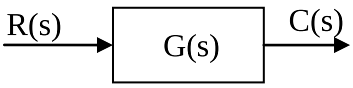
Feedback in Control

2.1 Introduction

- ✓ Feedback systems play an important role in modem engineering practice because they have the possibility for being adopted to perform their assigned tasks automatically.
- ✓ A non-feedback or **open-loop system** (represented by the block diagram and signal flow graph below) is activated by a single input signal for single-input systems.



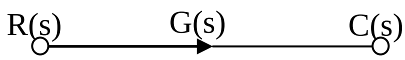
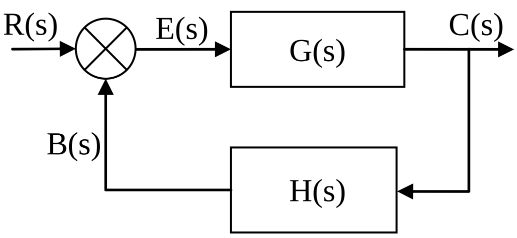


Fig. 2.1 Open loop system

- ✓ It can be seen from the above figs. that there is no provision within this system for supervision of the output and also no mechanism is provided to correct or compensate the system behaviour for any lack of proper performance of system components, changing environment, loading or ignorance of the exact value of process parameters.
- ✓ On the other hand, a feedback or **closed-loop** system (represented by the block diagram and signal flow graph in Fig. 2.2) is driven by two signals (more signals could be employed), one the input signal and the other, a signal called the feedback signal derived from the output of the system.



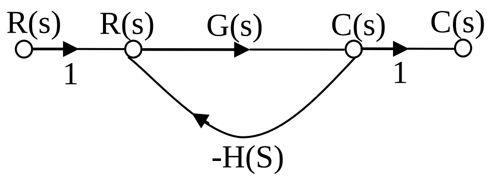


Fig. 2.2 Closed loop system

- ✓ The feedback signal gives this system the capability to act as self-correcting mechanism as explained below.
- ✓ The output signal C(s) is measured by a sensor H(s), which produces a feedback signal B(s).
- ✓ The comparator compares the feedback signal B(s) with the input or command signal R(S) generating the actuating signal E(s), which is as measure of discrepancy between R(s) and B(s).
- ✓ The actuating signal is applied to the process G(s) so as to influence the output C(s) in a manner which tends to reduce the errors.
- ✓ The beneficial effect of feedback in feedback system with high loop gain is given as follows;
  1. The controlled variable accurately follows the desired value.

2. Effect on the controlled variable of external disturbances other than those associated with the feedback sensor is greatly reduced.
3. Effect of variation in controller and process parameters (the forward path) on system performance is reduced to acceptable levels. These variations occur due to aging, environmental changes etc.

   Feedback in the control loop allows accurate control of the output (by means of the input signal) even when process or controlled plant parameters are not known accurately.
4. Feedback in a control system greatly improves the speed of its response compared to the response speed capability of the plant/components composing the system (forward path)

Limitations of Feedback

✓ These are: greater system complexity, need for much larger forward path gain and possibility of system instability (it means undesired or persistent oscillations of the output variable).

2.2 Minimization of the parameter variations using Feedback

✓ One of the primary purposes of using feedback in control systems is to reduce the sensitivity of the system to parameter variations.

✓ The parameters of a system may vary with age, with changing environment (e.g.; ambient temperature), etc.

✓ Conceptually, sensitivity is a measure of the effectiveness of feedback in reducing the influence of these variations on system performance.

✓ Let us define sensitivity on a quantitative basis. In the open-loop case

$$C(s) = G(s)R(s)$$

✓ Suppose due to parameter variations G(s) changes to $|G(s)+\Delta G(s)|$ where $|G(s)| >> |\Delta G(s)|$.

