Q1

'16D070027
Assignment - 1

1. $P(y_1 = \square)$

∴ Input symbol ~~$x_1 z_1$~~. $x_1 z_1, x_2 z_2, \ldots \; x_n z_n$

for $y_1 = \square$ $x_1 = \square$ and $z_1 = \square$

$$P(x_1 = \square, z_1 = \square) = P(x_1 = \square) P(z_1 = \square / x_1 = \square)$$

$$= \epsilon_0 \epsilon_1.$$

2. $P(y_2 = \square)$

$$\Rightarrow P(x_2 = \square, z_2 = \square) = P(x_2 = \square) \underbrace{P(z_2 = \square / x_2 = \square)}_{\epsilon_1}$$

$$= P(x_2 = \square) \epsilon_1$$

$$P(x_2 = \square) = P(x_2 = 0 / z_1 = \square) P(z_1 = \square) +$$

$$P(x_2 = \square / z_1 \neq \square) P(z_1 \neq \square)$$

$$= \epsilon_1 P(z_1 = \square) + \epsilon_0 (1 - P(z_1 = \square))$$

$$\Rightarrow (\epsilon_1 - \epsilon_0) P(z_1 = \square) + \epsilon_0.$$

$\Rightarrow$ ~~$P(z_i$~~

$$P(z_1 = \square) = (\epsilon_1 - \epsilon_0) \epsilon_0 + \epsilon_0$$

$$= \epsilon_1 \epsilon_0 - \epsilon_0^2 + \epsilon_0.$$

$$P(x_2 = \square) = (\epsilon_1 - \epsilon_0) [(\epsilon_1 - \epsilon_0) \epsilon_0 + \epsilon_0] + \epsilon_0.$$

$$= \epsilon_0 + (\epsilon_1 - \epsilon_0) \epsilon_0 + (\epsilon_1 - \epsilon_0)^2 \epsilon_0$$

$$P(y_2 = \square) = P(x_2 = \square) \epsilon_1$$

$$= \{ \epsilon_0 + (\epsilon_1 - \epsilon_0) \epsilon_0 + (\epsilon_1 - \epsilon_0)^2 \epsilon_0 \} \epsilon_1$$

~~$\epsilon_0 \epsilon_1 (\epsilon_1 - \epsilon_0)^2 + \epsilon_0 \epsilon_0 \ldots$~~)

$\varepsilon_1 \varepsilon_0 \left((\varepsilon_1 - \varepsilon_0)^2 + (\varepsilon_1 - \varepsilon_0) + 1 \right)$

$= \cancel{\varepsilon_1 \varepsilon_0}$

$\varepsilon_1 = 1 - \varepsilon_0.$

$P(y_n = \square) = (1 - \varepsilon_0)\varepsilon_0 \left(4\varepsilon_0^2 - 6\varepsilon_0 + 3 \right)$

(3) $\cancel{P(y_n = \square)} \Rightarrow \cancel{\ldots}$

(3) $\mathbb{P}(y_n = \square) = \mathbb{P}(x_n = \square, z_n = \square)$

$= \mathbb{P}(x_n = \square)\, \underbrace{\mathbb{P}(z_n = \square \mid x_n = \square)}_{\varepsilon_1}$

$= \varepsilon_1 \left[P(z_{n-1} = \square)\, \underbrace{\mathbb{P}(x_n = \square \mid z_{n-1} = \square)}_{\varepsilon_1} + \mathbb{P}(z_{n-1} \neq \square)\, \underbrace{\mathbb{P}(x_n = \square \mid z_{n-1} \neq \square)}_{\varepsilon_0} \right]$

$= \varepsilon_1 \left[\varepsilon_1 \mathbb{P}(z_{n-1} = \square) + \varepsilon_0 \left(1 - \mathbb{P}(z_{n-1} = \square)\right) \right]$

$= \varepsilon_1 \left[\varepsilon_0 + (\varepsilon_1 - \varepsilon_0)\, \mathbb{P}(z_{n-1} = \square) \right]$

$= \cancel{\ldots}$

$= \varepsilon_1 \left(\varepsilon_0 + (\varepsilon_1 - \varepsilon_0) \left[\varepsilon_0 + (\varepsilon_1 - \varepsilon_0)\, \mathbb{P}(x_{n-1} = \square) \right] \right)$

$= \varepsilon_1 \left[\varepsilon_0 + \varepsilon_0 (\varepsilon_1 - \varepsilon_0) + (\varepsilon_1 - \varepsilon_0)^2\, \mathbb{P}(x_{n-1} = \square) \right]$

$= \left[(\varepsilon_1 - \varepsilon_0)^2 \left[\varepsilon_0 + (\varepsilon_1 - \varepsilon_0) \left(P(z_{n-2} = \square) \right) \right] + \varepsilon_0 + \varepsilon_0 (1 - \varepsilon_0) \right] \varepsilon_1$ from above result

$\varepsilon_1 \left[\varepsilon_0 + \varepsilon_0 (1 - \varepsilon_0) + \varepsilon_0 (\varepsilon_1 - \varepsilon_0)^2 + (\varepsilon_1 - \varepsilon_0)^3\, \mathbb{P}(z_{n-2} = \square) \right]$

