Q5

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1 5(a)

Any two parallel lines from world coordinate remain parallel lines wrt. camera coordinates using rigid transformation. Now the projection on image plane will happen wrt. camera coordinates and the camera constant(c).
Any two parallel lines can be defined using :-

$$\frac{(x-x_0)}{l_1} = \frac{(y-y_0)}{m_1} = \frac{(z-z_0)}{n_1} = t_1$$

and

$$\frac{(x-x_1)}{l_1} = \frac{(y-y_1)}{m_1} = \frac{(z-z_1)}{n_1} = t_2$$

So line 1 can be represented as

$$(x_0 + l_1.t_1, y_0 + m_1.t_1, z_0 + n_1.t_1)$$

and line 2 can be represented as

$$(x_1 + l_1.t_2, y_1 + m_1.t_2, z_1 + n_1.t_2)$$

projecting this on the camera plane will give (x,y) coordinates of these lines as:-

$$(\frac{c(x_0 + l_1.t_1)}{z_0 + n_1.t_1}, \frac{c(y_0 + m_1.t_1)}{z_0 + n_1.t_1}), (\frac{c(x_1 + l_1.t_2)}{z_1 + n_1.t_2}, \frac{c(y_1 + m_1.t_2)}{z_1 + n_1.t_2})$$

And as t_1 and t_2 tends to infinity, the limit of these points will be:-

$$\lim_{t_1 \to \infty} (\frac{c(x_0 + l_1.t_1)}{z_0 + n_1.t_1}, \frac{c(y_0 + m_1.t_1)}{z_0 + n_1.t_1}) = (\frac{c.l_1}{n_1}, \frac{c.m_1}{n_1})$$

$$\lim_{t_2 \to \infty} (\frac{c(x_1 + l_1.t_2)}{z_1 + n_1.t_2}, \frac{c(y_1 + m_1.t_2)}{z_1 + n_1.t_2}) = (\frac{c.l_1}{n_1}, \frac{c.m_1}{n_1})$$

which is same. So These two parallel lines have an intersection point on image plane which is the vanishing point.

2 5(b)

Now, let the three sets of parallel lines have direction cosines as $\{(l_1, m_1, n_1),(l_2, m_2, n_2),(l_3, m_3, n_3)\}$

So the intersection points on image plane will have coordinates as :-

$$P_1 = (\frac{c.l_1}{n_1}, \frac{c.m_1}{n_1}), P_2 = (\frac{c.l_2}{n_2}, \frac{c.m_2}{n_2}), P_3 = (\frac{c.l_3}{n_3}, \frac{c.m_3}{n_3})$$

Now these point can be collinear points only if

$$\det \begin{vmatrix} \frac{c.l_1}{n_1} & \frac{c.m_1}{n_1} & 1 \\ \frac{c.l_2}{n_2} & \frac{c.m_2}{n_2} & 1 \\ \frac{c.l_3}{n_3} & \frac{c.m_3}{n_3} & 1 \end{vmatrix} = 0 \implies \frac{c^2}{n_1.n_2.n_3} \det \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

So, after simplifying the above matrix become

$$\det \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

Now, As all these three sets of parallel line are in a plane , the determinant of there direction cosine will also be zero :-

$$\det \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0$$

And the above equation confirms the condition for the points to become collinear points. So the vanishing points corresponding to three (different) sets of parallel lines on a 3D plane are collinear in the image plane