Wind energy

WIND ENERGY.

As the solar radiation falling in different angle depending upon the latitude, declination angle etc, there is a temperature gradient which causes the flow of wind.

Available power of the wind.

When an air mass flows with a vel. V m/s through an area A, mass flow rate $= \rho A V$ (kg/s)

$\therefore\ KE/_{sec} = \frac{1}{2} m v^2 = \frac{1}{2} \rho A V \cdot V^2 = \frac{1}{2} \rho A V^3$. Watt.

$\rho \rightarrow$ density, (kg/m³)

$A \rightarrow$ sweept area. (m²)

$V \rightarrow$ Undisturbed wind vel. in m/s.

Wind Power is directly proportional to height.

Factors under consideration

i. Roughness - The wind vel. slows down near the ground level due to effect of surface roughness. The wind speed increases with the increase in height. The rate of increase in wind velocity with height strongly depends upon the roughness of the terrain. For various terrain cond^ns, roughness factor Z_0 is determined.

The mathematical expression for ~~f~~ speed at any height z is given by -

$$\frac{V_z}{V_{zr}} = \ln\left(\frac{z}{z_0}\right) \Big/ \ln\left(\frac{z_r}{z_0}\right).$$

z_r = Reference level.

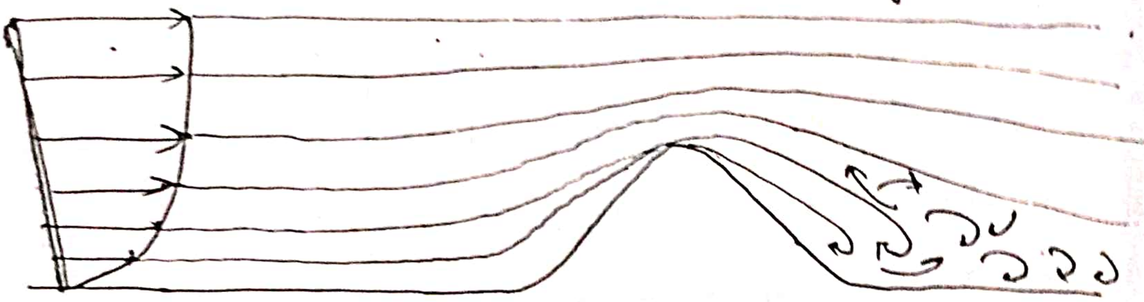
V_{zr} = Vel. at reference level.

It can also be calculated from the 1/7th law

$$\frac{V}{V_0} = \left(\frac{H}{H_0}\right)^{1/7}.$$

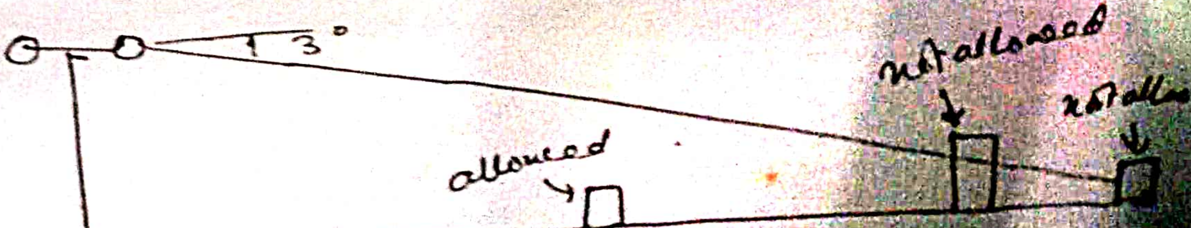
2) Turbulence - When wind flows over a building or a rough surface, it exhibits a change in wind velocity as well as in direction & leads to turbulence. This turbulence influence the power output of a wind machine.

3) Acceleration on ridges – The top of hills / ridges experience high wind speed due to the effect of wind shear at the ridges & also acts as a concentrator for air stream which causes the air stream to accelerate at the top of the ridges.



i) Direction of wind velocity – It is very important factor for a site selection. It depends on the wind velocity i.e. dirⁿ as well as obstruction, present in a particular region.

) 3° Freedom – This states that there should not be any obstacle on the line that makes an angle 3° at the point of rotation of the wind turbine.



