f(x)   (3 points)
    A. This equation has no real roots.
    B. The only real root is -3.
    C. Two of the real roots are 3 and -3.
    D. The only real roots are -3, -2 and 1.
    E. None of the statements above are true.

7. Line A is parallel to lines B and D, Line C is perpendicular to lines D and E. Which is not true? (2 points)

    A. Line A and E are parallel.
    B. Lines B and C are perpendicular.
    C. Lines B and D are parallel.
    D. Exactly two of the above statements A, B and C are true.
    E. None of the above statements is true.

8. A rectangular piece of paper measures 17 cm by 8 cm. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is the area, in square centimeters, of the new figure? We encourage you to ask for a paper to fold it. (3 points)


Before   After
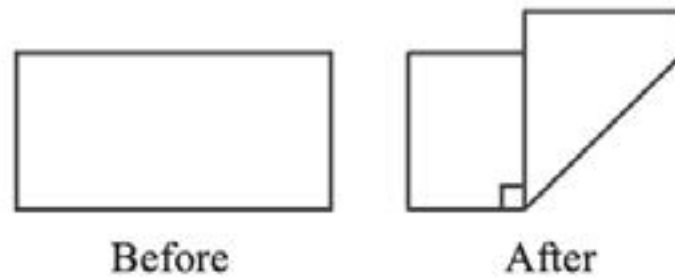
A. 104
B. 81
C. 72
D. 168
E. 64

9. A *palindrome* is a number that remains the same when its digits are reversed. (For example 79697) The number x and x+22 are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x? (2 points)

A. 21
B. 22
C. 23
D. 24
E. 25

10. Four right triangles with one leg 1 unit long and hypotenuse 4 units long are arranged in a square. What is the area of the square? (2 points)

A. 8
B. 9
C. 15
D. 17
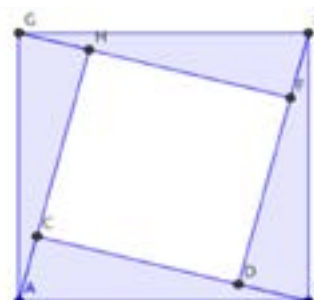
E. Between A and B.

11. Alex is planning to run a few miles today ahead of his running competition next week. He starts his run by heading due north for one mile. He then runs heading due northeast for another mile. He rests at the point where he stops for 5 minutes. After his rest, Alex heads due southeast for one mile and he finishes the last portion of the run in a straight line towards where he originally started.

How much distance, in miles, is the last portion of his run? (2 points)

    A. 1
    B. $\sqrt{2}$
   C. $\sqrt{3}$
    D. 2
    E. $2\sqrt{2}$

12. To measure the volume of a stone with an irregular shape, we put it in a cylindrical container of water. The radius of the container is 8 cm while the height of the water before putting the stone in is 20 cm. If after putting the stone in the container, the new water level is 23.5 cm, what is the volume, in cubic centimeters, of the stone? (2 points)

    a. 128π
    b. 224π
    c. 240π
    d. 282π

e. 320π
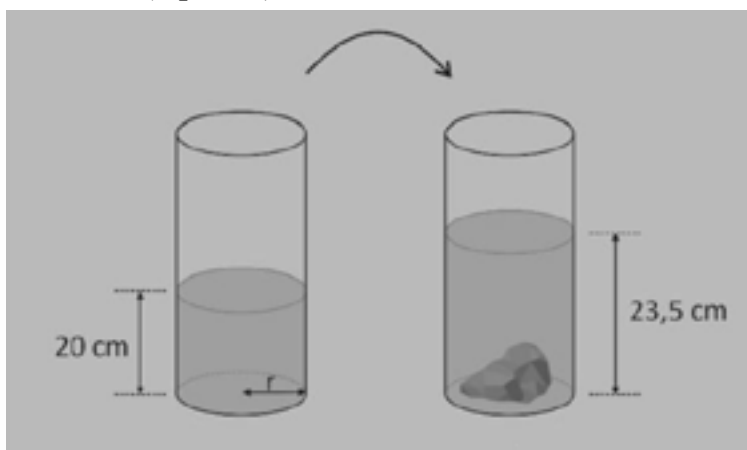
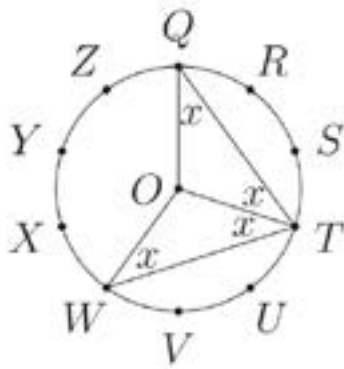13. Consider the expression A: $|x| + |y| \leq 1$
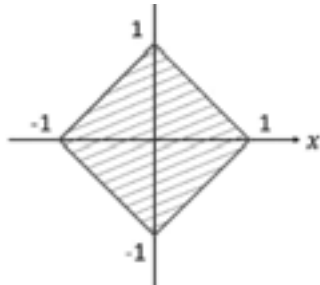
Given that expression B is $(x + 1)^2 + y^2 \geq 1$

