

## Confidence Intervals

Confidence intervals for the mean are interval estimators of the population mean. They take into account the variability of the sample statistic, in this case, the sample mean. The confidence interval shows a range of plausible values for the unknown population mean by reporting an upper and lower bound.

Here's a good way to think about confidence intervals. If you were to draw infinitely many samples and estimate your confidence interval exactly the same way each time, your confidence interval would represent the percentage of those intervals that would contain the true population mean. Of course, nobody draws infinitely many samples. It's rare that you would draw more than one. An important thing to remember is that the confidence interval calculated for a particular sample might or might not contain the value of the true population mean.

You get to choose your desired degree of confidence, or confidence level. A typical confidence level is 95%, meaning that 95% of those theoretically infinite number of intervals would contain the true population mean, and 5% would not.

Here's the formula for calculating the confidence interval for the mean.

$$\bar{x} \pm t \cdot S_{\bar{x}}$$

$\bar{x}$  is the sample mean.  $t$  is the  $t$  quantile value that is determined by the confidence level and the sample size.

$$S_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$S_{\bar{x}}$  is the standard error of the mean.

To construct a confidence interval for the mean, you must first select a significance level so that the appropriate value of  $t$  is used in the formula. Higher confidence levels are associated with larger  $t$  values, which, in turn, result in wider intervals. The formula calculates both the upper and lower bounds, equidistant from the sample mean estimate in the middle.

Why not raise the confidence level to 99.9%, so that any confidence interval you calculate contains the true value of the population mean? As you increase the confidence level, the width of the interval increases, making it less informative. In the extreme, a 100% confidence interval for the mean for any sample ranges from negative infinity to positive infinity, which would tell you nothing useful about where the true population mean lies.