✓ The output of the open-loop system then changes to

$$C(s)+\Delta C(s) = [G(s)+\Delta G(s)]R(s) \tag{2.1}$$

or, $$\Delta C(s) = \Delta G(s)R(s)$$

✓ Similarly, in the closed-loop case, the output

$$C(s) = \frac{G(s)}{1+G(s)H(s)}R(s)$$

changes to $$C(s)+\Delta C(s) = \frac{G(s)+\Delta G(s)}{1+G(s)H(s)+\Delta G(s)H(s)}R(s)$$

due to the variation $\Delta G(s)$ in G(s), the forward path transfer function.

✓ Since $|G(s)| >> |\Delta G(s)|$, we have the variation in the output as

$$\Delta C(s) = \frac{\Delta G(s)}{1+G(s)H(s)}R(s) \tag{3.2}$$

- ✓ From eqns. (3.1) and (3.2) it is seen that in comparison to the open-loop system, the change in the output of the closed-loop system due to variation G(s) is reduced by a factor of $[1+G(s)H(s)]$ which is much greater than unity in most practical cases over the frequency $(s = j\omega)$ of interest.
- ✓ The term ***system sensitivity*** is used to describe the relative variation in the overall transfer function $T(s) = C(s)/R(s)$ due to variation in G(s) and is defined below.

Sensitivity = percentage change in T(s) / percentage change in G(s)

- ✓ For small incremental variation in G(s), the sensitivity is written in the quantitative form as

$$S_G^T = \frac{\partial T/T}{\partial G/G} = \frac{\partial LnT}{\partial LnG} \tag{2.3}$$

where S_G^T denotes the sensitivity of T with respect to G.

- ✓ In accordance with the above definition, the sensitivity of the closed-loop system is

$$S_G^T = \frac{\partial T}{\partial G} \times \frac{G}{T} = \frac{(1+GH) - GH}{(1+GH)^2} \times \frac{G}{G/(1+GH)} = \frac{1}{1+GH} \tag{3.4}$$

- ✓ Similarly, the sensitivity of the open-loop system is

$$S \frac{\partial T}{\partial G} \times \frac{G}{T} = 1 \quad \text{(in this case T = G)} \tag{3.5}$$

- ✓ Thus, the sensitivity of a closed-loop system with respect to variation in G is reduced by a factor $(1+GH)$ as compared to that of an open-loop system.
- ✓ The sensitivity of T with respect to H, the feedback sensor, is given by

$$S_H^T = \frac{\partial T}{\partial T} \times \frac{H}{T} = G\left[\frac{-G}{(1+GH)^2}\right] \frac{H}{G/(1+GH)} = \frac{-GH}{1+GH} \tag{3.6}$$

- ✓ The above equation shows that for large values of GH, sensitivity of the feedback system with respect to H approaches unity.
- ✓ Thus, we see that the changes in H directly affect the system output.
- ✓ Therefore, it is important to use feedback elements which do not vary with environmental changes or can be maintained constant.
- ✓ Very often the system's sensitivity is to be determined with a particular parameter (or parameters) with the transfer function expressed in ratio of polynomial form i.e.,

$$T(s) = \frac{N(s,\alpha)}{D(s,\alpha)}; \ \alpha = \text{Parameter under consideration}$$

- ✓ From eqn. (3.3)

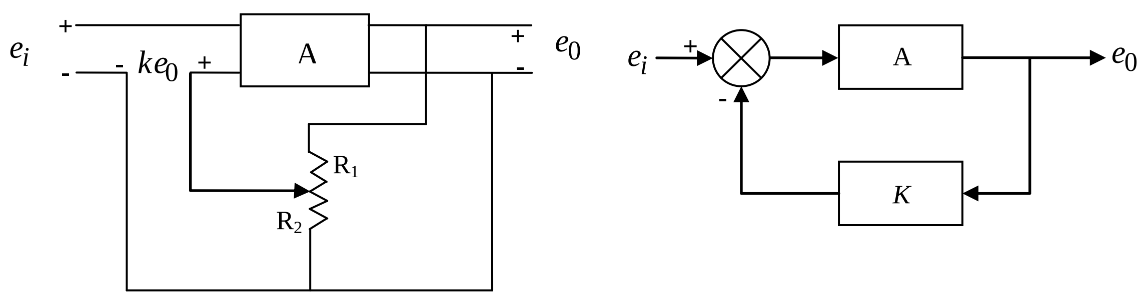
$$S_\alpha^T = \left.\frac{\partial LnN}{\partial Ln\alpha}\right|_{\alpha_0} - \left.\frac{\partial LnD}{\partial Ln\alpha}\right|_{\alpha_0} \tag{3.7}$$

