Musical Notes

A sound wave with a period of T will normally have maximum power content at a frequency $f = 1/T$. This is the *fundamental* frequency of this sound. Integral multiples of the fundamental frequency are called overtones. Two notes which have overtones at the same frequency are pleasant to hear together (harmony) or one after the other (melody). Thus two notes with frequencies in the ratio 2:1 sound 'in tune' in harmony or melody, because the fundamental frequency of the first is at the same frequency as the first overtone of the other. The interval between these two notes (at f and $2f$) defines an *octave*.

What other frequencies between these two will sound pleasant with them? The modern Indian scale and the Western musical scale divide this octave into 12 intervals. Each of the intervals is called a semi-tone. In the Western scale, each semi-tone carries the same frequency ratio: $2^{1/12}$. The intervals in the Indian scale are derived from a more complex consideration based on musicality. The twelve intervals are divided between the seven major notes. These notes are known as
do, re, me, fa, so, la, tee and *upper octave do* in the Western scale.
If you have noticed the keys on a piano or harmonium, each key represents an interval. The white keys are the major notes, while the black ones are the minor or sharp notes.

The seven major notes in the Indian scale are called
sa, re, ga, ma, pa, dha, ni followed by *taar saptak sa*.
(These are contractions for Shadaja or Sama, rishabh, gandhaar, madhyam, pancham, dhaivat and nishaad). Five of these carry a 'komal' or 'tivra' variant. Sa and Pa do not have a variant. Thus the twelve intervals in the Indian scale are: *Sa, Komal Re, Re, Komal Ga, Ga, Ma, Tivra Ma, Pa, Komal Dha, Dha, Komal Ni, Ni* and *Taar Saptak Sa*.

The frequencies of notes are fixed by musical considerations in the Indian scale. There are 22 choices (*shrutis*) based on resonances between harmonics of notes. The most appropriate 12 are chosen for the particular combination of notes employed in a composition, or *raaga*.

Since the lower overtones are stronger than higher overtones, notes with frequencies which have simple rational ratios with the base note at f will sound pleasant. Thus, the second harmonic of a note at $3f/2$ (which is at $3f$), will resonate with the third harmonic of the base note

at f. These are strong overtones (second and third) and therefore, this note sounds quite 'in tune' with the base note. In Indian musical notation, the base note (at f) is called 'sa', while this note at $3f/2$ is called 'pa'. The octave note at $2f$ is called the upper 'sa' or *taar saptak sa*.

To look for other resonances, we seek rational numbers between 1 and 2 which are ratios of small integers. The next rational number involving low integers gives a note at $4f/3$. The third harmonic of this (at $4f$), is at the same frequency as the fourth harmonic of f (and second harmonic of $2f$). This note is denoted as 'ma' in the Indian musical scale.

If we chose the frequency of the 'pa' note as the base note, then a note at $3/2 \times 3f/2$ will be at $9f/4$, which lies in the upper octave and has a ratio of 9/8 with the upper sa. Notice, that this is also the ratio of frequencies of 'ma' and 'pa'. Thus, this is a natural musical interval and defines the frequency of 're'.

The table below gives various low integer ratios whose value is between 1 and two and which lie close to some power of $2^{1/12}$.

Note	Power of 2	value	Ratio	Value
Sa	(0/12) =	1.0000	1/1=	1.0000
Komal re	(1/12) =	1.0595	16/15=	1.0667
re	(2/12) =	1.1225	9/8=	1.1250
Komal ga	(3/12) =	1.1892	6/5=	1.2000
ga	(4/12) =	1.2599	5/4=	1.2500
ma	(5/12) =	1.3348	4/3=	1.3333
Tivra ma	(6/12) =	1.4142	64/45=	1.4222
pa	(7/12) =	1.4983	3.2=	1.5000
Komal dha	(8/12) =	1.5874	8/5=	1.6000
dha	(9/12) =	1.6818	5/3=	1.6667
Komal ni	(10/12) =	1.7818	16/9=	1.7778
ni	(11/12) =	1.8877	15/8=	1.8750
sa	(12/12) =	2.0000	2/1=	2.0000