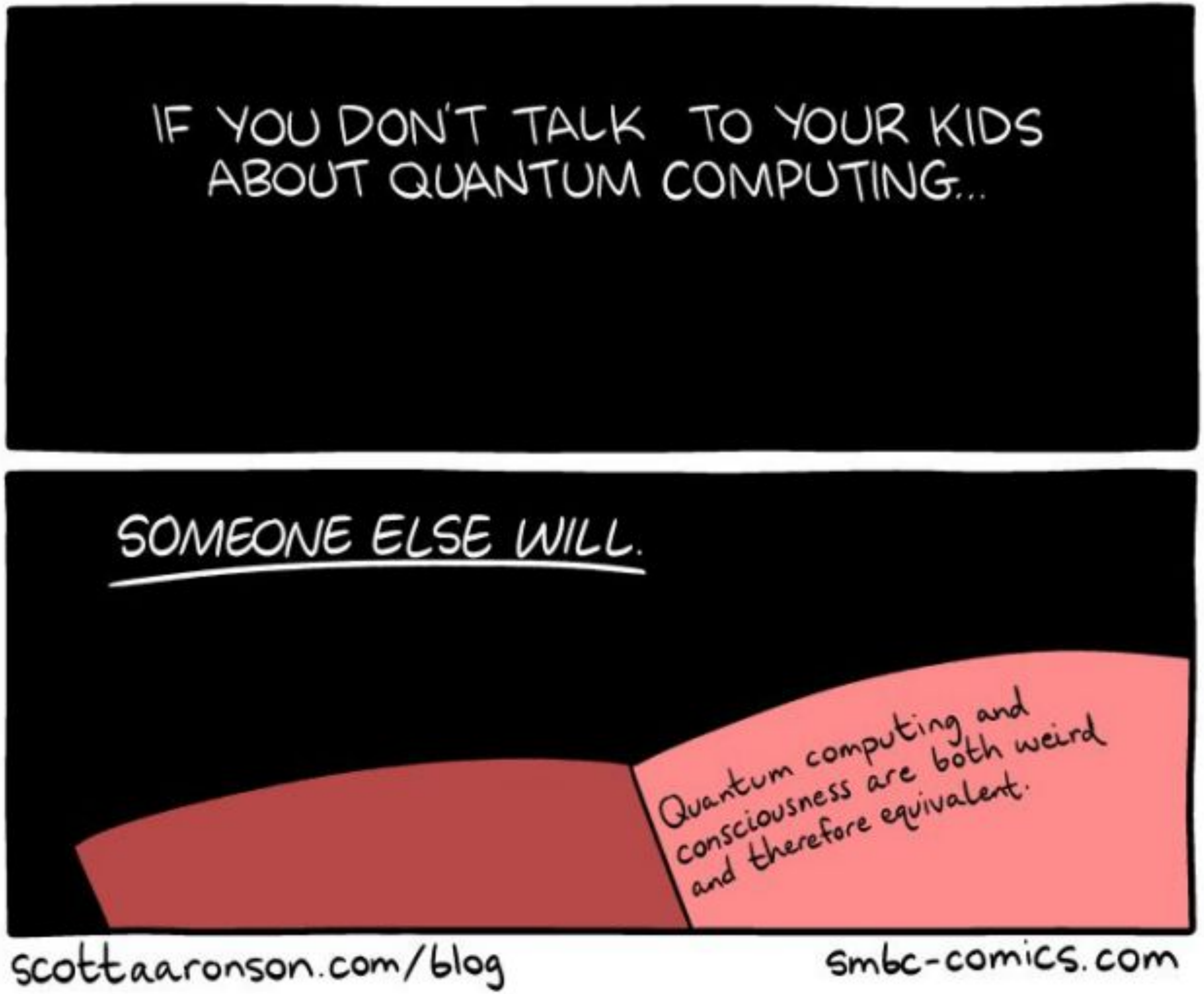
This is CS4084!

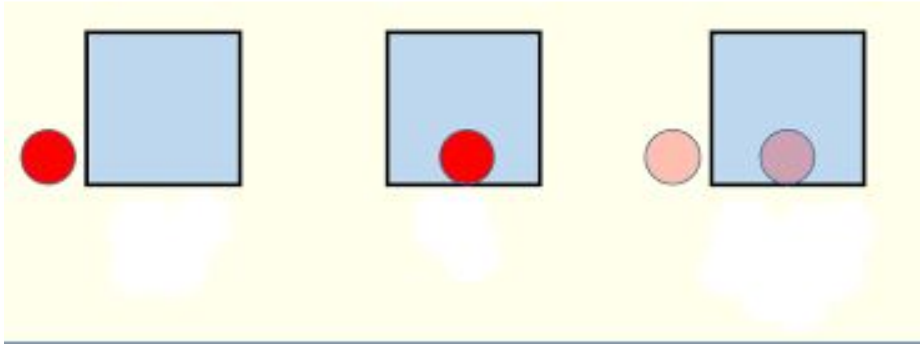
GCR:wzj3vua

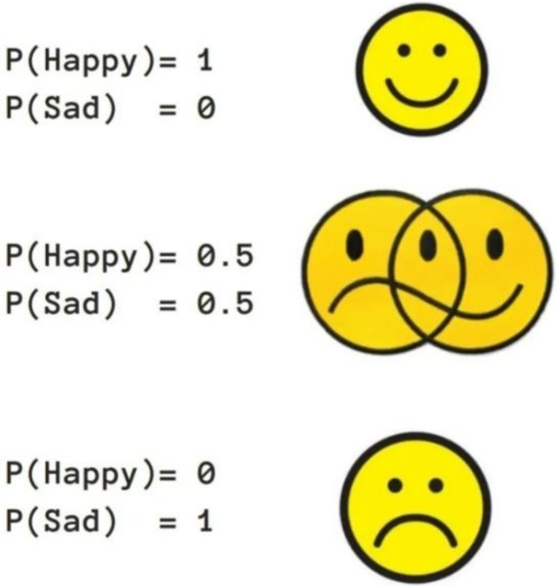

IF YOU DON'T TALK TO YOUR KIDS ABOUT QUANTUM COMPUTING...
SOMEONE ELSE WILL.
Quantum computing and consciousness are both weird and therefore equivalent.
scottaaronson.com/blog
smbc-comics.com

Single Qubit System

Superposition

Superposition is the ability of a quantum system to be in multiple states at the same time until it is measured.





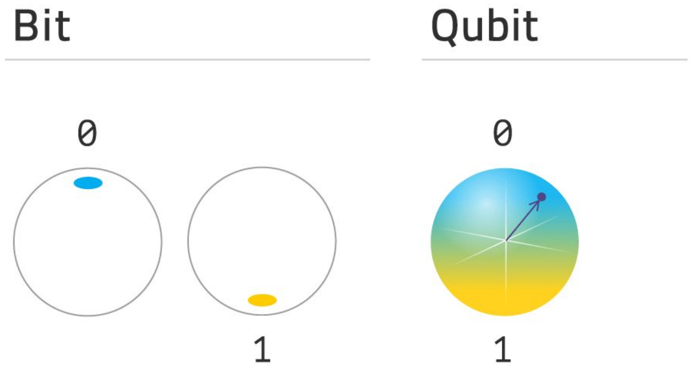
Quantum System

Any system that obeys the laws of quantum mechanics.

