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Performing a t Test

A one-sample t test compares the mean calculated from a sample to a hypothesized mean. You're testing the null hypothesis, μ is equal to μ_0, against the alternative hypothesis, μ is not equal to μ_0. When you don't know the true population standard deviation, σ, you must estimate it from the sample, and you must use Student's t distribution, rather than the normal distribution, for calculating p-values and confidence limits.

Student's t distribution is similar to the normal distribution, but it has more probability in the tails and is not as peaked as the normal distribution. Student's t distribution approaches the normal distribution as the sample size increases. You calculate the value of Student's t statistic using the equation $t = \frac{(\bar{x}-\mu_0)}{s_{\bar{x}}}$.

Let's calculate Student's t statistic for the Ames housing data to test our null hypothesis.

μ_0 is the hypothesized value of $135,000, $\bar{x}$ is the sample mean of SalePrice, $137,525, and $s_{\bar{x}}$ is the standard error of the mean, $2,172.1. The resulting t value is 1.16.

$$t = \frac{(137,525 - 135,000)}{2,172.1} = 1.16$$

What does this t value tell us? It measures how far the sample mean is from the hypothesized mean, in standard error units. The t value is positive when the sample mean is larger than the hypothesized mean, and negative when the sample mean is less than the hypothesized mean. If your null hypothesis is true, you'd expect $\bar{x}$ to be relatively close to μ_0, and the corresponding t value to be close to zero, providing evidence in favor of the null hypothesis. If $\bar{x}$ is far from μ_0 and the t value is large, you have evidence against the null hypothesis in favor of the alternative.

Before you can make a decision about your hypothesis, you need to know the probability of observing a test statistic of t=1.16 or more extreme, given that the null hypothesis is true. This probability is the p-value. It quantifies how likely we are to obtain the sample mean. If the p-value is less than α, you reject the null hypothesis in favor of the alternative. On the other hand, if your p-value is greater than α, evidence suggests that your null hypothesized value is statistically reasonable, so you fail to reject the null hypothesis.

Consider the Ames housing example. A p-value less than 0.05, our α, means there's less than a 5% chance you would have a sample mean of $137,525 if your population mean was in fact $135,000.

Let's discuss the t distribution. Our alternative hypothesis is an inequality, so this is a two-sided test. For a two-sided test, the rejection region is contained in both tails, and the area in each tail corresponds to $\alpha/2$, or in this case, 2.5%. If the t value falls in the rejection region, then you reject the null hypothesis. Otherwise, you fail to reject it. Our t value, 1.16, falls outside the rejection region, so we fail to reject our null hypothesis.

The α and t distribution mentioned here are directly related to those in confidence intervals. In our example, α is 0.05, or 5%. The area outside the confidence interval corresponds to the shaded region in the t distribution, or the rejection region.You can calculate the t value and p-value using t distribution tables, or with PROC TTEST in SAS.

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