Energy Pattern Factor

Because of the fluctuating trend in the wind velocity, the actual energy content in the fluctuating wind is higher than the energy computed with its mean velocity. This is because of the fact that energy in the wind follow CUBE LAW. This deviation is taken into account in the estimation of the actual power by considering a factor known as Energy Pattern Factor (EPF).

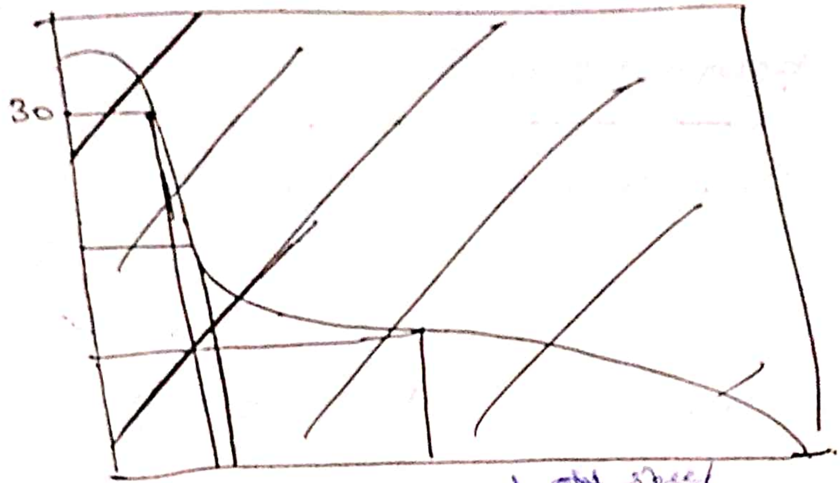
$$EPF = \frac{\text{Actual Energy in Wind}}{\text{Energy content at steady mean speed}} \geq 1$$

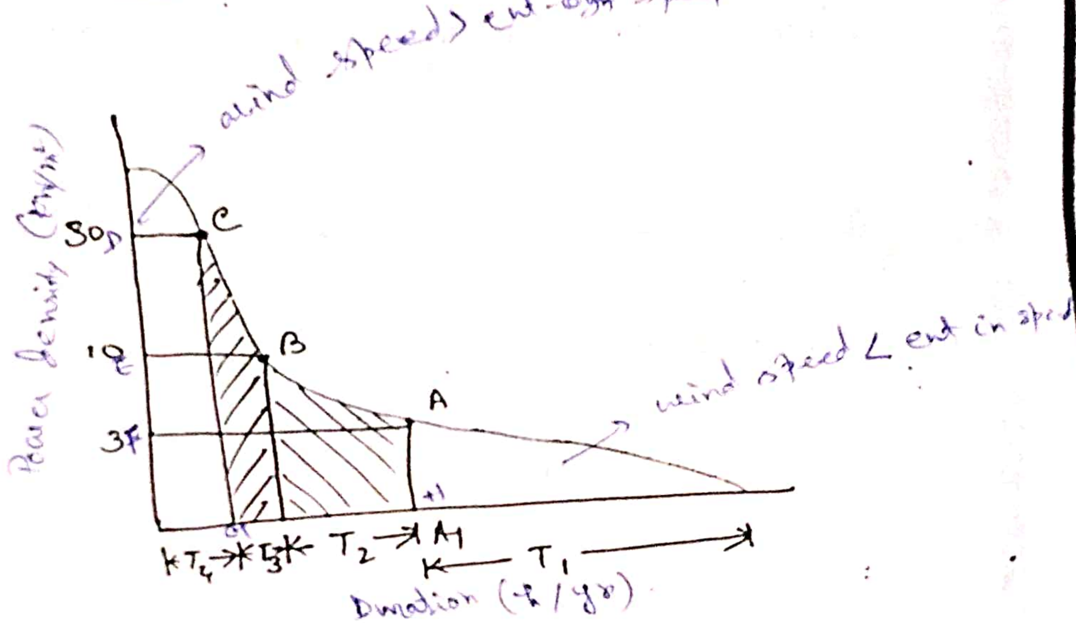
$$= \frac{\sum v^3}{n \bar{v}^3}$$

, $n \rightarrow$ no. of observation

$\bar{v} \rightarrow$ mean velocity

Cumulative distribution Curve.





T_1 = Zero output

T_2 = generating power but less than rated

T_3 = generating rated power

T_4 = (Non functional) Cut-off for safety.

