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Estimating a Mean

Michelson was trying to measure an unknown parameter, the constant speed of light, using equipment that provided imperfect measurements. He reasoned that the mean of his measurements would be a good estimate of the "true" speed of light.

In short, Michelson planned to use a sample statistic to draw a conclusion about an unknown population parameter. You can think of his measurements as a random sample of readings from his instruments.

Using the same instruments and the same procedure, he conducted five trials. For each trial, he recorded 20 measurements. For a continuous distribution, we are often interested in three characteristics: the center, the shape, and the spread, or dispersion of the distribution.

For this lesson, we're interested in only the center of a distribution: the average speed of light (or velocity), as measured by Michelson's experiment.

Consider the 20 measurements in Michelson's first trial. We could simply calculate the sample mean of these 20 measurements, and use this single number as our estimate of the population mean. We call this sample mean a point estimate.

It might be a reasonable approximation of the unknown parameter. However, the instrument readings are subject to random variation. If we have a different sample, drawn from the same population, we'd likely get a different estimate.

For the measurements in Michelson's first trial, the point estimate is 299,909 kilometers/second. For the measurements in his second trial, the sample mean is slightly lower. Why are these estimates, for samples drawn under the same conditions, different? This idea, that different subsets of observations will yield different estimates, is known as sampling variation.

To account for this variation, we commonly add a "cushion," or a margin of error, around the point estimate. This margin of error quantifies the uncertainty in our estimate of the true value.

Adding a margin of error produces a confidence interval. A confidence interval is a range of possible values for the parameter. (In our speed of light example, the parameter is the true mean, mu.)

The operational question is, "just how much of a cushion is sufficient?" Answering this question requires an understanding of the nature and extent of sampling variation.

In the Exploratory Data Analysis module, you learned about the Central Limit Theorem. You learned that the distribution of sample means of size n will be normal (or approximately normal) under a wide range of conditions.

This distribution will be centered at the true mean, mu. In this example, the true mean is the speed of light. You also learned how to calculate the standard deviation of sample means, which is a measure of the spread or dispersion of the sampling distribution.

The extent to which sample means vary depends on just two values: the dispersion of individual values in the population (the population standard deviation, or sigma) and the size of the sample. The larger the sample, the more similar the sample means.

Of course, we usually don't know the population standard deviation, so we estimate it using data. We use the standard error of the mean, or the standard error, as an estimate of the dispersion of the sampling distribution.

For the 20 measurements in Michelson's first trial, the standard error of the mean is 23.46. It's unlikely that any single sample mean will equal the true mean of the population. However, we can use the standard error of the mean to create a confidence interval for the true mean.

A confidence interval is an estimate that accounts for the likely range of the sampling error. Or, put another way, a confidence interval provides a range of plausible values for the theoretical or true population parameter.

Let's look again at the 20 velocity measurements in Michelson's first trial. The confidence interval for the true mean speed of light, based on these 20 measurements, is reported in the distribution analysis.

You can see that this confidence interval provides a cushion around the point estimate of the mean from this trial. This cushion is the margin of error that accounts for the uncertainty in the sample estimate. You learn more about how this confidence interval is constructed in an upcoming video.

In the next video, we revisit the idea of sampling variation using the Impurity data.

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