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Effect of Unequal Variances with Unequal Sample Sizes

To see the effect of unequal variances with unequal sample sizes, examine the expected value of the *F* test:

$$E(F) = E\left(\frac{MS(treatment)}{MS(error)}\right) \approx \frac{\frac{\sum_{i=1}^{I}(N-n_i)\sigma_i^2}{N(I-1)}}{\frac{\sum_{i=1}^{I}(n_i-1)\sigma_i^2}{N-I}}$$

where

N is the overall sample size,

n_i is the sample size for group *i*,

I is the number of groups, and

σ_i^2 is the variance for group *i*.

In the case where the larger variance is associated with the smaller sample size, the numerator is larger than the case for equal variances and the denominator is smaller. This results in a lower *p*-value and inflated Type I error rates. In the case where the larger variance is associated with the larger sample size, the numerator is smaller and the denominator is larger, resulting in a lower *p*-value and loss of power. The results are approximately the same because the expected value of a ratio is not exactly equal to the ratio of the expected values.

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