# Unit-2
## Linear Programming Problem

### Numericals.
### # Graphical Method

Q. A watch dealer wishes to buy new watch and has two models M1 and M2 and costs Rs 100 and Rs 200 respectively. In view of showcase of dealer, he want to buy watches not more than 30 and can spend upto Rs. 4000. The watch dealer can make a profit of Rs 20 in M1 and Rs 50 in M2. How many of each model should he buy in order to obtain maximum profit.

Solution:

By question,

| Watch | | | |
|---|---|---|---|
| | M1(X) | M2(Y) | Available |
| P(no. of watches) | 1 | 1 | 30 |
| Q (Total cost) | 100 | 200 | 4000 |
| Profit | Rs 20 | Rs 50 | |

Converting the above table into equations, we get,

Max Profit $(z) = 20x + 50y$ ⟶ objective function

$x + y \leq 30$ — ①

$100x + 200y \leq 4000$ — ②

constraint equation

⟶ available resources.

$x, y \geqslant 0$

For first equation ①, Assume $x + y = 30$

| | | |
|---|---|---|
| $x$ | 0 | 30 |
| $y$ | 30 | 0 |

Taking (0,0) as testing point for equation ①,

$$x+y \leq 30$$

or, $0+0 \leq 30$

or $0 \leq 30$ (True)

So, it lies towards the origin.

For equation ②, Assume $100x+200y=4000$

| | | |
|---|---|---|
| $x$ | 0 | 40 |
| $y$ | 20 | 0 |

Taking (0,0) as testing point for equation ②,

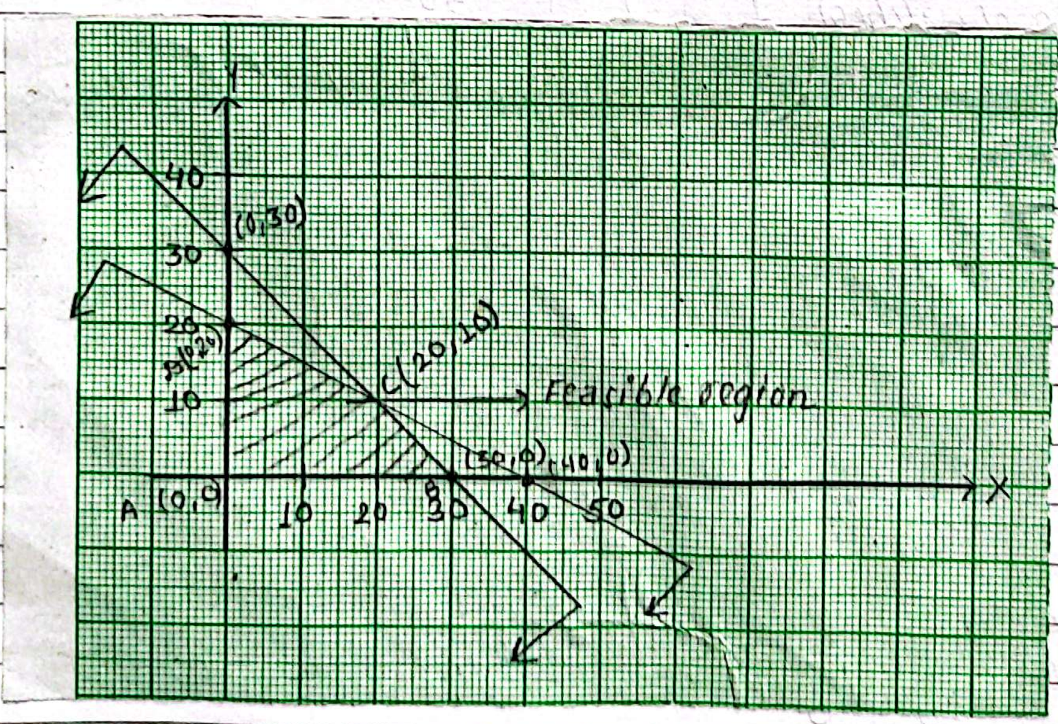
$$100x+200y \leq 4000$$

or, $100\times0+200\times0 \leq 4000$

or, $0 \leq 4000$ (True)

So, it lies towards the origin.

The graph of above constraints is given below:-



The feasible region is shown in the graph. The vertices of the feasible region are A(0,0), B(30,0), C(20,10) and D(0,20).

Analysis table

| Vertices | $M_1(x)$ | $M_2(y)$ | Objective function $Z = 20x + 50y$ |
|---|---|---|---|
| A | 0 | 0 | $Z = 20\times0 + 50\times0 = 0$ |
| B | 30 | 0 | $Z = 20\times30 + 50\times0 = 600$ |
| C | 20 | 10 | $Z = 20\times20 + 50\times10 = 900$ |
| D | 0 | 20 | $Z = 20\times0 + 50\times20 = 1000$ |

∴ From the above table, we conclude that the maximum profit will be Rs 1000 at $x = 0$ and $y = 20$.

# Simplex Method

1. Solve the following problem by simplex method.

e. Max $P = 3x_1 + 2x_2 + 5x_3$

subjected to constraint

$$x_1 + x_2 + x_3 \leq 9$$

$$2x_1 + 3x_2 + 5x_3 \leq 30$$

$$2x_1 - x_2 - x_3 \leq 8$$

Solution:

Formatting the problem in the standard form of the Linear Programming Problem (LPP), we get,

Max $P = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

st.

$$x_1 + x_2 + x_3 + 1s_1 + 0\cdot s_2 + 0\cdot s_3 = 9$$

$$2x_1 + 3x_2 + 5x_3 + 0s_1 + 1s_2 + 0\cdot s_3 = 30$$

$$2x_1 - x_2 - x_3 + 0s_1 + 0s_2 + 1s_3 = 8$$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

Now,

Developing initial simplex table,

| CB | $C_j \rightarrow$ | | 3 | 2 | 5 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | (XB) constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $\frac{XB}{\text{Keycolumn}}$ |
| $R_1: 0$ | $s_1$ | 9 | 1 | 1 | 1 | 1 | 0 | 0 | $9/1 = 9$ |
| $R_2: 0$ | $s_2$ | 30 | 2 | 3 | 5 | 0 | 1 | 0 | $30/5 = 6$ |
| $R_3: 0$ | $s_3$ | 8 | 2 | -1 | -1 | 0 | 0 | 1 | $8/-1 = -$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -3 | -2 | -5 | 0 | 0 | 0 | |

In above table, -5 is the most negative value in $z_j - c_j$ row so, the pivot column is $x_3$ column.

Also,

The minimum positive element in ratio column is 6, so, $S_2$ row is the key row.

From this, the intersection element of pivot colum and pivot row is 5 so 5 is the key element or pivot element.

So,

- new $R_2$ = (old $R_2$) / 5

  i.e $\frac{30}{5} = 6, \frac{2}{5}, \frac{3}{5}, \frac{5}{5} = 1, \frac{0}{5} = 0, \frac{1}{5}, \frac{0}{5} = 0$

- new $R_1$ = old $R_1$ − new $R_2$

  i.e $9 - 6 = 3, 1 - 2/5 = \frac{3}{5}, 1 - \frac{3}{5} = \frac{2}{5}, 1 - 1 = 0, 1 - 0 = 1, 0 - \frac{1}{5} = -\frac{1}{5}, 0$

- new $R_3$ = old $R_3$ + new $R_2$

  i.e $8 + 6 = 14, 2 + \frac{2}{5} = \frac{12}{5}, -1 + \frac{3}{5} = -\frac{2}{5}, -1 + 1 = 0, 0 + 0 = 0, 0 + \frac{1}{5} = \frac{1}{5},$

  $1 + 0 = 1$

| | $C_j \rightarrow$ | | 3 | 2 | 5 | 0 | 0 | 0 | Ratio = $\frac{XB}{\text{Key column}}$ |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | (XB) constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | |
| $R_1$: 0 | $s_1$ | 3 | 3/5 | 2/5 | 0 | 1 | -1/5 | 0 | 3/3/5 = 5 |
| $R_2$: 5 | $x_3$ | 6 | 2/5 | 3/5 | 1 | 0 | 1/5 | 0 | 6/2/5 = 15 |
| $R_3$: 0 | $s_3$ | 14 | 12/5 | -2/5 | 0 | 0 | 1/5 | 1 | 14/12/5 = 5.83 |
| | $z_j$ | 30 | 2 | 3 | 5 | 0 | 1 | 0 | |
| | $z_j - c_j$ | | -1 | 1 | 0 | 0 | 1 | 0 | |

In above table, -1 is the most negative value in $z_j - c_j$ row so, the pivot column is $x_1$ column.
Also,
The minimum positive element in the ratio column is 5, so, $s_1$ row is the key row.
From this, the intersection element of key column and key row is $\frac{3}{5}$ so $\frac{3}{5}$ is the key element or pivot element.
So,

- new $R_1$ = old$R_1 \times \frac{5}{3}$

i.e $3 \times \frac{5}{3} = 5$, $\frac{3}{5} \times \frac{5}{3} = 1$, $\frac{2}{5} \times \frac{5}{3} = \frac{2}{3}$, 0, $\frac{5}{3}$, $\frac{-1}{5} \times \frac{5}{3} = \frac{-1}{3}$, 0

- new $R_2$ = old$R_2 - \frac{2}{5}$ new$R_1$

i.e $6 - \frac{2}{5} \times 5 = 4$, $\frac{2}{5} - \frac{2}{5} \times 1 = 0$, $\frac{3}{5} - \frac{2}{5} \times \frac{2}{3} = \frac{1}{3}$, $1 - \frac{2}{5} \times 0 = 1$,

$0 - \frac{2}{5} \times \frac{5}{3} = -\frac{2}{3}$, $\frac{1}{5} - \frac{2}{5} \times \left(\frac{-1}{3}\right) = \frac{1}{3}$, $0 - \frac{2}{5} \times 0 = 0$

- new $R_3$ = old$R_3 - \frac{12}{5}$ new$R_1$

i.e $14 - \frac{12}{5} \times 5 = 2$, $\frac{12}{5} - \frac{12}{5} \times 1 = 0$, $-\frac{2}{5} - \frac{12}{5} \times \frac{2}{3} = -2$, $0 - \frac{12}{5} \times 0 = 0$, $0 - \frac{12}{5} \times \frac{5}{3}$

$= -4$, $\frac{1}{5} - \frac{12}{5} \times \left(\frac{-1}{3}\right) = 1$, $1 - \frac{12}{5} \times 0 = 1$

| | $C_j$ | | 3 | 2 | 5 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $\frac{xB}{\text{key colum}}$ |
| $R_1$: 3 | $x_1$ | 5 | 1 | 2/3 | 0 | 5/3 | -1/3 | 0 | |
| $R_2$: 5 | $x_3$ | 4 | 0 | 1/3 | 1 | -2/3 | 1/3 | 0 | |
| $R_3$: 0 | $S_3$ | 2 | 0 | -2 | 0 | -4 | 1 | 1 | |
| | $z_j$ | 35 | 3 | 11/3 | 5 | 5/3 | 8/3 | 0 | |
| | $z_j - c_j$ | | 0 | 5/3 | 0 | 5/3 | 2/3 | 0 | |

Since, all $(z_j - c_j) \geq 0$, so optimum solution is obtained.

Max Profit (z) = 35

Basic Variable : $x_1 = 5$, $x_3 = 4$

Non basic variable : $x_2 = 0$

---

8) Max $Z = 3x_1 + 2x_2$

s.t

$2x_1 + x_2 \leq 5$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in the standard form of the Linear Programming problem (LPP), we get,

Max $Z = 3x_1 + 2x_2 + 0s_1 + 0s_2 - MA_1$

s.t.c.

$2x_1 + x_2 + 1s_1 + 0s_2 + 0A_1 = 5$

$3x_1 + 4x_2 + 0s_1 - s_2 + A_1 = 12$

$x_1, x_2, s_1, s_2, A_1 \geq 0$

Now,

Developing the initial simplex table, we get.

| | $C_j \longrightarrow$ | | 3 | 2 | 0 | 0 | −M | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | XB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | A | XB / Key column |
| $R_1$: 0 | $s_1$ | 5 | 2 | 1 | 1 | 0 | 0 | $5/1 = 5$ |
| $R_2$: −M | A | 12 | 3 | 4 | 0 | −1 | 1 | $12/4 = 3$ |
| | $Z_j$ | −12M | −3M | −4M | 0 | M | −M | |
| | $Z_j - C_j$ | | −3m−3 | −4M−2 | 0 | M | 0 | |

In above table, $-4M-2$ is the most negative value in $Z_j - C_j$ row so, the pivot column is $x_2$ column. Since, 3 is the minimum ratio so $R_2$ is the key row and A is the outgoing variable and $x_2$ is incoming variable. Here, 4 is pivot element. So,

new $R_2$ = (old $R_2$) / 4

i.e $\frac{12}{4} = 3$, $\frac{3}{4}$, $\frac{4}{4} = 1$, $\frac{0}{4} = 0$, $\frac{-1}{4}$, $\frac{1}{4}$

new $R_1$ = old $R_1$ − new $R_2$

i.e $5 - 3 = 2$, $2 - \frac{3}{4} = \frac{5}{4}$, $1 - 1 = 0$, $1 - 0 = 1$, $0 - \left(\frac{-1}{4}\right) = \frac{1}{4}$, $\frac{0-1}{4} = \frac{-1}{4}$

Preparing second simplex table,

| | $C_j \longrightarrow$ | | 3 | 2 | 0 | 0 | −M | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | XB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | A | XB/key column |
| $R_1$: 0 | $s_1$ | 2 | 5/4 | 0 | 1 | 1/4 | −1/4 | $2/5/4 = 1.6$ |
| $R_2$: 2 | $x_2$ | 3 | 3/4 | 1 | 0 | −1/4 | 1/4 | $3/3/4 = 4$ |
| | $Z_j$ | 6 | 3/2 | 2 | 0 | −1/2 | 1/2 | |
| | $Z_j - C_j$ | | −3/2 | 0 | 0 | −1/2 | 1/2+M | |

In above table, −3/2 is the most negative value in $Z_j - C_j$ row so, the pivot column is $x_1$ column. Since, 1.6 is the minimum ratio so, $R_1$ is the key row and $S_1$ is the outgoing variable and $x_1$ is incoming variable. Hence, 5/4 is pivot element.

So,

- new $R_1$ = old $R_1 \times \frac{4}{5}$

i.e $2 \times \frac{4}{5} = \frac{8}{5}$, $\frac{5}{4} \times \frac{4}{5} = 1$, $0 \times \frac{4}{5} = 0$, $1 \times \frac{4}{5} = \frac{4}{5}$, $\frac{1}{4} \times \frac{4}{5} = \frac{1}{5}$, $-\frac{1}{4} \times \frac{4}{5} = \frac{-1}{5}$

- new $R_2$ = old $R_2 - \frac{3}{4}$ new $R_1$

i.e $3 - \frac{3}{4} \times \frac{8}{5} = \frac{9}{5}$, $\frac{3}{4} - \frac{3}{4} \times 1 = 0$, $1 - \frac{3}{4} \times 0 = 1$, $0 - \frac{3}{4} \times \frac{4}{5} = -\frac{3}{5}$,

$\frac{-1}{4} - \frac{3}{4} \times \frac{1}{5} = -\frac{2}{5}$, $\frac{1}{4} - \frac{3}{4} \times \left(\frac{-1}{5}\right) = \frac{2}{5}$

Preparing third simplex table,

| | $C_j \rightarrow$ | | 3 | 2 | 0 | 0 | −M | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | XB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | A | XB/Key column |
| $R_1$: 3 | $x_1$ | 8/5 | 1 | 0 | 4/5 | 1/5 | −1/5 | 8/5/1/5 = 8 |
| $R_2$: 2 | $x_2$ | 9/5 | 0 | 1 | −3/5 | −2/5 | 2/5 | 9/5/(−2/5) = −4.5 |
| | $z_j$ | 42/5 | 3 | 2 | 6/5 | −1/5 | 1/5 | |
| | $z_j - c_j$ | | 0 | 0 | 6/5 | −1/5 | $\frac{1}{5}$+M | |

In above table, −1/5 is the most negative value in $z_j - c_j$ row, so, the pivot column is $s_2$ column. Since 8 is the minimum ratio so, $R_1$ is the key row and $x_1$ is the outgoing variable and $s_2$ is incoming variable. Here, 1/5 is the pivot element.

Preparing fourth simplex table,

new $R_1$ = old $R_1 \times 5$

i.e 8, 5, 0, 4, 1, −1

new $R_2$ = old $R_2$ + $\frac{2}{5}$ new $R_1$

i.e. $\frac{9}{5} + \frac{2}{5} \times 8 = 5$, $0 + \frac{2}{5} \times 5 = 2$, $1 + \frac{2}{5} \times 0 = 1$, $-\frac{3}{5} + \frac{2}{5} \times 4 = 1$,

$-\frac{2}{5} + \frac{2}{5} \times 1 = 0$, $\frac{2}{5} + \frac{2}{5} \times (-1) = 0$

Preparing fourth simplex table, we get,

| | $c_j \rightarrow$ | | 3 | 2 | 0 | 0 | -M | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | $x_1$ | $x_2$ | $S_1$ | $S_2$ | A | xB/Keycolumn |
| $R_1$: 0 | $S_2$ | 8 | 5 | 0 | 4 | 1 | -1 | |
| $R_2$: 2 | $x_2$ | 5 | 2 | 1 | 1 | 0 | 0 | |
| | $z_j$ | 10 | 4 | 2 | 2 | 0 | 0 | |
| | $z_j - c_j$ | | 1 | 0 | 2 | 0 | M | |

Since, all $(z_j - c_j) \geq 0$, an optimum solution is obtained.
Max Profit $(z) = 10$
Basic variable : $x_2 = 5$
Non Basic Variable : $x_1 = 0$

---

4. A manufacture makes two types of products $P_1$ and $P_2$ using two machines $M_1$ and $M_2$. Product $P_1$ requires 5 hours on machine $M_1$ and no time on machine on $M_2$, product $P_2$ requires 1 hour on machine $M_1$ and 3 hours on machine $M_2$. There are 16 hours of time per day available on machine $M_1$ and 30 hours on machine $M_2$. Profit margin from $P_1$ and $P_2$ is Rs2 and Rs.10 per unit respectively. What should be the daily production mix to maximize the profit?

Solution:

Converting the above question in tabular form, we get,

| Machine | Product | | Available time |
|---|---|---|---|
| | $P_1(X_1)$ | $P_2(X_2)$ | |
| $M_1$ | 5 | 1 | 16 |
| $M_2$ | — | 3 | 30 |
| Profit | Rs 2 | Rs 10 | |

Mathematical formulation of LPP

$$\text{Max } z = 2x_1 + 10x_2$$

s.t.c

$$5x_1 + x_2 \leq 16$$
$$0x_1 + 3x_2 \leq 30$$
$$x_1, x_2 \geq 0$$

Formatting the problem in the standard form of LPP, we get,

$$\text{Max } z = 2x_1 + 10x_2 + 0s_1 + 0s_2$$
$$5x_1 + x_2 + 1s_1 + 0s_2 = 16$$
$$0x_1 + 3x_2 + 0s_2 + 1s_2 = 30$$
$$x_1, x_2 \geq 0$$

Developing initial simplex table, we get,

| | $c_j \longrightarrow$ | | 2 | 10 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | xB/key column |
| $R_1$: 0 | $s_1$ | 16 | 5 | 1 | 1 | 0 | $16/1 = 16$ |
| $R_2$: 0 | $s_2$ | 30 | 0 | 3 | 0 | 1 | $30/3 = 10$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -2 | -10 | 0 | 0 | |

In above table, -10 is the most negative value in $z_j - c_j$, so key column is $x_2$ column. Since, 10 is the minimum

ratio so, $R_2$ is the key row and $S_2$ is outgoing variable and $x_2$ is incoming variable. Here, 3 is the pivot element.
So,

- new $R_2$ = (old $R_2$) / 3
i.e $\frac{30}{3} = 10$, $\frac{0}{3} = 0$, $\frac{3}{3} = 1$, $0$, $\frac{1}{3}$

- new $R_1$ = old $R_1$ − new $R_2$
i.e $16 - 10 = 6$, $5 - 0 = 5$, $1 - 1 = 0$, $1 - 0 = 1$, $0 - \frac{1}{3} = -\frac{1}{3}$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 2 | 10 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 6 | 5 | 0 | 1 | −1/3 | 6/5 = 1.2 |
| $R_2$: 10 | $x_2$ | 10 | 0 | 1 | 0 | 1/3 | 10/0 = ∞ |
| | $z_j$ | 100 | 0 | 10 | 0 | 10/3 | |
| | $z_j - c_j$ | | −2 | 0 | 0 | 10/3 | |

In above table, −2 is the most negative value in $z_j - c_j$, so key column is $x_1$ column. Since, 1.2 is the minimum ratio so $R_1$ is the key row and $S_1$ is outgoing variable and $x_1$ is incoming variable. Here, 5 is pivot element.
So,
new $R_1$ = (old $R_1$) / 5
i.e $\frac{6}{5}$, $1$, $0$, $\frac{1}{5}$, $-\frac{1}{15}$

Developing third simplex table, we get,

| | $c_j \longrightarrow$ | | 2 | 10 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $x_B$/keycolumn |
| $R_1$: 2 | $x_1$ | 6/5 | 1 | 0 | 1/5 | -1/15 | |
| $R_2$: 10 | $x_2$ | 10 | 0 | 1 | 0 | 1/3 | |
| | $z_j$ | 512/5 | 2 | 10 | 2/5 | 16/5 | |
| | $z_j - c_j$ | | 0 | 0 | 2/5 | 16/5 | |

Since, all $(z_j - c_j) \geq 0$, an optimum solution is obtained.
Max Profit $(z) = 512/5$
Basic variable : $x_1 = 6/5$ & $x_2 = 10$. Ans.

5. A manufacturing house produces two articles X and Y, each of which is processed by two machines A and B. X requires 2 hours of A and 4 hours of B; Y requires 4 hours of A and 2 hours of B given that both machines can be run all the time in a day. If each article X yield a profit of Rs.60 and each article Y yield a profit of Rs 100, find how many of each article should be produced daily for maximum profit.

Solution:-
Converting the above question in tabular form, we get:

| Machine | Articles | | Available time |
|---|---|---|---|
| | X | Y | |
| A | 2 | 4 | 24 |
| B | 4 | 2 | 24 |
| Profit | Rs 60 | Rs 100 | |

Mathematical formulation of LPP is
$$\text{Max } z = 60x + 100y$$

s.t.c

$2x + 4y \le 24$

$4x + 2y \le 24$

$x, y \ge 0$

Formatting the problem in standard form of LPP, we get,

$$Max\ z = 60x + 100y + 0s_1 + 0s_2$$

$$2x + 4y + 1s_1 + 0s_2 = 24$$

$$4x + 2y + 0s_1 + 1s_2 = 24$$

Developing initial simplex table, we get,

| | $c_j \longrightarrow$ | | 60 | 100 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | x | y | $s_1$ | $s_2$ | xB/key column |
| $R_1$: 0 | $s_1$ | 24 | 2 | 4 | 1 | 0 | 24/4 = 6 |
| $R_2$: 0 | $s_2$ | 24 | 4 | 2 | 0 | 1 | 24/2 = 12 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -60 | -100 | 0 | 0 | |

In above table, -100 is the most negative value in $z_j - c_j$, so key column is y column. Since, 6 is the minimum ratio so $R_1$ is the key row. $s_1$ is the outgoing variable and y is the incoming variable.
Here, 4 is the pivot element.

So,

new $R_1$ = (old $R_1$)/4

i.e $\frac{24}{4} = 6$, $\frac{2}{4} = \frac{1}{2}$, $\frac{4}{4} = 1$, $\frac{1}{4}$, 0

new $R_2$ = old $R_2$ − 2 new $R_1$

i.e $24 - 2 \times 6 = 12$, $4 - 2 \times \frac{1}{2} = 3$, $2 - 2 \times 1 = 0$, $0 - 2 \times \frac{1}{4} = \frac{-1}{2}$

$1 - 2 \times 0 = 1$

Developing second simplex table, we get,

| | $c_j \rightarrow$ | | 60 | 100 | 0 | 0 | Ratio = $x_B$/key column |
|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x$ | $y$ | $s_1$ | $s_2$ | |
| $R_1$: 100 | $y$ | 6 | 1/2 | 1 | 1/4 | 0 | 6/1/2 = 12 |
| $R_2$: 0 | $s_2$ | 12 | 3 | 0 | -1/2 | 1 | 12/3 = 4 |
| | $z_j$ | 600 | 50 | 100 | 25 | 0 | |
| | $z_j - c_j$ | | -10 | 0 | 25 | 0 | |

In above table, -10 is the most negative value in $z_j - c_j$ so, $x$ column is key column. 4 is the minimum ratio so $R_2$ is the key row. $S_2$ is the outgoing variable and $x$ is the incoming variable.
Here, 3 is the pivot element.

So, new $R_2$ = (old $R_2$)/3
i.e 12/3 = 4, 3/3 = 1, 0/3 = 0, (-1/2)/3 = -1/6, 1/3

new $R_1$ = old $R_1$ − 1/2 new $R_2$
i.e $6 - \frac{1}{2} \times 4 = 4$, $\frac{1}{2} - \frac{1}{2} \times 1 = 0$, $1 - \frac{1}{2} \times 0 = 1$, $\frac{1}{4} - \frac{1}{2} \times \left(-\frac{1}{6}\right) = \frac{1}{3}$, $0 - \frac{1}{2} \times \frac{1}{3}$
$= -1/6$

Developing third simplex table, we get,

| | $c_j \rightarrow$ | | 60 | 100 | 0 | 0 | Ratio = $x_B$/key column |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x$ | $y$ | $s_1$ | $s_2$ | |
| $R_1$: 100 | $y$ | 4 | 0 | 1 | 1/3 | -1/6 | |
| $R_2$: 60 | $x$ | 4 | 1 | 0 | -1/6 | 1/3 | |
| | $z_j$ | 640 | 60 | 100 | 70/3 | 10/3 | |
| | $z_j - c_j$ | | 0 | 0 | 70/3 | 10/3 | |

Since, all $(z_j - c_j) \geq 0$, an optimum solution is obtained.
Max Profit (z) = 640
Basic variable : $x = 4$ and $y = 4$.

---

6. A firm makes two types of furniture; chairs and tables. The contribution for each product as calculated by accounting department is Rs. 20 per chair and Rs. 30 per table. Both the products are processed on three machines $M_1$, $M_2$ and $M_3$. The time required for each product and the total time available per week one each machine are as follows:

| Machine | Chair | Table | Available hours |
|---|---|---|---|
| $M_1$ | 3 | 3 | 36 |
| $M_2$ | 5 | 2 | 50 |
| $M_3$ | 2 | 6 | 60 |

a. Formulate this problem as L.P.P.
b. How should the manufacturer schedule his production in order to maximize contribution?

Solution:

Mathematical formulation of LPP is

$$\text{Max } z = 20x + 30y$$

s.t.c.

$$3x + 3y \leq 36$$
$$5x + 2y \leq 50$$
$$2x + 6y \leq 60$$
$$x, y \geq 0$$

Formatting the problem in standard form of LPP,

$$\text{Max } z = 20x + 30y + 0s_1 + 0s_2 + 0s_3$$

s.t.c

$$3x + 3y + 1s_1 + 0s_2 + 0s_3 = 36$$
$$5x + 2y + 0s_1 + 1s_2 + 0s_3 = 50$$
$$2x + 6y + 0s_1 + 0s_2 + 1s_3 = 60$$
$$x, y, s_1, s_2, s_3 \geq 0$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 30 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | x | y | $s_1$ | $s_2$ | $s_3$ | xB / key column |
| $R_1$: 0 | $s_1$ | 36 | 3 | 3 | 1 | 0 | 0 | 36/3 = 12 |
| $R_2$: 0 | $s_2$ | 50 | 5 | 2 | 0 | 1 | 0 | 50/2 = 25 |
| $R_3$: 0 | $s_3$ | 60 | 2 | 6 | 0 | 0 | 1 | 60/6 = 10 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -20 | -30 | 0 | 0 | 0 | |

In above table, -30 is the most negative value in $z_j - c_j$. so, y column is the key column. 10 is the minimum ratio so $R_3$ is the key row. Here, $s_3$ is the outgoing variable and y is the incoming variable. Here, 6 is the pivot element.