- Superposition
- Entanglement (We will study later)
- Quantization

Bit Vs Qubit

Quantum bits or qubits are similar to bits in that there are two measurable states called the 0 & 1.



However, qubits can also be in a superposition state of these 0 and 1 states.

Bit Vs Qubit

A classical bit can take two different values (0 or 1). It is discrete.

A qubit can “take” infinitely many different values.

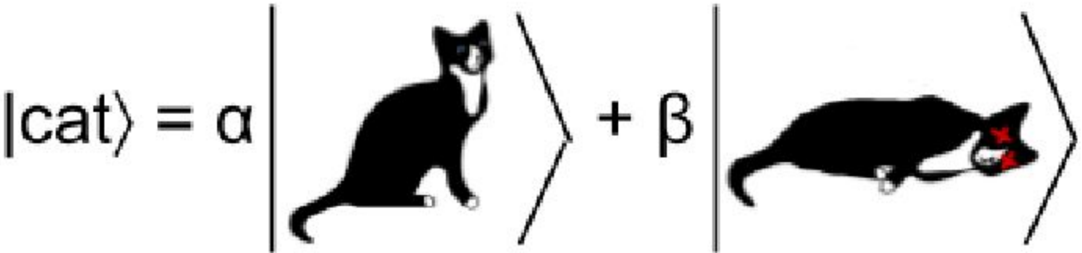
Dirac Bra-Ket Notation

Bra-ket notation is named after the symbols it uses:

"bra" ⟨ and "ket" ⟩

A quantum state is represented by a ket vector = $|\psi\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



Dirac Bra-Ket Notation

The symbol “|>” denotes a column vector, and is known as a “ket”.

The “bra” (<|) form of a vector is just the conjugate transpose of the original.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$\langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

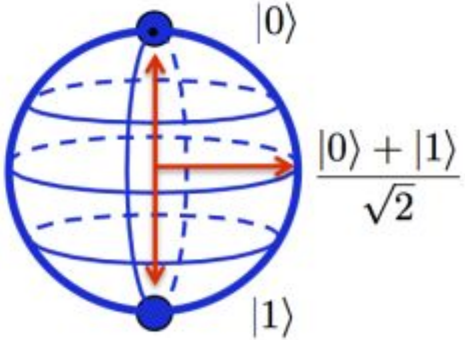
A generic qubit is in a **superposition**

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

where α and β are **complex numbers** such that

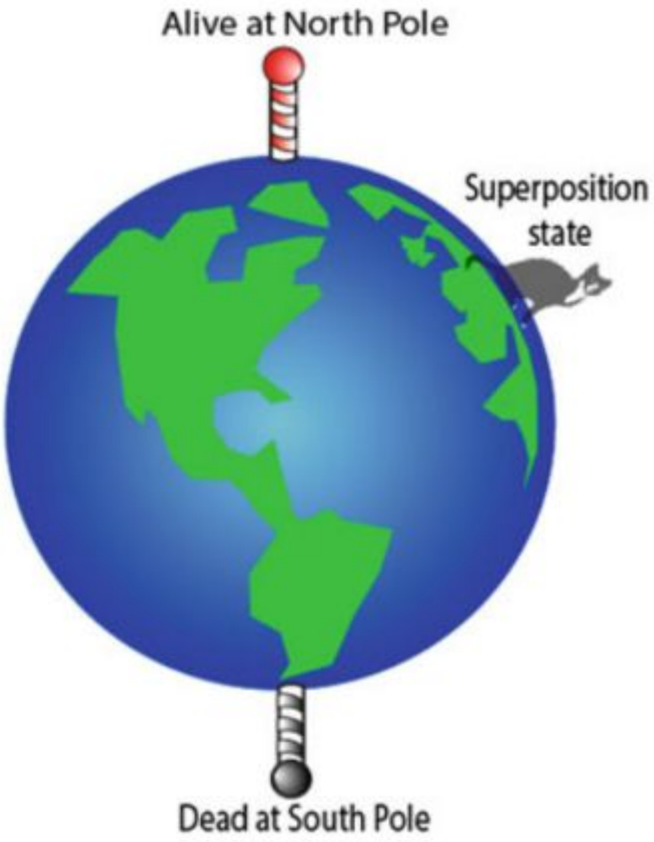
$$|\alpha|^2 + |\beta|^2 = 1$$

Classical Bit



Qubit

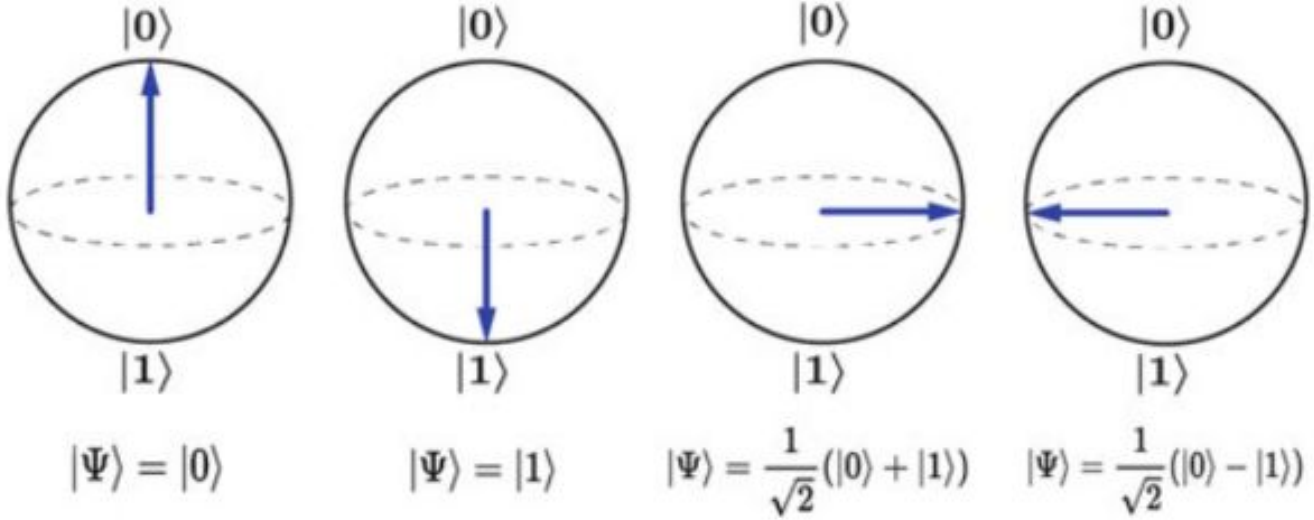
Bloch Sphere



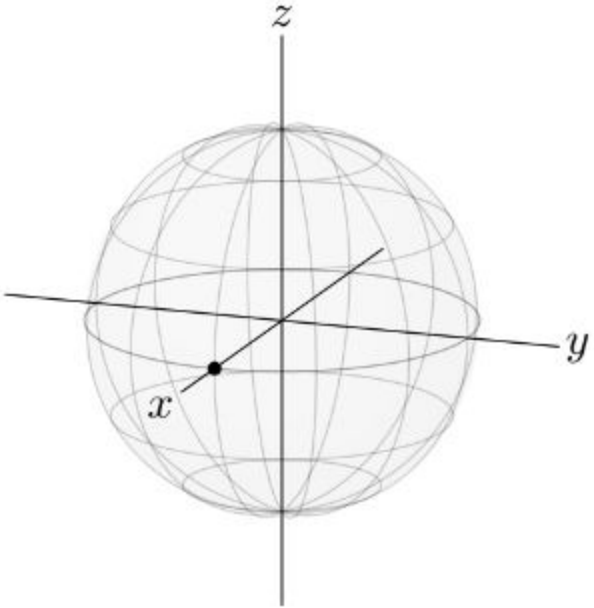
Bloch Sphere

A single qubit can be visualized using the Bloch sphere.

