Cartesian or rectangular coordinate system

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Coordinate Systems

Two types of co-ordinate systems:-

1. Orthogonal - Axes are mutually $\perp$ to each other.
2. Non-orthogonal- Axes are not mutually $\perp$ to each other.

Orthogonal coordinate systems :-

1. Cartesian or rectangular coordinate system
2. Circular cylindrical coordinate system
3. Spherical coordinate system

Different coordinate system is used for different given problem.

It is to be noted that a hard problem in one coordinate system may be easy in another system.

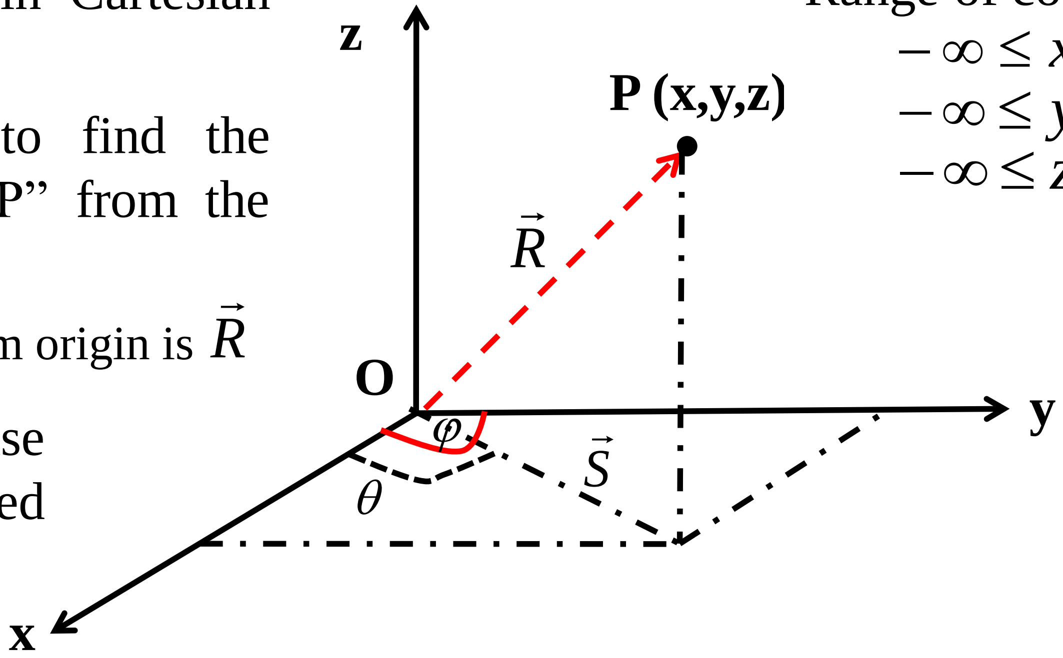
Let's revisit the **Cartesian**, the known coordinate system.

Suppose we want to find the position of the point, say "P", in Cartesian coordinate system.

Basically , we want to find the distance of the point "P" from the origin "O".

Let, the distance of "P" from origin is $\vec{R}$

Here, $\vec{R}$ is a vector because it has a direction, pointed towards "P".



Range of coordinates

$$-\infty \leq x \leq \infty$$
$$-\infty \leq y \leq \infty$$
$$-\infty \leq z \leq \infty$$

Prelims cont…..

To find "P", at first we have to find $\vec{S}$.

Thus, $\vec{S} = \vec{x} + \vec{y}$ [vector addition]; but to find the magnitude and phase of $\vec{S}$, we have

$\vec{S} = \left|\vec{S}\right| = \sqrt{x^2 + y^2 + 2xy\cos\varphi}$ and $\theta = \tan^{-}\dfrac{y}{x}$ [Parallelogram law]

Now, as we have the value of $\vec{S}$, we can have

$$\vec{R} = \vec{S} + \vec{Z} = \vec{x} + \vec{y} + \vec{z} \qquad (1)$$

This vector (1) can be represented as-

$$\boxed{\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}} \qquad (2)$$

where, $\hat{i}$, $\hat{j} and \hat{k}$ are known as unit vectors along x, y, z respectively.

Prelims cont…..

We can get the unit vectors as -

$$\hat{i} = \frac{\vec{x}}{|\vec{x}|};\ \hat{j} = \frac{\vec{y}}{|\vec{y}|} \text{ and } \hat{k} = \frac{\vec{k}}{|\vec{k}|}$$

So, now we can describe any point in this 3-dimensional space.