So,

- new $R_3$ = old $R_3$/6.

  i.e $\frac{60}{6} = 10$, $\frac{2}{6} = \frac{1}{3}$, $\frac{6}{6} = 1$, $\frac{0}{6} = 0$, $\frac{0}{6} = 0$, $\frac{1}{6}$

- new $R_1$ = old $R_1$ − 3 new $R_3$

  i.e $36 - 3\times10 = 6$, $3 - 3\times\frac{1}{3} = 2$, $3 - 3\times1 = 0$, $1 - 3\times0 = 1$, $0 - 3\times0 = 0$,

  $0 - 3\times\frac{1}{6} = -\frac{1}{2}$

- new $R_2$ = old $R_2$ − 2 new $R_3$

  i.e $50 - 2\times10 = 30$, $5 - 2\times\frac{1}{3} = \frac{13}{3}$, $2 - 2\times1 = 0$, $0 - 2\times0 = 0$,

$1 - 2\times 0 = 1, \; 0 - 2\times\frac{1}{6} = -\frac{1}{3}$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 30 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 6 | 2 | 0 | 1 | 0 | $-1/2$ | $6/2 = 3$ |
| $R_2$: 0 | $s_2$ | 30 | $13/3$ | 0 | 0 | 1 | $-1/3$ | $30/13/3 = 6.9$ |
| $R_3$: 30 | $y$ | 10 | $1/3$ | 1 | 0 | 0 | $1/6$ | $10/1/3 = 30$ |
| | $z_j$ | 300 | 10 | 30 | 0 | 0 | 5 | |
| | $z_j - c_j$ | | -10 | 0 | 0 | 0 | 5 | |

In the above table, -10 is the most negative value in $z_j - c_j$. So, $x$ column is the key column. 3 is the minimum ratio so $R_1$ is the key row. Here, $s_1$ is the outgoing variable and $x$ is the incoming variable. Here, 2 is pivot element. So,

- new $R_1$ = old $R_1 \times \frac{1}{2}$

i.e $6\times\frac{1}{2} = 3, \; 2\times\frac{1}{2} = 1, \; 0\times\frac{1}{2} = 0, \; 1\times\frac{1}{2} = \frac{1}{2}, \; 0\times\frac{1}{2} = 0, \; -\frac{1}{2}\times\frac{1}{2} = -\frac{1}{4}$

- new $R_2$ = old $R_2 - \frac{13}{3}$ new $R_1$

i.e $30 - \frac{13}{3}\times 3 = 17, \; \frac{13}{3} - \frac{13}{3}\times 1 = 0, \; 0 - \frac{13}{3}\times 0 = 0, \; 0 - \frac{13}{3}\times\frac{1}{2} = -\frac{13}{6},$

$1 - \frac{13}{3}\times 0 = 1, \; -\frac{1}{3} - \frac{13}{3}\times\left(-\frac{1}{4}\right) = \frac{3}{4}$

- new $R_3$ = old $R_3 - \frac{1}{3}$ new $R_1$

i.e $10 - \frac{1}{3}\times 3 = 9, \; \frac{1}{3} - \frac{1}{3}\times 1 = 0, \; 1 - \frac{1}{3}\times 0 = 1, \; 0 - \frac{1}{3}\times\frac{1}{2} = -\frac{1}{6},$

Date
Page

$0-\frac{1}{3}\times 0 = 0, \frac{1}{6}-\frac{1}{3}\times\left(\frac{-1}{4}\right)=\frac{1}{4}$

Developing third simplex table, we get,

| | $C_j \longrightarrow$ | | 20 | 30 | 0 | 0 | 0 | Ratio : |
|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | x | y | $S_1$ | $S_2$ | $S_3$ | xB/key column |
| $R_1$: 20 | x | 3 | 1 | 0 | 1/2 | 0 | -1/4 | |
| $R_2$: 0 | $S_2$ | 17 | 0 | 0 | -13/6 | 1 | 3/4 | |
| $R_3$: 30 | y | 9 | 0 | 1 | -1/6 | 0 | 1/4 | |
| | $z_j$ | 330 | 20 | 30 | 5 | 0 | 5/2 | |
| | $z_j - c_j$ | | 0 | 0 | 5 | 0 | 5/2 | |

Since, all $(z_j - c_j) \geq 0$, an optimum solution is obtained.
Max. Profit $(z) = 330$.
Basic Variable : $x = 3$, $y = 9$ Ans.

7. A watch manufacturing company produces two types of watches A and B by using three machines $M_1$, $M_2$ and $M_3$. The time required for each watch on each machine and the maximum time available on each machine are given below:

| Machine | Time required for each type A | Time required for each type B | Maximum time available |
|---|---|---|---|
| $M_1$ | 6 | 8 | 380 |
| $M_2$ | 8 | 4 | 300 |
| $M_3$ | 12 | 4 | 404 |

The profit on watch A and B are Rs.50 and Rs.50 respectively. What combination of watches should be produced to obtain the maximum profit?

## Solution:

Mathematical formulation of LPP is

$$\text{Max } z = 50x + 30y$$

s.t.c

$$6x + 8y \le 380$$
$$8x + 4y \le 300$$
$$12x + 4y \le 404$$
$$x, y \ge 0$$

Formatting the problem in standard form of LPP, we get,

$$\text{Max } z = 50x + 30y + 0s_1 + 0s_2 + 0s_3$$

s.t.c.

$$6x + 4y + 1s_1 + 0s_2 + 0s_3 = 380$$
$$8x + 4y + 0s_1 + 1s_2 + 0s_3 = 300$$
$$12x + 4y + 0s_1 + 0s_2 + 1s_3 = 404$$
$$x, y, s_1, s_2, s_3 \ge 0$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 50 | 30 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | Ratio = $x_B$/key column |
| $R_1$: 0 | $s_1$ | 380 | 6 | 4 | 1 | 0 | 0 | 380/6 = 63.33 |
| $R_2$: 0 | $s_2$ | 300 | 8 | 4 | 0 | 1 | 0 | 300/8 = 37.5 |
| $R_3$: 0 | $s_3$ | 404 | 12 | 4 | 0 | 0 | 1 | 404/12 = 33.67 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -50 | -30 | 0 | 0 | 0 | |

In above table, -50 is the most negative value in $z_j - c_j$ so $x$ column is the key column. 33.67 is the minimum ratio so R3 row is the key row. $s_3$ is the outgoing variable and $x$ is the incoming variable. 12 is the pivot element.

- new $R_3$ = (old $R_3$) / 12

i.e $\frac{404}{12} = \frac{101}{3}$, $\frac{12}{12} = 1$, $\frac{4}{12} = \frac{1}{3}$, 0, 0, $\frac{1}{12}$

- new $R_1$ = old $R_1$ − 6$R_3$

i.e $380 - 6 \times \frac{101}{3} = 178$, $6 - 6 \times 1 = 0$, $4 - 6 \times \frac{1}{3} = 2$, $1 - 6 \times 0 = 1$,

$0 - 6 \times 0 = 0$, $0 - 6 \times \frac{1}{12} = -\frac{1}{2}$

- new $R_2$ = old $R_2$ − 8 new $R_3$

i.e $300 - 8 \times \frac{101}{3} = \frac{92}{3}$, $8 - 8 \times 1 = 0$, $4 - 8 \times \frac{1}{3} = \frac{4}{3}$, $0 - 8 \times 0 = 0$,

$1 - 8 \times 0 = 1$, $0 - 8 \times \frac{1}{12} = -\frac{2}{3}$

Developing second simplex table, we get,

| | $c_j \rightarrow$ | | 50 | 30 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | x | y | $s_1$ | $s_2$ | $s_3$ | xB/key column |
| $R_1$: 0 | $s_1$ | 178 | 0 | 2 | 1 | 0 | −1/2 | 178/2 = 89 |
| $R_2$: 0 | $s_2$ | 92/3 | 0 | 4/3 | 0 | 1 | −2/3 | 92/3/4/3 = 23 |
| $R_3$: 50 | x | 101/3 | 1 | 1/3 | 0 | 0 | 1/12 | 101/3/1/3 = 101 |
| | $z_j$ | 5050/3 | 50 | 50/3 | 0 | 0 | 25/6 | |
| | $z_j - c_j$ | | 0 | −40/3 | 0 | 0 | 25/6 | |

In above table, $-\frac{40}{3}$ is the most negative value in $z_j - c_j$ so y column is the key column and 23 is the minimum ratio so $R_2$ is the key row. Here, $S_2$ is the outgoing variable and y is the incoming variable. Here, $\frac{4}{3}$ is the pivot element.

$50 \times 26 + 30 \times 23 = 1990$

Date ______
Page ______

- new $R_2$ = old $R_2 \times \frac{3}{4}$

i.e $\frac{92}{3} \times \frac{3}{4} = 23$, 0, $\frac{4}{3} \times \frac{3}{4} = 1$, 0, $\frac{3}{4}$, $\frac{-2}{3} \times \frac{3}{4} = -\frac{1}{2}$

- new $R_1$ = old $R_1$ − 2 new $R_2$

i.e $178 - 2 \times 23 = 132$, $0 - 2 \times 0 = 0$, $2 - 2 \times 1 = 0$, $1 - 2 \times 0 = 1$,
$0 - 2 \times \frac{3}{4} = -\frac{3}{2}$, $-\frac{1}{2} - 2 \times \left(-\frac{1}{2}\right) = \frac{1}{2}$

- new $R_3$ = old $R_3 - \frac{1}{3}$ new $R_2$

i.e $\frac{101}{3} - \frac{1}{3} \times 23 = 26$, $1 - \frac{1}{3} \times 0 = 1$, $\frac{1}{3} - \frac{1}{3} \times 1 = 0$, $0 - \frac{1}{3} \times 0 = 0$,
$0 - \frac{1}{3} \times \frac{3}{4} = -\frac{1}{4}$, $\frac{1}{12} - \frac{1}{3} \times \left(-\frac{1}{2}\right) = \frac{1}{4}$

$R_3 \to R_3 4$
$R_1 \to R_1 - \frac{1}{2} R_3$
$R_2 \to R_2 + \frac{1}{2} R_3$

Developing third simplex table, we get,

| | $C_j \longrightarrow$ | | 50 | 30 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | x | y | $S_1$ | $S_2$ | $S_3$ | xB/key column |
| $R_1$: 0 | $S_1$ | 132 | 0 | 0 | 1 | −3/2 | 1/2 | 132/ 1/2 = 264 |
| $R_2$: 30 | y | 23 | 0 | 1 | 0 | 3/4 | −1/2 | − |
| $R_3$: 50 | x | 26 | 1 | 0 | 0 | −1/4 | 1/4 | 26/1/4 = 104 |
| | $z_j$ | 1990 | 50 | 30 | 0 | 10 | −5/2 | |
| | $z_j - c_j$ | | 0 | 0 | 0 | 10 | −5/2 | |

| | $C_j \longrightarrow$ | | 50 | 30 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | x | y | S1 | S2 | S3 | |
| $R_1$: 0 | $S_1$ | 80 | −2 | 0 | 1 | −1 | 0 | |
| $R_2$: 30 | y | 75 | 2 | 1 | 0 | 1/4 | 0 | |
| $R_3$: 0 | $S_3$ | 104 | 4 | 0 | 0 | −1 | 1 | |
| | $z_j$ | 2250 | 60 | 30 | 0 | 15/2 | 0 | |
| | $z_j - c_j$ | | 10 | 0 | 0 | 15/2 | 0 | |

8. A firm produces three products that are presented on three different machines. The time required manufacturing one unit of each of the three products and daily capacity of the machines are given in the table below:

| Machine | Product (time in minutes) | | | Available time (in minutes) |
|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | |
| $M_1$ | 2 | 3 | 2 | 440 |
| $M_2$ | 4 | - | 3 | 470 |
| $M_3$ | 2 | 5 | - | 430 |

Determine the daily number of units to be manufactured of each product. The profit per unit for product $P_1, P_2, P_3$ is Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that all the products are consumed in the market.

Solution:

Mathematical formulation of LPP is

$$\text{Max } z = 4x_1 + 3x_2 + 6x_3$$

s.t.c

$$2x_1 + 3x_2 + 2x_3 \le 440$$

$$4x_1 + 0 \cdot x_2 + 3x_3 \le 470$$

$$2x_1 + 5x_2 + 0 \cdot x_3 \le 430$$

$$x_1, x_2, x_3 \geq 0$$

Formatting the problem in standard form of LPP, we get,

$$\text{Max } z = 4x_1 + 3x_2 + 6x_3 + 0s_1 + 0s_2 + 0s_3$$

s.t.c

$$2x_1 + 3x_2 + 2x_3 + 1s_1 + 0s_2 + 0s_3 = 440$$

$$4x_1 + 0x_2 + 3x_3 + 0s_1 + 1s_2 + 0s_3 = 470$$

$$2x_1 + 5x_2 + 0x_3 + 0s_1 + 0s_2 + 1s_3 = 430$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Developing initial simplex table, we get,

| | $c_j \longrightarrow$ | | 4 | 3 | 6 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 440 | 2 | 3 | 2 | 1 | 0 | 0 | 440/2 = 220 |
| $R_2$: 0 | $s_2$ | 470 | 4 | 0 | 3 | 0 | 1 | 0 | 470/3 = 156.67 |
| $R_3$: 0 | $s_3$ | 430 | 2 | 5 | 0 | 0 | 0 | 1 | 430/0 = ∞ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -4 | -3 | -6 | 0 | 0 | 0 | |

In the above table, -6 is the most negative value in $z_j - c_j$ so $x_3$ column is the key column and 156.67 is the minimum ratio so $R_2$ is the key row. Here, $s_2$ is the outgoing variable and $x_3$ is the incoming variable.
Here, 3 is the pivot element.

- new $R_2$ = old $R_2$ / 3
  i.e $\frac{470}{3}$, $\frac{4}{3}$, 0, 1, 0, $\frac{1}{3}$, 0

- new $R_1$ = old $R_1$ - 2 new $R_2$
  i.e $440 - 2 \times \frac{470}{3} = \frac{380}{3}$, $2 - 2 \times \frac{4}{3} = -\frac{2}{3}$, $3 - 2 \times 0 = 3$,

  $2 - 2 \times 1 = 0$, $1 - 2 \times 0 = 1$, $0 - 2 \times \frac{1}{3} = -\frac{2}{3}$, $0 - 2 \times 0 = 0$

Developing second simplex table, we get,

| CB | B.V | $x_B$ constant | $c_j \rightarrow$ 4 $x_1$ | 3 $x_2$ | 6 $x_3$ | 0 $s_1$ | 0 $s_2$ | 0 $s_3$ | Ratio = $x_B$/key column |
|---|---|---|---|---|---|---|---|---|---|
| $R_1$: 0 | $s_1$ | $\boxed{380/3}$ | $-2/3$ | 3 | 0 | 1 | $-2/3$ | 0 | $380/3/3 = 42.22$ |
| $R_2$: 6 | $x_3$ | 470/3 | 4/3 | 0 | 1 | 0 | 1/3 | 0 | $470/3/0 = \infty$ |
| $R_3$: 0 | $s_3$ | 430 | 2 | $\boxed{5}$ | 0 | 0 | 0 | 1 | $430/5 = 86$ |
| | $z_j$ | 940 | 8 | 0 | 6 | 0 | 6/3 | 0 | |
| | $z_j - c_j$ | | 4 | $-3$ | 0 | 0 | 6/3 | 0 | |

In above table, $-3$ is the most negative in $z_j - c_j$ so $x_2$ column is the key column and 42.22 is the positive minimum ratio so $R_1$ is the key row. $s_1$ is the outgoing variable and $x_2$ is the incoming variable. Here, 5 is the pivot element.

- new $R_1$ = old $R_1$ / 3

i.e $(380/3)/3 = \frac{380}{9}$, $\left(-\frac{2}{3}\right)/3 = -\frac{2}{9}$, 1, 0, $\frac{1}{3}$, $\left(\frac{-2}{3}\right)/3 = -\frac{2}{9}$, 0

- new $R_3$ = old $R_3$ $-$ 5 new $R_1$

i.e $430 - 5\times\frac{380}{9} = \frac{1970}{9}$, $2 - 5\times\left(\frac{-2}{9}\right) = \frac{28}{9}$, $5 - 5\times 1 = 0$,

$0 - 5\times 0 = 0$, $0 - 5\times\frac{1}{3} = -\frac{5}{3}$, $0 - 5\times\left(-\frac{2}{9}\right) = \frac{10}{9}$, $1 - 5\times 0 = 1$

Developing third simplex table, we get,

| CB | BV | $x_B$ constant | $c_j \rightarrow$ 4 $x_1$ | 3 $x_2$ | 6 $x_3$ | 0 $s_1$ | 0 $s_2$ | 0 $s_3$ | Ratio = $x_B$/key column |
|---|---|---|---|---|---|---|---|---|---|
| $R_1$: 3 | $x_2$ | 380/9 | $-2/9$ | 1 | 0 | 1/3 | $-2/9$ | 0 | |
| $R_2$: 6 | $x_3$ | 470/3 | 4/3 | 0 | 1 | 0 | 1/3 | 0 | |
| $R_3$: 0 | $s_3$ | 1970/9 | 28/9 | 0 | 0 | $-5/3$ | 10/9 | 1 | |
| | $z_j$ | $\frac{3200}{3}$ | 22/3 | 3 | 6 | 1 | 4/3 | 0 | |
| | $z_j - c_j$ | | 10/3 | 0 | 0 | 1 | 4/3 | 0 | |

Since, all $(z_j - c_j) \geq 0$, an optimum solution is obtained.

Max. Profit (z) : $\frac{5200}{3}$

Basic variable : $x_2 = \frac{380}{9}$, $x_3 = \frac{470}{3}$

Non Basic Variable : $x_1 = 0$. Ans.

---

9. A Gear Manufacturing Company received an order for three special types of gears for regular supply. The management is considering to devote the available excess capacity to one or more of the three types, say A, B and C the available capacity on the machines which might limit output and the no. of hours required for each unit of the respective gear is also given below:

| Machine type | Available machine hour per week | No. of hours required per unit | | |
|---|---|---|---|---|
| | | Gear A | Gear B | Gear C |
| Gear Hobbing Machine | 250 | 8 | 2 | 3 |
| Gear Shaping Machine | 150 | 4 | 3 | 0 |
| Gear Grinding Machine | 50 | 2 | - | 1 |

The unit profit would be Rs.20, Rs.6 and Rs.8 respectively for the gear A, B and C. Find how much of gear the company should produce in order to maximize the profit?

Solution:

Mathematical formulation of LPP is

$$\text{Max } z = 20A + 6B + 8C$$

s.t.c

$$8A + 2B + 3C \leq 250$$

$$4A + 3B + 0C \leq 150$$

$$2A + 0B + 1C \leq 50$$

$$A, B, C \geq 0$$

Formatting the problem in the standard form of LPP,

$$\text{Max } z = 20A + 6B + 8C + 0S_1 + 0S_2 + 0S_3$$

s.t.c

$$8A + 2B + 3C + 1S_1 + 0S_2 + 0S_3 = 250$$

$$4A + 3B + 0C + 0S_1 + 1S_2 + 0S_3 = 150$$

$$2A + 0B + 1C + 0S_1 + 0S_2 + 1S_3 = 50$$

$$A, B, C, S_1, S_2, S_3 \geq 0$$

Developing initial simplex table, we get.

| | $C_j \rightarrow$ | | 20 | 6 | 8 | 0 | 0 | 0 | Ratio : |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | A | B | C | $S_1$ | $S_2$ | $S_3$ | xB/keycolumn |
| $R_1$: 0 | $S_1$ | 250 | 8 | 2 | 3 | 1 | 0 | 0 | 250/8 = 31.25 |
| $R_2$: 0 | $S_2$ | 150 | 4 | 3 | 0 | 0 | 1 | 0 | 150/4 = 37.5 |
| $R_3$: 0 | $S_3$ | 50 | 2 | 0 | 1 | 0 | 0 | 1 | 50/2 = 25 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -20 | -6 | -8 | 0 | 0 | 0 | |

In above table, -20 is the most negative in $z_j - c_j$ so A column is the key column and 25 is the minimum ratio so $R_3$ is the key row and $S_3$ is the outgoing variable and A is the incoming variable. Here, 2 is the pivot element.

- new $R_3$ = old $R_3$ / 2

i.e $\frac{50}{2} = 25$, $\frac{2}{2} = 1$, 0, $\frac{1}{2}$, 0, 0, $\frac{1}{2}$

- new $R_1$ = old $R_1$ − 8 new $R_3$

i.e $250 - 8 \times 25 = 50$, $8 - 8 \times 1 = 0$, $2 - 8 \times 0 = 2$, $3 - 8 \times \frac{1}{2} = -1$, $1 - 8 \times 0 = 1$,

$0 - 8 \times 0 = 0$, $0 - 8 \times \frac{1}{2} = -4$

- new $R_2$ = old $R_2$ − 4 new $R_3$

i.e $150-4\times25=50$, $4-4\times1=0$, $3-4\times0=3$, $0-4\times\frac{1}{2}=-2$,

$0-4\times0=0$, $1-4\times0=1$, $0-4\times\frac{1}{2}=-2$.

Developing second simplex table, we get;

| | $C_j \rightarrow$ | | 20 | 6 | 8 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | A | B | C | $S_1$ | $S_2$ | $S_3$ | xB/key column |
| $R_1$: 0 | $S_1$ | 50 | 0 | 2 | −1 | 1 | 0 | −4 | 50/2 = 25 |
| $R_2$: 0 | $S_2$ | 50 | 0 | 3 | −2 | 0 | 1 | −2 | 50/3 = 16.67 |
| $R_3$: 20 | A | 25 | 1 | 0 | 1/2 | 0 | 0 | 1/2 | 25/0 = ∞ |
| | $z_j$ | 500 | 20 | 0 | 10 | 0 | 0 | 10 | |
| | $z_j - c_j$ | | 0 | −6 | 2 | 0 | 0 | 10 | |

In above table, −6 is the most negative in $z_j - c_j$ so B is the key column and 16.67 is the minimum ratio so $R_2$ is the key row. $S_2$ is the outgoing variable and B is the incoming variable. Here, 3 is the pivot element.

- new $R_2$ = old $R_2$/3

i.e $\frac{50}{3}$, 0, 1, $\frac{-2}{3}$, 0, $\frac{1}{3}$, $\frac{-2}{3}$

- new $R_1$ = old $R_1$ − 2 new $R_2$

i.e $50-2\times\frac{50}{3}=\frac{50}{3}$, $0-2\times0=0$, $2-2\times1=0$, $-1-2\times\left(\frac{-2}{3}\right)=\frac{1}{3}$,

$1-2\times0=1$, $0-2\times\frac{1}{3}=-\frac{2}{3}$, $-4-2\times\left(\frac{-2}{3}\right)=-\frac{8}{3}$

Developing third simplex table, we get,

| | $C_j \longrightarrow$ | | 20 | 6 | 8 | 0 | 0 | 0 | Ratio: |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | A | B | C | $S_1$ | $S_2$ | $S_3$ | $x_B$/key column |
| $R_1$: 0 | $S_1$ | 50/3 | 0 | 0 | 1/3 | 1 | -2/3 | -8/3 | 50/3/1/3 = 50 |
| $R_2$: 6 | B | 50/3 | 0 | 1 | -2/3 | 0 | 1/3 | -2/3 | 50/3/-2/3 = -25 |
| $R_3$: 20 | A | 25 | 1 | 0 | 1/2 | 0 | 0 | 1/2 | 25/1/2 = 50 |
| | $z_j$ | 600 | 20 | 6 | 6 | 0 | 2 | 6 | |
| | $z_j - c_j$ | | 0 | 0 | -2 | 0 | 2 | 6 | |

In above table, I take 50 from $R_1$ as the ratio. Here, -2 is the most negative in $z_j - c_j$ so C column is the key column and 50 in $R_1$ row is the key row.

new $R_1$ = old $R_1$ × 3 i.e 50, 0, 0, 1, 3, -2, -8

new $R_2$ = old $R_2$ + 2/3 new $R_1$ i.e 50, 0, 1, 0, 2, -1, -6

new $R_3$ = old $R_3$ $- \frac{1}{2}$ new $R_1$ i.e 0, 1, 0, 0, -3/2, 1, 9/2

| | $C_j \longrightarrow$ | | 20 | 6 | 8 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | C | $S_1$ | $S_2$ | $S_3$ | |
| $R_1$: 8 | C | 50 | 0 | 0 | 1 | 3 | -2 | -8 | 50/-8 = - |
| $R_2$: 6 | B | 50 | 0 | 1 | 0 | 2 | -1 | -6 | - |
| $R_3$: 20 | A | 0 | 1 | 0 | 0 | -3/2 | 1 | 9/2 | 0 |
| | $z_j$ | 700 | 20 | 6 | 8 | 6 | -2 | -10 | |
| | $z_j - c_j$ | | 0 | 0 | 0 | 6 | -2 | -10 | |

We cannot further continue the table so the possible solution is Max z = 700 at A = 0, B = 50 & C = 50.

10. A watch dealer wishes to buy new watch and has two models $M_1$ and $M_2$ and cost Rs.100 and Rs. 200 respectively. In view of showcase of dealer, he wants to buy watches not more than 30 and he can spend upto Rs. 4000. The watch dealer can make a profit of Rs.20 in $M_1$ and Rs.50 in $M_2$. How many of each model should he buy in order to obtain maximum profit?

Solution:

Converting the question in tabular form, we get.

| | Watch | | |
|---|---|---|---|
| | $M_1(X)$ | $M_2(Y)$ | Available |
| P(no. of watches) | 1 | 1 | 30 |
| Q (Total Cost) | 100 | 200 | 4000 |
| Profit | Rs20 | Rs50 | |

Mathematical formulation of LPP,

$$\text{Max } z = 20x + 50y$$

s.t.c

$$x+y \le 30$$

$$100x + 200y \le 4000$$

$$x, y \geq 0$$

Formatting the problem in standard form of LPP, we get.

$$\text{Max } z = 20x + 50y + 0s_1 + 0s_2$$

s.t.c

$$x+y+1s_1+0s_2 = 30$$

$$100x + 200y + 0s_1 + 1s_2 = 4000$$

$$x, y, s_1, s_2 \geq 0$$

Developing initial simplex table, we get,

| | $C_j \longrightarrow$ | | 20 | 50 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x$ | $y$ | $s_1$ | $s_2$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 30 | 1 | 1 | 1 | 0 | $30/1 = 30$ |
| $R_2$: 0 | $s_2$ | 4000 | 100 | 200 | 0 | 1 | $4000/200 = 20$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | −20 | −50 | 0 | 0 | |

In the above table, −50 is the most negative in $z_j - c_j$ so $y$ column is the key column and 20 is the minimum ratio so $R_2$ is key row. $S_2$ is outgoing variable and $y$ is incoming variable. Here, 200 is pivot element.

