Electrostatics: Part-1

Presented by

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To study **Electrostatics**, we need to know two laws -

1. Coulomb's law
2. Gauss's law

Because **Electrostatics** stands on to these two laws.

Let us see the **Coulomb's law** again

Statement :-

Coulomb's law states that the force 'F' between two point charges Q_1 and Q_2 along the line joining them is –

1. **Directly proportional** to the product of Q_1 and Q_2.
2. **Inversely proportional** to the square of the distance 'R' between them.

Mathematically, we can write -

$$\boxed{F \propto \frac{Q_1 Q_2}{R^2}} \qquad (1)$$

or, $$F = k\frac{Q_1 Q_2}{R^2} \qquad (2)$$

where k = proportionality constant, it value depends on the choice of systems of units.

Ex.- In SI unit, charges Q_1 and Q_2 are in Coulomb's (C), the distance 'R' in meters and the force 'F' in Newton's (N).

We should remember that-

$$\boxed{k = \frac{1}{4\pi\varepsilon_0}} \qquad (3)$$

where ε_0 = permittivity of the free space and its unit is Farad per meter.

The value of ε_0 is given as -

$$\varepsilon_0 = 8.854 \times 10^{-12} \cong \frac{10^{-9}}{36\pi} \quad \text{Farad/meter}$$

Hence, $k = \dfrac{1}{4\pi\varepsilon_0} \cong 9 \times 10^9$ Meter/ Farad

Finally, the **Coulomb's law** becomes - $\boxed{F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}}$

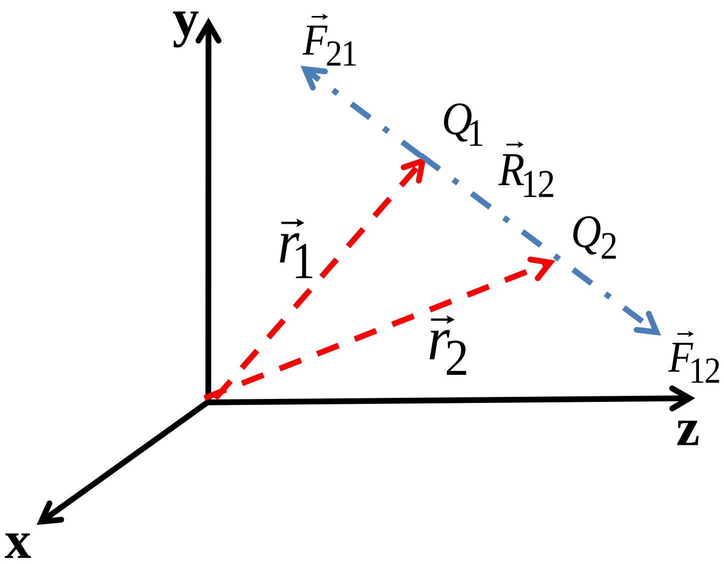
Let's represent the Coulomb's law in Cartesian Co-ordinate system.

Suppose, two point charges Q_1 and Q_2 are located at points having position vectors $\vec{r}_1$ and $\vec{r}_1$ respectively.

Now, we are going to find the force between them along the line joining them.

Here, the distance between Q_1 and Q_2 can be represented by the position vector and say it is $\vec{R}_{12}$.

Now, the force on Q_2 due to Q_1, say $\vec{F}_{12}$ can be written as-



$$\boxed{\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}} \qquad (5)$$

where $\hat{a}_{R_{12}}$ is the unit vector along $\vec{R}_{12}$ pointed towards Q_2 from Q_1.

Now,

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$

$$R = \left|\vec{R}_{12}\right| = \left|\vec{r}_2 - \vec{r}_1\right|$$

$$\hat{a}_{R_{12}} = \frac{\vec{R}_{12}}{\left|\vec{R}_{12}\right|} = \frac{\vec{R}_{12}}{R}$$

$$\frac{\hat{a}_{R_{12}}}{R^2} = \frac{\vec{R}_{12}}{R^3}$$

So we can write-

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\vec{R}_{12}}{R^3} \qquad (6)$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\left(\vec{r}_2 - \vec{r}_1\right)}{\left|\vec{r}_2 - \vec{r}_1\right|^3} \qquad (7)$$

From this vector representation, we can easily find out the force on Q_1 due to the charge Q_2 .

In this case, the direction of force will be in opposite direction along the line joining them.

The force on Q_1 due to Q_2 can be represented by $\vec{F}_{21}$, but the magnitude will be same, i.e., $Q_1 = Q_2$.

Hence, $\vec{F}_{21} = \left|\vec{F}_{12}\right| \hat{a}_{R_{21}} = \left|\vec{F}_{12}\right| \left(-\hat{a}_{R_{12}}\right) = -\left|\vec{F}_{12}\right|$ Since, $\hat{a}_{R_{12}} = \left(-\hat{a}_{R_{21}}\right)$

So we can write-

$$\vec{F}_{21} = -\frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\left(\vec{r}_2 - \vec{r}_1\right)}{\left|\vec{r}_2 - \vec{r}_1\right|^3} \tag{8}$$

From this discussion we can conclude that -

1. Like charges repel each other, while unlike charges attracts.
2. The distance $\vec{R}$ between the charged bodies Q_1 and Q_2 must be large compared to linear dimension of the bodies; i.e. Q_1 and Q_2 must be point charges.