It is a unit sphere which means radius=1



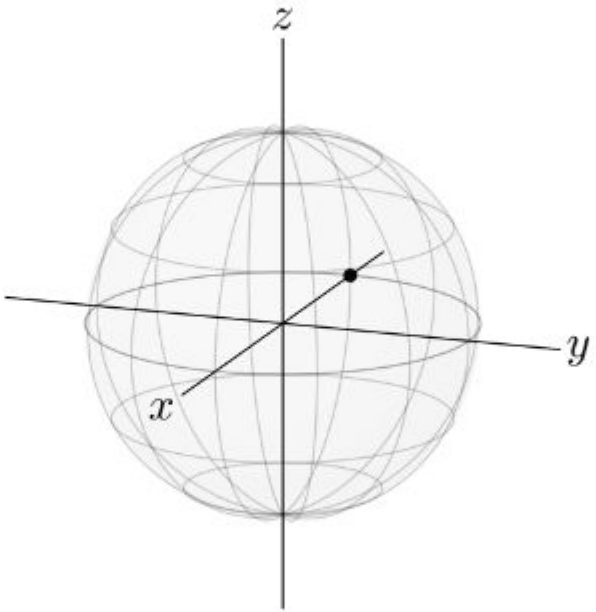
Bloch Sphere



$$\frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right).$$

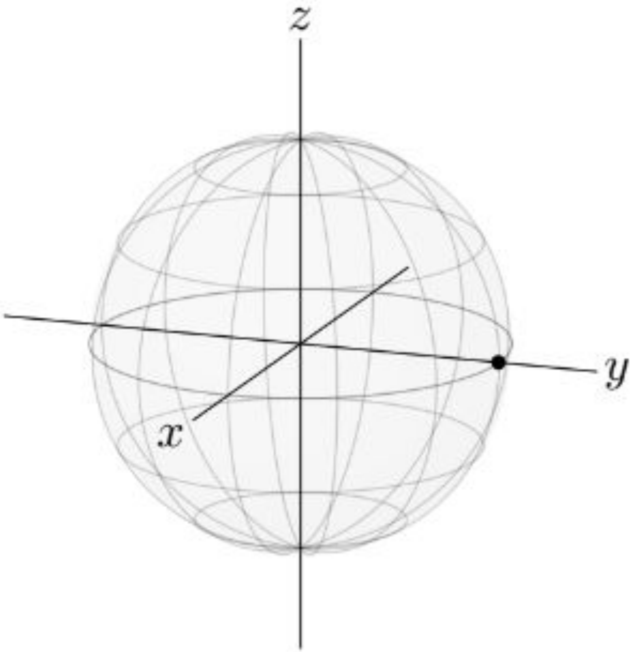
Bloch Sphere

$$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

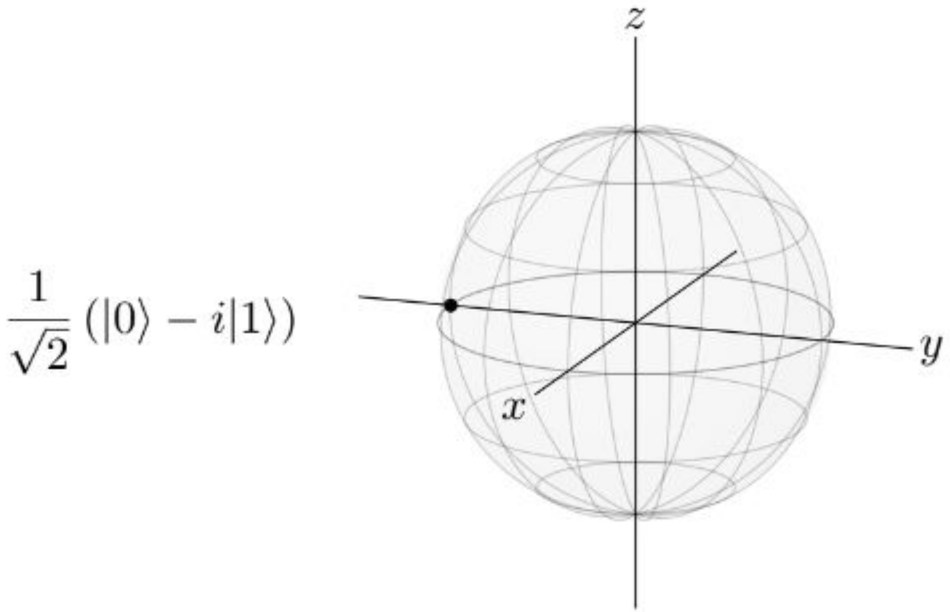


Bloch Sphere

$$\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$



Bloch Sphere



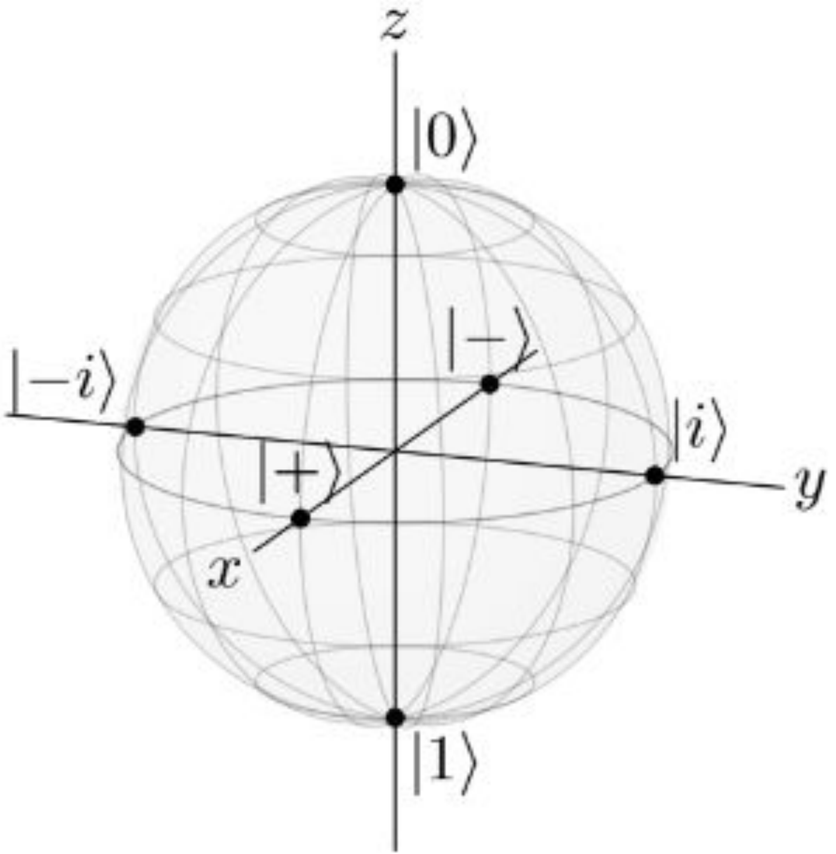
Bloch Sphere

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right),$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right),$$

$$|i\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i|1\rangle\right),$$

$$|-i\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right).$$



Review of Complex Numbers

$z = x+iy$ // Cartesian form of a complex number

In quantum computing, it is often useful to write a complex number as its length r times its complex phase $e^{i\theta}$