- new $R_2$ = old $R_2$ / 200

  i.e $\frac{4000}{200} = 20$, $\frac{100}{200} = \frac{1}{2}$, $\frac{200}{200} = 1$, $0$, $\frac{1}{200}$

- new $R_1$ = old $R_1$ − new $R_2$

  i.e $30 - 20 = 10$, $1 - \frac{1}{2} = \frac{1}{2}$, $1 - 1 = 0$, $1 - 0 = 1$, $0 - \frac{1}{200} = -\frac{1}{200}$

Developing second simplex table, we get,

| | $C_j \longrightarrow$ | | 20 | 50 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x$ | $y$ | $s_1$ | $s_2$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 10 | 1/2 | 0 | 1 | −1/200 | |
| $R_2$: 50 | $y$ | 20 | 1/2 | 1 | 0 | 1/200 | |
| | $z_j$ | 1000 | 25 | 50 | 0 | 1/4 | |
| | $z_j - c_j$ | | 5 | 0 | 0 | 1/4 | |

Since, all $(z_j - c_j) \geq 0$, the optimum solution is obtained.

Max Profit $(z) = 1000$

Basic variable : $y = 20$ and Non Basic variable : $x = 0$

Date ____
Page ____

11. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. A fan costs him Rs 360 and a sewing machine costs Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and sewing machine at profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize the profit?

Solution:

Converting the question in tabular form, we get,

| | Products | | Available |
|---|---|---|---|
| | Fans ($x_1$) | Sew machine ($x_2$) | |
| no. of fans & sew ma | 1 | 1 | 20 |
| Q (Total cost) | 360 | 240 | 5760 |
| Profit | Rs 22 | Rs 18 | |

Mathematical formulation of LPP,

$$\text{Max } z = 22x_1 + 18x_2$$

s.t.c

$$x_1 + x_2 \leq 20$$

$$360x_1 + 240x_2 \leq 5760$$

$$x_1, x_2 \geq 0$$

Formatting in standard form of LPP, we get,

$$\text{Max } z = 22x_1 + 18x_2 + 0s_1 + 0s_2$$

s.t.c

$$x_1 + x_2 + 1s_1 + 0s_2 = 20$$

$$360x_1 + 240x_2 + 0s_1 + s_2 = 5760$$

$$x_1, x_2, s_1, s_2 \geq 0$$

Developing initial simplex table, we get,

| | $c_j \longrightarrow$ | | 22 | 18 | 0 | 0 | Ratio: |
|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 20 | $\boxed{1}$ | 1 | 1 | 0 | $20/1 = 20$ |
| $R_2$: 0 | $s_2$ | 5760 | $\boxed{360}$ | 240 | 0 | 1 | $5760/360 = 16$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -22 | -18 | 0 | 0 | |

In the above table, -22 is the most negative in $z_j - c_j$ so $x_1$ column is key column and 16 is the minimum ratio so $R_2$ is the key row. $S_2$ is the outgoing variable and $x_1$ is the incoming variable. Here, 360 is pivot element.

- new $R_2$ = old $R_2$ / 360

  i.e $\frac{5760}{360} = 16$, 1, $\frac{240}{360} = \frac{2}{3}$, 0, $\frac{1}{360}$

- new $R_1$ = old $R_1$ − new $R_2$

  i.e $20 - 16 = 4$, $1 - 1 = 0$, $1 - \frac{2}{3} = \frac{1}{3}$, $1 - 0 = 1$, $0 - \frac{1}{360} = -\frac{1}{360}$

Developing second simplex table, we get,

| | $c_j \longrightarrow$ | | 22 | 18 | 0 | 0 | Ratio: |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 4 | 0 | $\boxed{1/3}$ | 1 | $-1/360$ | $4/1/3 = 12$ |
| $R_2$: 22 | $x_1$ | 16 | 1 | $\boxed{2/3}$ | 0 | $1/360$ | $16/2/3 = 24$ |
| | $z_j$ | 352 | 22 | 44/3 | 0 | 11/180 | |
| | $z_j - c_j$ | | 0 | -10/3 | 0 | 11/180 | |

In the above table, $-\frac{10}{3}$ is the most negative in $z_j - c_j$ so $x_2$ is key column and 12 is the minimum value/ratio so $R_1$ is the key row. $S_1$ is the outgoing variable and $x_2$ is the incoming variable. Here, $\frac{1}{3}$ is pivot element.

- new$R_1$ = old$R_1 \times 3$

i.e. $4 \times 3 = 12,\ 0,\ \frac{1}{3} \times 3 = 1,\ 3,\ -\frac{1}{360} \times 3 = -\frac{1}{120}$

- new$R_2$ = old$R_2 - \frac{2}{3}$ new$R_1$

i.e. $16 - \frac{2}{3} \times 12 = 8,\ 1 - \frac{2}{3} \times 0 = 1,\ \frac{2}{3} - \frac{2}{3} \times 1 = 0,\ 0 - \frac{2}{3} \times 3 = -2,$

$\frac{1}{360} - \frac{2}{3} \times \left(-\frac{1}{120}\right) = \frac{1}{120}$

Developing third simplex table, we get,

| | $C_j \longrightarrow$ | | 22 | 18 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $x_B$/key column |
| $R_1$: 18 | $x_2$ | 12 | 0 | 1 | 3 | $-1/120$ | |
| $R_2$: 22 | $x_1$ | 8 | 1 | 0 | $-2$ | $1/120$ | |
| | $z_j$ | 392 | 22 | 18 | 10 | $1/30$ | |
| | $z_j - c_j$ | | 0 | 0 | 10 | $1/30$ | |

Since, all $(z_j - c_j) \geq 0$ so, the optimum solution is obtained.

Max Profit $(z) = 392$.

Basic Variable : $x_1 = 8$ and $x_2 = 12$ Ans.

थाल चाहिन्छ          भाँडे चाहिन्छ raw material

12. A firm produces two types of clothes A and B and makes a profit of Rs 20 per unit on A and Rs 25 per unit on B. Three types of workers skilled, semi-skilled and un-skilled are available each for 8 hours a day. The production of the one unit of A requires 1/4 hour of skilled worker and 2/3 hour of un skilled worker. Production of one unit of B requires 1/3 hour of semi-skilled and 1/2 hour of un-skilled worker. What combination of the two types of clothes should be produced to maximized total profit?

Solution:

Converting the question in tabular form, we get,

| Workers | Clothes | | Available |
|---|---|---|---|
| | $A(x_1)$ | $B(x_2)$ | |
| Skilled | 1/4 | – | 8 |
| Semi skilled | – | 1/3 | 8 |
| Unskilled | 2/3 | 1/2 | 8 |
| Profit | Rs 20 | Rs 25 | |

Mathematical formulation of LPP,

$$\text{Max } z = 20x_1 + 25x_2$$

s.t.c

$$\frac{1}{4}x_1 + 0x_2 \leq 8$$

$$0x_1 + \frac{1}{3}x_2 \leq 8$$

$$\frac{2}{3}x_1 + \frac{1}{2}x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

Formatting the problem in standard form of LPP,

Date ______
Page ______

$$\text{Max } z = 20x_1 + 25x_2 + 0s_1 + 0s_2 + 0s_3$$

s.t.c

$$\frac{1}{4}x_1 + 0x_2 + 1s_1 + 0s_2 + 0s_3 = 8$$

$$0x_1 + \frac{1}{3}x_2 + 0s_1 + 1s_2 + 0s_3 = 8$$

$$\frac{2}{3}x_1 + \frac{1}{2}x_2 + 0s_1 + 0s_2 + 1s_3 = 8$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 25 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 8 | 1/4 | 0 | 1 | 0 | 0 | $8/0 = \infty$ |
| $R_2$: 0 | $s_2$ | 8 | 0 | 1/3 | 0 | 1 | 0 | $8/1/3 = 24$ |
| $R_3$: 0 | $s_3$ | 8 | 2/3 | 1/2 | 0 | 0 | 1 | $8/1/2 = 16$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -20 | -25 | 0 | 0 | 0 | |

In above table, -25 is the most negative in $z_j - c_j$ so $x_2$ column is the key column and 16 is the minimum ratio so R3 row is the key row. $s_3$ is the outgoing variable and $x_2$ is incoming variable. Here, $\frac{1}{2}$ is pivot element.

- new R3 = old R3 × 2

  i.e $8 \times 2 = 16$, $\frac{2}{3} \times 2 = \frac{4}{3}$, $\frac{1}{2} \times 2 = 1$, 0, 0, 2

- new R2 = old R2 $- \frac{1}{3}$ new R3

  i.e $8 - \frac{1}{3} \times 16 = \frac{8}{3}$, $0 - \frac{1}{3} \times \frac{4}{3} = -\frac{4}{9}$, $\frac{1}{3} - \frac{1}{3} \times 1 = 0$, $0 - \frac{1}{3} \times 0 = 0$, $1 - \frac{1}{3} \times 0 = 1$, $0 - \frac{1}{3} \times 2 = -\frac{2}{3}$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 25 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ (constant) | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ | $x_B$/key column |
| $R_1$: 0 | $S_1$ | 8 | 1/4 | 0 | 1 | 0 | 0 | |
| $R_2$: 0 | $S_2$ | 8/3 | 4/9 | 0 | 0 | 1 | -2/3 | |
| $R_3$: 25 | $x_2$ | 16 | 4/3 | 1 | 0 | 0 | 2 | |
| | $Z_j$ | 400 | 100/3 | 25 | 0 | 0 | 50 | |
| | $Z_j - C_j$ | | 40/3 | 0 | 0 | 0 | 50 | |

Since, $(Z_j - C_j) \geq 0$ so the optimum solution is obtained.
Max Profit $(Z) = 400$.
Basic variable : $x_2 = 16$.
Non Basic variable : $x_1 = 0$ Ans.

13. A manufacturing company produces three types of leather belts A, B and C which is produced on three machines $M_1$, $M_2$ and $M_3$. Belt A require 2 hours on machine $M_1$, 3 hours on machine $M_2$ and 2 hours on machine $M_3$. Belt C requires 5 hours on machine $M_2$ and 4 hours on machine $M_3$. There are 8 hours of time per day available on machine $M_1$, 10 hours on machine $M_2$ and 15 hours on machine $M_3$. The profit per unit gained from belts A, B and C are respectively Rs 3, Rs 5 and Rs. 4. What would be the daily production of belt A, B and C to maximize the profit.

Solution;

Converting the question in tabular form, we get,

| Machines | Leather Belts | | | Available |
|---|---|---|---|---|
| | $A(x_1)$ | $B(x_2)$ | $C(x_3)$ | |
| $M_1$ | 2 | 3 | - | 8 |
| $M_2$ | 3 | 2 | 5 | 10 |
| $M_3$ | 2 | 2 | 4 | 15 |
| Profit | Rs 3 | Rs 5 | Rs 4 | |

Mathematical formulation of LPP,

$$\text{Max } z = 3x_1 + 5x_2 + 4x_3$$

s.t.c

$$2x_1 + 3x_2 + 0x_3 \le 8$$

$$3x_1 + 2x_2 + 5x_3 \le 10$$

$$2x_1 + 2x_2 + 4x_3 \le 15$$

$$x_1, x_2, x_3 \geq 0$$

Formatting the problem in standard form of LPP, we get,

$$\text{Max } z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$$

s.t.c

$$2x_1 + 3x_2 + 0x_3 + 1s_1 + 0s_2 + 0s_3 = 8$$

$$3x_1 + 2x_2 + 5x_3 + 0s_1 + 1s_2 + 0s_3 = 10$$

$$2x_1 + 2x_2 + 4x_3 + 0s_1 + 0s_2 + 1s_3 = 15$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 3 | 5 | 4 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/keycolumn |
| $R_1$: 0 | $s_1$ | 8 | 2 | 3 | 0 | 1 | 0 | 0 | $8/3 = 2.667$ |
| $R_2$: 0 | $s_2$ | 10 | 3 | 2 | 5 | 0 | 1 | 0 | $10/2 = 5$ |
| $R_3$: 0 | $s_3$ | 15 | 2 | 2 | 4 | 0 | 0 | 1 | $15/2 = 7.5$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -3 | -5 | -4 | 0 | 0 | 0 | |

In the above table, $-5$ is the most negative in $z_j - c_j$ so $x_2$ column is key column and 2.667 is the minimum ratio so $R_1$ is the key row. Here, $s_1$ is the outgoing variable and $x_1$ is the incoming variable. Here, 3 is pivot element.

- new $R_1$ = old $R_1$ / 3

  i.e $\frac{8}{3}, \frac{2}{3}, 1, 0, \frac{1}{3}, 0, 0$

- new $R_2$ = old $R_2 - 2 \times$ new $R_1$

  i.e $10 - 2\times\frac{8}{3} = \frac{14}{3}$, $3 - 2\times\frac{2}{3} = \frac{5}{3}$, $2 - 2\times1 = 0$, $5 - 2\times0 = 5$,

  $0 - 2\times\frac{1}{3} = -\frac{2}{3}$, $1 - 2\times0 = 1$, $0 - 2\times0 = 0$

- new $R_3$ = old $R_3$ − 2 new $R_1$

  i.e $15 - 2\times\frac{8}{3} = \frac{29}{3}$, $2 - 2\times\frac{2}{3} = \frac{2}{3}$, $2 - 2\times1 = 0$, $4 - 2\times0 = 4$,

  $0 - 2\times\frac{1}{3} = -\frac{2}{3}$, $0 - 2\times0 = 0$, $1 - 2\times0 = 1$

Developing second simplex table, we get,

| | $c_j \longrightarrow$ | | 3 | 5 | 4 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 5 | $x_2$ | 8/3 | 2/3 | 1 | 0 | 1/3 | 0 | 0 | 8/3/0 = ∞ |
| $R_2$: 0 | $s_2$ | 14/3 | 5/3 | 0 | 5 | −2/3 | 1 | 0 | 14/3/5 = 0.933 |
| $R_3$: 0 | $s_3$ | 29/3 | 2/3 | 0 | 4 | −2/3 | 0 | 1 | 29/3/4 = 2.41 |
| | $z_j$ | 40/3 | 10/3 | 5 | 0 | 5/3 | 0 | 0 | |
| | $z_j - c_j$ | | 1/3 | 0 | −4 | 5/3 | 0 | 0 | |

In above table, $-4$ is the most negative in $z_j - c_j$ so $x_3$ column is key column and 0.933 is the minimum ratio so $R_2$ is keyrow. Here, $s_2$ is outgoing variable and $x_3$

is incoming variable. Here, 5 is the pivot element.

- new $R_2$ = old $R_2$ / 5

i.e $14/3/5 = \frac{14}{15}$, $5/3/5 = \frac{1}{3}$, 0, 1, $-2/3/5 = -\frac{2}{15}$, $\frac{1}{5}$, 0

- new $R_3$ = old $R_3$ − 4 new $R_2$

i.e $\frac{29}{3} - 4 \times \frac{14}{15} = \frac{89}{15}$, $\frac{2}{3} - 4 \times \frac{1}{3} = -\frac{2}{3}$, $0 - 4 \times 0 = 0$, $4 - 4 \times 1 = 0$,

$-\frac{2}{3} - 4 \times \left(-\frac{2}{15}\right) = -\frac{2}{15}$, $0 - 4 \times \frac{1}{5} = \frac{-4}{5}$, $1 - 4 \times 0 = 1$

Developing third simplex time, we get,

| | $C_j \rightarrow$ | | 3 | 5 | 4 | 0 | 0 | 0 | Ratio: |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 5 | $x_2$ | 8/3 | 2/3 | 1 | 0 | 1/3 | 0 | 0 | |
| $R_2$: 4 | $x_3$ | 14/15 | 1/3 | 0 | 1 | -2/15 | 1/5 | 0 | |
| $R_3$: 0 | $s_3$ | 89/15 | -2/3 | 0 | 0 | -2/15 | -4/5 | 1 | |
| | $z_j$ | $\frac{256}{15}$ | 14/3 | 5 | 4 | 17/15 | 4/5 | 0 | |
| | $z_j - c_j$ | | 5/3 | 0 | 0 | 17/15 | 4/5 | 0 | |

14. A manufacturing company manufactures two different products. The demand for both the products is strong enough so that the firm can sell as many units of either product, or both, as it can produce and at such a price as to realize a per unit profit contribution of Rs.16 on product. A and Rs 10 on product. B. Unfortunately, the production capacity of the company's plant is severely limited. This limitation stems from the fact that the manufacture of the products involves the utilization of three scarce resources; raw material, labor and machine time. Each of products A requires 4 units of raw material, 3 units of labor and 2 units of machine time. Each produt B requires 2 units of raw material, 3 units of labor and 5 units of machine time. The firm has a daily supply of 24 units of raw material, 21 units of labor and 30 units of machine time. Formulate a LPP model and determine how much of each product should be manufactured to maximize the total profit by using simplex method.

Solution:

Converting the problem in tabular form, we get,

| Resources | Products | | Available |
|---|---|---|---|
| | A ($x_1$) | B($x_2$) | |
| Raw material | 4 | 2 | 24 |
| Labor | 3 | 3 | 21 |
| Machine Time | 2 | 5 | 30 |
| Profit | Rs.16 | Rs.10 | |

Mathematical formulation of LPP,

$$\text{Max } Z = 16x_1 + 10x_2$$

s.t.c

$$4x_1 + 2x_2 \le 24$$

$3x_1 + 3x_2 \le 21$

$2x_1 + 5x_2 \le 30$

$x_1, x_2 \geq 0$

Formatting the problem in standard form of LPP,

$Max\ z = 16x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

s.t.c

$4x_1 + 2x_2 + 1s_1 + 0s_2 + 0s_3 = 24$

$3x_1 + 3x_2 + 0s_1 + 1s_2 + 0s_3 = 21$

$2x_1 + 5x_2 + 0s_1 + 0s_2 + 1s_3 = 30$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 16 | 10 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | xB/key column |
| $R_1$: 0 | $s_1$ | 24 | 4 | 2 | 1 | 0 | 0 | 24/4 = 6 |
| $R_2$: 0 | $s_2$ | 21 | 3 | 3 | 0 | 1 | 0 | 21/3 = 7 |
| $R_3$: 0 | $s_3$ | 30 | 2 | 5 | 0 | 0 | 1 | 30/2 = 15 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -16 | -10 | 0 | 0 | 0 | |

In the above table, -16 is the most negative value in $z_j - c_j$ so, $x_1$ is the key column and 6 is the minimum ratio so $R_1$ is the key row. Here, $s_1$ is outgoing variable and $x_1$ is incoming variable. Here, 4 is the pivot element.

- new $R_1$ = old$R_1$/4

  i.e 24/4 = 6, 4/4 = 1, 2/4 = 1/2, 1/4, 0, 0

- new$R_2$ = old$R_2$ - 3new$R_1$

i.e $21-3\times6=3,\ 3-3\times1=0,\ 3-3\times\frac{1}{2}=\frac{3}{2},\ 0-3\times\frac{1}{4}=-\frac{3}{4},\ 1-3\times0=0,0$

- new$R_3$ = old$R_3$ − 2new$R_1$

i.e $30-2\times6=18,\ 2-2\times1=0,\ 5-2\times\frac{1}{2}=4,\ 0-2\times\frac{1}{4}=-\frac{1}{2},\ 0-2\times0=0,\ 1$

Developing second simplex table, we get,

| | $c_j \longrightarrow$ | | 16 | 10 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 16 | $x_1$ | 6 | 1 | $\boxed{1/2}$ | 1/4 | 0 | 0 | $6/1/2=12$ |
| $R_2$: 0 | $s_2$ | 3 | 0 | $\boxed{3/2}$ | −3/4 | 0 | 0 | $3/3/2=2$ |
| $R_3$: 0 | $s_3$ | 18 | 0 | $\boxed{4}$ | −1/2 | 0 | 1 | $18/4=4.5$ |
| | $z_j$ | 96 | 16 | 8 | 4 | 0 | 0 | |
| | $z_j-c_j$ | | 0 | −2 | 4 | 0 | 0 | |

In the above table, −2 is most negative in $z_j-c_j$ so $x_2$ column is key column and 2 is minimum ratio so $R_2$ is the key row. Here, $s_2$ is outgoing variable and $x_2$ is incoming variable. Here, $\frac{3}{2}$ is pivot element.

- new$R_2$ = old$R_2$ × $\frac{2}{3}$

i.e $3\times\frac{2}{3}=2,\ 0,\ 1,\ -\frac{3}{4}\times\frac{2}{3}=-\frac{1}{2},\ 0,\ 0$

- new$R_1$ = old$R_1$ − $\frac{1}{2}$ new$R_2$

i.e $6-\frac{1}{2}\times2=5,\ 1-\frac{1}{2}\times0=1,\ \frac{1}{2}-\frac{1}{2}\times1=0,\ \frac{1}{4}-\frac{1}{2}\times\left(\frac{-1}{2}\right)=\frac{1}{2},$

$0-\frac{1}{2}\times0=0,\ 0-\frac{1}{2}\times0=0$

$newR_3 = oldR_3 - 4newR_2$

i.e $18-4\times2=10, 0-4\times0=0, 0, \frac{-1}{2}-4\times(\frac{-1}{2})=\frac{3}{2}, 0-4\times0=0, 1$

Developing third simplex table, we get,

| | $c_j \rightarrow$ | | 16 | 10 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | xB/key column |
| $R_1$: 16 | $x_1$ | 5 | 1 | 0 | 1/2 | 0 | 0 | |
| $R_2$: 10 | $x_2$ | 2 | 0 | 1 | -1/2 | 0 | 0 | |
| $R_3$: 0 | $s_3$ | 10 | 0 | 0 | 3/2 | 0 | 1 | |
| | $z_j$ | 100 | 16 | 10 | 3 | 0 | 0 | |
| | $z_j - c_j$ | | 0 | 0 | 3 | 0 | 0 | |

Since, all $(z_j - c_j) \geq 0$, the optimum solution is obtained.

Max Profit $Z = 100$.

Basic Variable: $x_1 = 5$, $x_2 = 2$. Ans//

---

15 Food X contains 6 units of vitamin A and 7 units of vitamin B and costs Rs.5 per gram. Food Y contains 8 units of vitamin A and 12 units of vitamin B and costs Rs.18 per gram. The daily minimum requirement of vitamins A and B are respectively 100 units and 138 units respectively.

a. Construct the problem as a LPP with the objective function minimizing the cost.

b. Find the minimum cost by using simplex method.

Solution:

Converting the problem in tabular form, we get,

| Vitamins | Food | | Daily requirements |
|---|---|---|---|
| | X | Y | |
| A | 6 | 8 | 100 |
| B | 7 | 12 | 138 |
| Cost | Rs 5 | Rs 18 | |

Mathematical formulation of LPP

$$\text{Min } z = 5x + 18y$$

s.t.c

$$6x + 8y \geq 100$$

$$7x + 12y \geq 138$$

$$x, y \geq 0$$

Formatting them in standard form of LPP, we get,

$$\text{Min } z = 5x + 18y + 0S_1 + 0S_2 + MA_1 + MA_2$$

s.t.c

$$6x + 18y - S_1 + 0S_2 + A_1 + 0A_2 = 100$$

$$7x + 12y + 0S_1 - S_2 + 0A_1 + A_2 = 138$$

Developing initial simplex table, we get

| | $c_j \rightarrow$ | | 5 | 18 | 0 | 0 | M | M | Ratio: |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | $x$ | $y$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | xB/key column |
| $R_1$: M | $A_1$ | 100 | 6 | 18 | -1 | 0 | 1 | 0 | 100/18 = 5.55 |
| $R_2$: M | $A_2$ | 138 | 7 | 12 | 0 | -1 | 0 | 1 | 138/12 = 11.5 |
| | $z_j$ | 238M | 13M | 30M | -M | -M | M | M | |
| | $z_j - c_j$ | | 13M-5 | 30M-18 | -M | -M | 0 | 0 | |

In the above table, 30M-18 is the most positive in $z_j - c_j$ so y column is the key column and 5.55 is the minimum ratio so $R_1$ is the key row. Here, $A_1$ is the outgoing variable in $R_1$ and $y$ is the incoming variable. Here, 18 is the pivot element.

- new $R_1$ = old $R_1$/18

i.e $\frac{50}{9}, \frac{6}{18} = \frac{1}{3}, 1, \frac{-1}{18}, 0, \frac{1}{18}, 0$

- new $R_2$ = old $R_2$ - 12 new $R_1$

i.e $138 - 12 \times \frac{50}{9} = \frac{214}{3}, 7 - 12 \times \frac{1}{3} = 3, 0, 0 - 12 \times \left(\frac{-1}{18}\right) = \frac{2}{3}, -1 - 12 \times 0 = -1,$

$0 - 12 \times \frac{1}{18} = \frac{-2}{3}, 1 - 12 \times 0 = 1.$

| | $C_j \longrightarrow$ | | 5 | 18 | 0 | 0 | M | M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: 18 | $y$ | 50/9 | 1/3 | 1 | -1/18 | 0 | 1/18 | 0 | 50/9/1/3 = 16.67 |
| $R_2$: 0 | $A_2$ | 214/3 | 3 | 0 | 2/3 | -1 | -2/3 | 1 | 214/3/3 = 23.67 |
| | $z_j$ | 100 | 9 | 18 | -1 | 0 | 1 | 0 | |
| | $z_j - c_j$ | | 4 | 0 | -1 | 0 | 1-M | 0 | |

In the above table, 4 is the most positive in $z_j - c_j$ so $x$ is the key column and 16.67 is the minimum ratio so $R_1$ is the key row. Here, $y$ is the outgoing variable and $x$ is incoming variable. Here, 1/3 is pivot element.

- new $R_1$ = old $R_1$ × 3

i.e $\frac{50}{9} \times 3 = \frac{50}{3}, 1, 3, \frac{-1}{18} \times 3 = \frac{-1}{6}, 0, \frac{1}{18} \times 3 = \frac{1}{6}, 0$

- new $R_2$ = old $R_2$ - 3 new $R_1$

i.e $\frac{214}{3} - 3 \times \frac{50}{3} = \frac{64}{3}, 0, 0 - 3 \times 3 = -9, \frac{2}{3} - 3 \times \left(\frac{-1}{6}\right) = \frac{-7}{6}, -1 - 3 \times 0 = -1,$

$\frac{-2}{3} - 3 \times \frac{1}{6} = \frac{-7}{6}, 1 - 3 \times 0 = 1$

| | $C_j \longrightarrow$ | | 5 | 18 | 0 | 0 | M | M | Ratio: |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: 5 | $x$ | 50/3 | 1 | 3 | -1/6 | 0 | 1/6 | 0 | $\frac{50}{3}/\frac{-1}{6} = -100$ |
| $R_2$: M | $A_2$ | 64/3 | 0 | -9 | 7/6 | -1 | -7/6 | 1 | $\frac{64}{3}/\frac{7}{6} = 18.28$ |
| | $z_j$ | $\frac{250}{3}+\frac{64}{3}M$ | 5 | 15-9M | $\frac{-5}{6}+\frac{7}{6}M$ | -M | $\frac{5}{6}-\frac{7M}{6}$ | M | |
| | $z_j - c_j$ | | 0 | -9M-3 | $\frac{7M-5}{6}$ | -M | $\frac{5-13M}{6}$ | 0 | |

In the above table, $\frac{7M-5}{6}$ is the most positive is $z_j - c_j$ so $s_1$ column is the key column and 18.28 is the minimum ratio so, $R_2$ is the key row and $A_2$ is the outgoing variable and $s_1$ is the incoming variable. Here, $\frac{7}{6}$ is the pivot element.

