

## Tolerance Intervals

In this lesson, you've learned about confidence intervals and prediction intervals.

Confidence intervals provide a range of plausible values for the true population parameter, such as the mean,  $\mu$ . With a confidence interval, you can make statements like this: We are 95% confident that the true mean is between 18 and 22.

A prediction interval provides a range of plausible values for the next observation, or for the next  $n$  observations. Here's a typical prediction interval: We are 95% confident that the next observation will fall between 16 and 24.

The final type of interval estimate that we'll discuss is a tolerance interval.

With a tolerance interval, you identify a range of values for the population that captures a specified proportion of observations, with a certain degree of confidence.

When you construct a tolerance interval, you make statements about the population such as this one: We are 95% confident that at least 90% of future values will fall between 15 and 25.

From a practical perspective, tolerance intervals enable you to answer this question: Given the data we have collected and our current process, what is the range of variation that we can expect to see in the process characteristic? This is, perhaps, a far more important question to engineers and scientists than estimating the mean or estimating the next value.

To construct a tolerance interval, we specify the confidence level ( $1 - \alpha$ ). We also specify  $p$ , the proportion of the population we want to include in the interval.

Let's return to the diameter example introduced in the previous video. Remember that the diameter of parts, in millimeters, is measured. You've used an X-Bar and R chart to determine that the process is stable. You can use these data to estimate the range of possible values for the population.

We'll construct a 95% tolerance interval to encompass at least 90% of future diameter values. Again, we do not include the details of how the interval is calculated. Instead, we focus on the interpretation. This interval is 16.03 to 16.25.

What does this mean? If the process hasn't changed, you can be 95% confident that at least 90% of the future diameter values will fall within this interval. This example involves a characteristic that is approximately normally distributed.

Let's look at another example, the impurity scenario. Remember that the upper specification limit is 7% impurity, and the goal is no more than 3% impurity. The range of values, for 100 batches, is 3.1 to 10.5.

From the histogram and the normal quantile plot, you can see that the distribution of impurity values is slightly right skewed. In fact, the distribution is lognormal.

For the diameter data, which are approximately normally distributed, we constructed what is known as a parametric tolerance interval.

When your distribution is not normal, you can construct a nonparametric tolerance interval instead. Nonparametric methods do not rely on assumptions about the underlying distribution of the data.

Here is the nonparametric tolerance interval for the Impurity data.

From this interval, we can be 95% confident that at least 90% of the future observations will fall between 3.4 and 9.5.

This interval is well above the target of 3% impurity, and many of the batches will fail to meet the upper specification for impurity of 7%.

To close, let's consider the practical use of tolerance interval. As you have learned in this video, a tolerance interval provides an estimated range of values for the process measure. Tolerance intervals can be particularly useful if you are developing a new process or if you want to estimate the range of variation you can expect from an existing process.

In both cases, you can use a tolerance interval to help set reasonable specification limits for your key process measures (assuming that the process is stable).

Note that the normal distribution, normal quantile plots, and the lognormal distribution were introduced in the Probability Concepts lesson in the Exploratory Data Analysis module of this course.

To learn more about using and interpreting parametric and nonparametric tolerance intervals, see the Read About It for this module.

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