For, example- any point vector $\vec{A}$ from the origin can be written as

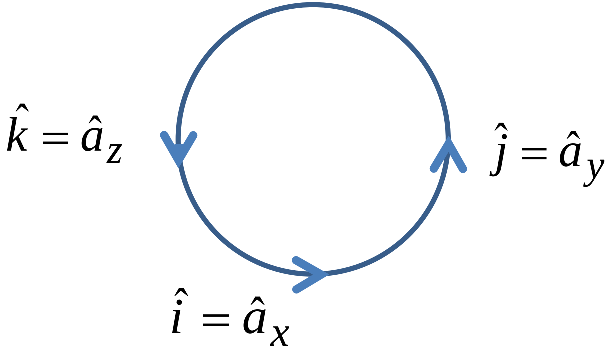
$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z \qquad (3)$$

Let's see the cross and dot product of two unit vectors -

$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{j} \times \hat{k} = \hat{i} \quad \text{or,} \quad \hat{a}_y \times \hat{a}_z = \hat{a}_x$$

$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{a}_z \times \hat{a}_x = \hat{a}_y$$

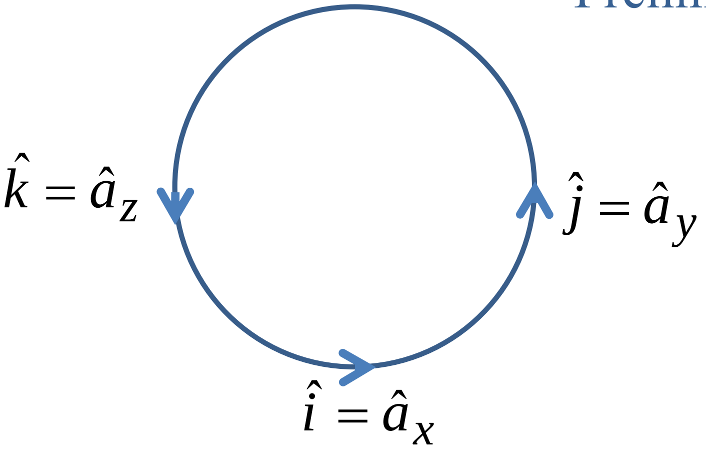


Similarly,

$$\hat{i} \times \hat{k} = -\hat{j} \qquad \hat{a}_x \times \hat{a}_z = -\hat{a}_y$$

$$\hat{k} \times \hat{j} = -\hat{i} \quad \text{or,} \quad \hat{a}_z \times \hat{a}_y = -\hat{a}_x$$

$$\hat{j} \times \hat{i} = -\hat{k} \qquad \hat{a}_y \times \hat{a}_x = -\hat{a}_z$$



For dot product :-

$$\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1$$

$$\hat{i} \bullet \hat{j} = \hat{j} \bullet \hat{k} = \hat{k} \bullet \hat{i} = 0$$

Similarly,

$$\hat{a}_x \bullet \hat{a}_x = \hat{a}_y \bullet \hat{a}_y = \hat{a}_z \bullet \hat{a}_z = 1$$

$$\hat{a}_x \bullet \hat{a}_y = \hat{a}_y \bullet \hat{a}_z = \hat{a}_z \bullet \hat{a}_x = 0$$

Let’s take another vector $\vec{B}$ along with vector $\vec{A}$, to find dot and cross products.

We know,

$$\vec{A} = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

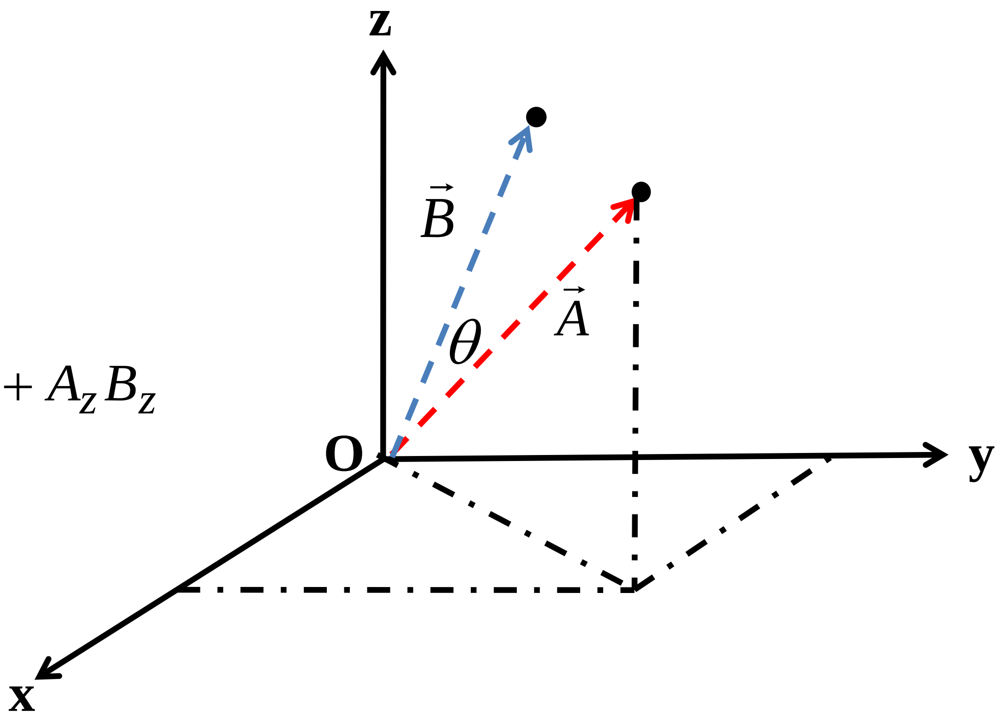
$$\vec{B} = B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z$$