$z = re^{i\theta}$ // Polar form of a complex number

Note: We are covering chapter 2 from Introduction to classical and quantum computing by Thomas G wong

Review of Complex Numbers

How to convert cartesian to polar?

$$r = \sqrt{x^2 + y^2},$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right).$$

$$|z| = \sqrt{zz^*} = \sqrt{(x+iy)(x-iy)}$$
$$= \sqrt{x^2 + y^2}$$

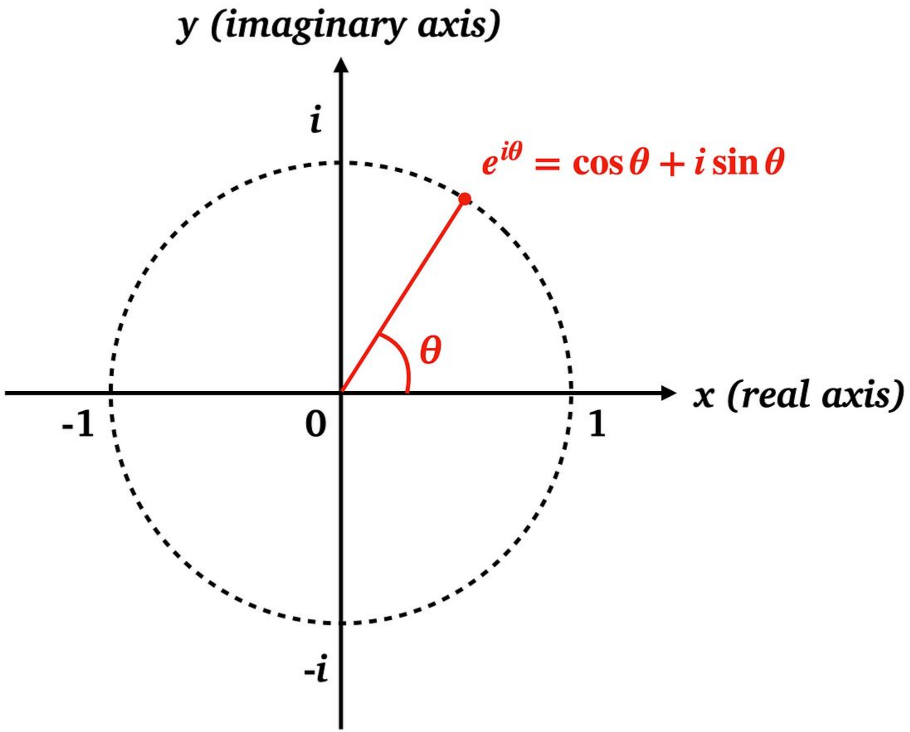
How to convert polar to cartesian?

$$x = r\cos\theta$$
$$y = r\sin\theta.$$

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$z = re^{i\theta} = r(\cos\theta + i\sin\theta) = \underbrace{r\cos\theta}_{x} + i\,\underbrace{r\sin\theta}_{y}.$$



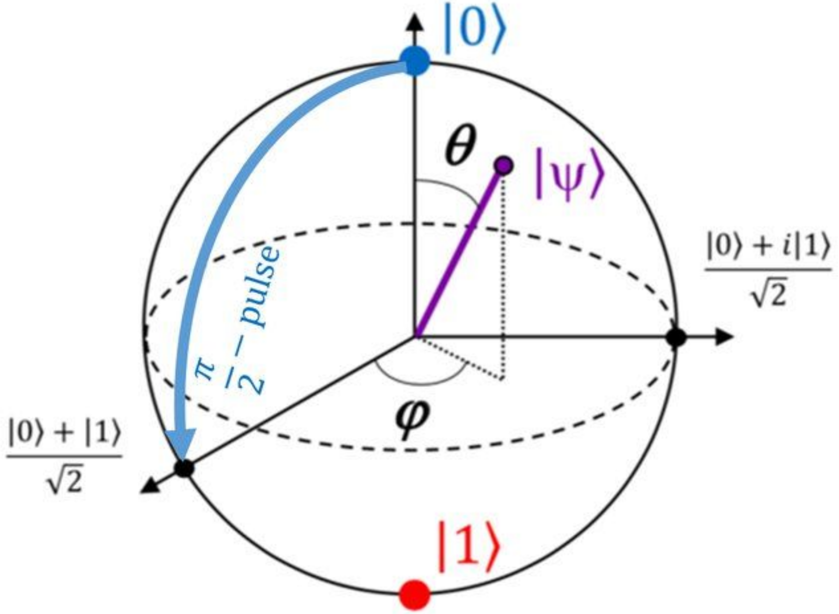
Euler formula is a bridge between trigonometric functions and exponential functions.

Bloch Sphere

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

$$\alpha = \cos\left(\frac{\theta}{2}\right), \quad \beta = e^{i\phi}\sin\left(\frac{\theta}{2}\right)$$

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$$



$0 \leq \theta \leq \pi$, θ is the angle wrt z axis

$0 \leq \phi < 2\pi$, ϕ is wrt x axis

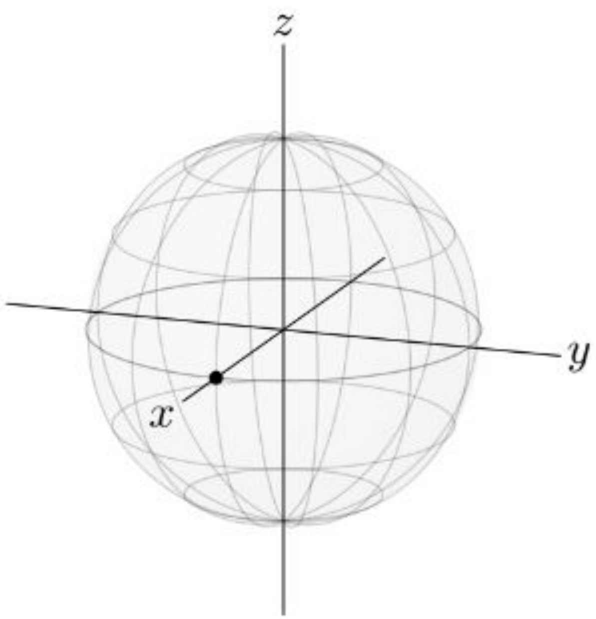
α is real and positive, β is complex, and the state is normalized.