- new $R_2$ = old $R_2 \times \frac{6}{7}$

i.e $\frac{64}{3} \times \frac{6}{7} = \frac{128}{7}, 0, \frac{-9\times 6}{7} = \frac{-54}{7}, 1, \frac{-6}{7}, \frac{-7}{6} \times \frac{6}{7} = -1, \frac{6}{7}$

- new $R_1$ = old $R_1 + \frac{1}{6}$ new $R_2$.

i.e $\frac{50}{3} + \frac{1}{6} \times \frac{128}{7} = \frac{138}{7}, 1 + \frac{1}{6} \times 0 = 1, 3 + \frac{1}{6} \times \left(\frac{-54}{7}\right) = \frac{12}{7}, 0, 0 + \frac{1}{6} \times \left(\frac{-6}{7}\right) = \frac{-1}{7}$

$\frac{1}{6} + \frac{1}{6} \times (-1) = 0, \; 0 + \frac{1}{6} \times \frac{6}{7} = \frac{1}{7}$

| | $C_j \longrightarrow$ | | 5 | 18 | 0 | 0 | M | M | Ratio: |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: 5 | $x$ | 138/7 | 1 | 12/7 | 0 | -1/7 | 0 | 1/7 | |
| $R_2$: 0 | $s_1$ | 128/7 | 0 | -54/7 | 1 | -6/7 | -1 | 6/7 | |
| | $z_j$ | $\frac{690}{7}$ | 5 | 60/7 | 0 | -5/7 | 0 | 5/7 | |
| | $z_j - c_j$ | | 0 | -66/7 | 0 | -5/7 | -M | 5/7-M | |

Since, all $(z_j - c_j) \leq 0$, the mimum solution is obtained.

Min p cost $(z) = \frac{690}{7} = 98.57$

Basic variable : $x = \frac{138}{7} = 19.71$

Non Basic variable : $y = 0$

16. The following table gives the three kinds of foods and three kinds of vitamin contained on them. Solve following problem for minimizing costs.

| Vitamin | Food | | | Daily Requirements |
|---|---|---|---|---|
| | $F_1$ | $F_2$ | $F_3$ | |
| $V_1$ | 20 | 10 | 10 | 300 |
| $V_2$ | 10 | 10 | 10 | 200 |
| $V_3$ | 10 | 20 | 10 | 240 |
| Cost for food | Rs.20 | Rs.24 | Rs.18 | |

Solution:

Mathematical formulation of LPP,

$$\text{Min } z = 20x_1 + 24x_2 + 18x_3$$

s.t.

$$20x_1 + 10x_2 + 10x_3 \geq 300$$

$$10x_1 + 10x_2 + 10x_3 \geq 200$$

$$10x_1 + 20x_2 + 10x_3 \geq 240$$

$$x_1, x_2, x_3 \geq 0$$

Formatting the problem in standard form of LPP, we get,

$$\text{Min } z = 20x_1 + 24x_2 + 18x_3 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2 + MA_3$$

$$20x_1 + 10x_2 + 10x_3 - S_1 + 0S_2 + 0S_3 + A_1 + 0A_2 + 0A_3 = 300$$

$$10x_1 + 10x_2 + 10x_3 + 0S_1 - S_2 + 0S_3 + 0A_1 + A_2 + 0A_3 = 200$$

$10x_1 + 20x_2 + 10x_3 + 0s_1 + 0s_2 - s_3 + 0A_1 + 0A_2 + A_3 = 240$

Developing initial simplex table, we get,

| | $C_j \longrightarrow$ | | 20 | 24 | 18 | 0 | 0 | 0 | M | M | M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | xB/key column |
| $R_1$: M | $A_1$ | 300 | 20 | 10 | 10 | -1 | 0 | 0 | 1 | 0 | 0 | 300/20 = 15 |
| $R_2$: M | $A_2$ | 200 | 10 | 10 | 10 | 0 | -1 | 0 | 0 | 1 | 0 | 200/10 = 20 |
| $R_3$: M | $A_3$ | 240 | 10 | 20 | 10 | 0 | 0 | -1 | 0 | 0 | 1 | 240/10 = 24 |
| | $z_j$ | 740M | 40M | 40M | 30M | -M | -M | -M | M | M | M | |
| | $z_j - C_j$ | | 40M-20 | 40M-24 | 30M-18 | -M | -M | -M | 0 | 0 | 0 | |

In the above table, 40M-20 is the most positive in $z_j - c_j$ so $x_1$ column is the key column. and 15 is the minimum ratio so $R_1$ is the key row. $A_1$ is the outgoing variable and $x_1$ is the incoming variable. Here, 20 is the pivot element.

- new $R_1$ = old $R_1$ / 20

  i.e $\frac{300}{20} = 15$, 1, $\frac{10}{20} = \frac{1}{2}$, $\frac{10}{20} = \frac{1}{2}$, $\frac{-1}{20}$, 0, 0, $\frac{1}{20}$, 0, 0

- new $R_2$ = old $R_2$ − 10 × new $R_1$

  i.e $200 - 10 \times 15 = 50$, 0, $10 - 10 \times \frac{1}{2} = 5$, $10 - 10 \times \frac{1}{2} = 5$, $0 - 10 \times \left(\frac{-1}{20}\right) = \frac{1}{2}$,

  $-1 - 10 \times 0 = -1$, $0 - 10 \times 0 = 0$, $1 - 10 \times \frac{1}{20} = \frac{1}{2}$, $1 - 10 \times 0 = 1$, $0 - 10 \times 0 = 0$

- new $R_3$ = old $R_3$ − 10 × new $R_1$

  i.e $240 - 10 \times 15 = 90$, $10 - 10 \times 1 = 0$, $20 - 10 \times \frac{1}{2} = 15$, $10 - 10 \times \frac{1}{2} = 5$, $0 - 10 \times \left(\frac{-1}{20}\right)$

  $= \frac{1}{20}$, $0 - 10 \times 0 = 0$, $-1 - 10 \times 0 = -1$, $0 - 10 \times \frac{1}{20} = -\frac{1}{2}$, $0 - 10 \times 0 = 0$,

  $1 - 10 \times 0 = 1$

Developing second simplex table, we get.

| | Cj → | | 20 | 24 | 18 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | XB | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 20 | $x_1$ | 15 | 1 | 1/2 | 1/2 | -1/20 | 0 | 0 | 1/20 | 0 | 0 | 15/1/2 = 30 |
| $R_2$: M | $A_2$ | 50 | 0 | 5 | 5 | 1/2 | -1 | 0 | -1/2 | 1 | 0 | 50/5 = 10 |
| $R_3$: M | $A_3$ | 90 | 0 | 15 | 5 | 1/2 | 0 | -1 | -1/2 | 0 | 1 | 90/15 = 6 |
| | $z_j$ | 300+140M | 20 | 10+20M | 10+10M | M-1 | -M | -M | 1-M | M | M | |
| | $z_j - c_j$ | | 0 | 20M-14 | 10N-8 | M-1 | -M | -M | 1-2M | 0 | 0 | |

In the above table, 20M-14 is most positive in zj-cj so $x_2$ column is key colum and 6 is the minimum ratio so $R_3$ is the key row. Here, 15 is pivot element. A3 is outgoing variable and $x_2$ is incoming variable.

- new R3 = old R3 / 15

i.e $\frac{90}{15} = 6, 0, 1, \frac{1}{3}, \frac{1}{2} \div 15 = \frac{1}{30}, 0, \frac{-1}{15}, \frac{-1}{30}, 0, \frac{1}{15}$

- new $R_1$ = old $R_1 - \frac{1}{2}$ new $R_3$

i.e $15 - \frac{1}{2} \times 6 = 12, 1 - \frac{1}{2} \times 0 = 1, \frac{1}{2} - \frac{1}{2} \times 1 = 0, \frac{1}{2} - \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}, \frac{-1}{20} - \frac{1}{2} \times \frac{1}{30} = -\frac{1}{15},$

$0 - \frac{1}{2} \times 0 = 0, 0 - \frac{1}{2} \times \left(\frac{-1}{15}\right) = \frac{1}{30}, \frac{1}{20} - \frac{1}{2} \times \left(\frac{-1}{30}\right) = \frac{1}{15}, 0 - \frac{1}{2} \times 0 = 0, 0 - \frac{1}{2} \times \frac{1}{15} = \frac{-1}{30}$

- new $R_2$ = old $R_2$ - 5 new $R_3$

i.e $50 - 5 \times 6 = 20, 0 - 5 \times 0 = 0, 5 - 5 \times 1 = 0, 5 - 5 \times \frac{1}{3} = \frac{10}{3}, \frac{1}{2} - 5 \times \frac{1}{30} = \frac{1}{3}, -1 - 5 \times 0 = -1,$

$0 - 5 \times \left(\frac{-1}{15}\right) = \frac{1}{3}, -\frac{1}{2} - 5 \times \left(\frac{-1}{30}\right) = -\frac{1}{3}, 1 - 5 \times 0 = 1, 0 - 5 \times \frac{1}{15} = -\frac{1}{3}$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 24 | 18 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 20 | $x_1$ | 12 | 1 | 0 | $1/3$ | $-1/15$ | 0 | $1/30$ | $1/15$ | 0 | $-1/30$ | $12/1/3 = 36$ |
| $R_2$: M | $A_2$ | 20 | 0 | 0 | $10/3$ | $1/3$ | $-1$ | $1/3$ | $-1/3$ | 1 | $-1/3$ | $20/10/3 = 6$ |
| $R_3$: 24 | $x_2$ | 6 | 0 | 1 | $1/3$ | $1/30$ | 0 | $-1/15$ | $-1/30$ | 0 | $1/15$ | $6/1/3 = 18$ |
| | $Z_j$ | $20M+384$ | 20 | 24 | $\frac{10}{3}M+\frac{44}{3}$ | $\frac{1}{3}M-\frac{8}{15}$ | $-M$ | $\frac{M}{3}-\frac{24}{15}$ | $-\frac{M}{3}+\frac{8}{15}$ | M | $\frac{-M}{3}+\frac{14}{15}$ | |
| | $Z_j - C_j$ | | 0 | 0 | $\frac{10}{3}M-\frac{10}{3}$ | $\frac{1}{3}M-\frac{8}{15}$ | $-M$ | $\frac{M}{3}-\frac{14}{15}$ | $-\frac{2}{3}M+\frac{8}{15}$ | 0 | $-\frac{4}{3}M+\frac{M}{15}$ | |

In above table, $\frac{10M}{3}-\frac{10}{3}$ is the most positive so, $x_3$ is the key column and 6 is the minimum ratio so $R_2$ is key row. Here, $A_2$ is outgoing variable and $x_3$ is incoming variable. Here, 10/3 is pivot element.

- $newR_2 = oldR_2 \times \frac{3}{10}$

i.e $20 \times \frac{3}{10} = 6, 0, 0, 1, \frac{1}{3} \times \frac{3}{10} = \frac{1}{10}, \frac{-3}{10}, \frac{1}{3} \times \frac{3}{10} = \frac{1}{10}, \frac{-1}{10}, \frac{3}{10}, \frac{-1}{10}$

- $newR_1 = oldR_1 - \frac{1\cancel{0}}{3} newR_2$

i.e $12 - \frac{1}{3} \times 6 = 10, 1 - \frac{1}{3} \times 0 = 1, 0 - \frac{1}{3} \times 0 = 0, 0, \frac{-1}{15} - \frac{1}{3} \times \frac{1}{10} = \frac{-1}{10}, 0 - \frac{-1}{\cancel{3}} \times \left(\frac{-3}{10}\right) = \frac{1}{10},$

$\frac{1}{30} - \frac{1}{3} \times \frac{1}{10} = 0, \frac{1}{15} - \frac{1}{3} \times \left(-\frac{1}{10}\right) = \frac{1}{10}, 0 - \frac{1}{3} \times \frac{3}{10} = \frac{-1}{10}, \frac{-1}{30} - \frac{1}{3} \times \left(\frac{-1}{10}\right) = 0$

- $newR_3 = oldR_3 - \frac{1}{3} newR_2$

i.e $6 - \frac{1}{3} \times 6 = 4, 0 - \frac{1}{3} \times 0 = 0, 1 - \frac{1}{3} \times 0 = 1, \frac{1}{3} - \frac{1}{3} \times 1 = 0, \frac{1}{30} - \frac{1}{3} \times \frac{1}{10} = 0,$

$0 - \frac{1}{3} \times \left(-\frac{3}{10}\right) = \frac{1}{10}, \frac{-1}{15} - \frac{1}{3} \times \frac{1}{10} = -\frac{1}{10}, \frac{-1}{30} - \frac{1}{3} \times \left(\frac{-1}{10}\right) = 0, \cancel{0}\frac{-1}{10}, \frac{1}{10}$

Developing fourth simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 24 | 18 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 20 | $x_1$ | 10 | 1 | 0 | 0 | $-1/10$ | $1/10$ | 0 | $1/10$ | $-1/10$ | 0 | |
| $R_2$:18 | $x_3$ | 6 | 0 | 0 | 1 | $1/10$ | $-3/10$ | $1/10$ | $-1/10$ | $3/10$ | $-1/10$ | |
| $R_3$:24 | $x_2$ | 4 | 0 | 1 | 0 | 0 | $1/10$ | $-1/10$ | 0 | $-1/10$ | $1/10$ | |
| | $z_j$ | 404 | 20 | 24 | 18 | $-1/5$ | $-1$ | $-3/5$ | $1/5$ | 1 | $3/5$ | |
| | $z_j - c_j$ | | 0 | 0 | 0 | $-1/5$ | $-1$ | $-3/5$ | $1/5 - M$ | $1 - M$ | $\frac{3}{5} - M$ | |

Since, all $(z_j - c_j) \leq 0$, the minimum solution is obtained.

Min $(z) = 404$.

Basic variable : $x_1 = 10$, $x_2 = 4$ and $x_3 = 6$.

---

17. A horticulturist wishes to mix fertilizer that will provide a minimum of 15 units of Potash, 20 units of Nitrates and 24 units of Phosphate. Brand I provides 3 units of Potash, 1 unit of Nitrates, and 3 units of Phosphate; it costs Rs 120. Brand II provides 1 unit of Potash, 5 units of Nitrates and 2 units of Phosphate; it costs Rs 60. Solve by using simplex method to determine the quantities of two brands, which should be mixed such that the cost is minimized.

Solution:

Converting the question in tabular form, we get,

| Fertilizers | Brand | | Minimum available |
|---|---|---|---|
| | I($x$) | II($y$) | |
| Potash | 3 | 1 | 15 |
| Nitrates | 1 | 5 | 20 |
| Phosphate | 3 | 2 | 24 |
| Cost | Rs 120 | Rs 60 | |

Mathematical formulation of LPP

$\text{Min } z = 120x + 60y$

s.t.c

$3x + y \geq 15$

$x + 5y \geq 20$

$3x + 2y \geq 24$

$x, y \geq 0$

Formatting the problem in standard form of LPP, we get,

$\text{Min } z = 120x + 60y + 0s_1 + 0s_2 + 0s_3 + MA_1 + MA_2 + MA_3$

s.t.c

$3x + y - s_1 + 0s_2 + 0s_3 + A_1 + 0A_2 + 0A_3 = 15$

$x + 5y + 0s_1 - s_2 + 0s_3 + 0A_1 + A_2 + 0A_3 = 20$

$3x + 2y + 0s_1 + 0s_2 - s_3 + 0A_1 + 0A_2 + A_3 = 24$

$x, y, s_1, s_2, s_3, A_1, A_2, A_3 \geq 0$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 120 | 60 | 0 | 0 | 0 | M | M | M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB constant | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | xB/keycolumn |
| $R_1$:M | $A_1$ | 15 | 3 | 1 | -1 | 0 | 0 | 1 | 0 | 0 | 15/1 = 15 |
| $R_2$:M | $A_2$ | 20 | 1 | 5 | 0 | -1 | 0 | 0 | 1 | 0 | 20/5 = 4 |
| $R_3$:M | $A_3$ | 24 | 3 | 2 | 0 | 0 | -1 | 0 | 0 | 1 | 24/2 = 12 |
| | $z_j$ | 59M | 7M | 8M | -M | -M | -M | M | M | M | |
| | $z_j - c_j$ | | 7M-120 | 8M-60 | -M | -M | -M | 0 | 0 | 0 | |

In above table, 8M-60 is the most positive value in $z_j - c_j$ so y column is key column and 4 is the minimum ratio so $A_2$ is the key row. Here, $A_2$ is the outgoing variable and y is incoming variable. Here, 5 is the pivot element.

- $new R_2 = old R_2/5$

i.e $\frac{20}{5} = 4, \frac{1}{5}, 1, 0, \frac{-1}{5}, 0, 0, \frac{1}{5}, 0$

- $new R_1 = old R_1 - new R_2$

i.e $15-4 = 11, 3-\frac{1}{5} = \frac{14}{5}, 0, -1-0 = -1, 0-\left(\frac{-1}{5}\right) = \frac{1}{5}, 0-0 = 0, 1-0 = 1,$

$0-\frac{1}{5} = \frac{-1}{5}, 0$

- $new R_3 = old R_3 - 2 new R_2$

i.e $24-2\times4 = 16, 3-2\times\frac{1}{5} = \frac{13}{5}, 0, 0-2\times0 = 0, 0-2\times\left(\frac{-1}{5}\right) = \frac{2}{5}$

$-1-2\times0 = -1, 0-2\times0 = 0, 0-2\times\frac{1}{5} = -\frac{2}{5}, 1-2\times0 = 1$

Developing second simplex table, we get,

| CB | BV | $x_B$ constant | x | y | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | Ratio = $x_B$/key column |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $c_j \rightarrow$ | | 120 | 60 | 0 | 0 | 0 | M | M | M | |
| $R_1$: M | $A_1$ | 11 | 14/5 | 0 | -1 | 1/5 | 0 | 1 | -1/5 | 0 | 11/14/5 = 3.92 |
| $R_2$: 60 | y | 4 | 1/5 | 1 | 0 | -1/5 | 0 | 0 | 1/5 | 0 | 4/1/5 = 20 |
| $R_3$: M | $A_3$ | 16 | 13/5 | 0 | 0 | 2/5 | -1 | 0 | -2/5 | 1 | 16/13/5 = 6.15 |
| | $Z_j$ | 27M+240 | $\frac{27}{5}M+\frac{60}{5}$ | 60 | -M | $\frac{3}{5}M-12$ | -M | M | $-\frac{3}{5}M+12$ | M | |
| | $Z_j - C_j$ | | $\frac{27}{5}M-108$ | 0 | -M | $\frac{3}{5}M-12$ | -M | 0 | $-\frac{8}{5}M+12$ | 0 | |

In the above table, $\frac{27}{5}M-108$ is the most positive so x is the key colum and 3.92 is the minimum ratio so $R_1$ is the key row. Here, $A_1$ is the outgoing variable and x is incoming variable. Here, $\frac{14}{5}$ is the pivot element.

- $new R_1 = old R_1 \times \frac{5}{14}$

i.e $\frac{55}{14}, 1, 0, \frac{-5}{14}, \frac{1}{14}, 0, \frac{5}{14}, -\frac{1}{14}, 0$

- $newR_2 = oldR_2 - \frac{1}{5} \times newR_1$

i.e $4 - \frac{1}{5} \times \frac{55}{14} = \frac{45}{14}$, $\frac{1}{5} - \frac{1}{5} \times 1 = 0$, $1 - \frac{1}{5} \times 0 = 1$, $0 - \frac{1}{5} \times \left(\frac{-5}{14}\right) = \frac{1}{14}$, $-\frac{1}{5} - \frac{1}{5} \times \frac{1}{14}$

$= -\frac{3}{14}$, $0 - \frac{1}{5} \times 0 = 0$, $0 - \frac{1}{5} \times \frac{5}{14} = \frac{-1}{14}$, $\frac{1}{5} - \frac{1}{5} \times \left(\frac{-1}{14}\right) = \frac{3}{14}$, $0 - \frac{1}{5} \times 0 = 0$

- $newR_3 = oldR_3 - \frac{13}{5} \times newR_1$

i.e $16 - \frac{13}{5} \times \frac{55}{14} = \frac{81}{14}$, $\frac{13}{5} - \frac{13}{5} \times 1 = 0$, $0 - \frac{13}{5} \times 0 = 0$, $0 - \frac{13}{5} \times \left(\frac{-5}{14}\right) = \frac{13}{14}$,

$\frac{2}{5} - \frac{13}{5} \times \frac{1}{14} = \frac{3}{14}$, $-1 - \frac{13}{5} \times 0 = -1$, $0 - \frac{13}{5} \times \frac{5}{14} = -\frac{13}{14}$, $\frac{-2}{5} - \frac{13}{5} \times \left(\frac{-1}{14}\right) = \frac{-3}{14}$,

$1 - \frac{13}{5} \times 0 = 1$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 120 | 60 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | x | y | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 120 | x | 55/14 | 1 | 0 | -5/14 | 1/14 | 0 | 5/14 | -1/14 | 0 | 55 |
| $R_2$: 60 | y | 45/14 | 0 | 1 | 1/14 | -3/14 | 0 | -1/14 | 3/14 | 0 | -15 |
| $R_3$: M | $A_3$ | 81/14 | 0 | 0 | 13/14 | 3/14 | -1 | -13/14 | -3/14 | 1 | 5.57 |
| | $z_j$ | $\frac{81M}{14} + \frac{4650}{7}$ | 120 | 60 | $\frac{13}{14}M - \frac{270}{7}$ | $\frac{3}{14}M - \frac{30}{7}$ | $-M$ | $-\frac{13}{14}M + \frac{270}{7}$ | $-\frac{3}{14}M + \frac{30}{7}$ | M | |
| | $z_j - c_j$ | | 0 | 0 | $\frac{13}{14}M - \frac{270}{7}$ | $\frac{13}{14}M - \frac{30}{7}$ | $-M$ | $-\frac{27M}{14} + \frac{270}{7}$ | $-\frac{17M}{14} + \frac{30}{7}$ | 0 | |

890 924.28

In above table, $\frac{13}{14}M - \frac{30}{7}$ is the positive value in $z_j - c_j$ so, $s_2$ column is the key column and 5.57 is the minimum ratio so $R_3$ is the key row. Here, $A_3$ is the outgoing variable and $s_2$ is incoming variable. Here, $\frac{3}{14}$ is the pivot element.

- $\text{new } R_3 = \text{old } R_3 \times \frac{14}{3}$

i.e $\frac{81}{14} \times \frac{14}{3} = 27, 0, 0, \frac{13}{3}, 1, -\frac{14}{3}, -\frac{13}{3}, -1, \frac{14}{3}$

- $\text{new } R_1 = \text{old } R_1 - \frac{1}{14} \text{ new } R_3$

i.e $\frac{55}{14} - \frac{1}{14} \times 27 = 2, 1 - \frac{1}{14} \times 0 = 1, 0 - \frac{1}{14} \times 0 = 0, -\frac{5}{14} - \frac{1}{14} \times \frac{13}{3} = -\frac{2}{3}$,

$0, 0 - \frac{1}{14} \times \left(-\frac{14}{3}\right) = \frac{1}{3}, \frac{5}{14} - \frac{1}{14} \times \left(-\frac{13}{3}\right) = \frac{2}{3}, -\frac{1}{14} - \frac{1}{14} \times (-1) = 0, -\frac{1}{3}$

- $\text{new } R_2 = \text{old } R_2 + \frac{3}{14} \text{ new } R_3$

i.e $\frac{46}{14} + \frac{3}{14} \times 27 = 9, 0 + \frac{1}{14} \times 0 = 0, 1 + \frac{3}{14} \times 0 = 1, \frac{1}{14} + \frac{3}{14} \times \frac{13}{3} = 1, 0$,

$0 + \frac{3}{14} \times \left(-\frac{14}{3}\right) = -1, -\frac{1}{14} + \frac{3}{14} \times \left(-\frac{13}{3}\right) = -1, \frac{3}{14} + \frac{3}{14} \times (-1) = 0$,

$0 + \frac{3}{14} \times \left(\frac{14}{3}\right) = 1.$

Developing fourth simplex table, we get,

| | $c_j \rightarrow$ | | 120 | 60 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 120 | $x$ | 2 | 1 | 0 | $-2/3$ | 0 | $1/3$ | $2/3$ | 0 | $-1/3$ | |
| $R_2$: 60 | $y$ | 9 | 0 | 1 | 1 | 0 | $-1$ | $-1$ | 0 | 1 | |
| $R_3$: 0 | $S_2$ | 27 | 0 | 0 | $13/3$ | 1 | $-14/3$ | $-13/3$ | $-1$ | $14/3$ | |
| | $z_j$ | 780 | 120 | 60 | $-20$ | 0 | $-20$ | 20 | 0 | 20 | |
| | $z_j - c_j$ | | 0 | 0 | $-20$ | 0 | $-20$ | $20-M$ | $-M$ | $20-M$ | |

Since, all $(z_j - c_j) \leq 0$, the optimal solution is obtained.

Min $z = 780$.

Basic variable: $x = 2$ and $y = 9$. i.e Brand I = 2 & II = 9

18. Suppose that 8, 12 and 9 units of Protein, Carbohydrate and Fat respectively are the minimum weekly requirements for a person. Food A contains 2, 6, 1 units of Protein, Carbohydrate and Fat respectively per kg. and Food B contains 1, 1, 3 units of Protein, Carbohydrate and Fat respectively per kg. If A costs Rs. 0.85 per kg and B costs Rs 0.40 per kg. How many kgs of each should be buy per week to minimize the cost and still to meet the minimum requirements.

Solution:

Converting the above question in a tabular form, we get,

| | Food | | Minimum Requirement |
|---|---|---|---|
| | A ($x$) | B ($y$) | |
| Protein | 2 | 1 | 8 |
| Carbohydrate | 6 | 1 | 12 |
| Fat | 1 | 3 | 9 |
| Cost | Rs. 0.85 | Rs. 0.40 | |

Mathematical formulation of LPP,

$$\text{Min } z = \text{Rs. } 0.85x + \text{Rs. } 0.40y$$

s.t.c

$$2x + y \geq 8$$

$$6x + y \geq 12$$

$$x + 3y \geq 9$$

$$x, y \geq 0$$

Formatting them in standard form of LPP, we get,

$$\text{Min } z = 0.85x + 0.40y + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2 + MA_3.$$

$$2x + y - S_1 + 0S_2 + 0S_3 + A_1 + 0A_2 + 0A_3 = 8$$

$$6x + y + 0S_1 - S_2 + 0S_3 + 0A_1 + A_2 + 0A_3 = 12$$

$$x + 3y + 0S_1 + 0S_2 - S_3 + 0A_1 + 0A_2 + A_3 = 9$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 0.85 | 0.4 | 0 | 0 | 0 | M | M | M | Ratio = $\frac{x_B}{\text{Key column}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: M | $A_1$ | 8 | $\boxed{2}$ | 1 | -1 | 0 | 0 | 1 | 0 | 0 | $8/2 = 4$ |
| $R_2$: M | $A_2$ | 12 | $\boxed{6}$ | 1 | 0 | -1 | 0 | 0 | 1 | 0 | $12/6 = 2$ |
| $R_3$: M | $A_3$ | 9 | $\boxed{1}$ | 3 | 0 | 0 | -1 | 0 | 0 | 1 | $9/1 = 9$ |
| | $z_j$ | 29M | 9M | 5M | -M | -M | -M | M | M | M | |
| | $z_j - c_j$ | | 9M-0.85 | 5M-0.4 | -M | -M | -M | 0 | 0 | 0 | |

In the above table, 9M-0.85 is the most positive in $z_j - c_j$ so, $x$ is the key column and 2 is the minimum ratio so $R_2$ is the key row. Here, $A_2$ is the outgoing variable and $x$ is the incoming variable. Here, 6 is the pivot element.

