

Sample Size for a Confidence Interval for the Mean

When you construct a confidence interval for a mean, you start with a point estimate, the sample mean.

Then you add a margin of error around the point estimate.

For the metal parts pilot study that is discussed throughout this module the sample mean is 40.345 hundredths of an inch and the 95% CI is 39.75 to 40.94. The margin of error is the halfwidth of this interval, 0.595 hundredths of an inch.

What does this mean?

With this sample of 20 parts and a 95% confidence level, you can estimate the true mean thickness within 0.595 hundredths of an inch.

But, what if you need a more precise estimate of the population mean?

For example, what if it is important to estimate the mean thickness within 0.3 hundredths of an inch.

To construct the margin of error, you need three values: the confidence level ($1-\alpha$), the standard error, and a critical value from the t distribution.

The confidence level, $1-\alpha$, is specified by you. For the metal parts example, a 95% confidence level was used by default.

The standard error, which is estimated from the sample data, is simply the standard deviation divided by the square root of the sample size.

The standard deviation plays a role in the margin of error.

The larger the standard deviation, for the given sample size, the wider your margin of error. The critical value from the t distribution is based on both the confidence level and the sample size.

Notice that the sample size influences both the critical t value and the standard error.

Suppose that we want to construct a 95% confidence interval. We can reorganize this formula to determine the sample size needed to produce a confidence interval with the specified margin of error.

However, we don't know the t value, because it's based on the sample size. If we assume that the sample size is large, we can use a value from the normal distribution, called a z value.

Earlier, you learned that approximately 95% of the area under the normal curve is within approximately two standard errors of the mean.

The precise value is 1.96 standard errors. We substitute this value, 1.96, into the formula. We also don't know the sample standard deviation. If we have a reasonable estimate of the population standard deviation, we can calculate the sample size required to produce the specified margin of error using this formula.

Let's return to our metal parts example. Remember that we want a margin of error of no more than 0.3 hundredths of an inch. If we use an estimate of the population standard deviation of 1.28, we'd need 70 parts to produce an interval with this degree of precision.

What if we could live with a margin of error of 0.4 instead of 0.3? In this case, only 40 parts are required. Of course, we rely on the software for this. You learn how to use the Sample Size for Confidence Intervals calculator in JMP in the next video.

Before we move on, here are some things to consider. First, if the precision of your interval estimate is of practical importance to you, you should determine the sample size prior to collecting the data.

You might also need to consider the cost of sampling. With larger sample sizes, you obtain more precise interval estimates. However, there is a trade-off. Sampling imposes costs.

For example, if your measurements are based on a destructive test, then you incur costs associated with both taking the measurements and with the loss of potentially expensive material. At some point, the added precision isn't worth the costs associated with taking larger samples.

For more information about sample size calculations for confidence intervals, see the Read About It for this module.

Statistical Thinking for Industrial Problem Solving

Copyright © 2020 SAS Institute Inc., Cary, NC, USA. All rights reserved.

Close