Bloch Sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

(1,0,0) θ=pi/2, ϕ=0

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

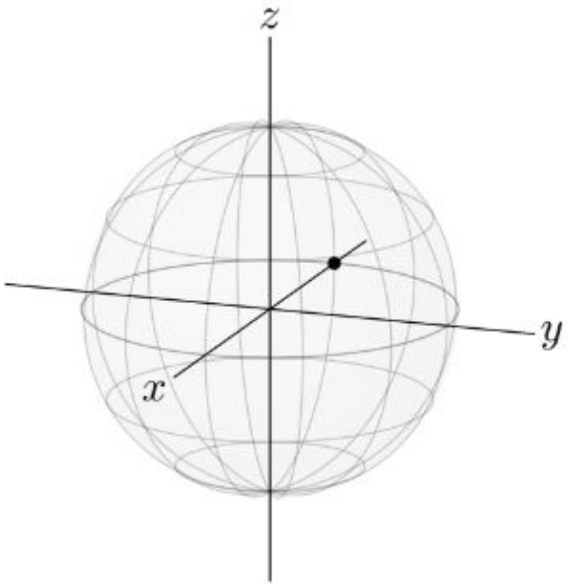


Bloch Sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

(-1,0,0) θ=pi/2, ϕ=pi

$$\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

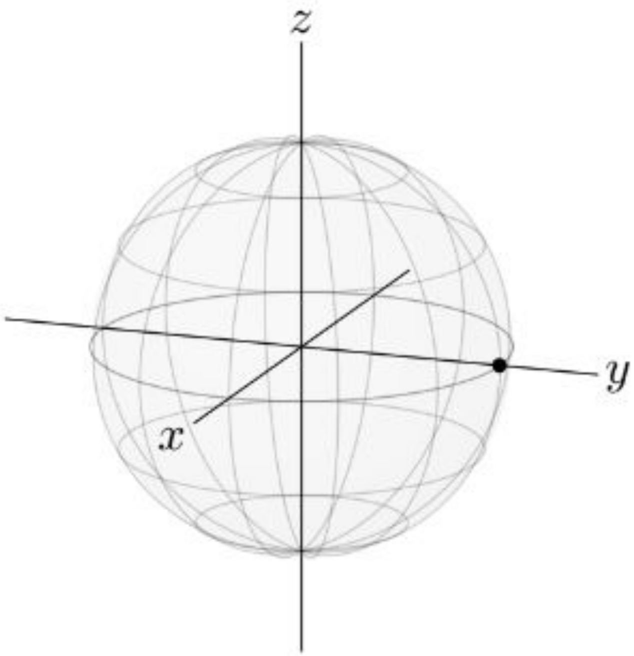


Bloch Sphere

$$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$$

(0,1,0) θ=pi/2, ϕ=pi/2

$$\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

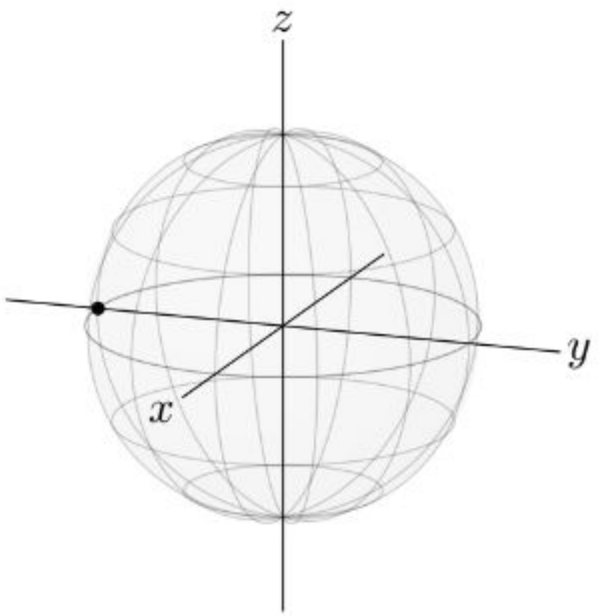


Bloch Sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

(0,1,0) θ=pi/2, ϕ=3pi/2

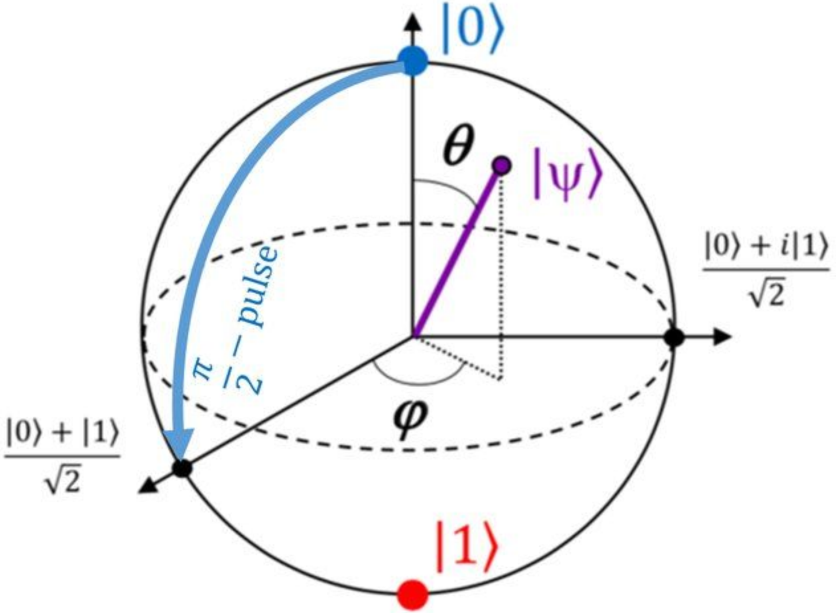
$$\frac{1}{\sqrt{2}}\left(|0\rangle - i|1\rangle\right)$$



For calculating anywhere on Bloch Sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

θ=?, ϕ=?



For calculating anywhere on Bloch Sphere

Calculations done in class

Example 1

1. The quantum state of a spinning coin can be written as a superposition of heads and tails. Using heads as $|1\rangle$ and tails as $|0\rangle$, the quantum state of the coin is

$$|\text{coin}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |0\rangle\right). \tag{2.3}$$

What is the probability of getting heads?

The amplitude of $|1\rangle$ is $\beta = 1/\sqrt{2}$, so $|\beta|^2 = \left(1/\sqrt{2}\right)^2 = 1/2$. So the probability is 0.5, or 50%.

Example 2

A weighted coin has twice the probability of landing on heads vs. tails. What is the state of the coin in “ket” notation?

Example 2

A weighted coin has twice the probability of landing on heads vs. tails. What is the state of the coin in “ket” notation?

$$P_{\text{heads}} + P_{\text{tails}} = 1 \quad \text{(Normalization Condition)}$$

$$P_{\text{heads}} = 2P_{\text{tails}} \quad \text{(Statement in Example)}$$

$$\to P_{\text{tails}} = \frac{1}{3} = \alpha^2$$

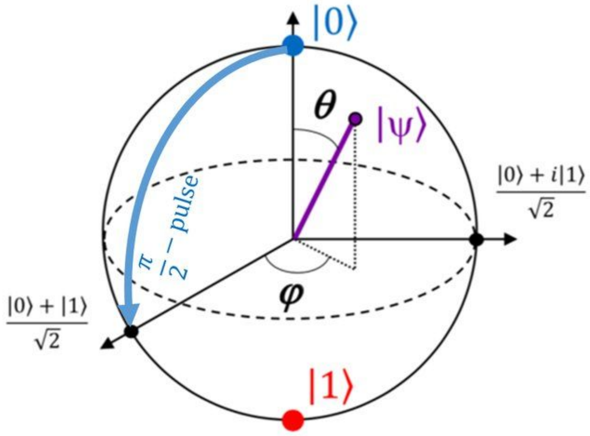
$$\to P_{\text{heads}} = \frac{2}{3} = \beta^2$$

$$\to \alpha = \sqrt{\frac{1}{3}},\ \beta = \sqrt{\frac{2}{3}}$$

$$\to |\text{coin}\rangle = \sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle.$$

For calculating anywhere on Bloch Sphere

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$



θ=?, ϕ=?