$$= S_\alpha^N - S_\alpha^D \tag{3.8}$$

where α_0 is the nominal value of the parameter around which the variation occurs.

- ✓ The use of feedback in reducing sensitivity to parameter variations is an important advantage of feedback control systems.
- ✓ To have a highly accurate open-loop system, the components of G(s) must be selected to meet the specifications rigidly in order to fulfil the overall goals of the system.
- ✓ On the other hand, in a closed-loop system G(s) may be less rigidly specified, since the effects of parameter variations are mitigated by the use of feedback.
- ✓ However, a closed-loop system requires careful selection of the components or the feedback sensor H(s).
- ✓ Since G(s) is made up of power elements and H(s) is made up of measuring elements which operate at low power levels, the selection of accurate H(s) is for less costly than that of G(s) to meet the exact specifications.
- ✓ The price for improvement in sensitivity by use of feedback is paid in terms of loss of' system gain.
- ✓ The open-loop system has a gain G(s), while the gain of the closed-loop system is $G(s)/[1+G(s)H(s)]$.
- ✓ Hence by use of feedback, the system gain is reduced by the same factor as by which the sensitivity of the system to parameter variations is reduced. Sufficient open-loop gain can, however, be easily built into a system so that we can afford to lose some gain to achieve improvement in sensitivity.

Example to understand the effect of feedback in reducing the system's sensitivity to parameter variations is described as follows



2.3 Feedback Amplifier

- ✓ The feedback amplifier with negative feedback, provided through a potential divider (with feedback gain <1) is shown in Fig. 2.3.
- ✓ Assuming the input impedance of the amplifier to be infinite and its output impedance as zero.
- ✓ Observe that both the forward gain A and feedback gain k (<1) are independent of the frequency (in the range of frequencies of our interest).
- ✓ It can be easily followed from the block diagram that overall gain of the amplifier current is

$$\frac{e_0}{e_i} = T = \frac{A}{1+kA}; \quad k = \frac{R_2}{R_1} \le 1 \tag{3.9}$$

$$S_A^T = \frac{dT}{dA}\frac{A}{T} = \frac{1}{1+kA} \tag{3.10}$$

For $A = 10^4, k = 0.1$; $\qquad S_A^T = \frac{1}{1+10^3} = 0.001$

- ✓ While the feedback reduces the sensitivity to variation in forward gain to a very low figure (0.001), it also reduces the overall gain to

$$T = \frac{10^4}{1+10^3} = 10\text{; compare with forward gain of } 10^4$$

- ✓ Now sensitivity to feedback gain is given by

$$S_k^T = \frac{dT}{dk}\frac{k}{T} = \frac{-kA}{1+kA} = \frac{-10^3}{1+10^3} = -1 \tag{3.11}$$

- ✓ S_k^T being equal to unity, the feedback constant $k = R_2/R_1$ must not vary i.e., the resistor ratio R_2/R_1 must be accurate and stable.
- ✓ In fact for such large $A(=10^4)$, $kA >> 1$ and so from eqn. (3.9)

$$T = \frac{1}{k} = \frac{R_1}{R_2} = 10; \quad \text{independent of A}$$

2.3 Control Over System Dynamics using Feedback

- ✓ Let us consider the elementary single-loop feedback system of fig. 3.4.
- ✓ The open-loop transfer function of this system is