- new$R_2$ = old$R_2$/6

  i.e $\frac{12}{6} = 2, 1, \frac{1}{6}, 0, \frac{-1}{6}, 0, 0, \frac{1}{6}, 0$

- new$R_1$ = old$R_1$ - 2new$R_2$

  i.e $8-2\times2=4, 2-2\times1=0, 1-2\times\frac{1}{6}=\frac{2}{3}, -1-2\times0=-1, 0-2\times\left(\frac{-1}{6}\right)=\frac{1}{3},$

  $0-2\times0=0, 1-2\times0=1, 0-2\times\frac{1}{6}=\frac{-1}{3}, 0-2\times0=0$

- new$R_3$ = old$R_3$ - new$R_2$

  i.e $9-2=7, 0, 3-\frac{1}{6}=\frac{17}{6}, 0-0=0, 0-\left(\frac{-1}{6}\right)=\frac{1}{6}, -1-0=-1, 0-0=0, 0-\frac{1}{6}=\frac{-1}{6},$

  $1-0=1$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 0.85 | 0.4 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: M | $A_1$ | 4 | 0 | 2/3 | −1 | 1/3 | 0 | 1 | −1/3 | 0 | 4/2/3 = 6 |
| $R_2$: 0.85 | $x$ | 2 | 1 | 1/6 | 0 | −1/6 | 0 | 0 | 1/6 | 0 | 2/1/6 = 12 |
| $R_3$: M | $A_3$ | 7 | 0 | 17/6 | 0 | 1/6 | −1 | 0 | −1/6 | 1 | 7/17/6 = 2.47 |
| | $z_j$ | $11M+\frac{17}{10}$ | 0.85 | $\frac{7}{2}M+\frac{17}{120}$ | −M | $\frac{1}{2}M-\frac{17}{120}$ | −M | M | $-\frac{1}{2}M+\frac{17}{120}$ | M | |
| | $z_j - c_j$ | | 0 | $\frac{7}{2}M-\frac{31}{120}$ | −M | $\frac{1}{2}M-\frac{17}{120}$ | −M | 0 | $-\frac{3}{2}M+\frac{17}{120}$ | 0 | |

In above table, $\frac{7}{2}M - \frac{31}{120}$ is the most positive in $z_j - c_j$ so $y$ is the key column and 2.47 is minimum ratio so $R_3$ is key row.
Here, $A_3$ is outgoing variable and $y$ is incoming variable.
Here, $\frac{17}{6}$ is pivot element.

- new $R_3$ = old $R_3 \times \frac{6}{17}$

i.e $7 \times \frac{6}{17} = \frac{42}{17}, 0, 1, 0, \frac{1}{6} \times \frac{6}{17} = \frac{1}{17}, \frac{-6}{17}, 0, \frac{-1}{6} \times \frac{6}{17} = \frac{-1}{17}, \frac{6}{17}$

- new $R_1$ = old $R_1 - \frac{2}{3} \times$ new $R_3$

i.e $4 - \frac{2}{3} \times \frac{42}{17} = \frac{40}{17}, 0 - \frac{2}{3} \times 0 = 0, 0, -1 - \frac{2}{3} \times 0 = -1, \frac{1}{3} - \frac{2}{3} \times \frac{1}{17} = \frac{5}{17}, 0 - \frac{2}{3} \times \left(\frac{-6}{17}\right)$

$= \frac{4}{17}, 1 - \frac{2}{3} \times 0 = 1, -\frac{1}{3} - \frac{2}{3} \times \left(\frac{-1}{17}\right) = -\frac{5}{17}, 0 - \frac{2}{3} \times \frac{6}{17} = -\frac{4}{17}$

- new $R_2$ = old $R_2 - \frac{1}{6}$ new $R_3$

i.e $2 - \frac{1}{6} \times \frac{42}{17} = \frac{27}{17}, 1 - \frac{1}{6} \times 0 = 1, \frac{1}{6} - \frac{1}{6} \times 1 = 0, 0 - \frac{1}{6} \times 0 = 0, -\frac{1}{6} - \frac{1}{6} \times \frac{1}{17}$

$= -\frac{3}{17}, 0 - \frac{1}{6} \times \left(\frac{-6}{17}\right) = \frac{1}{17}, 0 - \frac{1}{6} \times 0 = 0, \frac{1}{6} - \frac{1}{6} \times \left(\frac{-1}{17}\right) = \frac{3}{17},$

$0 - \frac{1}{6} \times \frac{6}{17} = -\frac{1}{17}$

Developing third simplex table, we get,

| | $c_j \rightarrow$ | | 0.85 | 0.4 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: M | $A_1$ | $40/17$ | 0 | 0 | $-1$ | $5/17$ | $4/17$ | 1 | $-5/17$ | $-4/17$ | $\frac{40/17}{5/17}=8$ |
| $R_2$: 0.85 | $x$ | $27/17$ | 1 | 0 | 0 | $-3/17$ | $1/17$ | 0 | $3/17$ | $-1/17$ | $\frac{27/17}{-3/17}=-9$ |
| $R_3$: 0.4 | $y$ | $42/17$ | 0 | 1 | 0 | $1/17$ | $-6/17$ | 0 | $-1/17$ | $6/17$ | $\frac{42/17}{1/17}=42$ |
| | $z_j$ | $\frac{40}{17}M+\frac{159}{68}$ | 0.85 | 0.4 | $-M$ | $\frac{5}{17}M-\frac{43}{340}$ | $\frac{4}{17}M-\frac{31}{340}$ | M | $\frac{-5}{17}M+\frac{43}{340}$ | $\frac{-4}{17}M+\frac{31}{340}$ | |
| | $z_j-c_j$ | | 0 | 0 | $-M$ | $\frac{5}{17}M-\frac{43}{340}$ | $\frac{4}{17}M-\frac{31}{340}$ | 0 | $-\frac{22M}{17}+\frac{43}{340}$ | $-\frac{21}{17}M+\frac{31}{340}$ | |

In above table, $\frac{5}{17}M-\frac{43}{340}$ is the most positive in $z_j-c_j$ so $s_2$ column is the key column and 8 is the minimum ratio so $R_1$ is the key row. $A_1$ is the outgoing variable and $s_2$ is incoming variable. Here, $\frac{5}{17}$ is the pivot element.

- $\text{new}R_1 = \text{old}R_1 \times \frac{17}{5}$

i.e. $\frac{40}{17}\times\frac{17}{5}=8, 0, 0, \frac{-17}{5}, 1, \frac{4}{17}\times\frac{17}{5}=\frac{4}{5}, \frac{17}{5}, \frac{-5}{17}\times\frac{17}{5}=-1, \frac{-4}{17}\times\frac{17}{5}=\frac{-4}{5}$

- $\text{new}R_2 = \text{old}R_2 + \frac{3}{17}\,\text{new}R_1$

i.e. $\frac{27}{17}+\frac{3}{17}\times 8=3, 1+\frac{3}{17}\times 0=1, 0+\frac{3}{17}\times 0=0, 0+\frac{3}{17}\times\left(\frac{-17}{5}\right)=\frac{-3}{5}, 0, \frac{1}{17}+\frac{3}{17}\times\frac{4}{5}=\frac{1}{5},$

$0+\frac{3}{17}\times\frac{17}{5}=\frac{3}{5}, \frac{3}{17}+\frac{3}{17}\times(-1)=0, -\frac{1}{17}+\frac{3}{17}\times\left(\frac{-4}{5}\right)=-\frac{1}{5}$

- $\text{new}R_3 = \text{old}R_3 - \frac{1}{17}\,\text{new}R_1$

i.e. $\frac{42}{17}-\frac{1}{17}\times 8=2, 0-\frac{1}{17}\times 0=0, 1, 0-\frac{1}{17}\times\left(\frac{-17}{5}\right)=\frac{1}{5}, 0, \frac{-6}{17}-\frac{1}{17}\times\frac{4}{5}=\frac{-2}{5}, 0-\frac{1}{17}\times\frac{17}{5}$

$=\frac{-1}{5}, \frac{-1}{17}-\frac{1}{17}\times(-1)=0, \frac{6}{17}-\frac{1}{17}\times\left(\frac{-4}{5}\right)=\frac{2}{5}$

Developing fourth simplex table, we get,

| | $C_j \longrightarrow$ | | 0.85 | 0.4 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | XB | $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 0 | $s_2$ | 8 | 0 | 0 | $-17/5$ | 1 | $4/5$ | $17/5$ | $-1$ | $-4/5$ | $8/4/5 = 10$ |
| $R_2$: 0.85 | $x$ | 3 | 1 | 0 | $-3/5$ | 0 | $1/5$ | $3/5$ | 0 | $-1/5$ | $3/1/5 = 15$ |
| $R_3$: 0.4 | $y$ | 2 | 0 | 1 | $1/5$ | 0 | $-2/5$ | $-1/5$ | 0 | $2/5$ | $2/-2/5 = -5$ |
| | $z_j$ | 3.35 | 0.85 | 0.4 | $-\frac{43}{100}$ | 0 | $1/100$ | $43/100$ | 0 | $-1/100$ | |
| | $z_j - c_j$ | | 0 | 0 | $-\frac{43}{100}$ | 0 | $\frac{1}{100}$ | $\frac{43}{100} - M$ | $-M$ | $-\frac{1}{100} - M$ | |

In above table, $\frac{1}{100}$ is the most positive in $z_j - c_j$ so $s_3$ is the key column and 10 is the minimum ratio so $R_1$ is the key row. Here, $s_2$ is the outgoing variable and $s_3$ is incoming variable. Here, $4/5$ is pivot element.

- new $R_1$ = old $R_1 \times \frac{5}{4}$

i.e $8 \times \frac{5}{4} = 10, 0, 0, \frac{-17}{5} \times \frac{5}{4} = \frac{-17}{4}, \frac{5}{4}, 1, \frac{17}{5} \times \frac{5}{4} = \frac{17}{4}, \frac{-5}{4}, \frac{-4}{5} \times \frac{5}{4} = -1$

- new $R_2$ = old $R_2 - \frac{1}{5}$ new $R_1$

i.e $3 - \frac{1}{5} \times 10 = 1, 1 - \frac{1}{5} \times 0 = 1, 0 - \frac{1}{5} \times 0 = 0, \frac{-3}{5} - \frac{1}{5} \times \left(\frac{-17}{4}\right) = \frac{1}{4}, 0 - \frac{1}{5} \times \frac{5}{4} = \frac{-1}{4},$

$0, \frac{3}{5} - \frac{1}{5} \times \frac{17}{4} = \frac{-1}{4}, 0 - \frac{1}{5} \times \left(\frac{-5}{4}\right) = \frac{1}{4}, \frac{-1}{5} - \frac{1}{5} \times (-1) = 0$

- new $R_3$ = old $R_3 + \frac{2}{5}$ new $R_1$

i.e $2 + \frac{2}{5} \times 10 = 6, 0 + \frac{2}{5} \times 0 = 0, 1 + \frac{2}{5} \times 0 = 1, \frac{1}{5} + \frac{2}{5} \times \left(\frac{-17}{4}\right) = -\frac{3}{2}, 0 + \frac{2}{5} \times \frac{5}{4} = \frac{1}{2},$

$0, \frac{-1}{5} + \frac{2}{5} \times \left(\frac{17}{4}\right) = \frac{3}{2}, 0 + \frac{2}{5} \times \left(\frac{-5}{4}\right) = \frac{-1}{2}, \frac{2}{5} + \frac{2}{5} \times (-1) = 0$

Developing fifth simplex table, we get,

| | $C_j \rightarrow$ | | 0.85 | 0.4 | 0 | 0 | 0 | M | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x$ | $y$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | $A_3$ | |
| $R_1$: 0 | $S_3$ | 10 | 0 | 0 | $-17/4$ | $5/4$ | 1 | $17/4$ | $-5/4$ | -1 | |
| $R_2$: 0.85 | $x$ | 1 | 1 | 0 | $1/4$ | $-1/4$ | 0 | $-1/4$ | $1/4$ | 0 | |
| $R_3$: 0.4 | $y$ | 6 | 0 | 1 | $-3/2$ | $1/2$ | 0 | $3/2$ | $-1/2$ | 0 | |
| | $z_j$ | 3.25 | 0.85 | 0.4 | -0.38 | -0.0125 | 0 | 0.3875 | 0.0125 | 0 | |
| | $z_j - c_j$ | | 0 | 0 | -0.38 | -0.0125 | 0 | 0.3875-M | 0.0125-M | -M | |

Since, all $(z_j - c_j) \leq 0$, so minimum solution is obtained.
Min $(z) = 3.25$ at $x = A = 1$ and $y = B = 6$. Ans.

19 The XYZ company combines factors A and B to form a product which must weight 50 pounds. At least 20 pounds of A and no more than 40 pounds of B can be used. The cost of A is Rs 25 per pound and of B is Rs 10 per pound. Use the simplex method to find the amount of factors A and B which should be used to minimize total costs.

Solution:
Converting the question to tabular form, we get,

| Weight | Factor product | | Available |
|---|---|---|---|
| | A($x_1$) | B($x_2$) | |
| | 1 | 1 | 50 |
| | 1 | – | 20 |
| | – | 1 | 40 |
| Cost | 25 | 10 | |

Mathematical formulation of LPP is,

$Min\ z = 25x_1 + 10x_2$

s.t.c

$x_1 + x_2 = 50$

$x_1 \geq 20$

$x_2 \leq 40$

$x_1, x_2 \geq 0$

Converting into the standard form of LPP, we get,

$\therefore Min\ z = 25x_1 + 10x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

$x_1 + x_2 + 0S_1 + 0S_2 + A_1 + 0A_2 = 50$

$x_1 + 0x_2 - S_1 + 0S_2 + 0A_1 + 1A_2 = 20$

$0x_1 + x_2 + 0S_1 + S_2 + 0A_1 + 0A_2 = 40$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Developing initial simplex table,

| | $C_j \rightarrow$ | | 25 | 10 | 0 | 0 | M | M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: M | $A_1$ | 50 | 1 | 1 | 0 | 0 | 1 | 0 | 50/1 = 50 |
| $R_2$: M | $A_2$ | $\boxed{20}$ | $\boxed{1}$ | 0 | -1 | 0 | 0 | 1 | 20/1 = 20 |
| $R_3$: 0 | $S_2$ | 40 | 0 | 1 | 0 | 1 | 0 | 0 | 40/0 = ∞ |
| | $z_j$ | 70M | 2M | M | -M | 0 | M | M | |
| | $z_j - c_j$ | | 2M-25 | M-10 | -M | 0 | 0 | 0 | |

Note: In case of minimization, for choosing pivot or key column we have to look most positive number in $z_j - c_j$ row and to obtain the optimum solution every $z_j - c_j$ row should be $\leq 0$

In above table, 2M-25 is the most positive in $z_j - c_j$ so $x_1$ is the key column and 20 is the minimum ratio so $R_2$ is the key row. Here, $A_2$ is the outgoing variable and $x_1$ is the incoming variable. Here, 1 is the pivot element.

- $new R_2 = old R_2$
- $new R_3 = old R_3$
- $new R_1 = old R_1 - new R_2$

i.e $50-20=30, 1-1=0, 1-0=1, 0-(-1)=1, 0, 1-0=1, 0-1=-1$

Developing second simplex table, we get,

| | | $C_j \rightarrow$ | 25 | 10 | 0 | 0 | M | M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: M | $A_1$ | 30 | 0 | 1 | 1 | 0 | 1 | -1 | 30/1 = 30 |
| $R_2$: 25 | $x_1$ | 20 | 1 | 0 | -1 | 0 | 0 | 1 | 20/0 = ∞ |
| $R_3$: 0 | $S_2$ | 40 | 0 | 1 | 0 | 1 | 0 | 0 | 40/1 = 40 |
| | zj | 30M+500 | 25 | M | M-25 | 0 | M | -M+25 | |
| | zj-cj | | 0 | M-10 | M-25 | 0 | 0 | -2M+25 | |

In above table, M-10 is the most positive in $z_j - c_j$ so $x_2$ is the key column and 30 is the minimum ratio so $R_1$ is the key row. Here, $A_1$ is the outgoing variable and $x_2$ is incoming variable.

- $new R_1 = old R_1$
- $new R_2 = old R_2$
- $new R_3 = old R_3 - old R_1$

i.e $40-30=10, 0, -1, 1, -1, 1.$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 25 | 10 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 10 | $x_2$ | 30 | 0 | 1 | 1 | 0 | 1 | -1 | |
| $R_2$: 25 | $x_1$ | 20 | 1 | 0 | -1 | 0 | 0 | 1 | |
| $R_3$: 0 | $S_2$ | 10 | 0 | 0 | -1 | 1 | -1 | 1 | |
| | $z_j$ | 800 | 25 | 10 | -15 | 0 | 10 | 15 | |
| | $z_j - c_j$ | | 0 | 0 | -15 | 0 | 10-M | 15-M | |

$\forall$ $(z_j - c_j) \leq 0$ so, the optimal solution is obtained.

Min $z = 800$

Basic variable: $x_1 = A = 20$ and $x_2 = B = 30$, $S_2 = 10$.

20. A firm must produce 200 kgs of a mixture consisting of the ingrediants P and Q which cost Rs 3 and Rs 8 per kg respectively. Not more than 80 kgs of P and not less than 60 kgs of Q are used. Find how much of each ingrediant should be used to minimize total cost.

Solution:

Converting the question to tabular form, we get,

| | Ingrediants | | Available |
|---|---|---|---|
| | P (x) | Q (y) | |
| | 1 | 1 | 200 |
| | 1 | - | 80 |
| | - | 1 | 60 |
| cost | Rs 3 | Rs 8 | |

Formulation of LPP,

Min $z = 3x + 8y$

s.t.c $\quad x + y = 200$

$x + y + a_1$

$x + s_1$

$y - s_2 + a_2$

Date ______
Page ______

$x \le 80$

$y \ge 60$

$x \ge 0, y \ge 0$

Formatting the problem in standard form of LPP, we get,

$\text{Min } z = 3x + 8y + 0s_1 + 0s_2 + MA_1 + MA_2$

$x + y + 0s_1 + 0s_2 + A_1 + 0A_2 = 200$

$x + 0 \cdot y + 1s_1 + 0s_2 + 0A_1 + 0A_2 = 80$

$0x + y + 0s_1 - s_2 + 0A_1 + A_2 = 60$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 3 | 8 | 0 | 0 | M | M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: M | $A_1$ | 200 | 1 | 1 | 0 | 0 | 1 | 0 | 200/1 = 200 |
| $R_2$: 0 | $S_1$ | 80 | 1 | 0 | 1 | 0 | 0 | 0 | 80/0 = ∞ |
| $R_3$: M | $A_2$ | 60 | 0 | 1 | 0 | -1 | 0 | 1 | 60/1 = 60 |
| | $z_j$ | 260M | M | 2M | 0 | -M | M | M | |
| | $z_j - c_j$ | | M-3 | 2M-8 | 0 | -M | 0 | 0 | |

In the above table, 2M-8 is the most positive in $z_j - c_j$ so y is the key column and 60 is the minimum ratio so $R_3$ is the key row. Here, $A_2$ is outgoing variable and y is the incoming variable.

- $newR_3 = oldR_3$
- $newR_2 = oldR_2$
- $newR_1 = oldR_1 - newR_3$

i.e $200 - 60 = 140$, $1 - 0 = 1$, $0$, $0$, $0 - (-1) = 1$, $1 - 0 = 1$, $0 - 1 = -1$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 3 | 8 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x$ | $y$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 140 | 1 | 0 | 0 | -1 | 1 | -1 | 140/1 = 140 |
| $R_2$: 0 | $S_1$ | 80 | 1 | 0 | 1 | 0 | 0 | 0 | 80/1 = 80 |
| $R_3$: 8 | $y$ | 60 | 0 | 1 | 0 | -1 | 0 | 1 | 60/0 = ∞ |
| | $Z_j$ | 140M+480 | M | 8 | 0 | -M-8 | M | -M+8 | |
| | $Z_j - C_j$ | | M-3 | 0 | 0 | -M-8 | 0 | -2M+8 | |

In above table, M-3 is the most positive in $z_j - c_j$ so $x$ is the key column and 80 is the minimum ratio so R2 is the key row. Here, $S_1$ is outgoing variable and $x$ is incoming variable.

- new R2 = old R2
- new R3 = old R3
- new R1 = old R1 - new R2

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 3 | 8 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 60 | 0 | 0 | -1 | 1 | 1 | -1 | 60/1 = 60 |
| $R_2$: 3 | $x$ | 80 | 1 | 0 | 1 | 0 | 0 | 0 | 80/0 = ∞ |
| $R_3$: 8 | $y$ | 60 | 0 | 1 | 0 | -1 | 0 | 1 | 60/-1 = - |
| | $Z_j$ | 60M+720 | 3 | 8 | -M+3 | M-8 | M | -M+8 | |
| | $Z_j - C_j$ | | 0 | 0 | -M+3 | M-8 | 0 | -2M+8 | |

In above table, M-8 is the most positive in $z_j - c_j$ so $S_2$ is the key column and 60 is the minimum ratio so R1 is the key row. Here, $A_1$ is outgoing variable and $S_2$ is incoming variable.

- new R1 = old R1
- new R2 = old R2
- new R3 = old R3 + new R1

Developing fourth simplex table, we get,

| | $C_j \rightarrow$ | | 3 | 8 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x$ | $y$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $S_2$ | 60 | 0 | 0 | -1 | 1 | 1 | -1 | |
| $R_2$: 3 | $x$ | 80 | 1 | 0 | 1 | 0 | 0 | 0 | |
| $R_3$: 8 | $y$ | 120 | 0 | 1 | -1 | 0 | 1 | 0 | |
| | $z_j$ | 1200 | 3 | 8 | -5 | 0 | 8 | 0 | |
| | $z_j - c_j$ | | 0 | 0 | -5 | 0 | 8-M | -M | |

$\forall (z_j - c_j) \leq 0$, the optimal solution is obtained.

Min $z = 1200$ at $x = P = 80$ and $y = Q = 120$.

1. Solve the following problem by simplex method.

a. Max $Z = 60x_1 + 50x_2$

Subject to

$2x_1 + 4x_2 \leq 80$

$3x_1 + 2x_2 \leq 60$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

$$\text{Max } Z = 60x_1 + 50x_2 + 0S_1 + 0S_2$$

s.t.

$$2x_1 + 4x_2 + S_1 + 0S_2 = 80$$

$$3x_1 + 2x_2 + 0S_1 + 1S_2 = 60$$

$$x_1, x_2, S_1, S_2 \geq 0$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 60 | 50 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|
| CB | B.V | xB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | xB/key column |
| $R_1$: 0 | $s_1$ | 80 | 2 | 4 | 1 | 0 | 80/2 = 40 |
| $R_2$: 0 | $s_2$ | 60 | 3 | 2 | 0 | 1 | 60/3 = 20 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | −60 | −50 | 0 | 0 | |

In the above table, −60 is the most negative in $z_j - c_j$ so $x_1$ column is the key column and 20 is the minimum ratio so $R_2$ is the key row. Here, $s_2$ is the outgoing variable and $x_1$ is the incoming variable. Here, 3 is the pivot element.

- new$R_2$ = old$R_2$ / 3

i.e $\frac{60}{3} = 20$, $\frac{3}{3} = 1$, $\frac{2}{3}$, 0, $\frac{1}{3}$

- new$R_1$ = old$R_1$ − 2 new$R_2$

i.e $80 - 2\times 20 = 40$, $2 - 2\times 1 = 0$, $4 - 2\times\frac{2}{3} = \frac{8}{3}$, $1 - 2\times 0 = 1$, $0 - 2\times\frac{1}{3} = -\frac{2}{3}$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 60 | 50 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|
| CB | BV | xB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | |
| $R_1$: 0 | $s_1$ | 40 | 0 | 8/3 | 1 | −2/3 | 40/8/3 = 15 |
| $R_2$: 60 | $x_1$ | 20 | 1 | 2/3 | 0 | 1/3 | 20/2/3 = 30 |
| | $z_j$ | 1200 | 60 | 40 | 0 | 20 | |
| | $z_j - c_j$ | | 0 | −10 | 0 | 20 | |

In the above table, −10 is the most negative in $z_j - c_j$ so $x_2$ is the key column and ~~8~~15 is the minimum ratio so $R_1$ is the key row. Here, $\frac{8}{3}$ is the pivot element.

- new$R_2$ = old$R_2$ − $\frac{2}{3}$ new$R_1$

i.e 10, 1, 0, $-\frac{1}{4}$

$new R_1 = old R_1 \times \frac{3}{8}$

i.e $40 \times \frac{3}{8} = 15, 0, 1, \frac{3}{8}, \frac{-2}{3} \times \frac{-3}{8} = \frac{-1}{4}$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 60 | 50 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|
| CB | BV | xB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | |
| $R_1$: 50 | $x_2$ | 15 | 0 | 1 | 3/8 | -1/4 | |
| $R_2$: 60 | $x_1$ | 10 | 1 | 0 | -1/4 | 1/2 | |
| | $z_j$ | 1350 | 60 | 50 | 15/4 | 35/2 | |
| | $z_j - C_j$ | | 0 | 0 | 15/4 | 35/2 | |

Since, all $(z_j - C_j) \geq 0$ so the optimal solution is obtained.
Max $z = 1350$
Basic variable: $x_1 = 10$ and $x_2 = 15$.

---

b) Max $z = 2x_1 + 3x_2$
subject to
$x_1 + 2x_2 \leq 13$
$2x_1 + x_2 \leq 14$
$x_1 \geq 0, x_2 \geq 0$

solution:
Formatting the problem in standard form of LPP, we get,

$Max\ z = 2x_1 + 3x_2 + 0s_1 + 0s_2$

$x_1 + 2x_2 + 1s_1 + 0s_2 = 13$

$2x_1 + x_2 + 0s_1 + 1s_2 = 14$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 2 | 3 | 0 | 0 | Ratio = xB/key column |
|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | |
| $R_1$: 0 | $s_1$ | 13 | 1 | $\boxed{2}$ | 1 | 0 | $13/2 = 6.5$ |
| $R_2$: 0 | $s_2$ | 14 | 2 | 1 | 0 | 1 | $14/1 = 14$ |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -2 | -3 | 0 | 0 | |

In the above table, -3 is the most negative in $z_j - c_j$ so $x_2$ is the key column and 6.5 is the minimum ratio so $R_1$ is the key row. Here, $s_1$ is the outgoing variable and $x_2$ is the incoming variable. Here, 2 is the pivot element.

- new$R_1$ = old$R_1$ / 2
  i.e $\frac{13}{2}, \frac{1}{2}, 1, \frac{1}{2}, 0$

- new$R_2$ = old$R_2$ - new$R_1$
  i.e $14 - \frac{13}{2} = \frac{15}{2}$, $2 - \frac{1}{2} = \frac{3}{2}$, $1 - 1 = 0$, $0 - \frac{1}{2} = -\frac{1}{2}$, $1 - 0 = 1$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 2 | 3 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|
| CB | B.V | xB constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | |
| $R_1$: 3 | $x_2$ | $13/2$ | $1/2$ | 1 | $1/2$ | 0 | $13/2 / 1/2 = 13$ |
| $R_2$: 0 | $s_2$ | $15/2$ | $\boxed{3/2}$ | 0 | $-1/2$ | 1 | $15/2 / 3/2 = 5$ |
| | $z_j$ | $\frac{39}{2}$ | $3/2$ | 3 | $3/2$ | 0 | |
| | $z_j - c_j$ | | $-1/2$ | 0 | $3/2$ | 0 | |

Here, $-\frac{1}{2}$ is the most negative in $z_j - c_j$ so $x_1$ is the key column and 5 is the minimum ratio so $R_2$ is key row. Here, $s_2$ is outgoing variable and $x_1$ is incoming variable. Here $\frac{3}{2}$ is the pivot element

Developing third simplex table, we get,

- new $R_2$ = old $R_2 \times \frac{2}{3}$

i.e $\frac{15}{2} \times \frac{2}{3} = 5$, $\frac{3}{2} \times \frac{2}{3} = 1$, $0$, $-\frac{1}{2} \times \frac{2}{3} = -\frac{1}{3}$, $\frac{2}{3}$

- new $R_1$ = old $R_1 - \frac{1}{2} \times$ new $R_2$

i.e $\frac{13}{2} - \frac{1}{2} \times 5 = 4$, $\frac{1}{2} - \frac{1}{2} \times 1 = 0$, $1 - \frac{1}{2} \times 0 = 1$, $\frac{1}{2} - \frac{1}{2} \times (-\frac{1}{3}) = \frac{2}{3}$,

$0 - \frac{1}{2} \times \frac{2}{3} = -\frac{1}{3}$

| | $C_j \rightarrow$ | | 2 | 3 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | |
| $R_1$: 3 | $x_2$ | 4 | 0 | 1 | 2/3 | -1/3 | |
| $R_2$: 2 | $x_1$ | 5 | 1 | 0 | -1/3 | 2/3 | |
| | $z_j$ | 22 | 2 | 3 | 4/3 | 1/3 | |
| | $z_j - C_j$ | | 0 | 0 | 4/3 | 1/3 | |

Since, all $(z_j - c_j) \geq 0$, the optimal solution is obtained.