What is the area of which the expressions intersect? (3 points)

    a. $4 - \dfrac{\pi}{4}$

    b. $4 - \dfrac{\pi}{2}$

    c. $2 - \dfrac{\pi}{4}$

    d. $2 - \dfrac{\pi}{2}$

    e. $\sqrt{2} - \dfrac{\pi}{2}$

14. The points Q,R,S,T,U,V,W,X,Y and Z are equally and consecutively spread in a circle. What is the size, in degrees, of 2x? (2 points)

a. 36
b. 54
c. 60
d. 72
e. 75

15. This is a question about logic: (2 points)

Students Greg and Mark were invited to participate in a guessing game, where they are given clues to guess which object (out of the five above) contains the precious prize. Specifically, Greg is given the shape of the object while Mark is given the color of the object. (Note that both students know what type of clue is given to each other and they are incredibly smart to the extent that they are capable of predicting what the others are thinking.)

When the host asks, for the first time, if any of them knows which object contains the prize, they both respond as no.

The hosts ask again, they still respond as no.

Third time, though, some ideas flew through their heads and they were astonished, yelling, "yes!"

Which one of the five objects contained the prize?
   a. Blue square.
   b. Green triangle,
   c. Yellow circle,
   d. Blue triangle.
   e. Green circle.

16. A rectangle is divided into 4 smaller rectangles, labeled W, X, Y, and Z, as shown. The perimeters of rectangles W, X and Y are 5, 6, 7, respectively. What is the perimeter of rectangle Z? (2 points)

W	X
Y	Z
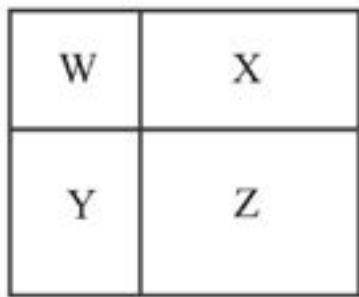
   A. 5
   B. 6
   C. 7
   D. 8
   E. 9

17. James and Joe played a knowledge game. Every time the winner is awarded 2 points for winning and the loser is given 1 point. James won exactly 4 games while Joe got 10 points in total. How many games did they play? (3 points)

   A. 5
   B. 6
   C. 7
   D. 8
   E. 9

18. Define a@b as $a^2b - ab^2$. Which of the following describes the set of points (x,y) for which x@y=y@x? (3 points)

A. A finite set of points.
B. One line.
C. Two parallel lines.
D. Two intersecting lines.
E. Three lines.

19. A semicircle is tangent to both legs of a right triangle and has its center on the hypotenuse. The hypotenuse is partitioned into 4 segments, with lengths 3, 12, 12, and x, as shown. What is the value of x? (3 points)

Some values provided: $27^2 = 729,\ 21^2 = 441,\ 12^2 = 144.$ (Others approaches might not require it)

Also note that one of the sides of the square + the "right" side of the triangle that has the hypotenuse 12+x = 28

A. 8
B. 6
C. 4
D. 10
E. 12

20.. A square has side length 5. How many different ways can x be placed so that the distance from x to the four sides of the square are 1, 2, 3, and 4? (3 points)

A. 0
B. 12
C. 8
D. 4
E. 16