Ok so Abhiram talked about Vandermonde Matrix, we have some other good computation approaches also to find an interpolating polynomial. One of them is Lagrange Polynomials.

Now suppose we have n no. of discrete points which belong to a function f(x) and we don’t know the exact function. So we will derive a polynomial of order n-1, known as Lagrange polynomial L(x) which will pass through those n points and approximates the function f(x) within those points.

The formula for computing Lagrange polynomial is given below. We can expand the formula and see that each of the term of L(x) actually represents a single point as the other terms automatically equates to zero when we substitute any x from those n points in the formula.

Here, all the n-1 terms denoted by Pj(x) forms the Lagrange Basis which spans the polynomial space.

Now we will look at an example to get more clarity.

In this example, the left image shows a parabolic function 5-x(square). We have taken 3 discrete points from it. Now as we can see in the image on the right side, the resultant Lagrange polynomial actually gives us the exact parabolic function. Let’s check the computations below for each of the points. You will notice that for each particular x, the rest of the terms equates to zero and we get L(x) for it.

It is an ideal case. Generally, we do not get the exact overlapping function as we got in this example. So, the error in interpolation can be computed by taking the absolute difference between the actual function f(x) and the interpolating polynomial L(x) for each x.

As we can see in this example shown on the right, L(x) does not overlaps f(x) exactly. So it is not a fully correct approximation.

Generally, as the order of Lagrange polynomials gets higher OR the points are far away from each other, it becomes more prone to errors. In these cases, it is better to use piecewise lagrange or splines for better results.

so now Amanpreet will carry on.. with one of the applications of polynomial interpolation