Max $z = 22$

Basic Variable: $x_1 = 5$ and $x_2 = 4$ Ans.

c) Max $z = 10x_1 + 20x_2$

Subject to:

$4x_1 + 2x_2 \leq 60$

$4x_1 + 10x_2 \leq 100$

$2x_1 + 3x_2 \leq 38$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

$$\text{Max } z = 10x_1 + 20x_2 + 0s_1 + 0s_2 + 0s_3$$

$$4x_1 + 2x_2 + 1s_1 + 0s_2 + 0s_2 = 60$$

$$4x_1 + 10x_2 + 0s_1 + 1s_2 + 0s_3 = 100$$

$$2x_1 + 3x_2 + 0s_1 + 0s_2 + 1s_3 = 38$$

Developing initial simplex table, we get,

| | | $C_j \rightarrow$ | 10 | 20 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | Ratio = $x_B$/key column |
| $R_1$: 0 | $s_1$ | 60 | 4 | 2 | 1 | 0 | 0 | 60/2 = 30 |
| $R_2$: 0 | $s_2$ | 100 | 4 | 10 | 0 | 1 | 0 | 100/10 = 10 |
| $R_3$: 0 | $s_3$ | 38 | 2 | 3 | 0 | 0 | 1 | 38/3 = 12.667 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -10 | -20 | 0 | 0 | 0 | |

In above table, -20 is the most negative in $z_j - c_j$ so $x_2$ is the key column and 10 is the minimum ratio so $R_2$ is the key row. Here, $s_2$ is the outgoing variable and $x_2$ is the incoming variable. Here, 10 is pivot element.

- new $R_2$ = old $R_2$ / 10

  i.e $100/10 = 10$, $\frac{4}{10} = \frac{2}{5}$, 1, 0, $\frac{1}{10}$, 0

- new $R_1$ = old $R_1$ − 2 new $R_2$

  i.e $60 - 2\times10 = 40$, $4 - 2\times\frac{2}{5} = \frac{16}{5}$, 0, $1 - 2\times0 = 1$, $0 - 2\times\frac{1}{10} = \frac{-1}{5}$, 0

- new $R_3$ = old $R_3$ − 3 new $R_2$

  i.e $38 - 3\times10 = 8$, $2 - 3\times\frac{2}{5} = \frac{4}{5}$, 0, $0 - 3\times0 = 0$, $0 - 3\times\frac{1}{10} = \frac{-3}{10}$, $1 - 3\times0 = 1$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 10 | 20 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | |
| $R_1$: 0 | $s_1$ | 40 | $16/5$ | 0 | 1 | $-1/5$ | 0 | $40/16/5 = 12.5$ |
| $R_2$: 20 | $x_2$ | 10 | $2/5$ | 1 | 0 | $1/10$ | 0 | $10/2/5 = 25$ |
| $R_3$: 0 | $s_3$ | 8 | $4/5$ | 0 | 0 | $-3/10$ | 1 | $8/4/5 = 10$ |
| | $z_j$ | 200 | 8 | 20 | 0 | 2 | 0 | |
| | $z_j - c_j$ | | $-2$ | 0 | 0 | 2 | 0 | |

In above table, $-2$ is the most negative in $z_j - c_j$ so $x_1$ is the key column and 10 is the minimum ratio so $s_3$ is the key row. Here, $s_3$ is outgoing variable and $x_1$ is incoming variable. Here, $\frac{4}{5}$ is pivot element.

- $\text{new}R_3 = \text{old}R_3 \times \frac{5}{4}$

  i.e $8 \times \frac{5}{4} = 10, \frac{4}{5} \times \frac{5}{4} = 1, 0, 0, \frac{-3}{10} \times \frac{5}{4} = -\frac{3}{8}, \frac{5}{4}$

- $\text{new}R_1 = \text{old}R_1 - \frac{16}{5}\,\text{new}R_3$

  i.e $40 - \frac{16}{5} \times 10 = 8, \frac{16}{5} - \frac{16}{5} \times 1 = 0, 0 - \frac{16}{5} \times 0 = 0, 1 - \frac{16}{5} \times 0 = 1,$

  $\frac{-1}{5} - \frac{16}{5} \times \left(-\frac{3}{8}\right) = 1, 0 - \frac{16}{5} \times \frac{5}{4} = -4$

- $\text{new}R_2 = \text{old}R_2 - \frac{2}{5}\,\text{new}R_3$

  i.e $10 - \frac{2}{5} \times 10 = 6, 0, 1 - \frac{2}{5} \times 0 = 1, 0 - \frac{2}{5} \times 0 = 0, \frac{1}{10} - \frac{2}{5} \times \left(\frac{-3}{8}\right) = \frac{1}{4},$

  $0 - \frac{2}{5} \times \frac{5}{4} = -\frac{1}{2}$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 10 | 20 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | |
| $R_1$: 0 | $s_1$ | 8 | 0 | 0 | 1 | 1 | -4 | |
| $R_2$: 20 | $x_2$ | 6 | 0 | 1 | 0 | -1/4 | -1/2 | |
| $R_3$: 10 | $x_1$ | 10 | 1 | 0 | 0 | -3/8 | 5/4 | |
| | $z_j$ | 220 | 10 | 20 | 0 | 5/4 | 5/2 | |
| | $z_j - c_j$ | | 0 | 0 | 0 | 5/4 | 5/2 | |

Since, all $(z_j - c_j) \geq 0$ so the optimal solution is obtained.

Max $z = 220$

Basic Variable: $x_1 = 10$ and $x_2 = 6$

d) Max $z = 400x_1 + 320x_2$

Subject to

$4x_1 + 10x_2 \leq 100$

$20x_1 + 10x_2 \leq 300$

$2x_1 + 3x_2 \leq 38$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

Max $z = 400x_1 + 320x_2 + 0s_1 + 0s_2 + 0s_3$

$4x_1 + 10x_2 + s_1 + 0s_2 + 0s_3 = 100$

$20x_1 + 10x_2 + 0s_1 + s_2 + 0s_3 = 300$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 38$

Developing the initial simplex table, we get,

| | $C_j \rightarrow$ | | 400 | 320 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 100 | 4 | 10 | 1 | 0 | 0 | 100/4 = 25 |
| $R_2$: 0 | $s_2$ | 300 | 20 | 10 | 0 | 1 | 0 | 300/20 = 15 |
| $R_3$: 0 | $s_3$ | 38 | 2 | 3 | 0 | 0 | 1 | 38/2 = 19 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -400 | -320 | 0 | 0 | 0 | |

In the above table, -400 is the most negative in $z_j - c_j$ so $x_1$ is the key column and 15 is the minimum ratio so $R_2$ is the key row. Here, $s_2$ is the outgoing variable and $x_1$ is the incoming variable. Here, 20 is pivot element.

- new$R_2$ = old$R_2$ / 20

i.e $\frac{300}{20} = 15, 1, \frac{10}{20} = \frac{1}{2}, 0, \frac{1}{20}, 0$

- new$R_1$ = old $R_1$ − 4 new$R_2$

i.e $100 - 4\times15 = 40, 4 - 4\times1 = 0, 10 - 4\times\frac{1}{2} = 8, 1 - 4\times0 = 1, 0 - 4\times\frac{1}{20} = -\frac{1}{5},$

$0 - 4\times0 = 0$

- new$R_3$ = old$R_3$ − 2new$R_2$

i.e $38 - 2\times15 = 8, 2 - 2\times1 = 0, 3 - 2\times\frac{1}{2} = 2, 0 - 2\times0 = 0, 0 - 2\times\frac{1}{20} = -\frac{1}{10}, 1 - 2\times0 = 1$

Developing the second simplex table, we get,

| | $C_j \rightarrow$ | | 400 | 320 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | |
| $R_1$: 0 | $s_1$ | 40 | 0 | 8 | 1 | $-1/5$ | 0 | $40/8 = 5$ |
| $R_2$: 400 | $x_1$ | 15 | 1 | $1/2$ | 0 | $1/20$ | 0 | $15/1/2 = 30$ |
| $R_3$: 0 | $s_3$ | 8 | 0 | 2 | 0 | $-1/10$ | 1 | $8/2 = 4$ |
| | $z_j$ | 6000 | 400 | 200 | 0 | 20 | 0 | |
| | $z_j - c_j$ | | 0 | -120 | 0 | 20 | 0 | |

In the above table, -120 is the most negative in $z_j - c_j$ so $x_2$ is the key column and 4 is the minimum ratio so $R_3$ is the key row. Here, $s_3$ is the outgoing variable and $x_2$ is the incoming variable.

- new $R_3$ = old $R_3/2$

  i.e $\frac{8}{2} = 4, 0, 1, 0, -\frac{1}{20}, \frac{1}{2}$

- new $R_1$ = old $R_1$ - 8 new $R_3$

  i.e $40 - 8 \times 4 = 8$, $0 - 8 \times 0 = 0$, $0$, $1 - 8 \times 0 = 1$, $-\frac{1}{5} - 8 \times \left(-\frac{1}{20}\right) = \frac{1}{5}$, $0 - 8 \times \frac{1}{2} = -4$

- new $R_2$ = old $R_2 - \frac{1}{2}$ new $R_3$

  i.e $15 - \frac{1}{2} \times 4 = 13$, $1 - \frac{1}{2} \times 0 = 1$, $0$, $0 - \frac{1}{2} \times 0 = 0$, $\frac{1}{20} - \frac{1}{2} \times \left(-\frac{1}{20}\right) = \frac{3}{40}$, $0 - \frac{1}{2} \times \frac{1}{2} = \frac{-1}{4}$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 400 | 320 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $s_3$ | |
| $R_1$: 0 | $s_1$ | 8 | 0 | 0 | 1 | $1/5$ | $-4$ | |
| $R_2$: 400 | $x_1$ | 13 | 1 | 0 | 0 | $3/40$ | $-1/4$ | |
| $R_3$: 320 | $x_2$ | 4 | 0 | 1 | 0 | $-1/20$ | $1/2$ | |
| | $z_j$ | 6480 | 400 | 320 | 0 | 14 | 60 | |
| | $z_j - c_j$ | | 0 | 0 | 0 | 14 | 60 | |

Since, all $(z_j - c_j) \geq 0$ so the optimal solution is obtained.
Max $Z = 6480$.
Basic variable: $x_1 = 13$, $x_2 = 4$ Ans.

---

f. Max $Z = 2x_1 + 4x_2 + 3x_3$
Subject to
$3x_1 + 4x_2 + 2x_3 \leq 60$
$2x_1 + x_2 + 2x_3 \leq 40$
$x_1 + 3x_2 + 2x_3 \leq 80$
$x_1, x_2, x_3 \geq 0$

Solution:
Formatting the problem in standard form of LPP, we get,

$$\text{Max } Z = 2x_1 + 4x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3$$
$$3x_1 + 4x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 60$$
$$2x_1 + x_2 + 2x_3 + 0s_1 + 1s_2 + 0s_3 = 40$$
$$x_1 + 3x_2 + 2x_3 + 0s_1 + 0s_2 + s_3 = 80$$

Developing initial simplex table, we get,

| | $c_j \rightarrow$ | | 2 | 4 | 3 | 0 | 0 | 0 | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $x_B$/key column |
| $R_1$: 0 | $s_1$ | 60 | 3 | 4 | 2 | 1 | 0 | 0 | 60/4 = 15 |
| $R_2$: 0 | $s_2$ | 40 | 2 | 1 | 2 | 0 | 1 | 0 | 40/1 = 40 |
| $R_3$ 0 | $s_3$ | 80 | 1 | 3 | 2 | 0 | 0 | 1 | 80/3 = 26.67 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | -2 | -4 | -3 | 0 | 0 | 0 | |

In above table, -4 is the most negative in $z_j - c_j$ so $x_2$ is the key column and 15 is the minimum ratio so $R_1$ is the key row. Here, $s_1$ is outgoing variable and $x_2$ is incoming variable. Here, 4 is the pivot element.

- newR1 = oldR1 / 4

i.e $\frac{60}{4} = 15, \frac{3}{4}, 1, \frac{2}{4} = \frac{1}{2}, \frac{1}{4}, 0, 0$

- newR2 = oldR2 − newR1

i.e $40 - 15 = 25, 2 - \frac{3}{4} = \frac{5}{4}, 1 - 1 = 0, 2 - \frac{1}{2} = \frac{3}{2}, 0 - \frac{1}{4} = \frac{-1}{4}, 1 - 0 = 1, 0 - 0 = 0$

- newR3 = oldR3 − 3newR1

i.e $80 - 3 \times 15 = 35, 1 - 3 \times \frac{3}{4} = -\frac{5}{4}, 0, 2 - 3 \times \frac{1}{2} = \frac{1}{2}, 0 - 3 \times \frac{1}{4} = -\frac{3}{4}, 0 - 3 \times 0 = 0, 1 - 3 \times 0 = 1$

Developing second simplex table, we get,

| | $c_j \rightarrow$ | | 2 | 4 | 3 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | Ratio |
| $R_1$: 4 | $x_2$ | 15 | 3/4 | 1 | 1/2 | 1/4 | 0 | 0 | 15/1/2 = 30 |
| $R_2$: 0 | $s_2$ | 25 | 5/4 | 0 | 3/2 | −1/4 | 1 | 0 | 25/3/2 : 16.67 |
| $R_3$: 0 | $s_3$ | 35 | −5/4 | 0 | 1/2 | −3/4 | 0 | 1 | 35/1/2 : 70 |
| | $z_j$ | 60 | 3 | 4 | 2 | 1 | 0 | 0 | |
| | $z_j - c_j$ | | 1 | 0 | −1 | 1 | 0 | 0 | |

In above table, −1 is the most negative in $z_j - c_j$ so $x_3$ is the key column and 16.67 is the minimum ratio so R2 is the key row. Here, $s_2$ is outgoing variable and $x_3$ is incoming variable. Here, $\frac{3}{2}$ is pivot element.

- newR2 = oldR2 × $\frac{2}{3}$

i.e $25 \times \frac{2}{3} = \frac{50}{3}, \frac{5}{4} \times \frac{2}{3} = \frac{5}{6}, 0, 1, -\frac{1}{4} \times \frac{2}{3} = -\frac{1}{6}, \frac{2}{3}, 0$

- newR1 = oldR1 − $\frac{1}{2}$ newR2

i.e $15 - \frac{1}{2} \times \frac{50}{3} = \frac{20}{3}$, $\frac{3}{4} - \frac{1}{2} \times \frac{5}{6} = \frac{1}{3}$, $1 - \frac{1}{2} \times 0 = 1$, $0$, $\frac{1}{4} - \frac{1}{2} \times \left(\frac{-1}{6}\right) = \frac{1}{3}$,

$0 - \frac{1}{2} \times \frac{2}{3} = -\frac{1}{3}$, $0 - \frac{1}{2} \times 0 = 0$

- $newR_3 = oldR_3 - \frac{1}{2} newR_2$

i.e $35 - \frac{1}{2} \times \frac{50}{3} = \frac{80}{3}$, $-\frac{5}{4} - \frac{1}{2} \times \frac{5}{6} = -\frac{5}{3}$, $0 - \frac{1}{2} \times 0 = 0$, $0$, $-\frac{3}{4} - \frac{1}{2} \times \left(\frac{-1}{6}\right) = -\frac{2}{3}$

$0 - \frac{1}{2} \times \frac{2}{3} = -\frac{1}{3}$, $1 - \frac{1}{2} \times 0 = 1$.

Developing third simplex table, we get,

| | $c_j \rightarrow$ | | 2 | 4 | 3 | 0 | 0 | 0 | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | |
| $R_1$: 4 | $x_2$ | 20/3 | 1/3 | 1 | 0 | 1/3 | −1/3 | 0 | |
| $R_2$: 3 | $x_3$ | 50/3 | 5/6 | 0 | 1 | −1/6 | 2/3 | 0 | |
| $R_3$: 0 | $s_3$ | 80/3 | −5/3 | 0 | 0 | −2/3 | −1/3 | 1 | |
| | $z_j$ | 230/3 | 23/6 | 4 | 3 | 5/6 | 2/3 | 0 | |
| | $z_j - c_j$ | | 11/6 | 0 | 0 | 5/6 | 2/3 | 0 | |

Since, all $(z_j - c_j) \geq 0$ so the optimal solution is obtained.

$Max\, z = \frac{230}{3}$

Basic Variable: $x_2 = \frac{20}{3}$, $x_3 = \frac{50}{3}$

Non Basic Variable: $x_1 = 0$.

---

h. $Max.\ Z = 8x_1 - 3x_2$

Subject to:

$x_1 + 2x_2 \leq 7$

$2x_1 + 4x_2 \geq 14$

$x_1 = 5$ and $x_2 \geq 0$

Solution:

Formating the equation in standard form of LPP, we get,

$$Max\ z = 8x_1 - 3x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

$$1x_1 + 2x_2 + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 7$$

$$2x_1 + 4x_2 + 0S_1 - 1S_1 + 1A_1 + 0A_2 = 14$$

$$1x_1 + 0x_2 + 0S_1 + 0S_2 + 0A_1 + 1A_2 = 5$$

$$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 8 | -3 | 0 | 0 | -M | -M | |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | Ratio: $x_B$/key column |
| $R_1$: 0 | $S_1$ | 7 | 1 | 2 | 1 | 0 | 0 | 0 | 7/2 = 3.5 |
| $R_2$: -M | $A_1$ | 14 | 2 | 4 | 0 | -1 | 1 | 0 | 14/4 = 3.5 |
| $R_3$: -M | $A_2$ | 5 | 1 | 0 | 0 | 0 | 0 | 1 | 5/0 = ∞ |
| | $z_j$ | -19M | -3M | -4M | 0 | M | -M | -M | |
| | $z_j - C_j$ | | -3M-8 | -4M+3 | 0 | M | 0 | 0 | |

In the above table, -4M+3 is the most negative in $z_j - C_j$ so $x_2$ column is pivot column and 3.5 is the smallest ratio. Here, the smallest ratio is tie. so I chose $R_1$ as the pivot row. Here, $S_1$ is outgoing variable and $x_2$ is incoming variable. Here, 2 is pivot element.

- new $R_1$ = old $R_1$ / 2

  i.e $\frac{7}{2}, \frac{1}{2}, 1, \frac{1}{2}, 0, 0, 0$

- new $R_2$ = old $R_2$ - 4 new $R_1$

  i.e $14 - 4\times\frac{7}{2} = 0,\ 2 - 4\times\frac{1}{2} = 0,\ 0,\ 0 - 4\times\frac{1}{2} = -2,\ -1 - 4\times 0 = -1,\ 1,\ 0$

- new $R_3$ = old $R_3$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 8 | -3 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: 8 | $x_2$ | 7/2 | 1/2 | 1 | 1/2 | 0 | 0 | 0 | $\frac{7}{2}/\frac{1}{2} = 7$ |
| $R_2$: -M | $A_1$ | 0 | 0 | 0 | -2 | -1 | 1 | 0 | 0/0 |
| $R_3$: -M | $A_2$ | 5 | 1 | 0 | 0 | 0 | 0 | 1 | 5/1 = 5 |
| | $z_j$ | 28-5M | 4-M | 8 | 4+2M | M | -M | -M | |
| | $z_j - c_j$ | | -M-4 | 11 | 4+2M | M | 0 | 0 | |

In above table, -M-4 is the most negative in $z_j - c_j$ so $x_1$ is the key column and 5 is the minimum ratio so R3 is the key row. Here, $A_2$ is the outgoing variable and $x_1$ is the incoming variable. Here, 1 is the pivot element.

- new R3 = old R3
- new R2 = old R2
- new $R_1$ = old $R_1 - \frac{1}{2} \times$ new R3

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 8 | -3 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: -3 | $x_2$ | 1 | 0 | 1 | 1/2 | 0 | 0 | -1/2 | |
| $R_2$: -M | $A_1$ | 0 | 0 | 0 | -2 | -1 | 1 | 0 | |
| $R_3$: 8 | $x_1$ | 5 | 1 | 0 | 0 | 0 | 0 | 1 | |
| | $z_j$ | 37 | 8 | -3 | $2M - \frac{3}{2}$ | M | -M | 19/2 | |
| | $z_j - c_j$ | | 0 | 0 | $2M - \frac{3}{2}$ | M | 0 | $\frac{19}{2} + M$ | |

Since all $(z_j - c_j) \geq 0$ so the optimal solution is obtained.

Max $z = 37$ at $x_1 = 5$ and $x_2 = 1$

i. Max $P = -2x_1 - 8x_2$

subjected to :

$0 \leq x_1 \leq 20$

$5x_1 + 10x_2 = 150$

$x_2 \geq 14$

Solution:

The given problem is

$\text{Max } P = -2x_1 - 8x_2$

s.t.c

$x_1 \geq 0$

$x_1 \leq 20$

$5x_1 + 10x_2 = 150$

$x_2 \geq 14$

Converting the problem in standard form of LPP,

$\text{Max } P = -2x_1 - 8x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

$1x_1 + 0x_2 + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 20$

$5x_1 + 10x_2 + 0S_1 + 0S_2 + 1A_1 + 0A_2 = 150$

$0x_1 + 1x_2 + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 14$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | -2 | -8 | 0 | 0 | -M | -M | Ratio = |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ constant | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | $x_B$/keycolumn |
| $R_1$: 0 | $S_1$ | 20 | 1 | 0 | 1 | 0 | 0 | 0 | 20/0 = ∞ |
| $R_2$: -M | $A_1$ | 150 | 5 | 10 | 0 | 0 | 1 | 0 | 150/10 = 15 |
| $R_3$: -M | $A_2$ | 14 | 0 | 1 | 0 | -1 | 0 | 1 | 14/1 = 14 |
| | $z_j$ | -164M | -5M | -11M | 0 | M | -M | -M | |
| | $z_j - c_j$ | | -5M+2 | -11M+8 | 0 | M | 0 | 0 | |

- newR3 = oldR3
- newR1 = oldR1
- newR2 = oldR2 - 10newR3

i.e 150 - 10×14 = 10, 5 - 10×0 = 5, 0, 0 - 10×0 = 0, 0 - 10×(-1) = 10,
1 - 10×(0) = 1, 0 - 10×1 = -10

Developing second simplex table,

| | $C_j \rightarrow$ | | -2 | -8 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $s_1$ | 20 | 1 | 0 | 1 | 0 | 0 | 0 | 20/0 = ∞ |
| $R_2$: -M | $A_1$ | 10 | 5 | 0 | 0 | 10 | 1 | -10 | 10/10 = 1 |
| $R_3$: -8 | $x_2$ | 14 | 0 | 1 | 0 | -1 | 0 | 1 | 14/-1 = - |
| | $z_j$ | -10M-112 | -5M | -8 | 0 | -10M+8 | -M | 10M-8 | |
| | $z_j - c_j$ | | -5M+2 | 0 | 0 | -10M+8 | 0 | 11M-8 | |

- newR2 = oldR2/10

i.e $\frac{10}{10} = 1$, $\frac{5}{10} = \frac{1}{2}$, 0, 0, 1, $\frac{1}{10}$, -1

- newR1 = oldR1
- newR3 = oldR3 + newR2

i.e 14 + 1 = 15, $0 + \frac{1}{2} = \frac{1}{2}$, 1 + 0 = 1, 0 + 0 = 0, -1 + 1 = 0, $0 + \frac{1}{10} = \frac{1}{10}$, 1 - 1 = 0

Developing third simplex table,

| | $C_j \rightarrow$ | | -2 | -8 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $s_1$ | 20 | 1 | 0 | 1 | 0 | 0 | 0 | 20/1 = 20 |
| $R_2$: 0 | $s_2$ | 1 | 1/2 | 0 | 0 | 1 | 1/10 | -1 | 1/1/2 = 2 |
| $R_3$: -8 | $x_2$ | 15 | 1/2 | 1 | 0 | 0 | 1/10 | 0 | 15/1/2 = 30 |
| | $z_j$ | -120 | -4 | -8 | 0 | 0 | -4/5 | 0 | |
| | $z_j - c_j$ | | -2 | 6 | 0 | 0 | $-\frac{4}{5}+M$ | M | |

- $newR_2 = oldR_2 \times 2$

i.e $2, 1, 0, 0, 2, \frac{1}{10} \times 2 = \frac{1}{5}, -2$

- $newR_1 = oldR_1 - newR_2$

i.e $20 - 2 = 18, 1 - 1 = 0, 0 - 0 = 0, 1 - 0 = 1, 0 - 2 = -2, 0 - \frac{1}{5} = -\frac{1}{5}, 0 - (-2) = 2$

- $newR_3 = oldR_3 - \frac{1}{2} newR_2$

i.e $15 - \frac{1}{2} \times 2 = 14, 0, 1 - \frac{1}{2} \times 0 = 1, 0 - \frac{1}{2} \times 0 = 0, 0 - \frac{1}{2} \times 2 = -1, \frac{1}{10} - \frac{1}{2} \times \frac{1}{5} = 0,$

$0 - \frac{1}{2} \times (-2) = 1$

| | $C_j \longrightarrow$ | | -2 | -8 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $s_1$ | 18 | 0 | 0 | 1 | -2 | $-1/5$ | 2 | |
| $R_2$: -2 | $x_1$ | 2 | 1 | 0 | 0 | 2 | $1/5$ | -2 | |
| $R_3$: -8 | $x_2$ | 14 | 0 | 1 | 0 | -1 | 0 | 1 | |
| | $z_j$ | -116 | -2 | -8 | 0 | 4 | $-2/5$ | 4 | |
| | $z_j - c_j$ | | 0 | 0 | 0 | 4 | $M - 2/5$ | $4+M$ | |

Since, all $(z_j - c_j) \geq 0$ so optimal solution is obtained.
Max $z = -116$ at $x_1 = 2$ and $x_2 = 14$.

---

j. Max $Z = 4A + 8B + 10C$

Subject to:

$6A + 8B + 5C \leq 56$

$2A + 3B + 4C \geq 29$

$2A + B + C \geq 11$ & $A, B, C \geq 0$

Solution:

Formatting the problem in standard form of LPP,

$Max\ Z = 4A + 8B + 10C + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_2$

$6A + 8B + 5C + 1S_1 + 0S_2 + 0S_3 + 0A_1 + 0A_2 = 56$

$2A + 3B + 4C + 0S_1 - 1S_2 + 0S_3 + A_1 + 0A_2 = 29$

$2A + B + C + 0S_1 + 0S_2 - 1S_3 + 0A_1 + A_2 = 11$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 4 | 8 | 10 | 0 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | XB | A | B | C | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $S_1$ | 56 | 6 | 8 | 5 | 1 | 0 | 0 | 0 | 0 | 56/5 = 11.2 |
| $R_2$: -M | $A_1$ | 29 | 2 | 3 | 4 | 0 | -1 | 0 | 1 | 0 | 29/4 = 7.25 |
| $R_3$: -M | $A_2$ | 11 | 2 | 1 | 1 | 0 | 0 | -1 | 0 | 1 | 11/1 = 11 |
| | $Z_j$ | -40M | -4M | -4M | -5M | 0 | M | M | -M | -M | |
| | $Z_j - C_j$ | | -4M-4 | -4M-8 | -5M-10 | 0 | M | M | 0 | 0 | |

In the above table, -5M-10 is the most negative in $Z_j - C_j$ row so C column is the pivot column and 7.25 is the minimum ratio so $R_2$ is the pivot row. Here $A_1$ is the outgoing variable and C is the incoming variable. Here, 4 is the pivot element.

