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Linear Mixed Model with Nested Classification

For the school study, let's look at the the equation for the linear mixed model with nested classification and two factors: $y_{ijk} = \mu + \alpha_i + b(\alpha)_{ij} + \varepsilon_{ijk}$.

y_{ijk} represents the test score for the i^{th} material, the j^{th} teacher nested within the i^{th} material, and the k^{th} student within the j^{th} teacher for the i^{th} material, where i = 1 to 4, and j = 1 to 5, and k = 1 to 6.

μ represents the overall mean score.

α_i represents the fixed effects associated with the treatment variable, **Material**. Specifically, this term specifies the i^{th} material.

$b(\alpha)_{ij}$ represents the random effects associated with **Teacher** nested within **Material**, specifically, the j^{th} teacher nested within the ith material.

ε_{ijk} represents the random error associated with **Student**.

Now let's look at the assumptions that underlie the random components (that is, the variance). First, the random effects $b(\alpha)_{ij}$ are assumed to be independently and normally distributed with a mean of zero and variance σ_t^2. Because the random components are independent and share a common distribution, they are typically referred to as "independent and identically distributed"—that is, i.i.d. The variance σ_t^2 is the parameter to be estimated in the mixed model for this random, nested effect.

Second, the random errors are assumed to be independently and normally distributed with a mean of zero and variance σ^2. The variance σ^2 is the parameter to be estimated in the mixed model for random error.

Finally, the effects $b(\alpha)_{ij}$ and ε_{ijk} are assumed to be independent random variables. Therefore, the expected value of y_{ijk} is the mean test score (that is, μ) for the i^{th} material (that is, α_i) averaged across all teachers and students in the population. And the variance of $y_{ijk} = \sigma^2 + \sigma_t^2$. In this example, the variance of an observation is the sum of the variances due to **Teacher** and **Student** (the random errors).

Now let's consider hypothesis testing. In mixed model analysis, the hypotheses about the fixed effects are the same as those in the fixed-effects model, that is, whether there are significant treatment effects. So, the null hypothesis is $\alpha_i = 0$. The hypotheses about the random effects are whether the variance components associated with the random effects equal zero; in other words, whether there are significant variations due to these random variables, so, the null hypothesis for random effects is $\sigma_t^2 = 0$. Often, inferences about random effects are of little interest. The primary role of random effects is to model sources of variation so that the fixed effects can be more accurately estimated and tested.

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