θ is the polar angle (latitude) on the Bloch sphere, ranging from 0 to π

ϕ is the azimuthal angle (longitude), ranging from 0 to 2π

Relative Phase

On the Bloch sphere, the relative phase ϕ \phi determines the qubit’s longitude.

Changing the relative phase ϕ rotates the qubit state around the Z-axis.

The relative phase of a quantum state is a measure of the angle in the complex plane.

Relative Phase

Two superpositions states whose amplitudes have the same magnitudes but that differ in a relative phase represent different states.

Relative phase is a physically important quantity.

Global Phase

Applying a global phase is like rotating the entire carousel by a certain angle. It doesn't change the relative positions of where you are on the carousel; it just shifts everything uniformly.

Global Phase

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

$$|\psi'\rangle = e^{i\phi_{\text{global}}}|\psi\rangle$$

$$|\psi'\rangle = e^{i\phi_{\text{global}}}\left[\cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle\right]$$

Global Phase

$$|\psi'\rangle = e^{i\phi_{\text{global}}}\left[\cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle\right]$$

$$|\psi'\rangle = \cos\left(\frac{\theta}{2}\right)e^{i\phi_{\text{global}}}|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i(\phi+\phi_{\text{global}})}|1\rangle$$

$$\alpha' = \cos\left(\frac{\theta}{2}\right)e^{i\phi_{\text{global}}}$$

$$\beta' = \sin\left(\frac{\theta}{2}\right)e^{i(\phi+\phi_{\text{global}})}$$

Global Phase

Global phases are physically irrelevant.

Just like Upgrading

Global Phase

$$e^{i\phi} = \cos(\phi) + i\sin(\phi)$$

$$|e^{i\phi}| = \sqrt{\cos^2(\phi) + \sin^2(\phi)}$$

$$|e^{i\phi}| = \sqrt{1} = 1$$

Global Phase

$$e^{i\theta}\left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right),$$

Calculate probabilities

Global phases can be dropped/ignored.

States that differ by a global phase are actually the same state; they correspond to the same point on the Bloch sphere.

Exercise 2.6. A qubit is in the state

$$\frac{1+i\sqrt{3}}{3}|0\rangle+\frac{2-i}{3}|1\rangle.$$

If you measure the qubit, what is the probability of getting

(a) $|0\rangle$?

(b) $|1\rangle$?

Exercise 2.7. A qubit is in the state

$$\frac{2}{3}|0\rangle + \frac{1+2i}{3}|1\rangle.$$

Say you measure the qubit and get $|0\rangle$. If you measure the qubit a *second time*, what is the probability of getting

(a) $|0\rangle$?

(b) $|1\rangle$?

Normalization

$$A\left(\sqrt{2}|0\rangle + i|1\rangle\right).$$

$$\begin{aligned}1 &= (A\sqrt{2})(A\sqrt{2})^* + (Ai)(Ai)^* \\ &= 2|A|^2 + |A|^2 \\ &= 3|A|^2 \\ |A|^2 &= \frac{1}{3}.\end{aligned}$$

$$A = \frac{1}{\sqrt{3}},$$

$$\frac{1}{\sqrt{3}}\left(\sqrt{2}|0\rangle + i|1\rangle\right).$$

Exercise 2.8. A qubit is in the state

$$\frac{e^{i\pi/8}}{\sqrt{5}}|0\rangle + \beta|1\rangle.$$

What is a possible value of β?

Exercise 2.9. A qubit is in the state

$$A\left(2e^{i\pi/6}|0\rangle - 3|1\rangle\right).$$

(a) Normalize the state (i.e., find A).
(b) If you measure the qubit, what is the probability that you get $|0\rangle$?
(c) If you measure the qubit, what is the probability that you get $|1\rangle$?

Electron as a quantum system

Electron as Qubit

An electron is a prototype for a qubit.

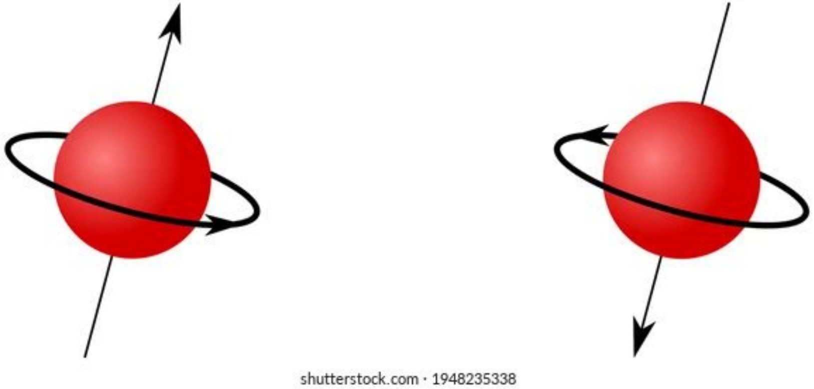
An electron has many measurable properties such as energy, mass, momentum.

But, for the purposes of creating a qubit, we want to focus on a property with only two measurable values. An electron has a two-state property which is called **spin**.

Electron as Qubit

The property was called spin because it can be described mathematically just like orbital momentum (angular momentum), but spin does not actually correspond to the electron physically rotating.

Just like a lot of quantum phenomena, spin can be confusing at first.



Electron Spin

$|\uparrow\rangle$ = spin up → clockwise

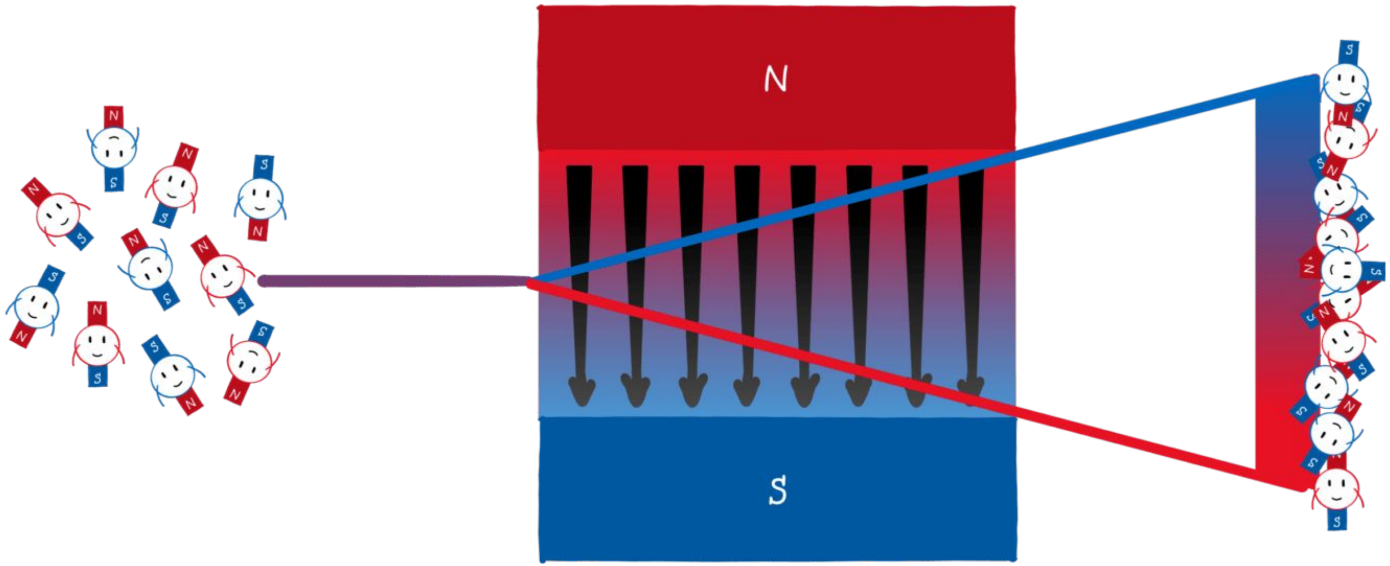
$|\downarrow\rangle$ = spin down → anticlockwise