- $newR_2 = oldR_2 / 4$

i.e $\frac{29}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0, \frac{-1}{4}, 0, \frac{1}{4}, 0$

- $newR_1 = oldR_1 - 5\,newR_2$

i.e $56 - 5\times\frac{29}{4} = \frac{79}{4}$, $6 - 5\times\frac{1}{2} = \frac{7}{2}$, $8 - 5\times\frac{3}{4} = \frac{17}{4}$, $5 - 5\times1 = 0$, $1 - 5\times0 = 1$,

$0 - 5\times\left(\frac{-1}{4}\right) = \frac{5}{4}$, $0$, $0 - 5\times\frac{1}{4} = \frac{-5}{4}$, $0 - 5\times0 = 0$

- $newR_3 = oldR_3 - newR_2$

i.e $11 - \frac{29}{4} = \frac{15}{4}$, $2 - \frac{1}{2} = \frac{3}{2}$, $1 - \frac{3}{4} = \frac{1}{4}$, $1 - 1 = 0$, $0 - 0 = 0$, $0 + \frac{1}{4} = \frac{1}{4}$,

$-1-0=-1$, $0-\frac{1}{4}=-\frac{1}{4}$, $1-0=1$

Developing second simplex table, we get,

| | $C_j \longrightarrow$ | | 4 | 8 | 10 | 0 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | A | B | C | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $s_1$ | 79/4 | 7/2 | 17/4 | 0 | 1 | 5/4 | 0 | -5/4 | 0 | 79/4 / 7/2 = 5.6 |
| $R_2$: 10 | C | 29/4 | 1/2 | 3/4 | 1 | 0 | -1/4 | 0 | 1/4 | 0 | 29/4 / 1/2 = 14.5 |
| $R_3$: -M | $A_2$ | 15/4 | 3/2 | 1/4 | 0 | 0 | 1/4 | -1 | -1/4 | 1 | 15/4 / 3/2 = 2.5 |
| | $z_j$ | $-\frac{15}{4}M+\frac{145}{2}$ | $-\frac{3}{2}M+5$ | $-\frac{1}{4}M+\frac{15}{2}$ | 10 | 0 | $\frac{1}{4}M-\frac{5}{2}$ | M | $\frac{1}{4}M+\frac{5}{2}$ | -M | |
| | $z_j-c_j$ | | $-\frac{3}{2}M+1$ | $-\frac{1}{4}M-\frac{1}{2}$ | 0 | 0 | $\frac{1}{4}M-\frac{5}{2}$ | M | $\frac{5}{4}M+\frac{5}{2}$ | 0 | |

In above table $-\frac{3}{2}M+1$ is the most negative in $z_j-c_j$ so A is pivot column and 2.5 is the minimum ratio so $R_3$ is the pivot row. Here, $A_2$ is the outgoing variable and A is incoming variable. Here, $\frac{3}{2}$ is pivot element.

- $newR_3 = oldR_3 \times 2/3$

i.e $\frac{5}{2}, 1, \frac{1}{6}, 0, 0, \frac{1}{6}, -\frac{2}{3}, -\frac{1}{4}\times\frac{2}{3}=\frac{-1}{6}, \frac{2}{3}$

- $newR_1 = oldR_1 - \frac{7}{2} newR_3$

i.e $11, 0, \frac{11}{3}, 0, 1, \frac{2}{3}, \frac{7}{3}, -\frac{2}{3}, \frac{-7}{3}$,

- $newR_2 = oldR_2 - \frac{1}{2} newR_3$

i.e $6, 0, \frac{2}{3}, 1, 0, -\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{3}$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 4 | 8 | 10 | 0 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | C | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $S_1$ | 11 | 0 | $11/3$ | 0 | 1 | $2/3$ | $7/3$ | $-2/3$ | $-7/3$ | $11/2/3 = 16.5$ |
| $R_2$: 10 | C | 6 | 0 | $2/3$ | 1 | 0 | $-1/3$ | $1/3$ | $1/3$ | $-1/3$ | $6/(-1/3) = -18$ |
| $R_3$: 4 | A | $\frac{5}{2}$ | 1 | $1/6$ | 0 | 0 | $1/6$ | $-2/3$ | $-1/6$ | $2/3$ | $5/2/1/6 = 15$ |
| | $z_j$ | 70 | 4 | $\frac{22}{3}$ | 10 | 0 | $-8/3$ | $2/3$ | $8/3$ | $-2/3$ | |
| | $z_j - c_j$ | | 0 | $-2/3$ | 0 | 0 | $-8/3$ | $2/3$ | $\frac{8}{3}+M$ | $-\frac{2}{3}+M$ | |

In above table, $-8/3$ is the most negative in $z_j - c_j$ so $S_2$ is the key column and 15 is the minimum ratio so $R_3$ is the key row. Here, A is outgoing variable and $S_2$ is the incoming variable. Here, $1/6$ is the pivot element.

- new$R_3$ = old$R_3$ × 6
  i.e. 15, 6, 1, 0, 0, 1, -4, -1, 4, 6

- new$R_2$ = old$R_2$ + $\frac{1}{3}$ new$R_3$
  i.e. 11, 2, 1, 1, 0, 0, -1, 0, 1

- new$R_1$ = old$R_1$ − $\frac{2}{3}$ new$R_3$
  i.e. 1, $\frac{-4}{4}$, $\frac{3}{4}$, 0, 1, 0, 5, 0, $\frac{-5}{4}$

Developing fourth simplex table.

| | | $C_j \rightarrow$ | 4 | 8 | 10 | 0 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | C | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | |
| $R_1$:0 | $S_1$ | 1 | -4 | 3 | 0 | 1 | 0 | $\boxed{5}$ | 0 | -5 | 1/5 = 0.2 |
| $R_2$:10 | C | 11 | 2 | 1 | 1 | 0 | 0 | -1 | 0 | 1 | 11/-1 = - |
| $R_3$:0 | $S_2$ | 15 | 6 | 1 | 0 | 0 | 1 | -4 | -1 | 4 | 15/-4 = |
| | $z_j$ | -110 | 20 | 10 | 10 | 0 | 0 | -10 | 0 | 10 | |
| | $z_j - c_j$ | | 16 | 2 | 0 | 0 | 0 | -10 | M | M+10 | |

In the above table -10 is the most negative in $Z_j - C_j$ row so $S_3$ column is key column and 0.2 is the minimum ratio so $R_1$ is the key row. Here, $S_1$ is outgoing variable and $S_3$ is incoming variable.

- new$R_1$ = old$R_1$/5

i.e $\frac{1}{5}, \frac{-4}{5}, \frac{3}{5}, 0, \frac{1}{5}, 0, 1, 0, -1$

- new$R_2$ = old$R_2$ + new$R_1$

i.e $\frac{56}{5}, \frac{6}{5}, \frac{8}{5}, 1, \frac{1}{5}, 0, 0, 0, 0$

- new$R_3$ = old$R_3$ + new $R_1$

i.e $\frac{79}{5}, \frac{14}{5}, \frac{17}{5}, 0, \frac{4}{5}, 1, 0, -1, 0$

| | | $C_j \rightarrow$ | 4 | 8 | 10 | 0 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | C | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | |
| $R_1$:0 | $S_3$ | 1/5 | -4/5 | 3/5 | 0 | 1/5 | 0 | 1 | 0 | -1 | |
| $R_2$:10 | C | 56/5 | 6/5 | 8/5 | 1 | 1/5 | 0 | 0 | 0 | 0 | |
| $R_3$:0 | $S_2$ | 79/5 | 14/5 | 17/5 | 0 | 4/5 | 1 | 0 | -1 | 0 | |
| | $z_j$ | 112 | 12 | 16 | 10 | 2 | 0 | 0 | 0 | 0 | |
| | $z_j - c_j$ | | 8 | 8 | 0 | 2 | 0 | 0 | M | M | |

$\therefore$ Max $z = 112$ at $C = \frac{56}{5}$, $S_1 = 0$, $S_2 = \frac{79}{5}$, $S_3 = 1/5$

# Five Special Cases [In rough and in Book]

1. Degeneracy

→ It always happens if there is the tie in the minimum ratio for the selection of key row. In this case we need to break the tie, either leaving $s_1$ or $s_2$ as both the variable are slack variable.

2. Alternative Optimum

→ This happens when any non basic variable column have zero coefficient in $z_j - c_j$ row in final table. This indicates that the variable with zero coefficient in $z_j - c_j$ row can be brought into the solution without changing the total cost or profit.

3. Unboundedness

→ If no any row can be selected as key row due to negative or infinite ratio, then the solution becomes unbounded solution.

4. Infeasible solution

→ If indicator $z_j - c_j$ row indicates that the solution is optimum but even there is one or more artificial variable in the basic variable column of final table, then the solution becomes infeasible or no solution.

5. Redundancy

→ A redundancy is one in which omission of some constraint of the LPP doesn't affect the problem and the optimal value of the problem.

Date
Page

2. Solve the following problem by simplex method.

a) Minimize $Z = 4x_1 + 6x_2$

Subject to

$x_1 + 2x_2 \geq 80$

$3x_1 + 2x_2 \geq 75$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

Min $Z = 4x_1 + 6x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

s.t

$x_1 + 2x_2 - S_1 + 0S_2 + A_1 + 0A_2 = 80$

$3x_1 + 2x_2 + 0S_1 - S_2 + 0A_1 + A_2 = 75$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 4 | 6 | 0 | 0 | M | M | Ratio: |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | $x_B$/key column |
| $R_1$: M | $A_1$ | 80 | $\boxed{1}$ | 2 | -1 | 0 | 1 | 0 | 80/1 = 80 |
| $R_2$: M | $A_2$ | 75 | $\boxed{3}$ | 2 | 0 | -1 | 0 | 1 | 75/3 = 25 |
| | $Z_j$ | 155M | 4M | 4M | -M | -M | M | M | |
| | $Z_j - C_j$ | | 4M-4 | 4M-6 | -M | -M | 0 | 0 | |

In the above table, 4M-4 is the most positive in $Z_j - C_j$ so $x_1$ is the key column and 25 is the minimum ratio so $R_2$ is the key row. Here, $A_2$ is outgoing variable and $x_1$ is incoming variable. Here, 3 is the pivot element.

New $R_2$ = Old $R_2$/3

i.e $25, 1, \frac{2}{3}, 0, -\frac{1}{3}, 0, \frac{1}{3}$

- $newR_1 = oldR_1 - newR_2$

i.e $55, 0, \frac{4}{3}, -1, \frac{1}{3}, 1, -\frac{1}{3}$

Developing second simplex table.

| | $c_j \longrightarrow$ | | 4 | 6 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 55 | 0 | $\boxed{4/3}$ | -1 | 1/3 | 1 | -1/3 | $55/4/3 = 41.25$ |
| $R_2$: 4 | $x_1$ | 25 | 1 | $\boxed{2/3}$ | 0 | -1/3 | 0 | 1/3 | $25/2/3 = 37.5$ |
| | $z_j$ | $55M+100$ | 4 | $\frac{4}{3}M+\frac{8}{3}$ | -M | $\frac{1}{3}M-\frac{4}{3}$ | M | $-\frac{1}{3}M+\frac{4}{3}$ | |
| | $z_j - c_j$ | | 0 | $\frac{4}{3}M-\frac{10}{3}$ | -M | $\frac{1}{3}M-\frac{4}{3}$ | 0 | $-\frac{4}{3}M+\frac{4}{3}$ | |

In the above table, $\frac{4}{3}M-\frac{10}{3}$ is the most positive in $z_j - c_j$ so $x_2$ is the key column and 37.5 is the minimum ratio so $R_2$ is the key row. Here, $x_1$ is the outgoing variable and $x_2$ is the incoming variable. Here, 1 is pivot element.

- $newR_2 = oldR_2 \times 3/2$
- $newR_1 = oldR_1 - \frac{4}{3} newR_2$

Developing third simplex table,

| | $c_j \longrightarrow$ | | 4 | 6 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 5 | -2 | 0 | -1 | $\boxed{1}$ | 1 | -1 | $5/1 = 5$ |
| $R_2$: 6 | $x_2$ | 75/2 | 3/2 | 1 | 0 | $\boxed{-1/2}$ | 0 | 1/2 | $75/2 / -1/2 = -$ |
| | $z_j$ | $5M+225$ | $-2M+9$ | 6 | -M | M - 3 | M | -M + 3 | |
| | $z_j - c_j$ | | $-2M+5$ | 0 | -M | M - 3 | 0 | -2M + 3 | |

In above table, M - 3 is the most positive in $z_j - c_j$ so $s_2$ is the key column and 5 is minimum ratio so $R_1$ is key row.

- $newR_1 = oldR_1$
- $newR_2 = oldR_2 + \frac{1}{2} newR_1$

| | $C_j \rightarrow$ | | 4 | 6 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $S_2$ | 5 | -2 | 0 | -1 | 1 | 1 | -1 | |
| $R_2$: 6 | $x_2$ | 40 | 1/2 | 1 | -1/2 | 0 | 1/2 | 0 | |
| | zj | 240 | 3 | 6 | -3 | 0 | 3 | 0 | |
| | $z_j - c_j$ | | -1 | 0 | -3 | 0 | 3-M | -M | |

Since, all $(z_j - c_j) \leq 0$, an optimal solution is obtained.
Min $z = 240$ at $x_1 = 0$ and $x_2 = 40$ Ans.

---

b. Minimize $Z = 20x_1 + 30x_2$
Subject to
$3x_1 + 6x_2 \geq 45$
$2x_1 + x_2 \geq 20$
$x_1, x_2 \geq 0$

Solution:
Formatting the problem in standard form of LPP, we get,
Min $z = 20x_1 + 30x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$
s.t
$3x_1 + 5x_2 - S_1 + 0S_2 + A_1 + 0A_2 = 45$
$2x_1 + x_2 + 0S_1 - S_2 + 0A_1 + A_2 = 20$
$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Developing initial simplex table, we get.

| | $C_j \rightarrow$ | | 20 | 30 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 45 | 3 | $\boxed{5}$ | -1 | 0 | 1 | 0 | 45/5 = 9 |
| $R_2$: M | $A_2$ | 20 | 2 | 1 | 0 | -1 | 0 | 1 | 20/1 = 20 |
| | $z_j$ | 65M | 5M | 6M | -M | -M | M | M | |
| | $z_j - c_j$ | | 5M-20 | 6M-30 | -M | -M | 0 | 0 | |

In above table, 6M-30 is the most positive in $z_j - c_j$ so $x_2$ is the key column and 9 is the minimum ratio so $R_1$ is the key row. Here, $A_1$ is outgoing variable and $x_2$ is incoming variable.

- new$R_1$ = old$R_1$/5

i.e $9, \frac{3}{5}, 1, \frac{-1}{5}, 0, \frac{1}{5}, 0$

- new$R_2$ = old$R_2$ - new$R_1$

i.e $20-9 = 11, 2-\frac{3}{5} = \frac{7}{5}, 0, \frac{1}{5}, -1, -\frac{1}{5}, 1$

Developing second simplex table, we get,

| | $C_j \rightarrow$ | | 20 | 30 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: 30 | $x_2$ | 9 | 3/5 | 1 | -1/5 | 0 | 1/5 | 0 | 9/3/5 = 15 |
| $R_2$: M | $A_2$ | 11 | 7/5 | 0 | 1/5 | -1 | -1/5 | 1 | 11/7/5 = 7.85 |
| | $z_j$ | 11M+270 | $\frac{7}{5}M+18$ | 30 | $\frac{1}{5}M-6$ | -M | $-\frac{1}{5}M+6$ | M | |
| | $z_j - c_j$ | | $\frac{7}{5}M-2$ | 0 | $\frac{1}{5}M-6$ | -M | $-\frac{6}{5}M+6$ | 0 | |

In above table, $\frac{7}{5}M-2$ is the most positive in $z_j - c_j$ so $x_1$ is the key column and 7.85 is the most minimum ratio so $R_2$ is the key row. Here, $A_2$ is outgoing variable and $x_1$ is incoming variable.

- $\text{new}R_2 = \text{old}R_2 \times \frac{5}{7}$

i.e $11 \times \frac{5}{7} = \frac{55}{7}$, $\frac{7}{5} \times \frac{5}{7} = 1$, $0$, $\frac{1}{5} \times \frac{5}{7} = \frac{1}{7}$, $\frac{-5}{7}$, $-\frac{1}{5} \times \frac{5}{7} = -\frac{1}{7}$, $\frac{5}{7}$

- $\text{new}R_1 = \text{old}R_1 - \frac{3}{5}\,\text{new}R_2$

i.e $9 - \frac{3}{5} \times \frac{55}{7} = \frac{30}{7}$, $\frac{3}{5} - \frac{3}{5} \times 1 = 0$, $1 - \frac{3}{5} \times 0 = 1$, $-\frac{1}{5} - \frac{3}{5} \times \frac{1}{7} = -\frac{2}{7}$,

$0 - \frac{3}{5} \times \left(-\frac{5}{7}\right) = \frac{3}{7}$, $\frac{1}{5} - \frac{3}{5} \times \left(-\frac{1}{7}\right) = \frac{2}{7}$, $0 - \frac{3}{5} \times \frac{5}{7} = -\frac{3}{7}$

Developing third simplex table, we get,

| | $c_j \rightarrow$ | | 20 | 30 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | |
| $R_1$: 30 | $x_2$ | $30/7$ | 0 | 1 | $-2/7$ | $3/7$ | $2/7$ | $-3/7$ | |
| $R_2$: 20 | $x_1$ | $55/7$ | 1 | 0 | $1/7$ | $-5/7$ | $-1/7$ | $5/7$ | |
| | $z_j$ | $\frac{2000}{7}$ | 20 | 30 | $-40/7$ | $-10/7$ | $40/7$ | $10/7$ | |
| | $z_j - c_j$ | | 0 | 0 | $-40/7$ | $-10/7$ | $40/7 - M$ | $10/7 - M$ | |

Since, all $(z_j - c_j) \leq 0$, an optimal solution is obtained.
Min $z = \frac{2000}{7}$ at $x_1 = \frac{55}{7}$ and $x_2 = \frac{30}{7}$

c) Min $Z = 20A + 10B$
Subject to
$A + 2B \leq 40$
$4A + 3B \geq 60$
$3A + B \geq 30$
$A, B \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

$$Min\ z = 20A + 10B + 0s_1 + 0s_2 + 0s_3 + MA_1 + MA_2$$

s.t.

$$A + 2B + s_1 + 0s_2 + 0s_3 + 0A_1 + 0A_2 = 40$$

$$4A + 3B + 0s_1 - s_2 + 0s_3 + A_1 + 0A_2 = 60$$

$$3A + B + 0s_1 + 0s_2 - s_3 + 0A_1 + A_2 = 30$$

$$A, B, C, s_1, s_2, s_3, A_1, A_2 \geq 0$$

Developing initial simplex table, we get,

| | $c_j \rightarrow$ | | 20 | 10 | 0 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | $s_1$ | $s_2$ | $s_3$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $s_1$ | 40 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 40/1 = 40 |
| $R_2$: M | $A_1$ | 60 | 4 | 3 | 0 | -1 | 0 | 1 | 0 | 60/4 = 15 |
| $R_3$: M | $A_2$ | 30 | 3 | 1 | 0 | 0 | -1 | 0 | 1 | 30/3 = 10 |
| | $z_j$ | 90M | 7M | 4M | 0 | -M | -M | M | M | |
| | $z_j - c_j$ | | 7M-20 | 4M-10 | 0 | -M | -M | 0 | 0 | |

In above table, 7M-20 is the most positive in $z_j - c_j$ so A is the key column and 10 is the minimum ratio so R3 is the key row. Here, A2 is outgoing variable and A is incoming variable.

- new R3 = old R3 / 3

i.e $10, 1, \frac{1}{3}, 0, 0, \frac{-1}{3}, 0, \frac{1}{3}$

- new R1 = old R1 - new R3

i.e $-9, 0, \frac{5}{3}, 1, 0, \frac{1}{3}, 0, -\frac{1}{3}$

- new R2 = old R2 - 4 new R3

i.e $20, 0, \frac{5}{3}, 0, -1, \frac{4}{3}, 0, -\frac{4}{3}$

Developing second simplex table, we get,

| CB | BV | $x_B$ | A | B | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| | $C_j \rightarrow$ | | 20 | 10 | 0 | 0 | 0 | M | M | |
| $R_1$: 0 | $S_1$ | -9 | 0 | $5/3$ | 1 | 0 | $1/3$ | 0 | $-1/3$ | $-9/5/3 = -$ |
| $R_2$: M | $A_1$ | 20 | 0 | $5/3$ | 0 | -1 | $4/3$ | 0 | $-4/3$ | 12 |
| $R_3$: 20 | A | 10 | 1 | $1/3$ | 0 | 0 | $-1/3$ | 0 | $1/3$ | 30 |
| | $z_j$ | 20M+200 | 20 | $\frac{5}{3}M+\frac{20}{3}$ | 0 | -M | $\frac{4}{3}M-\frac{20}{3}$ | 0 | $-\frac{4}{3}M+\frac{20}{3}$ | |
| | $z_j - c_j$ | | 0 | $\frac{5}{3}M-\frac{10}{3}$ | 0 | -M | $\frac{4}{3}M-\frac{20}{3}$ | -M | $-\frac{7}{3}M+\frac{20}{3}$ | |

In above table, $\frac{5M}{3}-\frac{10}{3}$ is the positive value so B is key column and 12 is minimum ratio so $R_2$ is key row.

- new $R_2$ = old $R_2 \times \frac{3}{5}$

i.e $12, 0, 1, 0, -\frac{3}{5}, \frac{-4}{5}, 0, -\frac{4}{5}$

- new $R_1$ = old $R_1 - \frac{5}{3}$ new $R_2$

i.e $-29, 0, 0, 1, 1, \frac{5}{3}, 0, 1$

- new $R_3$ = old $R_3 - \frac{1}{3}$ new $R_2$

i.e $6, 1, 0, 0, \frac{1}{5}, -\frac{1}{15}, 0, -\frac{1}{15}$

Developing third simplex table, we get,

| CB | BV | $x_B$ | A | B | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| | $C_j \rightarrow$ | | 20 | 10 | 0 | 0 | 0 | M | M | |
| $R_1$: 0 | $S_1$ | -29 | 0 | 0 | 1 | 1 | $5/3$ | 0 | 1 | |
| $R_2$: 10 | B | 12 | 0 | 1 | 0 | $-3/5$ | $-4/5$ | 0 | $-4/5$ | |
| $R_3$: 20 | A | 6 | 1 | 0 | 0 | $1/5$ | $-1/15$ | 0 | $-1/15$ | |
| | $z_j$ | 240 | 20 | 10 | 0 | -2 | $-\frac{28}{3}$ | 0 | $-28/3$ | |
| | $z_j - c_j$ | | 0 | 0 | 0 | -2 | $-\frac{28}{3}$ | -M | $-\frac{28}{3}-M$ | |

Since, all $(z_j - c_j) \leq 0$ so the optimal solution is obtained.
Min $z = 240$ at $A = 6$ and $B = 12$.

d. Min $z = x_1 + 2x_2 + 3x_3$

Subject to

$x_1 + x_2 + x_3 \geq 30$

$10x_1 + 15x_2 + 20x_3 \leq 600$

$5x_2 + 6x_3 \leq 120$

$x_1, x_2, x_3 \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

Min $z = x_1 + 2x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3 + MA_1$

s.t

$x_1 + x_2 + x_3 - s_1 + 0s_2 + 0s_3 + A_1 = 30$

$10x_1 + 15x_2 + 20x_3 + 0s_1 + s_2 + 0s_3 + 0A_1 = 600$

$0x_1 + 5x_2 + 6x_3 + 0s_1 + 0s_2 + s_3 + 0A_1 = 120$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 1 | 2 | 3 | 0 | 0 | 0 | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | |
| $R_1$: M | $A_1$ | 30 | 1 | 1 | 1 | -1 | 0 | 0 | 1 | 30/1 = 30 ← |
| $R_2$: 0 | $s_2$ | 600 | 10 | 15 | 20 | 0 | 1 | 0 | 0 | 600/10 = 60 |
| $R_3$: 0 | $s_3$ | 120 | 0 | 5 | 6 | 0 | 0 | 1 | 0 | 120/0 = ∞ |
| | $z_j$ | 30M | M | M | M | -M | 0 | 0 | M | |
| | $z_j - c_j$ | | M-1 ↑ | M-2 | M-3 | -M | 0 | 0 | 0 | |

- new $R_1$ = old $R_1$
- new $R_2$ = old $R_2$ − 10 new $R_1$
- new $R_3$ = old $R_3$

| | $C_j \rightarrow$ | | 1 | 2 | 3 | 0 | 0 | 0 | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | $A_1$ | |
| $R_1$: 1 | $x_1$ | 30 | 1 | 1 | 1 | -1 | 0 | 0 | 1 | |
| $R_2$: 0 | $s_2$ | 300 | 0 | 5 | 10 | 10 | 1 | 0 | -10 | |
| $R_3$: 0 | $s_3$ | 120 | 0 | 5 | 6 | 0 | 0 | 1 | 0 | |
| | $z_j$ | 30 | 1 | 1 | 1 | -1 | 0 | 0 | 1 | |
| | $z_j - c_j$ | | 0 | -1 | -2 | -1 | 0 | 0 | 1-M | |

∴ Min $z = 30$ at $x_1 = 30$ Ans.

c. Min $z = x_1 + 5x_2 + 6x_3$

s.t

$x_1 + x_2 + x_3 = 40$

$5x_1 + 6x_2 + 7x_3 = 250$

$x_1, x_2, x_3 \geq 0$

Solution:

Formatting the problem in standard form of LPP,

Min $z = x_1 + 5x_2 + 6x_3 + MA_1 + MA_2$

s.t

$x_1 + x_2 + x_3 + A_1 + 0A_2 = 40$

$5x_1 + 6x_2 + 7x_3 + 0A_1 + A_2 = 250$

$x_1, x_2, x_3, A_1, A_2 \geq 0$

Developing initial simplex table, we get,

| | $C_j \longrightarrow$ | | 1 | 5 | 6 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 40 | 1 | 1 | 1 | 1 | 0 | $40/1 = 40$ |
| $R_2$: M | $A_2$ | 250 | 5 | 6 | 7 | 0 | 1 | $250/7 = 35.7$ ← |
| | $Z_j$ | 290M | 6M | 7M | 8M | M | M | |
| | $Z_j - C_j$ | | 6M-1 | 7M-5 | 8M-6 | 0 | 0 | |
| | | | | | ↑ | | | |

- new $R_2$ = old $R_2/7$

i.e $\frac{250}{7}, \frac{5}{7}, \frac{6}{7}, 1, 0, \frac{1}{7}$

- new $R_1$ = old $R$ - new $R_2$

i.e $40 - \frac{250}{7} = \frac{30}{7}, \frac{2}{7}, \frac{1}{7}, 0, 1, \frac{-1}{7}$

| | $C_j \longrightarrow$ | | 1 | 5 | 6 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | $30/7$ | $2/7$ | $1/7$ | 0 | 1 | $-1/7$ | 15 ← |
| $R_2$: 6 | $x_3$ | $250/7$ | $5/7$ | $6/7$ | 1 | 0 | $1/7$ | 50 |
| | $z_j$ | $\frac{30}{7}M + \frac{1500}{7}$ | $\frac{2}{7}M + \frac{30}{7}$ | $\frac{1}{7}M + \frac{36}{7}$ | 6 | M | $-\frac{1}{7}M + \frac{6}{7}$ | |
| | $Z_j - C_j$ | | $\frac{2}{7}M + \frac{23}{7}$ | $\frac{1}{7}M + \frac{1}{7}$ | 0 | 0 | $-\frac{8}{7}M + \frac{6}{7}$ | |
| | | | ↑ | | | | | |

• new $R_1$ = old $R_1 \times \frac{7}{2}$

i.e $15, 1, \frac{1}{2}, 0, \frac{7}{2}, -\frac{1}{2}$

• new $R_2$ = old $R_2 - \frac{5}{7}$ new $R_1$

i.e $25, 0, \frac{1}{2}, 1, -\frac{5}{2}, \frac{1}{2}$

| | $C_j \rightarrow$ | | 1 | 5 | 6 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $x_3$ | $A_1$ | $A_2$ | |
| $R_1$: 1 | $x_1$ | 15 | 1 | 1/2 | 0 | 7/2 | -1/2 | |
| $R_2$: 6 | $x_3$ | 25 | 0 | 1/2 | 1 | -5/2 | 1/2 | |
| | $z_j$ | 165 | 1 | 7/2 | 6 | -23/2 | 5/2 | |
| | $z_j - c_j$ | | 0 | -3/2 | 0 | $-\frac{23}{2} - M$ | $\frac{5}{2} - M$ | |

Since all $(z_j - c_j) \leq 0$, the optimal solution is obtained
Min $z = 165$ at $x_1 = 15$, $x_3 = 25$ and $x_2 = 0$.