Hence, the **dot product**

$$\vec{A} \bullet \vec{B} = \left[|\vec{A}||\vec{B}|\cos\theta \right] = A_x B_x + A_y B_y + A_z B_z$$

and

$$|\vec{A}||\vec{B}|\cos\theta = A_x B_x + A_y B_y + A_z B_z$$



$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}||\vec{B}|}$$

where, $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$

- **Cross product**

$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta_{AB}\hat{a}_n = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{a}_x + (A_zB_x - A_xB_z)\hat{a}_y + (A_xB_y - A_yB_x)\hat{a}_z$$

[$\hat{a}_n$ = unit vector, normal to the plane containing two vectors]

Scalar triple product

Suppose, we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$; we define scalar triple product as

$$\vec{A}\bullet(\vec{B}\times\vec{C}) = \vec{B}\bullet(\vec{C}\times\vec{A}) = \vec{C}\bullet(\vec{A}\times\vec{B}) \qquad (4)$$

If $\vec{A} = (A_x, A_y, A_z)$, $\vec{B} = (B_x, B_y, B_z)$ and $\vec{C} = (C_x, C_y, C_z)$; then $\vec{A}\bullet(\vec{B}\times\vec{C})$ is the volume of a parallelogram having $\vec{A}$, $\vec{B}$, and $\vec{C}$ as edges and can be obtained easily by finding the determinant of these three vectors as

$$\vec{A}\bullet\left(\vec{B}\times\vec{C}\right)=\begin{bmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{bmatrix} \qquad (5)$$

Since the results of this vector multiplication is scalar, so (4) or (5) is called scalar triple product.

Vector triple product

We define the vector triple product as

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}\bullet\vec{C}\right)-\vec{C}\left(\vec{A}\bullet\vec{B}\right)$$

Note : $\left(\vec{A}\bullet\vec{B}\right)\vec{C}\neq\vec{A}\left(\vec{B}\bullet\vec{C}\right)$ but $\left(\vec{A}\bullet\vec{B}\right)\vec{C}=\vec{C}\left(\vec{A}\bullet\vec{B}\right)$

Components of a vector product :-

A direct application of vector product is, its use in determining the projection (or components) of a vector in a given direction.

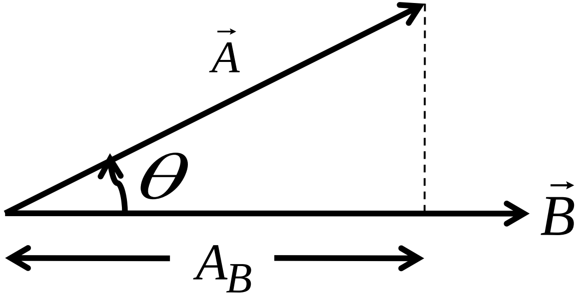
The projection can be scalar or vector.

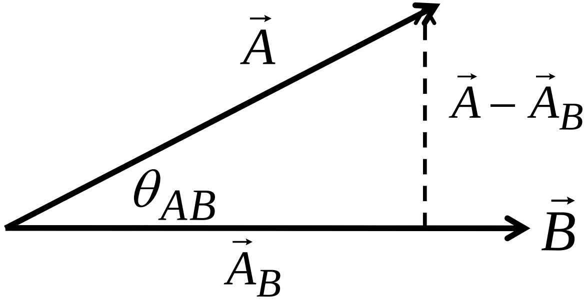
Let a vector $\vec{A}$, we define the scalar component A_B of $\vec{A}$ along $\vec{B}$ as

$$A_B = A\cos\theta_{AB}$$

$$= \left|\vec{A}\right|\left|\vec{a}_B\right|\cos\theta_{AB}$$

$$A_B = \vec{A}\bullet\vec{a}_B$$





The vector component $\vec{A}_B$ of $\vec{A}$ along $\vec{B}$ is simply the scalar.

Notice from the 2nd fig. that the vector can be resolved into two orthogonal components; one component $\vec{A}_B$ parallel to $\vec{B}$, another one is $\vec{A}-\vec{A}_B$; $\perp$ to $\vec{B}$

Thank you