If our electron — our quantum system — is just left alone then it is said to be in a superposition of both these states, In other words, the electron isn't $|\uparrow\rangle$ or $|\downarrow\rangle$, it's $|\uparrow\rangle$ and $|\downarrow\rangle$.

Stern-Gerlach Experiment

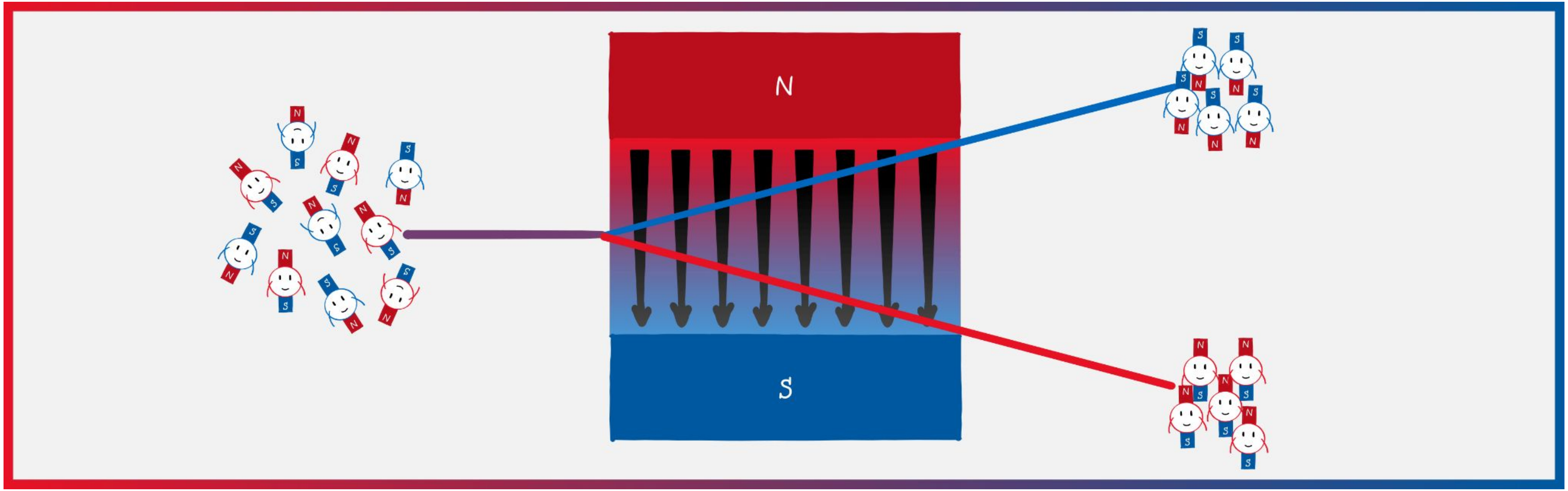
Otto Stern and Walther Gerlach in 1921

Atoms behave like mini-magnets



Stern-Gerlach Experiment

Atoms behave like mini-magnets



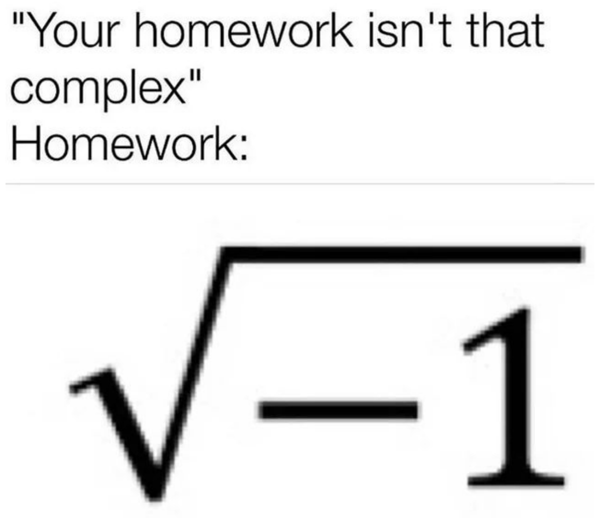
Qubit Rotations

Complex Conjugate

The complex conjugate of a complex number is obtained by changing the sign of its imaginary part.

Z = x + iy

Z’ = x – iy



HERMITIAN MATRIX

A matrix H is called Hermitian if it is equal to its own conjugate transpose (or Hermitian adjoint)

$$H = \begin{pmatrix} 2 & i \\ -i & 3 \end{pmatrix}$$

$$H = \begin{pmatrix} 4 & 1+2i & 3-i \\ 1-2i & 5 & 2+4i \\ 3+i & 2-4i & 6 \end{pmatrix}$$

Unitary Matrix

Changing a qubit's state through a physical action mathematically corresponds to multiplying the qubit vector by some unitary matrix U so that after the operation the state is now

$$|\psi'\rangle = U|\psi\rangle$$

Unitary is a mathematical term which expresses that U can only act on the qubit in such a way that the total probability remains same

Unitary Matrix

A matrix U is unitary if the matrix product of U and its conjugate transpose U† (called U-dagger) multiply to give the identity matrix:

$$U^\dagger U = U U^\dagger = I$$

One fundamental assumption is that each (matrix) operator must be unitary in all mathematical constructions of quantum mechanics

Single Qubit Gates

Analogous to logical gates in classical computing, single-qubit gates act on individual qubits, modifying their quantum states.

PAULI GATES

PHYSICIST WOLFGANG PAULI

When a Pauli gate is applied to a qubit, the state of the qubit is rotated around the corresponding axis of the Bloch sphere

These gates are important for manipulating the phase of a qubit

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli X-Gate

```
     ┌───────┐
q_0: ┤ Rx(Θ) ├
     └───────┘
```

Pauli X-gate perform a rotation of 180 degrees around the X axis

The gate is called NOT gate as it flips the qubit from |1> to |0> and vice versa

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

Pauli Y-Gate

Pauli y-gate perform a rotation of 180 degrees around the y axis

The gate is a combination of bit flip and phase flip

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Y|0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i|1\rangle$$

$$Y|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i|0\rangle$$

Pauli Z-Gate

Pauli y-gate perform a rotation of 180 degrees around the z axis

The gate leaves state |0> as such but flips the state |1> to -|1>

The gate is called phase flip

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$Z|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$$

Exercise

Prove each of the pauli matrices satisfies the unitary condition

$$X^\dagger X = I, \quad Y^\dagger Y = I, \quad Z^\dagger Z = I$$

Prove that Pauli matrices are Hermitian

Exercise 2.7. A qubit is in the state

$$\frac{2}{3}|0\rangle + \frac{1+2i}{3}|1\rangle.$$

Apply the X gate to this qubit state.