$$\frac{C(s)}{R(s)} = G(s) = \frac{K'}{s+\alpha} \tag{3.12}$$

$$= \frac{K}{\tau s+1}; \; K = K'/\alpha, \; \tau = 1/\alpha \tag{3.13}$$

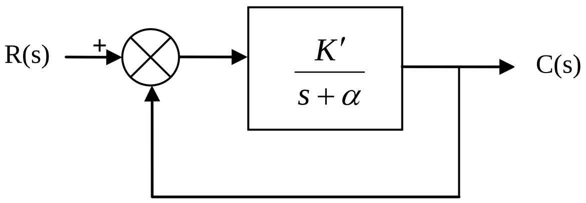


Fig. 2.4 A simple feedback system

- ✓ These are two alternative forms of expressing a transfer function.
- ✓ At $s = -\alpha$, G(s) tends to infinity so is known as the pole of the system, while $\tau = 1/\alpha$ is known as its time constant.
- ✓ The dc gain of the system is

$$G(0) = K = K'/\alpha$$

- ✓ With the feedback loop closed the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K}{\tau s + (1+K)} = \frac{K'}{s+(\alpha+K')} \tag{3.14}$$

$$= \frac{K/(1+K)}{\tau_c s+1}; \quad \tau_c = \tau/(1+K) \tag{3.15}$$

- ✓ We find from eqns. (3.14) and (3.15) that the effect of closing the loop (that is introduction of negative feedback) is to shift the system's pole from $-\alpha$ to $(\alpha + K')$ or $-\alpha(1+K)$.
- ✓ Alternatively to reduce the system time constant from τ to $\tau/(1+K)$.
- ✓ Of course in the mean time the dc gain has reduced to $K(1+K)$.

We shall now examine the effect of these changes in system transfer function on its dynamic response.

- ✓ For this purpose we shall assume that the system is excited (disturbed) by an impulse input $r(t) = \delta(t)$.
- ✓ The Laplace transform of an impulse excitation is $R(s) = 1$.
- ✓ Taking the inverse Laplace transform of eqn. (3.12) and (3.14) with R(s) = 1, we have

$$c(t) = L^{-1}\frac{K'}{s+\alpha} = K'e^{-\alpha t}$$
$$= K'e^{-t/\tau} \qquad \text{(for open-loop system)} \qquad (3.16)$$

- ✓ The location of pole and the dynamic response of open-loop and feedback system are shown in Fig. 2.5.
- ✓ These responses decay in accordance with respective time constants.

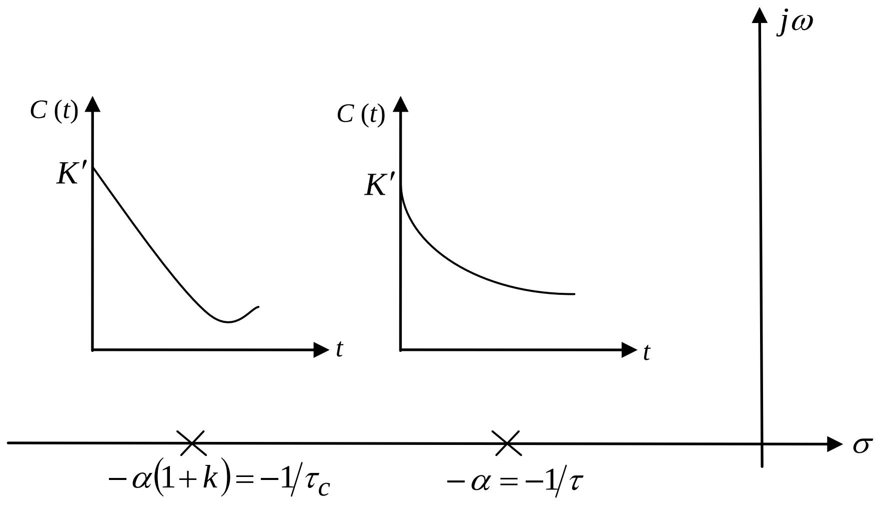


Fig. 2.5 Impulse response of open and closed loop system

- ✓ As the closed-loop time constant $\tau_c = \tau/(1+K)$, its response decays **much faster** which means that the speed of the system's response with loop closed is faster by a factor of (1+*K*) compared to the open-loop system.

