Electrostatics: Part-3

Presented by

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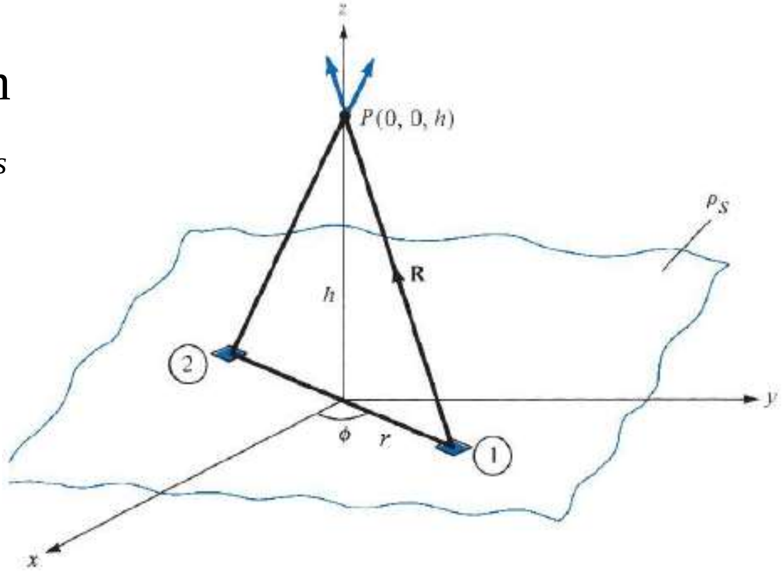
(B) Surface Charge



Let us consider an infinite sheet of charge in the x-y plane having uniform charge density ρ_s

Now consider an elemental area dS_1

Thus, we can write the charge associated with this area is-

$$dQ_s = \rho_s dS \qquad (24)$$

Now we want to find electric field intensity $(\vec{E})$ at point (0, 0. h) due to charge dQ on the elemental surface dS_1

So, we can write - $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$ or $d\vec{E} = \dfrac{dQ}{4\pi\varepsilon_0 R^2}\hat{a}_R$ (25)

From the fig. we can write -

$$\vec{R} = r(-\hat{a}_r) + h\hat{a}_z$$

and

$$R = |\vec{R}| = \sqrt{r^2(-\hat{a}_r)^2 + h^2\hat{a}^2{}_z} = (r^2 + h^2)^{\frac{1}{2}}$$

$$\hat{a}_R = \frac{\vec{R}}{R} = \frac{-r\hat{a}_r + h\hat{a}_z}{[r^2 + h^2]^{\frac{1}{2}}}$$

Now, $dQ = \rho_s dS = \rho_s r d\varphi dr$

Substituting these terms in (25) results in -

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0}\frac{\vec{R}}{R^3}$$

$$d\vec{E} = \frac{\rho_s r d\varphi dr}{4\pi\varepsilon_0}\frac{[-r\hat{a}_r + h\hat{a}_z]}{[r^2 + h^2]^{\frac{3}{2}}}$$

Here one thing should be known that, only z-component of $\vec{E}$ is exists. *x* and *y* components will be zero. **Why ?**

Let us see the fig. the electric field intensity $\vec{E}$ due to dS_1 has three components $E_x(-\hat{a}_x)$ along (-*x*) direction, $E_y(-\hat{a}_y)$ along (-*y*) and $E_z(\hat{a}_z)$ along z-axis.

But electric field intensity of the same magnitude will exists at point due to dS_2 , whose distance from the origin is same as dS_1 but in opposite site.

Due to the elementary charge dQ present in the elementary surface dS_2 we will also get $\vec{E}$ at point '*P*'

Now if we divide this (**2nd one**) into three components, we will get $E_x(\hat{a}_x)$ along $+x$-axis, $E_y(\hat{a}_y)$ along $+y$-axis and $E_z(\hat{a}_z)$ along *z*-axis.

As both the field intensity in equal in magnitude then all the components will cancel out except z-component.

Thus, in (26) $\hat{a}_r$ component will be zero.

Hence,

$$d\vec{E}_z = \frac{\rho_s r dr d\varphi h \hat{a}_z}{4\pi\varepsilon_0 \left[r^2 + h^2\right]^{\frac{3}{2}}}$$

$$\vec{E} = \int_s d\vec{E}_z = \int_s \frac{\rho_s r dr d\varphi h}{4\pi\varepsilon_0 \left[r^2 + h^2\right]^{3/2}} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_{\varphi=0}^{2\pi} \int_{r=0}^{\infty} \frac{r dr d\varphi h}{\left[r^2 + h^2\right]^{3/2}} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \int_{\varphi=0}^{2\pi} d\varphi \int_{r=0}^{\infty} \frac{r dr}{\left[r^2 + h^2\right]^{3/2}} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \left[\varphi\right]_0^{2\pi} \int_{r=0}^{\infty} \frac{r dr}{\left[r^2 + h^2\right]^{3/2}} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \left[2\pi - 0\right] \int_{r=0}^{\infty} \frac{r dr}{\left[r^2 + h^2\right]^{3/2}} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0} \int_{r=0}^{\infty} \frac{r dr}{\left[r^2 + h^2\right]^{3/2}} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0} \left[-\frac{1}{\sqrt{r^2 + h^2}}\right]_0^{\infty} \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0} \cdot \frac{1}{h} \hat{a}_z$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0} \hat{a}_z} \qquad (27)$$

i.e. the electric field intensity has only z-component if the charge distributed in *x*-*y* plane.

The equation (27) is valid only for $h > 0$; but if $h < 0$ then we need to replace $\hat{a}_z$ with $-\hat{a}_z$.

So in general, for infinite sheet of charge -

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_n} \tag{28}$$

From, (27) and (28) we can see that the electric field is normal to the sheet and it is surprisingly independent of the distance between the sheet and the point of observation '*P*'.

Electric flux density :-

We can calculate flux due to the electric field $\vec{E}$ using the general definition of flux as -

$$\psi = \int_S \vec{E} \bullet dS \tag{29}$$

But for practical reasons, this quantity ψ is not considered as most useful parameter in electrostatics.

Because we have seen that the electric field intensity $\vec{E}$ is dependent on the medium in which the charge is placed. (**In this case free space**)

Hence, we need a new parameter that is not dependent on the medium and that **is electric flux density** $\vec{D}$**,** which is defined as-

$$\boxed{\vec{D} = \varepsilon_0 \vec{E}} \tag{30}$$

Now, we can write the electric flux in terms of $\vec{D}$ as -

$$\boxed{\psi = \int_S \vec{D} \bullet dS} \tag{31}$$

In other way we can see -

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \Rightarrow \varepsilon_0\vec{E} = \frac{Q}{4\pi r^2}\hat{a}_r \Rightarrow \vec{D} = \frac{Charge}{Area}$$

As the electric flux is measured in Coulomb’s, so the electric flux density should be measured in Coulomb’s/ m^2

Finally, we can say that all the calculations for $\vec{E}$ can be expressed in terms of $\vec{D}$

Example:- we can say that for an infinite sheet of charge -

$\vec{D} = \frac{\rho_s}{2}\hat{a}_n$ (32) ; and for volume charge distribution - $\vec{D} = \int_v \frac{\rho_v dv}{4\pi R^2}\hat{a}_R$ (33)

The equations (32) and (33) shows that, they are independent of the medium.

Thank you