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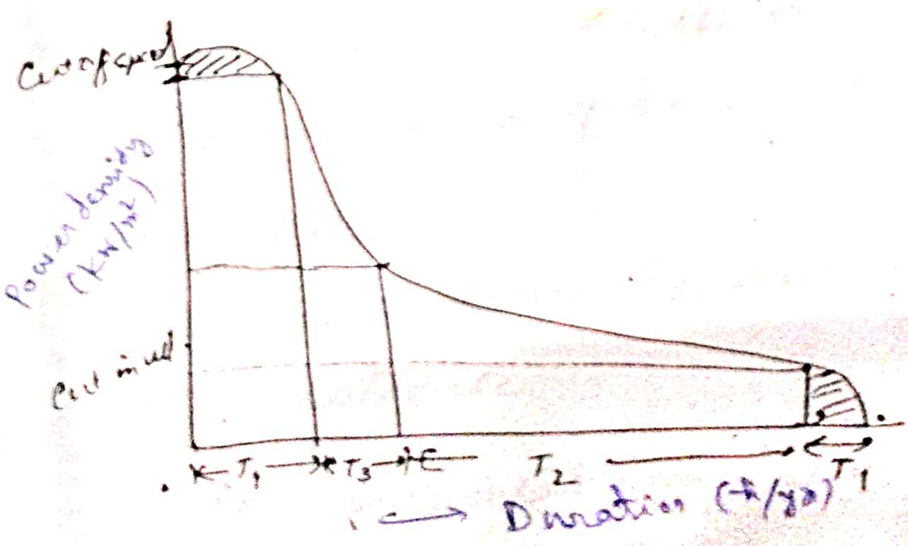
Specification for Wind Turbine

1. <u>Cut in speed</u> – It is the speed at which a wind turbine starts developing useful power.

2. <u>Rated speed</u> – It is the speed at which the wind turbine produces its rated power.

3. <u>Cut off speed</u> – It is the speed beyond which a wind turbine is made inoperative for the reason of safety.



T_1 = zero output
T_2 = generating power but less than rated
T_3 = generating rated power
T_4 = cut off for safety

A wind turbine is classified into 2 categories based on — 1) Orientation of the rotating shaft.

2) By the direction of flow fed on the turbine.

<u>Wind turbine</u>

1. Horizontal Axis Rotor
2. Vertical Axis Rotor.

<u>Horizontal Axis Rotor</u> — In the horizontal axis rotor, the turbine has to rotate $\perp^r$ to the direction of the wind.

The tail orients the turbine perpendicular to the direction of the wind. It also balances the moment.

eg - High Speed Propeller Rotor, multiblade rotor etc.

The horizontal axis rotor is again classified into — 1) Upwind Rotor
2) Down wind Rotor.

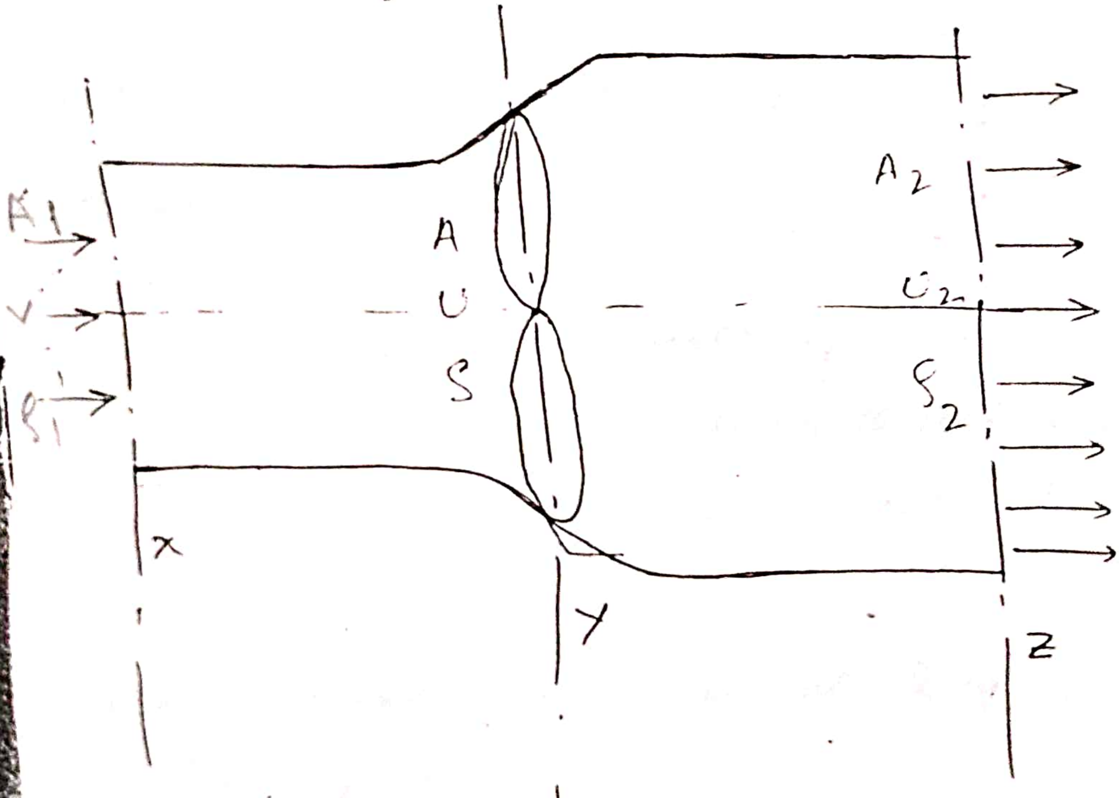
Upwind Rotor - In case of upwind rotor, the wind turbine first faces the wind.