The response decay to $e^{-5} = 0.0067$ or 0.67% of the value immediately after application of impulse when time is 5τ.

- ✓ From this example, it is concluded that feedback controls the dynamics of the system by adjusting the location of its poles.
- ✓ It is, however, important to note here that feedback introduces the possibility of instability, i.e., a closed-loop system may be unstable even though the open-loop is stable.

- **Effect of feedback on bandwidth**

✓ A control system is a low-pass filter-it responds to frequencies from dc to a certain value ω_b at which the gain drops to $1/\sqrt{2}$ of its dc value.

✓ This frequency ω_b is the bandwidth of the system.

✓ A large bandwidth implies that the system responds accurately to higher frequencies i.e., fast changing signals, which is another way (frequency domain) of looking at the speed of response of a control system.

✓ Let us consider the system in fig. 2.4 again.

✓ The open- and closed-loop transfer functions (eqns.3.13 and 3.15) can be expressed in the frequency domain ($s = j\omega$) as

$$G(s) = \frac{K}{j\omega\tau + 1}; \qquad \text{open-loop} \qquad (3.17)$$

$$G(s) = \frac{K/(1+K)}{j\omega\tau_c + 1}; \qquad \text{closed-loop} \qquad (3.18)$$

✓ Their bandwidth is determined as below

$$(\omega_b\tau)^2 + 1 = (\sqrt{2})^2 \text{ or } \omega_b(OL) = 1/\tau$$

$$(\omega_b\tau_c)^2 + 1 = (\sqrt{2})^2 \text{ or } \omega_b(CL) = 1/\tau_c$$

$$\frac{\omega_b(CL)}{\omega_b(CL)} = \tau/\tau_c = (1+K)$$

✓ Thus the closed-loop system has a bandwidth (1+*K*) times the bandwidth of the open-loop system this implies increased speed to response.

2.4 Control of the Effects of Disturbance Signals using Feedback

✓ Fig 2.6 shows the signal flow graph of a closed-loop system with the disturbance signal T_D in the forward path.

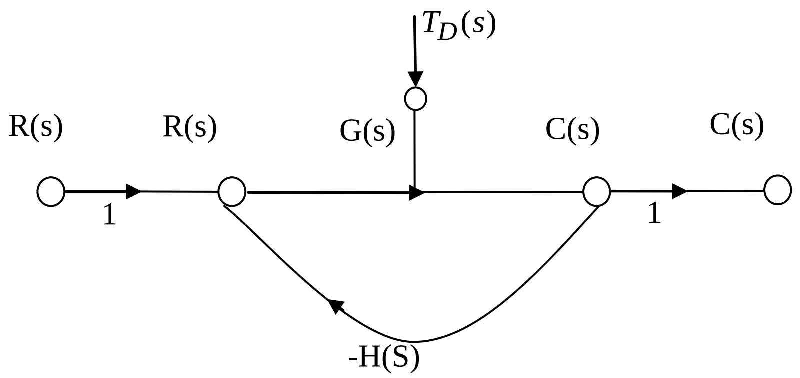


Fig. 2.6 A closed loop-system with added disturbance

✓ The ratio of the output C(s) to the disturbance signal $T_D(s)$, when $R(s) = 0$, is obtained by applying the signal flow gain formula to the graph of fig. 2.6, and is given by

$$\frac{C_D(s)}{T_D(s)} = \frac{-G_2(s)}{1+G_1(s)G_2(s)H(s)} \tag{3.19}$$

- ✓ If $|G_1G_2H(s)| >> 1$ over the working range of 's', then from eqn. (3.19) we have

$$\frac{C_D(s)}{T_D(s)} \approx \frac{-1}{G_1(s)H(s)} \tag{3.20}$$

- ✓ Thus, it can be seen that if $G_1(s)$ is made sufficiently large, the effect of disturbance can be decreased by feedback.
- ✓ Generally, feedback is introduced by a set of additional elements, called the measurement sensor '*H*' which may itself generate some noise.
- ✓ Let us evaluate the effect of this noise on the system performance.
- ✓ Fig. 2.7 shows the signal flow graph of a system with the noise signal $N(s)$ in the feedback path.

