Electrostatics: Part-2

Presented by

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Let's consider that the density of charges along a line is ρ_L (in C/m), on the surface ρ_s (in C/m^2) and in the volume ρ_v (in C/m^3) .

As the density of line charge, surface charge and volume charge are given, then the charge element 'dQ' and the total charge 'Q' due to these charge distribution can be found.

So if we consider a small length 'dl' in the line charge where we can say charge present is 'dQ'. Then we can write-

$$dQ = \rho_L dl$$

Hence, the total charge in the line is-

$$\boxed{Q = \int_L \rho_L dl} \qquad (13)$$

Similarly we can find,

$$dQ = \rho_s dS$$ The total surface Charge

$$\boxed{Q = \int_S \rho_s dS} \quad (14)$$

and, $$dQ = \rho_v dv$$ The total volume Charge

$$\boxed{Q = \int_v \rho_v dv} \quad (15)$$

Now we can find the field intensity for various charge distributions as -

For point charge $\Rightarrow$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R} \quad (16)$$

For line charge $\Rightarrow$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \int_L \frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\hat{a}_R} \quad (17)$$

For surface charge ⇒
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \int_S \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R \qquad (18)$$

For volume charge ⇒
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \int_v \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R \qquad (19)$$

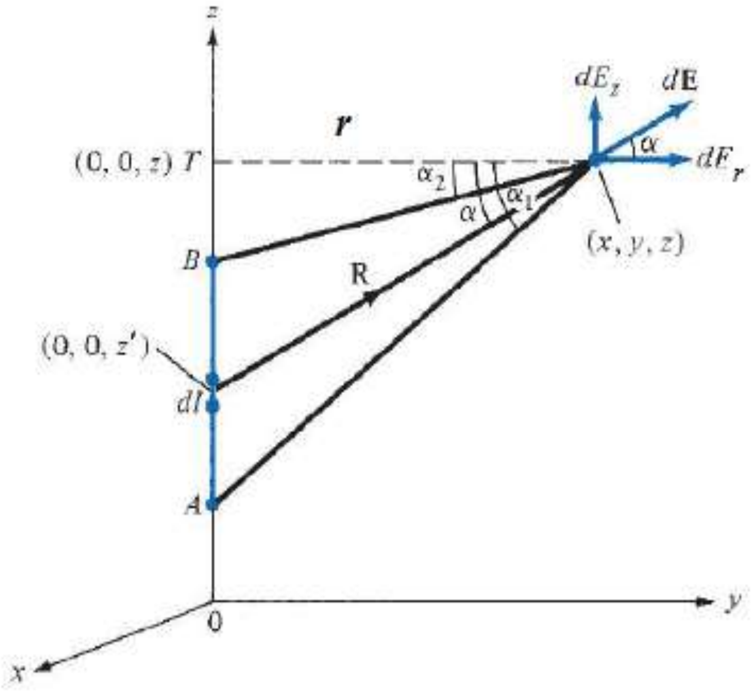
One point to be noted that R^2 and $\hat{a}_R$ will vary as the integrals in (13)-(19) are evaluated.

As we have calculated the electric field intensity due to line charge, surface charge and volume charge; let us now apply these formulae to some specific charge distribution.

(A) A line Charge

Let us consider a line of charge with uniform density ρ_L is extended along z-axis and its limit is A to B.

Now consider a small length '*dl*' from A-B where charge elements present is '*dQ*'.



So now we can write -

$$dQ = \rho_L dl = \rho_L dz'$$

Hence the total charge present in the line A-B is-

$$\boxed{Q = \int_A^B \rho_L dz'} \qquad (20)$$

Now suppose we want to find the electric field intensity $\left(\vec{E}\right)$ at the point (x, y, z) .

Let's now draw the line of force $\vec{F}$ between 'dl' and the point (x, y, z).

The distance is denoted by a vector .