Down wind rotor :- In this case the wind turbine faces the wind after it crosses the tower & the assembly.

Betz Limit.

For extracting power from the wind, the extracting device (turbine rotor) should reduce the ~~wind~~ velocity of the flowing wind. To extract the complete energy, the device should bring the wind velocity to 0 ie at a complete rest ~~sat~~ state. But this type of situation will change the flow path of air & thus reduce further the extractable power. Therefore there is a point of optimum level of which the velocity of the free stream is slowed down to achieve the maximum efficiency. The max. value was found out by Betz (1926) and is called the Betz limit & is ~~[illegible]~~ the Betz limit is 59.3%.

Derivation of Betz Limit.



Conditions assumed :

1) Flow is purely axial
2) Fluid is a non-viscous medium
3) Fluid is an in-compressible flow.

Let us assume a slip stream as shown in fig.

Section	X	Y	Z
Velocity ——	V	U	U_2
Area ——	A_1	A	A_2
Density ——	ρ_1	ρ	ρ_2

V = free stream velocity

U = axial flow velocity through rotor

U_2 = axial flow vel. in fully developed wake

For a constant mass flow rate & eq^n of continuity,

$$\dot{m} = \rho_1 A_1 V = \rho A V = \rho_2 A_2 V_2 \quad \text{—— (1)}$$

Force (rate of change of momentum)

$$F = \dot{m}\Delta V = \dot{m}(V - U_2) \quad \text{—— (2)}$$

Power (Rate of change of kinetic energy)

$$P = \frac{1}{2}\dot{m}(V^2 - U_2^2) \quad \text{—— (3)}$$

At the rotor the force does work at velocity U.

$$P = FU \quad \text{——— (4)}$$

Substituting the value of A_2^{P} in eq (4).

$$\frac{1}{2}\dot{m}(v^2 - u_2^2) = \dot{m}(v - u_2)U$$

$$\Rightarrow \quad U = \frac{v^2 - u_2^2}{2(v - u_2)}$$

$$= \frac{(v + u_2)\cancel{(v - u_2)}}{2\cancel{(v - u_2)}}$$

$$U = \frac{1}{2}(v + u_2) \quad \text{——— (5)}$$

from eq^n (2) $\quad F = \dot{m}(v - u_2)$

$$= \rho U A (v - u_2) \quad \text{(Substituting (1) in (2))}$$

Now, $F'_{\theta'} = \frac{F}{A} = \rho U (v - u_2)$.

$$= \frac{1}{2}\rho (v + u_2)(v - u_2)$$

$$= \frac{1}{2}\rho v^2 \left(1 - \frac{u_2^2}{v^2}\right)$$

Let $\frac{u_2}{v} = K$.

$$\therefore F'_{\theta'} = \frac{1}{2}\rho v^2 (1 - K^2) \quad \text{——— (6)}$$

rotor

envelope

&

(1)

