Poisson Regression Models

You know that the mean for the Poisson distribution must be positive. Because linear predictor variables can become negative for certain parameter combinations, an additive model for the mean is not workable. To ensure that the mean remains positive for all linear predictors, as well as for all parameter and covariate combinations, Poisson regression models use a log link function that relates the expected value of the response variable to the linear combination of predictors.

For *k* explanatory variables, the Poisson model has the form Log (μ)= β_0+ $\beta_1 X_1$+ $\beta_2 X_2$+...+$\beta_k X_k$. We estimate the parameters of Poisson regression models by using the method of maximum likelihood. This method finds the parameter estimates that are most likely to occur given the data. These parameter estimates maximize the likelihood function, which expresses the probability of the observed data as a function of the unknown parameters. The mean satisfies the exponential relationship, and the fitted values are the exponentiation of the linear predictor, that is, μ = e($\beta_1 X_1$+ $\beta_2 X_2$+...+$\beta_k X_k$).

You can see that the effect of the predictors on the mean is multiplicative, not additive, and that you need to consider how the mean will change based on the changing value of a predictor. To see the effect of the change in the original scale, we must exponentiate the parameter estimates. Regardless of the value of the parameter estimate (beta-hat), the exponentiation will be positive. The question is whether the exponentiation be greater than or less than 1. Exponentiations greater than 1 yield an increase of the mean for each one-unit increase in X. Exponentiations less than 1 yield a decrease of the mean for each one-unit increase in X. (Of course, this assumes that all other predictor variables are held constant.)

Changes in the mean are measured by percent of change. This percent of change can be calculated by subtracting 1 from the exponentiation of the parameter estimate and then multiplying by 100. Let's see an example. If the exponentiation of $\beta\hat{}_1$ is equal to 1.20, then a one-unit increase in X_1 yields a 20% increase in the estimated mean. If the exponentiation of $\beta\hat{}_2$ is equal to 0.80, then a one-unit increase in X_2 yields a 20% decrease in the estimated mean.