Now we can use (17) to find $\vec{E}$.

$$\boxed{\vec{E} = \int_A^B \frac{\rho_L dz'}{4\pi\varepsilon_0 R^2}\hat{a}_R} \tag{21}$$

Let's derive this equation in another form. From the fig. we can write -

$$dl = dz'$$

and

$$\begin{aligned}\vec{R} &= (x, y, z) - (0, 0, z') \\ &= \{(x-0)\hat{a}_x + (y-0)\hat{a}_y + (z-z')\hat{a}_z\} \\ &= x\hat{a}_x + y\hat{a}_y + (z-z')\hat{a}_z\end{aligned}$$

where

$$r\hat{a}_r = x\hat{a}_x + y\hat{a}_y$$

So we can write -

$$\vec{R} = r\hat{a}_r + (z - z')\hat{a}_z$$

and

$$R = |\vec{R}| = \sqrt{(r\hat{a}_r)^2 + \{(z - z')\hat{a}_z\}^2}$$

$$R = \sqrt{r^2 + (z - z')^2}$$

where,

$$r = \sqrt{x^2 + y^2}$$

Now,

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{r\hat{a}_r + (z - z')\hat{a}_z}{\{r^2 + (z - z')^2\}^{1/2}}$$

or,

$$\frac{\hat{a}_R}{R^2} = \frac{\vec{R}}{R^2|\vec{R}|} = \frac{\vec{R}}{R^3} = \frac{r\hat{a}_r + (z - z')\hat{a}_z}{\{r^2 + (z - z')^2\}^{3/2}}$$

Substituting this expression into (21) we get -

$$\boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_A^B \frac{r\hat{a}_r + (z - z')\hat{a}_z}{\{r^2 + (z - z')^2\}^{3/2}}\, dz'} \quad (22)$$

Now instead of taking limits A-B, if we consider the limit α_1 and α_2.
Thus we can write,

$$\boxed{\left|\vec{R}\right| = \sqrt{r^2 + (z - z')^2} = r\sec\alpha}$$

and $\boxed{r = \vec{R}\cos\alpha}$ $\boxed{(z - z') = \vec{R}\sin\alpha}$

because,

$$\left[\begin{array}{l} \dfrac{r}{\vec{R}} = \cos\alpha \\ \dfrac{\vec{R}}{r} = \sec\alpha \\ \vec{R} = r\sec\alpha \end{array}\right]$$

again

$$Z' = OT - TZ' = OT - r\tan\alpha$$

$$\tan\alpha = \frac{TZ'}{r}$$

So, we can write -

$$dZ' = -r\sec^2\alpha d\alpha$$

Now, let's modify (22)

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{(\vec{R}\cos\alpha\hat{a}_r + \vec{R}\sin\alpha\hat{a}_z)}{r^3\sec^3\alpha}\left(-r\sec^2\alpha\right)d\alpha$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2}\frac{\vec{R}(\cos\alpha\hat{a}_r + \sin\alpha\hat{a}_z)}{r^2\sec\alpha}d\alpha$$

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0 r}\int_{\alpha_1}^{\alpha_2}(\cos\alpha\hat{a}_r + \sin\alpha\hat{a}_z)d\alpha$$

Hence, for a finite line charge

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0 r}\left[\int_{\alpha_1}^{\alpha_2}\cos\alpha\hat{a}_r d\alpha + \int_{\alpha_1}^{\alpha_2}\sin\alpha\hat{a}_z d\alpha\right]$$

$$= \frac{-\rho_L}{4\pi\varepsilon_0 r}\left[\left|\sin\alpha\right|_{\alpha_1}^{\alpha_2}\hat{a}_r + \left|-\cos\alpha\right|_{\alpha_1}^{\alpha_2}\hat{a}_z\right]$$

$$\boxed{\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0 r}\left[(\sin\alpha_1 - \sin\alpha_2)\hat{a}_r + (\cos\alpha_2 - \cos\alpha_1)\hat{a}_z\right]}$$

Now, as a special case, if we consider an infinite line of charge

Then we can say point '*B*' is at $(0.0,\infty)$ and point '*A*' is at $(0.0,-\infty)$

so that **(From the axis anti-clockwise)**

$$\alpha_1 = \pi/2$$

$$\alpha_2 = -\pi/2$$ **(From the axis clockwise)**

Hence, putting the values of α_1 and, α_2 we get-

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0}\hat{a}_r} \quad (23)$$

So the equation we got here is for infinite line of charge along z-axis.

Thank you