)

(2)

energy)

(3)

ut-

From eqⁿ (3)

$$P = \frac{1}{2} \dot{m} (V^2 - U_2^2)$$

$$= \frac{1}{2} \rho U A (V^2 - U_2^2) \quad \text{--- (7)}$$

$$P' = \frac{P}{A} = \frac{1}{2} \rho U (V^2 - U_2^2)$$

$$= \frac{1}{2} \rho \left(\frac{V + U_2}{2}\right) (V^2 - U_2^2)$$

$$= \frac{1}{4} \rho (V + U_2) V^2 \left(1 - \frac{U_2^2}{V^2}\right)$$

$$= \frac{1}{4} \rho V^2 \cdot V \left(1 + \frac{U_2}{V}\right) \left(1 - \frac{U_2^2}{V^2}\right)$$

$$= \frac{1}{4} \rho V^3 (1 + k)(1 - k^2) \quad \text{--- (8)}$$

Power Coefficient; $C_p = \frac{\text{Output Power}}{\text{Input Power}}$

$$= \frac{\frac{1}{4} \rho V^3 (1 + k)(1 - k^2)}{\frac{1}{2} \rho_1 V^3}$$

$$C_p = \frac{1}{2} \left(\frac{\rho}{\rho_1}\right) (1 + k)(1 - k^2) \quad \text{--- (9)}$$

$$C_p = \frac{1}{2} (1 - k^2 + k - k^3)$$

[For subsonic & incompressible flow $\frac{\rho}{\rho_1} = 1$

Wave energy

$$C_p = \frac{1}{2}\left(1+k-k^2-k^3\right).$$

$$\therefore \frac{dC_p}{dk} = \frac{1}{2}\left(1-2k-3k^2\right).$$

$$\therefore \frac{d^2C_p}{dk^2} = \frac{1}{2}\left(-2-6k\right)$$

$$= -(1+3k)$$

Now for $C_{p\,max}$ $\frac{dC_p}{dk} = 0.$

$$\therefore \frac{1}{2}\left(1-2k-3k^2\right) = 0.$$

$$\Rightarrow 1-2k-3k^2 = 0.$$

$$\Rightarrow 3k^2+2k-1=0$$

$$\Rightarrow 3k^2+3k-k-1=0$$

$$\Rightarrow 3k(k+1)-1(k+1)=0$$

$$\Rightarrow (k+1)(3k-1)=0.$$

$$k=-1 \quad , \quad k=\frac{1}{3}.$$

For $k=-1$.

$$\frac{d^2C_p}{dk^2} = -(1-3) = 2 > 0$$

& for $k=\frac{1}{3}$, $\frac{d^2C_p}{dk^2} = -(1+1) = -2 < 0.$

Wave energy

$\therefore$ For $K = \frac{1}{3}$, C_p is max &

$$C_{p\,max} = \frac{1}{2}\left(1+\frac{1}{3}\right)\left(1-\frac{1}{9}\right)$$

$$= \frac{1}{2}\left(\frac{4}{3}\right)\left(\frac{8}{9}\right)$$

$$= \frac{16}{27} = 0.593$$

$$= 59.3\%$$

Axial Momentum Theory

The axial momentum theory was first given by Rankine in 1865 & later improved by Froude. This theory provides a relationship between the force acting on a rotor and the velocity of the fluid. It also predicts the ideal efficiency of the rotor.

Now a new term called axial induction factor 'a' is introduced

interference factor

$a = \frac{V-U}{V}$

such that $U = V(1-a)$

$U \rightarrow$ Axial flow velocity through rotor

$V \rightarrow$ free stream velocity

defined as fractional wind speed decrease at the turbine.

