

Calculating the Variance in Generalized Linear Models

From the SAS online documentation, the general form for a density or probability function of the exponential family can be expressed as:

$$f(y | \theta) = \exp \left\{ \frac{y\theta - b(\theta)}{a(\phi)} + c(y, \phi) \right\}$$

where θ is the natural location parameter, and ϕ is the dispersion parameter.

For this family of distributions, the variance of y , $Var(y)$, can be expressed as a function of the mean of y as $Var(y) = V(\mu)\alpha(\phi)$, where $V(\mu)$ denotes the variance function, and $\alpha(\phi)$ denotes a function of the dispersion parameter. For the normal distribution, $V(\mu)$ is the identity function and $\alpha(\phi) = \sigma^2$, so the relationship between the mean and variance can be expressed as $Var(Y) = V(\mu)\alpha(\phi) = \mu^0 \cdot \sigma^2$.

An alternative parameterization for the general form for a density or probability mass function for an exponential family distribution is:

$$f(y | \theta) = h(y) \cdot c(\theta) \cdot e^{\sum t_i(y_i)(W_i(\theta))}$$

When written in this form, the canonical link can be identified as $W_i(\phi)$. (Casella and Berger 1990)

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