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Generalized Versus Ordinary Least Squares for Linear Mixed Models

For a linear mixed model $\mathbf{Y} = \mathbf{X\beta} + \mathbf{Z\gamma} + \boldsymbol{\varepsilon}$, assuming that the random effect γ and the residuals ε are independently and normally distributed with the following:

$$E\begin{bmatrix}\gamma \\ \varepsilon\end{bmatrix} = \mathbf{0} \text{ and } Var\begin{bmatrix}\gamma \\ \varepsilon\end{bmatrix} = \begin{bmatrix}\mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}\end{bmatrix}$$

It can be shown that for the observed response variable **Y**, you have the following:

$E(\mathbf{Y}) = \mathbf{X}\boldsymbol{\beta}$ and $Var(\mathbf{Y}) = \mathbf{ZGZ'} + \mathbf{R} = \mathbf{V}$

PROC GLIMMIX enables you to specify various covariance structures for both the **G** and **R** matrices. The default structure models a different variance component for each random effect.

The generalized least squares (GLS) estimates take into account the covariance matrices **G** and **R**. When you use this estimation method, it can be shown that the parameter estimates and variance are computed as follows:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'}\hat{\mathbf{V}}^{-1}\mathbf{X})^{-}\mathbf{X'}\hat{\mathbf{V}}^{-1}\mathbf{Y}\text{, and } Var(\hat{\boldsymbol{\beta}}) = (\mathbf{X'}\hat{\mathbf{V}}^{-1}\mathbf{X})^{-}$$

The ordinary least squares (OLS) solution for a fixed effect model $\mathbf{Y} = \mathbf{X\beta} + \boldsymbol{\varepsilon}$ is given by $\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'Y}$, and the standard errors are computed based on $\mathrm{var}(\hat{\boldsymbol{\beta}}) = \sigma^2(\mathbf{X'X})^{-1}$. It can be seen that OLS is a special case of the GLS solution with $\mathbf{V} = \sigma^2 I_n$. The variance of the OLS solution is also a special case of the variance of the GLS solution.

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