Now $U = \frac{V+U_2}{2} \Rightarrow \frac{V+U_2}{2} = V(1-a)$

or $v + U_2 = 2v(1-a)$

$$U_2 = -v + 2v(1-a)$$

$$= -v + 2v - 2va$$

$$= v - 2va$$

$$U_2 = v(1-2a) \text{ ——— } a$$

Now substituting the value of U & U_2 in eqn (7).

$$P = \frac{1}{2} \rho U A (v^2 - U_2^2)$$

$$= \cancel{\frac{1}{2} \rho U A v}$$

$$= \frac{1}{2} \rho A . v(1-a) \left[v^2 - v^2(1-2a)^2 \right]$$

$$= \frac{1}{2} \rho A (1-a) v^3 \left[1 - (1-2a)^2 \right]$$

$$= \frac{1}{2} \rho A (1-a) v^3 \left[\cancel{1} - \cancel{1} + 4a - 4a^2 \right]$$

$$= \frac{1}{2} \rho A (1-a) v^3 . 4a(1-a)$$

$$= \frac{1}{2} \rho A (1-a)^2 . v^3 . 4a .$$

$\therefore a = \frac{1}{3}$, $\therefore$ The power developed P reaches the Betz limit, i.e. 59.3%

$$\frac{dP}{da} = \frac{1}{2} \rho A v^3 . 4(1-a)^2 + \frac{1}{2} \rho A v^3 4a . 2(1-a)(-1) = 0$$

or $(1-a)^2 = 2a(1-a)$

$(1-a) = 2a$ or $1 = 3a \Rightarrow a = \frac{1}{3}$

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Now, the general thrust eq^n dT

$$dT = \frac{1}{2}\rho A(v^2 - u_2^2)$$

$$\cancel{dT = 4a(1-a)}$$

$$dT = \frac{1}{2}\rho A\left(v^2 - v^2(1-2a)^2\right)$$

$$= \frac{1}{2}\rho A v^2\left[1 - (1-2a)^2\right]$$

$$= \frac{1}{2}\rho A v^2\left[\cancel{1} - \cancel{1} - 4a^2 + 4a\right]$$

$$= \frac{1}{2}\rho A v^2 \cdot 4a(1-a)$$

$$\therefore 4a(1-a) = \frac{dT}{\frac{1}{2}\rho A v^2}$$

$$= \frac{dT}{\frac{1}{2}\rho\, 2\pi r\, dr \cdot v^2}$$

$$4a(1-a) = \frac{dT}{\frac{1}{2}\, 2\pi r \cdot dr \cdot v^2 \cdot \rho}$$

$$4a'(1-a') = \frac{dT}{\frac{1}{2}\rho\, 2\pi r\, dr\, \Omega^2 r^2}$$

$$\therefore \frac{4a(1-a)}{4a'(1-a')} = \frac{\Omega^2 r^2}{V^2} = \lambda^2.$$

By realising the conservation of angular momentum, it is visualised that the torque exerted is equal to the angular momentum of the rotating wake.

$$dQ = ~~4a~~\ u\rho(2\pi r\,dr).\omega r.\ ~~r~~\ r.$$

Power generated $dP = \Omega\,dQ$.

$$P = \int_0^R \Omega . dQ.$$

$$\Rightarrow P = \frac{1}{2}\rho A V^3 . \frac{8}{\lambda^2}\int_0^\lambda a'(1-a)\,r^3 \lambda\, d\lambda.$$

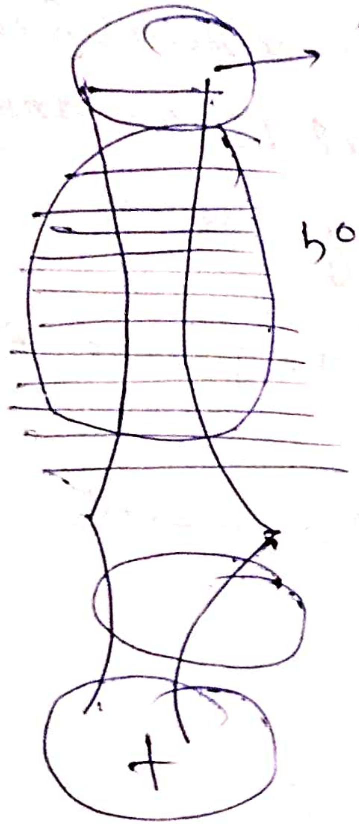
$$\therefore a' = \frac{1-3a}{4a-1}$$

$$~~dQ~~ \Rightarrow$$

$$dT = \frac{1}{2}\rho A(V^2 - u_2^2)$$

$\lambda \rightarrow$ tip speed ratio

$$\therefore \lambda^2 = \frac{\Omega^2 r^2}{V^2} = \frac{4a(1-a)}{4a'(1+a')}$$



Tip loss model factor

$$F = \frac{2}{\pi} \cos^{-1}\left[\exp\left(-\frac{1}{2} B \frac{R-r}{r \sin\phi}\right)\right]$$