So from the trend, we can write

$$P(y_n = B) = \epsilon_1 \epsilon_0 [1 + (\epsilon_1 - \epsilon_0) + (\epsilon_1 - \epsilon_0)^2 + \cdots]$$

therefore it form a G.P with $|r| < 1$

as $\epsilon_0 \in [0, 1)$

$(\epsilon_1 - \epsilon_0) \in [-1, 1)$

$$\lim_{n \to \infty} P(y_n = B) = \frac{\epsilon_0 \epsilon_1}{1 - (\epsilon_1 - \epsilon_0)}$$

$$\epsilon_0 = 1 - \epsilon_1$$

$$\therefore P(y_n = B) = \frac{\cancel{(1-\epsilon_1)\,\epsilon_1}}{\cancel{1 - \epsilon_1 - 1 + \epsilon_1}}$$

$$= \frac{(1-\epsilon_1)\,\epsilon_1}{2 - 2\epsilon_1} = \frac{\epsilon_1}{2}$$

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{Q2}

a)

(i) $\mathbb{P}(q_{t+1} = k \mid q_t = j, O_1, \dots, O_T)$

current state will not depend on past output

$\mathbb{P}(q_{t+1} = k \mid q_t = j, O_1 \dots O_T) = \mathbb{P}(q_{t+1} = k \mid q_t = j, O_{t+1} \dots O_T)$

$$= \frac{\mathbb{P}(q_{t+1} = k, O_{t+1} \dots O_T \mid q_t = j)}{\mathbb{P}(O_{t+1} \dots O_T \mid q_t = j)}$$

$$= \frac{\mathbb{P}(q_{t+1} = k \mid q_t = j)\, \mathbb{P}(O_{t+1} \dots O_T \mid q_{t+1} = k, q_t = j)}{\beta_t(j)}$$

$\to$ So then

$$\frac{a_{jk}\, \mathbb{P}(O_{t+1} \dots O_T \mid q_{t+1} = k)}{\beta_t(j)}$$

using the property

Since $\Pr(O_{t+1} \dots O_T \mid q_t = j, q_{t+1} = k) = \Pr(O_{t+1} \dots O_T \mid q_{t+1} = k)$

$$\frac{a_{jk}}{\beta_t(j)}\, \mathbb{P}(O_{t+1} \mid q_{t+1} = k) \cdot \mathbb{P}(O_{t+2} \dots O_T \mid q_{t+1} = k)$$

$$= \frac{a_{jk}}{\beta_t(j)}\, b_k(O_{t+1})\, \beta_{t+1}(k)$$

$$\boxed{\mathbb{P}(q_{t+1} = k \mid q_t = j, O_1 \dots O_T) = \frac{\beta_{t+1}(k)}{\beta_t(j)} \times a_{jk} \times b_k(O_{t+1})}$$

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(iii) $\mathbb{P}(q_{t-1} = i, q_t = j, q_{t+1} = k / O_1, \dots, O_T) =$

$\mathbb{P}(q_{t-1} = i / O_1 \dots O_T)\, \mathbb{P}(q_t = j / q_{t-1} = k, O_1 \dots O_T)$

$\mathbb{P}(q_{t+1} / q_t, q_{t-1} = i, O_1, \dots, O_T)$

q_{t+1} does not depend on q_{t-1} ~~given~~, & using part (i) of question 2.

$$\Rightarrow \mathbb{P}(q_{t-1} = i / O_1 \dots O_T) \times \frac{a_{ij}\, b_j(O_t)\, \beta_t(j)}{\beta_{t-1}(i)} \times$$

$$\frac{a_{jk}\, b_k(O_{t+1})\, \beta_{t+1}(k)}{\beta_t(j)}$$

$$\frac{\mathbb{P}(O_1 \dots O_T, q_{t-1} = i)}{P(O_1 \dots O_T)} \times a_{ij}\, a_{jk}\, b_j(O_t)\, b_k(O_{t+1}) \times \frac{\beta_{t+1}(k)}{\beta_{t-1}(i)} \times \cancel{P(O_1)}$$

$=$ [illegible]

$$\Rightarrow \frac{\alpha_{t-1}(i)\, \cancel{\beta_{t-1}(i)}}{\sum_{j=1}^{N} \alpha_t(j)\, \beta_t(j)}\, a_{ij}\, a_{jk}\, b_j(O_t)\, \frac{b_k(O_{t+1})\, \beta_{t+1}(k)}{\cancel{\beta_{(t-1)}(i)}}$$

$$= \frac{\alpha_{(t-1)}(i)\, a_{ij}\, a_{jk}\, b_j(O_t)\, b_k(O_{t+1})\, \beta_{t+1}}{\sum_{j=1}^{N} \alpha_t(j)\, \beta_t(j)}$$

(b)

Viterbi algorithm Consist of three sub-parts.

i) is Initialization which takes $O(N)$ time

(ii) Recursion

Original recursion takes $O(N^2T)$ time due to use of two nested loop on each iteration of all $t-1$ state to current node. $[1 \le j \le N, 1 \le t \le T]$

Given $a_{ii} = p$, $a_{ij} = 2$ $\forall i \neq j$ & $p > 2$

But now if it is modified such that we only have to compare

$V_t(j)$ $\max\left(p \times \text{Viterbi}[j, t-1] , 2 \times \max_{i=1}^{N} \text{Viterbi}[i, t-1]\right) \times b_j(o_t)$

So we only have to make Const ~~Comparision~~ Comparison. $O(C)$ ~~and~~ and $O(N)$ in order to get. i.e will be.

$\max_{i=1}^{N} \text{Viterbi}[i, t-1]$ &

Common for each node j, so we don't have to run for loop in (1 to N)

therefore total time taken will be.

$$T[O(N) + C\,O[N]] = O(NT)$$