3. The charges Q_1 and Q_2 must be static (at rest).

4. The sign of Q_1 and Q_2 must be taken as- for like charges $Q_1 Q_2 > 0$; for unlike charges $Q_1 Q_2 < 0$

Now, suppose instead of two point charges, there are N no. of charges present as $Q_1, Q_2, \cdots, Q_N$.

And their location is described by the position vectors $\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_N$.

Now if we want to find out the force $\vec{F}$ on a charge 'Q' due to N no. of charges, **What should we do**?

We can use the **Principle of Superposition** to determine the force on the charge 'Q'.

Hence,

$$\vec{F} = \frac{QQ_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^3} + \frac{QQ_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3} + \cdots + \frac{QQ_N(\vec{r}-\vec{r}_N)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_N|^3}$$

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\vec{r}-\vec{r}_k)}{|\vec{r}-\vec{r}_k|^3} \qquad (9)$$

Let's find now the field intensity $\left(\vec{E}\right)$

What is field intensity ?

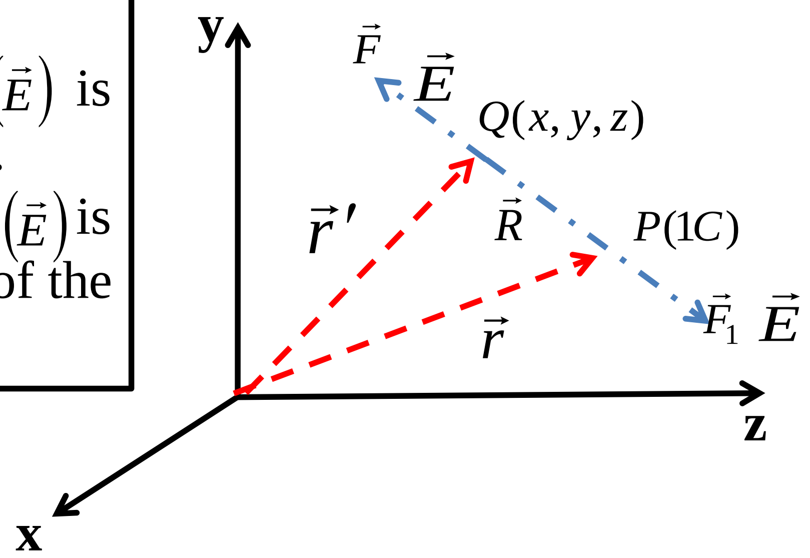
Definition :- The electric field intensity or electric field strength $\left(\vec{E}\right)$ is the force per unit charge when placed in an electric field.

Mathematically, we can write -

$$\vec{E} = \lim_{Q\to 0}\frac{\vec{F}}{Q} \quad \text{or, Simply} \quad \boxed{\vec{E} = \frac{\vec{F}}{Q}} \qquad (10)$$

Note :-

1. For $Q > 0$, the electric field intensity $\left(\vec{E}\right)$ is obviously in the direction of the force $\vec{F}$.
2. When $Q < 0$, the electric field intensity $\left(\vec{E}\right)$ is in the direction opposite to the direction of the force $\vec{F}$.



Let's see the graphical representation of the electric field intensity

Suppose, a point charge 'Q' is placed at some place of the coordinate system, whose position vector is $\vec{r}\,'$.

Now if we place an unit charge in any where in the coordinate system, it experiences a force due to charge 'Q'.

Again say, the unit charge is placed at point '*P*', whose position vector is $\vec{r}$.

Now, according to the Coulomb's law, the force on the unit charge due to point charge '*Q*' can be written as-

$$\vec{F} = \frac{Q \cdot 1}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{R}}{R^3}$$

Hence,

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0}\frac{\vec{R}}{R^3} = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{a}_R}{R^2}$$

where, $\vec{R} = \left(\vec{r} - \vec{r}\,'\right)$

$$R = \left|\vec{R}\right| = \left|\vec{r} - \vec{r}\,'\right|$$

$$\hat{a}_R = \frac{\vec{R}}{\left|\vec{R}\right|} \quad \text{or} \quad \frac{\hat{a}_R}{R^2} = \frac{\vec{R}}{\left|\vec{R}\right|^3}$$

Thus,

$$\vec{E} = \frac{Q(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^3} \qquad (11)$$

If we consider two point charges having same charge 'Q' , then the force on one charge due to another can be written as-

$$\vec{F} = \frac{QQ(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^3} \qquad \text{so,} \qquad \vec{E} = \frac{\vec{F}}{Q} = \frac{Q(\vec{r} - \vec{r}\,')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}\,'|^3}$$

Again consider 'N' no. charges present in the space specified in Cartesian coordinate system.

Now, if we place an unit charge among them, the intensity at that point where unit charge is placed can be calculated using **Superposition Theorem** . i.e.

$$\vec{E} = \frac{Q_1(\vec{r}-\vec{r}_1')}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1'|^3} + \frac{Q_2(\vec{r}-\vec{r}_2')}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2'|^3} + \cdots + \frac{Q_N(\vec{r}-\vec{r}_N')}{4\pi\varepsilon_0|\vec{r}-\vec{r}_N'|^3}$$

or,

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\vec{r}-\vec{r}_k')}{|\vec{r}-\vec{r}_k'|^3}} \qquad (12)$$

Upto this point of discussion, we derived all the formulae considering point charges.

But instead of point charge, if the line of charge or surface of charge or volume of charge is given then what will happen if we want to calculate the field intensity ?

Thank you