$$dT = 4aF(1-aF)\frac{1}{2}\rho V^2 2\pi r\,dr$$

$$dT = 4a'F(1-aF)\frac{1}{2}\rho V \Omega r r^2 2\pi r\,dr$$

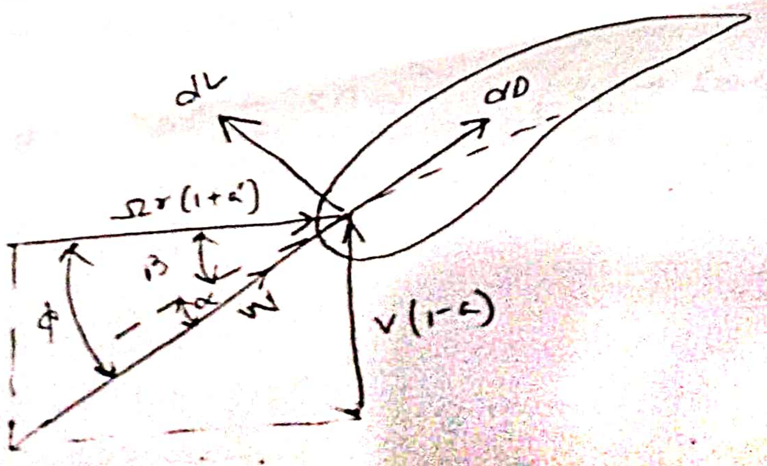
Blade Element Theory

The momentum theory doesnot provide any information about the design of blade of a rotor. This is obtained from the blade element theory in conjunction with the momentum theory.

The basic assumptions of blade element theory are –

1. There is no interference between adjacent blade element along each blade.
2. Forces acting on the elemental blade profile is solely due to its lift & drag coefficient.

In this method the forces are calculated on each differential element of the blade & subsequently integrated over its entire length to find the total for cases such as thrust, torque etc.



W → relative velocity flow

The aerodynamic factors effecting blade elements are as under –

<u>Angle of attack</u> (α) → It is the angle betⁿ the sectional chord & the relative velocity flow path.

<u>Blade setting angle</u> : (β) → It is the physical angle between the sectional chord & the rotor plane (plane of rotation)

<u>Flow angle</u> (ϕ) – It is the angle betⁿ the relative velocity of flow & the rotor plane.

<u>Lift force</u> :– The component of the total aerodynamic force perpendicular to the incoming relative velocity factor (W)

$$F_L = C_L \times \frac{1}{2} \rho A W^2$$

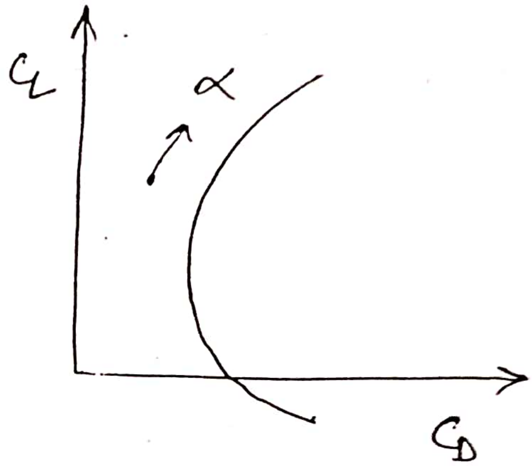
(C_L → Lift coefficient)

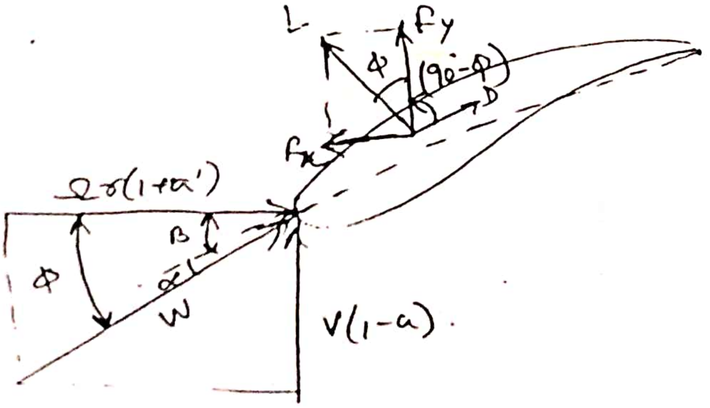
<u>Drag force</u> → The component of the total force parallel to the direction of the relative velocity.

$$F_D = C_D \times \frac{1}{2} \rho A W^2$$

(C_D → coeff. of drag)

Profile Polar Effect – The characteristics curve represents the C_L, C_D, α characteristics of a profile.





