

Name : Sumaiya Tarannum Noor

ID : 2425410650

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Assignment - 2

Answer to the Question No. 1

- ① Step-by-step dynamic programming explanation:
we use the classic DP approach where
 $dp[i]$ = length of the longest increasing subsequence
ending at index i .

Input : $A = [10, 9, 2, 5, 3, 7, 10, 18]$

Indices 0 → 7

1. Initialize $dp = [1, 1, 1, 1, 1, 1, 1, 1]$
(every element alone is an LIS of length 1).

2. Also keep a prev array initialized to -1
to reconstruct one LIS later.

$$prev = [-1, -1, -1, -1, -1, -1, -1, -1]$$

Now, iterate over i from 0 to 7, and for each
 i check all $j < i$.

If $A[j] < A[i]$ then $A[i]$ can extend the
LIS that ends at j , so consider $dp[j] + 1$.

Now, updates are shown:

- $i=0$ ($A[0] = 10$): no $j < 0 \rightarrow dp[0] = 1$
- $i=1$ ($A[1] = 9$): check $j=0$ ($10 < 9$? no) $\rightarrow dp[1] = 1$.
- $i=2$ ($A[2] = 2$): $j=0$ ($10 < 2$? no), $j=1$ ($9 < 2$? no)
 $dp[2] = 1$
- $i=3$ ($A[3] = 5$):
 $= j=0$ ($10 < 5$? no), $j=1$ ($9 < 5$? no), $j=2$
($2 < 5$? yes) \rightarrow candidate $dp[2] + 1 = 2$.
so, $dp[3] = 2$, $prev[3] = 2$
- $i=4$ ($A[4] = 3$):
 $= j=2$ ($2 < 3$? yes) $\rightarrow dp[2] + 1 = 2 \rightarrow dp[4] = 2$,
 $prev[4] = 2$
 $=$ other j 's give no larger value.
- $i=5$ ($A[5] = 7$):
 $= j=2$ ($2 < 7 \rightarrow 2$), $j=3$ ($5 < 7 \rightarrow dp[3] + 1 = 3$),
 $j=4$ ($3 < 7 \rightarrow dp[4] + 1 = 3$).

Best is 3 $\rightarrow dp[5] = 3$, $prev[5] = 3$
(or 4 - either works)

- $i=6$ ($A[6] = 101$):

= many j with ($A[j] < 101$), best extension is from $j=5$ with

$$dp[5] = 3 \rightarrow dp[6] = 4, prev[6] = 5.$$

- $i=7$ ($A[7] = 18$):

= best extension also from $j=5$ ($dp[5] = 3$) \Rightarrow

$$dp[7] = 4, prev[7] = 5$$

final dp array:

$$dp = [1, 1, 1, 2, 2, 3, 4, 4]$$

maximum value is 4 (so LIS length = 4).

Reconstruction (example for index 7): use
prev chain: $7 \leftarrow 5 \leftarrow 3 \leftarrow 2 \rightarrow$

sequence indices $[2, 3, 5, 7] \rightarrow$ values $[2, 3, 7, 18]$.

For index 6 chain $6 \leftarrow 5 \leftarrow 3 \leftarrow 2 \rightarrow$
 $[2, 3, 7, 101]$. Another valid LIS is
 $[2, 3, 7, 18]$ if $prev[5]$ chosen as 4.

So LIS length = 4 (e.g. $[2, 3, 7, 18]$ or
 $[2, 3, 7, 101]$).