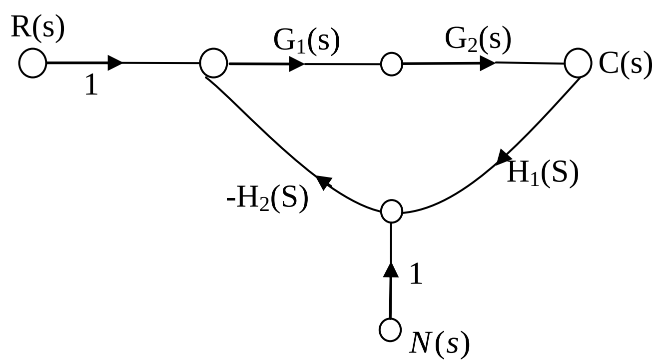


Fig. 2.7 Closed-loop system with measurement noise

- ✓ Using the gain formula for the signal flow graph, the following result is obtained.

$$\left.\frac{C(s)}{N(s)}\right|_{R(s)=0} = \frac{C_n(s)}{N(s)} = \frac{-G_1(s)G_2(s)H_2(s)}{1+G_1(s)G_2(s)H_1(s)H_2(s)}$$

- ✓ For large values of loop gain ($|G_1G_2H_1(s)H_2(s)| >> 1$), the above equation reduces to

$$\frac{C_n(s)}{N(s)} \approx -\frac{1}{H_1(s)}$$

- ✓ Therefore, the effect of noise on output is

$$C_n(s) = -\frac{N(s)}{H_1(s)} \tag{3.21}$$

- ✓ Thus, for optimum performance of the system, the measurement sensor should be designed such that $H_1(s)$ is maximum, which is equivalent to maximizing the signal-to-noise ratio of the sensor.
- ✓ The design specifications of the feedback sensor are far more stringent than those of the forward path transfer function.

- ✓ The feedback sensor must have low parameter variations as these are directly reflected in system response (the sensitivity $S_H^T \approx -1$).
- ✓ Further the signal-to-noise ratio for the sensor must be high as explained above.
- ✓ Usually it is possible to design and construct the sensor with such stringent specifications and at reasonable cost because the feedback elements operate at low power level.
- ✓ To conclude, the use of feedback has the advantages of reducing sensitivity, improving transient response and minimizing the effects of disturbance signals in control systems.
- ✓ On the other hand, the use of feedback increases the number of components of the system, thereby increasing its complexity.
- ✓ Further it reduces the gain of the system and also introduces the possibility of instability.
- ✓ However, in most cases the advantages outweigh the disadvantages and therefore the feedback systems are commonly employed in practice.

2.5 Linearizing Effect of Feedback

- ✓ Yet another property of feedback is its linearizing effect which is illustrated by means of the simple single-loop static system of Fig. 2.8.

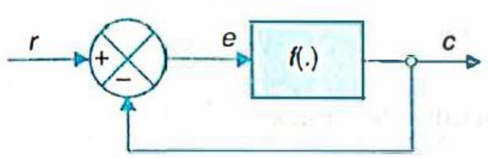


Fig. 2.8 single-loop static system

- ✓ In a static system various gains (transmittances) are independent of time.
- ✓ We shall assume that the forward block function is nonlinear expresses as

$$e = f(e) = e^2 \text{; square law function}$$

- ✓ When the feedback loop is open

$$e = r \Rightarrow c = r^2$$

this is plotted in fig.2.9 (i).