We have expression for lift & drag as

$$F_L = C_L \frac{1}{2} \rho W^2 c\,dr \qquad (c\,dr = \text{Area } A)$$

$$F_D = C_D \cdot \frac{1}{2} \rho W^2 c\,dr .$$

~~From above fig,~~ $\cancel{\phi = \beta + \alpha}$

$\cancel{\tan\phi = \frac{}{\Omega r(1+a')}}$

from fig, $\phi = \beta + \alpha$.

$$\tan\phi = \frac{V(1-a)}{\Omega r(1+a')}$$

$$C_y = C_L \cos\phi + C_D \sin\phi$$

$$C_x = C_L \sin\phi - C_D \cos\phi$$

Now the thrust & torque for an elemental area of the rotor can be written as -

$$dT = B.C \tfrac{1}{2} \rho W^2 C_y dr$$

$$dQ = B.C . \tfrac{1}{2} \rho W^2 C_x . r dr.$$

where B = no. of blade

C = chord of the profile.

By combining blade element theory with momentum theory, we can derive the following equation by eliminating drag term

$$\frac{a}{1-a} = \frac{\sigma C_L \cos\phi}{4\sin^2\phi}$$

$$\frac{a'}{1+a'} = \frac{C_L \sigma}{4\cos\phi} \qquad \sigma = \frac{BC}{2\pi r}.$$

The term σ is commonly known as Solidity. For actual calculation C_L-α & C_D-α characteristics are needed. Further correction for tip loss has to be introduced as in practice blade are of finite no. The tip loss correction introduced by Prandtl is introduced in the momentum theory for determining the actual thrust & torque component.

Design Criteria

$$C_{P(max)}^{*} = \left[1 - \frac{1.386}{B} \sin\frac{\phi}{2}\right]^2 \times \frac{16}{27} [K]$$

where $K = \left[e^{-6.35\lambda^{-1.29}} - \frac{C_L}{C_d}\lambda\right]$

$$R = \sqrt{\frac{2P}{\pi \rho_a V_a^2 C_p^*}}$$

$$\text{Length}_{(blade)} = R_{(Tip)} - R_{(Hub)}$$

$$\lambda_r = \lambda\left[\frac{r}{R}\right]$$

$$\phi = \frac{2}{3}\tan^{-1}\left(\frac{1}{\lambda_r}\right)$$

$$c = \frac{8\pi r}{B C_L}(1 - \cos\phi)$$

~~Vane~~

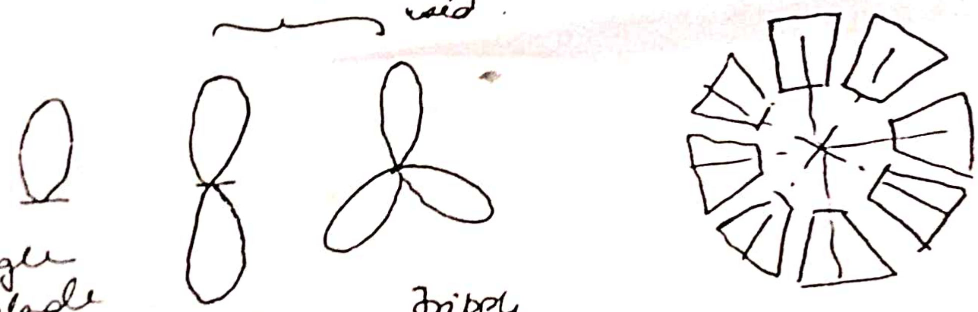
The Tail Vane – This device is mainly used to orient slow wind turbines upto 6 m diameter.

High Speed - low torque turbine

Suitable for electricity generation.

Low Speed - high torque turbine

most popularly used



Single blade

Double blade

Tripple blade

Multi blade system.

High Speed low torque.

very Low speed high torque

used Particularly because of high inertia.

Torque - force that tend to cause rotation.