Exercise 2.7. A qubit is in the state

$$\frac{2}{3}|0\rangle + \frac{1+2i}{3}|1\rangle.$$

Apply the Y gate to this qubit state.

Exercise 2.7. A qubit is in the state

$$\frac{1+i\sqrt{3}}{3}|0\rangle + \frac{2-i}{3}|1\rangle.$$

Apply the X gate to this qubit state.

Exercise 2.7. A qubit is in the state

$$\frac{1+i\sqrt{3}}{3}|0\rangle + \frac{2-i}{3}|1\rangle.$$

Apply the Y gate to this qubit state.

Exercise 2.7. A qubit is in the state

$$\frac{1+i\sqrt{3}}{3}|0\rangle + \frac{2-i}{3}|1\rangle.$$

Apply the Z gate to this qubit state.

Rotation Operator

Rotation around the x-axis

$$R_x(\theta) = \cos\left(\frac{\theta}{2}\right) I - i \sin\left(\frac{\theta}{2}\right) \sigma_x$$

$$R_x(\theta) = \cos\left(\frac{\theta}{2}\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$R_x(\theta) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -i \sin\left(\frac{\theta}{2}\right) \\ -i \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

Rotation Operator

Rotation around the y-axis

$$R_y(\theta) = \cos\left(\frac{\theta}{2}\right) I - i\sin\left(\frac{\theta}{2}\right)\sigma_y$$

$$R_y(\theta) = \cos\left(\frac{\theta}{2}\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - i\sin\left(\frac{\theta}{2}\right)\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix}\cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right)\\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right)\end{pmatrix}$$

Rotation Operator

Rotation around the z-axis

$$R_z(\theta) = \cos\left(\frac{\theta}{2}\right) I - i \sin\left(\frac{\theta}{2}\right) \sigma_z$$

$$R_z(\theta) = \cos\left(\frac{\theta}{2}\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i \sin\left(\frac{\theta}{2}\right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{pmatrix}$$

Task

Prove this

$$\sigma_x = iR_x(-180^\circ) = -iR_x(-180^\circ)$$

Task

Prove this

$$\sigma_x = iR_x(-180^\circ) = -iR_x(-180^\circ)$$

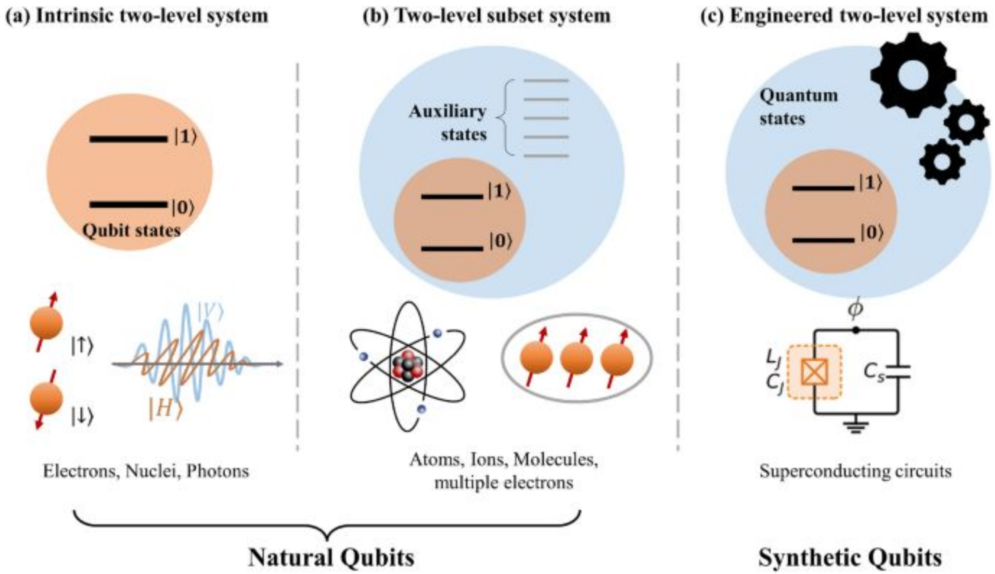
$$\sigma_y = -iR_y(180^\circ) = iR_y(-180^\circ)$$

$$\sigma_z = -iR_z(180^\circ) = iR_z(-180^\circ)$$

Types of Qubits

There are many kinds of qubits, some occurring naturally and others that are engineered. Some of the most common types include:

- Spin
- Trapped Atoms and Ions
- Photons
- Superconducting Circuits



What technologies are used to build quantum computers?

	Superconducting loops	Trapped ions	Silicon quantum dots	Topological qubits	Diamond vacancies
Company support	Google, IBM, Quantum Circuits	ionQ	Intel	Microsoft, Bell Labs	Quantum Diamond Technologies
Pros	Fast working. Build on existing semiconductor industry.	Very stable. Highest achieved gate fidelities.	Stable. Build on existing semiconductor industry.	Greatly reduce errors.	Can operate at room temperature.
Cons	Collapse easily and must be kept cold.	Slow operation. Many lasers are needed.	Only a few entangled. Must be kept cold.	Existence not yet confirmed.	Difficult to entangle.

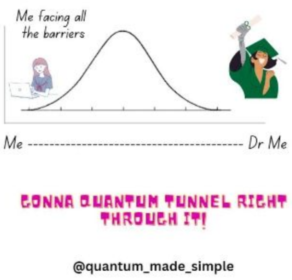
quantum_made_simple Following Message

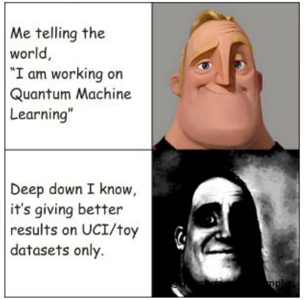
17 posts **67** followers **2** following

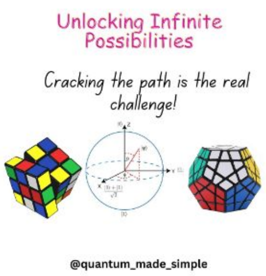
Quantum Made Simple
Quantum for everyone!

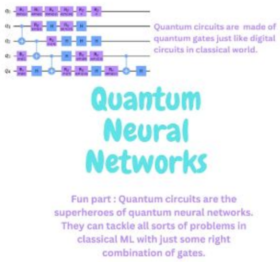
Followed by javeria.ilyas.21, _saffia_baloch_ + 40 more

POSTS TAGGED









References

https://www.youtube.com/watch?v=UjaAxUO6-Uw

https://physicus-minimus.com/en/the-stern-gerlach-experiment/

https://medium.com/@satoshihgsn/e-i%CF%80-1-0-the-most-beautiful-theorem-in-mathematics-493a7f4e7332

https://javafxpert.github.io/grok-bloch/