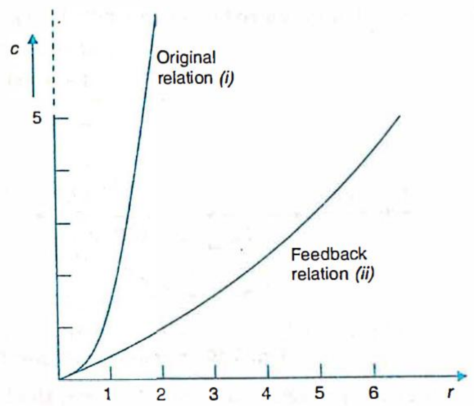


Fig. 2.9 Plot of output (c) when feedback is open

✓ On the other hand when the loop is closed, we have

$$e = r - c$$

$$\text{and so, } e = f(e) = (r - c)^2$$

which is shown in fig. 2.9(*ii*).

✓ It is easily seen by comparison of the graphs 2.9(*i*) and 2.9(*ii*) that the input-output relation [*c*(*r*)] is approximately linear over a much wider range for the closed-loop system compared to its open-loop behaviour.

2.6 Linearizing Effect of Feedback

✓ The preceding material in this chapter has emphasized a negative or degenerative type of feedback.

✓ In regenerative feedback, the output is feedback with positive sign as shown in Fig. 2.10.

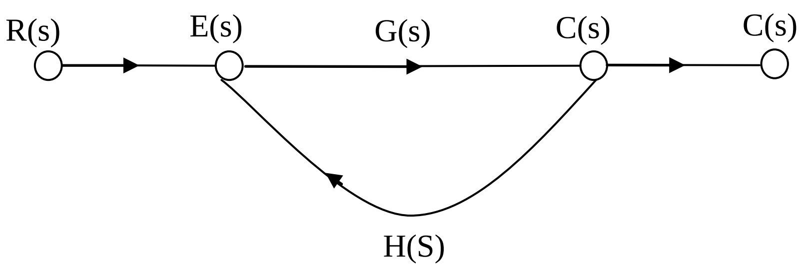


Fig. 2.10 A regenerative feedback control system

✓ In this case, the transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} \tag{3.25}$$

✓ There is a negative sign in the denominator of eqn. (3.25) which indicates the possibility of denominator becoming equal to zero thereby giving an infinite output for a finite input which is the condition of instability.

✓ The regenerative feedback is sometimes used for increasing the loop gain of feedback systems.

✓ Fig. 2.11 shows a feedback system with an inner loop having regenerative feedback.

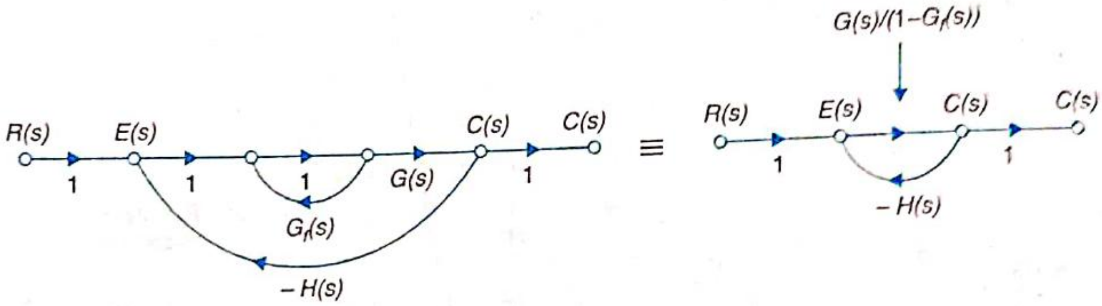


Fig. 2.11 Increasing loop gain by regenerative feedback

✓ This signal flow graph reduces to a single-loop graph whose loop gain is

$$\frac{-G(s)H(s)}{1-G_f(s)}$$

- ✓ If $G_f(s)$ is selected to be nearly unity, the loop gain becomes very high and the closed-loop transfer function approximates to

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G_f(s)+G(s)H(s)} \approx \frac{1}{H(s)} \qquad (3.26)$$

- ✓ Thus due to high loop gain provided by the inner regenerative feedback loop, the closed-loop transfer function becomes insensitive to G(s).