Date
Page

# Unit-3

## Transportation and Assignment Problem

### # North-West Corner Method (NWCM)

| Warehouse / Factory | P | Q | R | S | Supply |
|---|---|---|---|---|---|
| A | 19 [5] | 30 [2] | 50 | 10 | 7→2→0 |
| B | 70 | 30 [6] | 40 [3] | 60 | 9→3→0 |
| C | 40 | 8 | 70 [4] | 20 [14] | 18→14→0 |
| Demand | 5→0 | 8→6→0 | 7→4→0 | 14→0 | 34 |

Here, We have, $\Sigma D = 5+8+7+14 = 34$

$\Sigma S = 7+9+18 = 34$

We have to ensure whether the initial solution is feasible or not.

Since, the no. of occupied cell (allocated cell) = m+n-1 then our solution is found.

Here, m = no. of row

n = no. of column

Now,

no. of occupied cell = m+n-1

or, 6 = 3+4-1

or, 6 = 6

∴ no. of occupied cell = m+n-1

The total transportation cost of initial solution can be calculated as :

T.C = Σ unit cell cost × allocated quantity

$= 19\times5 + 30\times2 + 30\times6 + 40\times3 + 70\times4 + 20\times14$

$= 1015$ Ans.

# Least Cost Method

| Factory \ Warehouse | P | Q | R | S | Supply |
|---|---|---|---|---|---|
| A | 19 | 30 | 50 | 10 [7] | 7→0 |
| B | 70 [2] | 30 | 40 [7] | 60 | 9→2→0 |
| C | 40 [3] | 8 [8] | 70 | 20 [7] | 18→~~10~~→3→0 |
| Demand | 5→2→0 | 8→0 | 7→0 | 14→7→0 | 34 |

Here, We have, $\Sigma D = 5+8+7+14 = 34$

$\Sigma S = 7+9+18 = 54$

Since, $\Sigma D = \Sigma S$ so the given problem is balanced.
If it was not balanced then we have to add dummy row or dummy column.
We have to ensure whether the initial solution is feasible or not.
Since, the no. of occupied cell $= m+n-1$ then our solution is formed.

no. of occupied cell $= 6$

or, $m+n-1 = 6$

or, $3+4-1 = 6$

$\therefore 6 = 6$

The total transportation cost of initial solution can be calculated as

$TC = \Sigma$ unit cell cost × allocated quantity

$= 70\times2 + 40\times3 + 8\times8 + 40\times7 + 10\times7 + 20\times7$

$= 814$ Ans.

Note: cost diff [illegible]

# Vogel's Approximation Method (VAM)

| Warehouse / Factory | P | Q | R | S | Supply | Cost Difference I | II | III | IV | V |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 19 [5] | 30 | 50 | 10 [2] | 7→2→0 | 9 | 9 | 40 | 40 | – |
| B | 10 | 30 | 40 [7] | 60 [2] | 9→2→0 | 10 | 20 | 20 | 20 | 20 |
| C | 40 | 8 [8] | 10 | 20 [10] | 18→10 | 12 | 20 | 50 | – | – |
| Demand | $D_1=5\to0$ | $D_2=8\to0$ | $D_3=7\to0$ | $D_4=14\to4\to2\to0$ | | | | | | |
| Cost Difference I | 21 | 22↑ | 10 | 10 | | | | | | |
| II | 21↑ | – | 10 | 10 | | | | | | |
| III | – | – | 10 | 10 | | | | | | |
| IV | – | – | 10 | 50↑ | | | | | | |
| V | – | – | 40 | 60↑ | | | | | | |

Here, we have, $\sum D = 5+8+7+14 = 34$

$\sum S = 7+9+18 = 34$

Since, $\sum D = \sum S$, So given problem is balanced.

Note:

1. To find the cost difference in 1st row: Least element of 1st row - second least element of same row.
   To find the cost difference in 1st column: Least element of 1st column - second least element of same column.

2. Look the highest difference either in row or in column.
   Exceptional Case:
   - If the cost difference (below demand line) are found to be equal then we have to look least cost in above table to allocate the quantity. If the least element are also found to be equal then look maximum demand, if again demand value are found to be equal we can choose

anyone.

We have to ensure whether the initial solution is feasible or not.

Since, the no. of occupied cell = m+n-1 then our solution is found.

Here, no. of occupied cell = 6

or, $m+n-1 = 6$

or, $3+4-1 = 6$

$\therefore\ 6 = 6$

So, the initial solution is feasible or non degeneracy

Now,

$TC = \sum$ unit cell cost × allocated quantity

$= 19\times5 + 8\times8 + 40\times7 + 10\times2 + 60\times2 + 20\times10$

$= 779$ Ans.

Note:

If we want to reduce the cost from VAM method then there are other two methods inside the VAM method.

i) Stepping Stone Method
ii) MODI Method.

Test for optimality

Imp # Test of Optimality By MODI Method

| Warehouse / Factory | P | Q | R | S | Supply | Ri |
|---|---|---|---|---|---|---|
| A | 19 [5] | 30 | 50 | 10 [2] | 7 | $R_1 = 10$ |
| B | 70 | 30 + | 40 [7] | 60 − [2] | 9 | $R_2 = 60$ |
| C | 40 | 8 − [8] | 70 | 20 + [10] | 18 | $R_3 = 20$ |
| Demand | 5 | 8 | 7 | 14 | 34 | |
| Kj | $K_1 = 9$ | $K_2 = -12$ | $K_3 = -20$ | $K_4 = 0$ | | |

Date
Page

Here, we have,

no. of occupied cells = 6

or, $m+n-1 = 6$

or, $3+4-1 = 6$

$\therefore 6 = 6$

so, the solution is feasible.

To calculate the row value and column value, let us assume $k_4 = 0$. We have to assume that $R_i$ or $k_j$ element where most of the cells are allocated.

Note: If most allocated cells of $R_i$ or $k_j$ are ties we can choose anyone from them.

To calculate, row value and column value we have the formula:

$$C_{ij} = R_i + k_j$$

Note: Further calculation by this formula, we have to take a help of allocated cell [$C_{ij}$ should be allocated cell]

so,

$$C_{ij} = R_i + k_j$$

i.e $C_{14} = R_1 + k_4 \Rightarrow 10 = R_1 + 0 \Rightarrow R_1 = 10$

$C_{11} = R_1 + k_1 \Rightarrow 19 = 10 + k_1 \Rightarrow k_1 = 9$

$C_{24} = R_2 + k_4 \Rightarrow 60 = R_2 + 0 \Rightarrow R_2 = 60$

$C_{23} = R_2 + k_3 \Rightarrow 40 = 60 + k_3 \Rightarrow k_3 = -20$

$C_{32} = R_3 + k_2 \Rightarrow 8 = (-20) + k_2 \Rightarrow k_2 = -12$

$C_{34} = R_3 + k_4 \Rightarrow 20 = R_3 + 0 \Rightarrow R_3 = 20$

Now, we have to calculate the improvement index of each unoccupied cell by using the relation:

$$\Delta_{ij} = C_{ij} - (R_i + k_j)$$

i.e $\Delta_{12} = C_{12} - (R_1 + k_2) = 30 - (10 + (-12)) = 32$

$\Delta_{13} = C_{13} - (R_1 + k_3) = 50 - (10 + (-20)) = 60$

$\Delta_{21} = C_{21} - (R_2 + k_1) = 70 - (60 + 9) = 1$

$\Delta_{22} = C_{22} - (R_2 + k_2) = 30 - (60 + (-12)) = -18$

$\Delta_{31} = C_{31} - (R_3 + k_1) = 40 - (20 + 9) = 11$

$\Delta_{33} = C_{33} - (R_3 + k_3) = 70 - (20 \div 20) = 70$

Note:

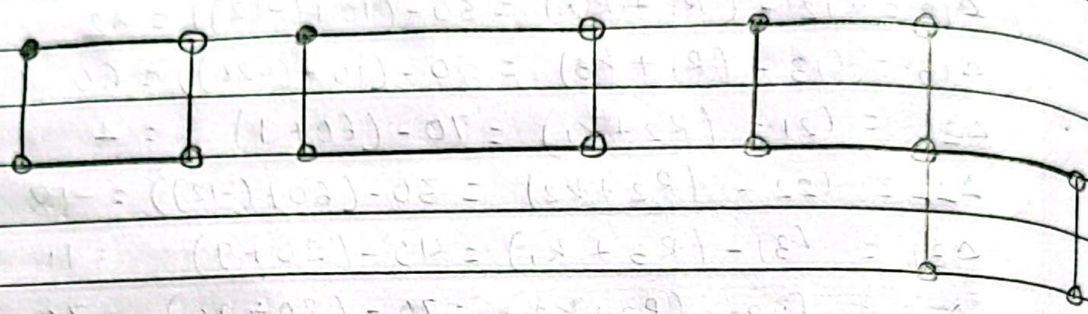
- The solution is optimal if all the improvement index are found to be greater than or equals to zero ($\Delta_{ij} \geq 0$).
- If $\Delta_{ij}$ are not greater than or equals to zero (found to be negative) then we have to select most negative value.

OR

- If $\Delta_{ij} \geq 0$, optimal solution is found so we can calculate the minimum total transportation cost.
- If $\Delta_{ij} < 0$, optimal solution is not found so we cannot calculate the minimum total transportation cost.
- The steps will further extend towards loop method by selecting most negative value from improvement index.

The above process already gets towards the loop. as improvement index $\Delta_{22} = -18 < 0$, the solution is not optimal. so a loop starts from the cell (2,2) and passes through the cells (2,4), (3,4) and (3,2).

# How to create a loop?

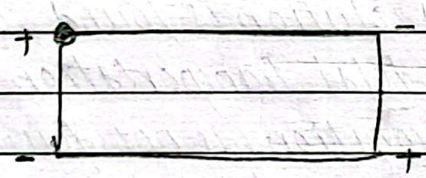


Here,

- ● = starting point
- o = Turning point (from allocated cell)

- Starting point inside the VAM table is the most negative values of the improvement index.
- Turning point should be allocated cell.
- Loop should be drawn either vertical or horizontal from the starting point.
- Loop starting point should be kept as positive of further turning should be kept at alternative sign.

i.e



- Select the minimum values from the negative corner (allocated quantity).

i.e Min (8,2) = 2

| Warehouse \ Factory | P | Q | R | S | Supply | Ri |
|---|---|---|---|---|---|---|
| A | 19 [5] | 30 | 50 | 10 [2] | 7 | $R_1 = 0$ |
| B | 70 | 30 [2] | 40 [7] | 60 | 9 | $R_2 = 32$ |
| C | 40 | 8 [6] | 70 | 20 [12] | 18 | $R_3 = 10$ |
| Demand | 5 | 8 | 7 | 14 | 34 | |
| $k_j$ | $k_1 = 19$ | $k_2 = -2$ | $k_3 = 8$ | $k_4 = 10$ | | |

Compute the minimum element (i.e 2) as linked with the sign of loop.

no. of occupied cells = 6

or, $m+n-1 = 6$

or, $3+4-1 = 6$

or, $7-1 = 6$

$\therefore 6 = 6$

So, the solution is still not degenerate.

Note: If the solution is degenerate then only we can calculate row values ($R_i$) and column values ($k_j$) and improvement index.

Calculating row value and column value, we get,

$$C_{ij} = R_i + k_j$$

i.e $C_{22} = R_2 + k_2 \Rightarrow 30 = R_2 - 2 \Rightarrow R_2 = 32$

$C_{23} = R_2 + k_3 \Rightarrow 40 = 32 + k_3 \Rightarrow k_3 = 8$

$C_{14} = R_1 + k_4 \Rightarrow 10 = 0 + k_4 \Rightarrow k_4 = 10$

$C_{11} = R_1 + k_2 \Rightarrow 19 = 0 + k_1 \Rightarrow k_1 = 19$

$C_{34} = R_3 + k_4 \Rightarrow 20 = R_3 + 10 \Rightarrow R_3 = 10$

$C_{32} = R_3 + k_2 \Rightarrow 8 = 10 + k_2 \Rightarrow k_2 = -2$

Now, Calculating Improvement Index for each unoccupied cell by

$$\Delta_{ij} = C_{ij} - (R_i + k_j)$$

i.e $\Delta_{12} = C_{12} - (R_1 + k_2) = 30 - (0 + (-2)) = 32$

$\Delta_{13} = C_{13} - (R_1 + k_3) = 50 - (0+8) = 42$

$\Delta_{21} = C_{21} - (R_2 + k_1) = 70 - (32+19) = 19$

$\Delta_{24} = C_{24} - (R_2 + k_4) = 60 - (32+10) = 18$

$\Delta_{31} = C_{31} - (R_3 + k_1) = 40 - (10+19) = 11$

$\Delta_{33} = C_{33} - (R_3 + k_3) = 70 - (10+8) = 52$

Every $\Delta_{ij} >, 0$. Hence optimal solution is obtained. If again we had found some negative element at $\Delta_{ij}$ then again we have to start loop by selecting most negative value from $\Delta_{ij}$ as the starting point of the loop.

Then,

Minimum Total Cost = $\Sigma$ cell cost × Quantity Allocated

$$= C_{11}X_{11} + C_{14}X_{14} + C_{22}X_{22} + C_{23}X_{23} + C_{32}$$
$$+ C_{34}X_{34}$$
$$= 19\times5 + 10\times2 + 30\times2 + 40\times7 + 8\times6 + 20\times12$$
$$= Rs743 \text{ Ans//.}$$

# Test of Optimality By Stepping Stone Method

| Warehouse / Factory | P | Q | R | S | Supply |
|---|---|---|---|---|---|
| A | 19 [5] | 30 | 50 | 10 [2] | 7 |
| B | 70 | 30 + | 40 [7] | 60 − [2] | 9 |
| C | 40 | 8 − [8] | 70 | 20 + [10] | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

We have,

no. of occupied cell = 6

or, m+n-1 = 6

or, 3+4-1 = 6

or, 6 = 6

So, the solution is feasible

To test the optimality select the unoccupied cell (A,Q) for which the loop is formed by joining the cell (A,S), (C,S) & (

and (A,Q). The improvement index for (A,Q) is 30-10+20-8 = 32

| Unoccupied cells | Loops | Improvement Index |
|---|---|---|
| (A,Q) | +(A,Q)-(A,S)+(C,S)-(C,Q) | 30-10+20-8 = 32 |
| (A,R) | +(A,R)-(A,S)+(B,S)-(B,R) | 50-10+60-40 = 60 |
| (B,P) | +(B,P)-(A,P)+(A,S)-(B,S) | 70-19+10-60 = 1 |
| (B,Q) | +(B,Q)-(B,S)+(C,S)-(C,Q) | 30-60+20-8 = -18 |
| (C,P) | +(C,P)-(A,P)+(A,S)-(C,S) | 40-19+10-20 = 11 |
| (C,R) | +(C,R)-(B,R)+(B,S)-(C,S) | 70-40+60-20 = 70 |

Here, there is a negative value -18 in the improvement index so a loop is formed.

Min negative values = min(8,2) = 2

| Warehouse / Factory | P | Q | R | S | Supply |
|---|---|---|---|---|---|
| A | 19 [5] | 30 | 50 | 10 [2] | 7 |
| B | 70 | 30 [2] | 40 [7] | 60 | 9 |
| C | 40 | 8 [6] | 70 | 20 [12] | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

Making improvement table, we get,

| Unoccupied cell | Loops | Improvement Index |
|---|---|---|
| (A,Q) | +(A,Q) − (A,S) + (C,S) − (C,Q) | 30 − 10 + 20 − 8 = 32 |
| (A,R) | +(A,P) − (A,S) + (C,S) − (C,Q) +(B,Q) − (B,R) | 50 − 10 + 20 − 8 + 30 − 40 = 42 |
| (B,P) | +(B,P) − (A,P) + (A,S) − (C,S) + (C,Q) − (B,Q) | 70 − 19 + 10 − 20 + 8 − 30 = 19 |
| (B,S) | +(B,S) − (C,S) + (C,Q) − (B,Q) | 60 − 20 + 8 − 30 = 18 |
| (C,P) | +(C,P) − (A,P) + (A,S) − (C,S) | 40 − 19 + 10 − 20 = 11 |
| (C,R) | +(C,R) − (B,R) + (B,Q) − (C,Q) | 70 − 40 + 30 − 8 = 52 |

Hence, all the improvement index are found to be ≥ 0 so our optimal solution is obtained.

Total Minimum Transportation Cost = Σ cell cost × Quantity Allocated

= 19×5 + 10×2 + 30×2 + 40×7 + 8×6 + 20×12

= Rs 743.

# Unit-2

2. Solve the following problem by simplex method.

f. Min $C = 200x + 800y$

Subject to

$x+y = 200$

$0 \leq x \leq 40$

$0 \leq y \geq 30$

Solution:

The given problem can be written as:

Min $C = 200x + 800y$

s.t

$x+y = 200$

$x \leq 40$

$y \geq 30$

$x \geq 0, y \geq 0$

The given equations can be written in standard form of LPP.

Min $C = 200x + 800y + 0S_1 + 0S_2 + MA_1 + MA_2$

s.t

$x+y+0S_1+0S_2+A_1+0A_2 = 200$

$x+0y+S_1+0S_2+0A_1+0A_2 = 40$

$0x+y+0S_1-S_2+0A_1+A_2 = 30$

$x, y, S_1, S_2, A_1, A_2 \geq 0$

Developing the initial simplex table, we get,

| | $C_j \longrightarrow$ | | 200 | 800 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | x | y | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 200 | 1 | 1 | 0 | 0 | 1 | 0 | 200/1 = 200 |
| $R_2$: 0 | $s_1$ | 40 | 1 | 0 | 1 | 0 | 0 | 0 | 40/0 = ∞ |
| $R_3$: M | $A_2$ | 30 | 0 | 1 | 0 | -1 | 0 | 1 | 30/1 = 30 |
| | $z_j$ | 230M | M | 2M | 0 | -M | M | M | |
| | $z_j - c_j$ | | M-200 | 2M-800 | 0 | -M | 0 | 0 | |

- new$R_3$ = old$R_3$
- new$R_2$ = old$R_2$
- new$R_1$ = old$R_1$ - new$R_3$
  i.e 170, 1, 0, 0, 1, 1, -1

Developing second simplex table, we get,

| | $C_j \longrightarrow$ | | 200 | 800 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | x | y | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 170 | 1 | 0 | 0 | 1 | 1 | -1 | 170/1 = 170 |
| $R_2$: 0 | $s_1$ | 40 | 1 | 0 | 1 | 0 | 0 | 0 | 40/1 = 40 |
| $R_3$: 800 | y | 30 | 0 | 1 | 0 | -1 | 0 | 1 | 30/0 = ∞ |
| | $z_j$ | 170M+24000 | M | 800 | 0 | M-800 | M | -M+800 | |
| | $z_j - c_j$ | | M-200 | 0 | 0 | M-800 | 0 | -2M+800 | |

- new$R_2$ = old$R_2$
- new$R_3$ = old$R_3$
- new$R_1$ = old$R_1$ - new$R_2$
  i.e 130, 0, 0, -1, 1, 1, -1

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 200 | 800 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | x | y | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: M | $A_1$ | 130 | 0 | 0 | -1 | $\boxed{1}$ | 1 | -1 | 130/1 = 130 ← |
| $R_2$: 200 | x | 40 | 1 | 0 | 1 | 0 | 0 | 0 | 40/0 = ∞ |
| $R_3$: 800 | y | 30 | 0 | 1 | 0 | -1 | 0 | 1 | 30/-1 = - |
| | $z_j$ | 130M+32000 | 200 | 800 | -M | M-800 | M | -M+800 | |
| | $z_j - c_j$ | | 0 | 0 | -M+200 | M-800 | 0 | -2M+800 | |

- new $R_1$ = old $R_1$
- new $R_2$ = old $R_2$
- new $R_3$ = old $R_3$ + new $R_1$

i.e 160, 0, 1, -1, 0, 1, 0

Developing fourth simplex table, we get,

| | $C_j \rightarrow$ | | 200 | 800 | 0 | 0 | M | M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | xB | x | y | $s_1$ | $s_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $s_2$ | 130 | 0 | 0 | -1 | 1 | 1 | -1 | |
| $R_2$: 200 | x | 40 | 1 | 0 | 1 | 0 | 0 | 0 | |
| $R_3$: 800 | y | 160 | 0 | 1 | -1 | 0 | 1 | 0 | |
| | $z_j$ | 136000 | 200 | 800 | -600 | 0 | 800 | 0 | |
| | $z_j - c_j$ | | 0 | 0 | -600 | 0 | 800-M | -M | |

Here, all $(z_j - c_j) \leq 0$, the optimal solution is obtained.

Min $Z = 136000$ at $x = 40, y = 160$.

3. Use simplex method to solve:

a) Max $Z = 4x_1 + 2x_2$

subject to

$x_1 - 2x_2 \geq 2$

$-M \times (-2) + 1 \times (-M)$

$2M - M$

$2x_1 + x_2 \geq 8$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

$\text{Max } Z = 4x_1 + 2x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

s.t

$x_1 - 2x_2 - S_1 + 0S_2 + A_1 + 0A_2 = 2$

$2x_1 + x_2 + 0S_1 - S_2 + 0A_1 + A_2 = 8$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

Developing initial simplex table, we get,

| | $c_j \rightarrow$ | | 4 | 2 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | B.V | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: -M | $A_1$ | 2 | $\boxed{1}$ | -2 | -1 | 0 | 1 | 0 | 2/1 = 2 ← |
| $R_2$: -M | $A_2$ | 8 | 2 | 1 | 0 | -1 | 0 | 1 | 8/2 = 4 |
| | $z_j$ | -10M | -3M | M | M | M | -M | -M | |
| | $z_j - c_j$ | | -3M-4 ↑ | M-2 | M | M | 0 | 0 | |

- new $R_1$ = old $R_1$
- new $R_2$ = old $R_2$ - 2 new $R_1$

i.e 4, 0, 5, 2, -1, -2, 1

Developing second simplex table,

| | $c_j \rightarrow$ | | 4 | 2 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 4 | $x_1$ | 2 | 1 | $\boxed{-2}$ | -1 | 0 | 1 | 0 | 2/-2 = - |
| $R_2$: -M | $A_2$ | 4 | 0 | $\boxed{5}$ | 2 | -1 | -2 | 1 | 4/5 ← |
| | $z_j$ | -4M+8 | 4 | -8-5M | -4-2M | M | 4+2M | -M | |
| | $z_j - c_j$ | | 0 | -10-5M | -4-2M | M | 4+3M | 0 | |

- new $R_2$ = old $R_2$ / 5
- new $R_1$ = old $R_1$ + 2 new $R_2$

Developing third simplex table, we get,

| | $C_j \rightarrow$ | | 4 | 2 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 4 | $x_1$ | 18/5 | 1 | 0 | -1/5 | -2/5 | 1/5 | 2/5 | - |
| $R_2$: 2 | $x_2$ | 4/5 | 0 | 1 | 2/5 | -1/5 | -2/5 | 1/5 | - |
| | $z_j$ | 16 | 4 | 2 | 0 | -2 | 0 | 2 | |
| | $z_j - c_j$ | | 0 | 0 | 0 | -2 | M | 2+M | |

Since, no row can be selected as both of them are negative the solution is unbounded.

---

6. Max $z = 10x_1 + 5x_2$

Subject to:

$x_1 - 2x_2 \leq 6$

$x_1 \leq 10$

$x_2 \geq 1$ & $x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP,

Max $z = 10x_1 + 5x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1$

s.t.

$x_1 - 2x_2 + S_1 + 0S_2 + 0S_3 + 0A_1 = 6$

$x_1 + 0x_2 + 0S_1 + S_2 + 0S_3 + 0A_1 = 10$

$0x_1 + x_2 + 0S_1 + 0S_2 - S_3 + A_1 = 1$

$x_1, x_2, S_1, S_2, S_3, A_1 \geq 0$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 10 | 5 | 0 | 0 | 0 | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | XB | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | |
| $R_1$: 0 | $S_1$ | 6 | 1 | $\boxed{-2}$ | 1 | 0 | 0 | 0 | $6/-2$ : - |
| $R_2$: 0 | $S_2$ | 10 | 1 | $\boxed{0}$ | 0 | 1 | 0 | 0 | $1/0 = \infty$ |
| $R_3$: -M | $A_1$ | 1 | 0 | $\boxed{1}$ | 0 | 0 | -1 | 1 | $0/1 = 0$ |
| | $Z_j$ | -M | 0 | -M | 0 | 0 | M | -M | |
| | $Z_j - C_j$ | | -10 | -M-5 | 0 | 0 | M | 0 | |

Here, the ratio are negative, infinite and zero so the solution is unbounded solution.

c) $Max\ Z = 6x_1 - 4x_2$

Subject to:

$2x_1 + 4x_2 \leq 4$

$4x_1 + 8x_2 \geq 16$

$x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP,

$Max\ Z = 6x_1 - 4x_2 + 0S_1 + 0S_2 - MA_1$

s.t.

$2x_1 + 4x_2 + S_1 + 0S_2 + 0A_1 = 4$

$4x_1 + 8x_2 + 0S_1 - S_2 + A_1 = 16$

Developing initial simplex table, we get,

| | $C_j \rightarrow$ | | 6 | -4 | 0 | 0 | -M | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | XB | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | |
| $R_1$: 0 | $S_1$ | 4 | 2 | $\boxed{4}$ | 1 | 0 | 0 | $4/4 = 1$ |
| $R_2$: -M | $A_1$ | 16 | 4 | $\boxed{8}$ | 0 | -1 | 1 | $16/8 = 2$ |
| | $Z_j$ | -16M | -4M | -8M | 0 | M | -M | |
| | $Z_j - C_j$ | | -4M-6 | -8M+4 | 0 | M | 0 | |

↑

- $newR_1 = oldR_1/4$
- $newR_2 = oldR_2 - 8\, newR_1$

| | $C_j \rightarrow$ | | 6 | -4 | 0 | 0 | -M | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ | |
| $R_1$: -4 | $x_2$ | 1 | 1/2 | 1 | 1/4 | 0 | 0 | 1/1/2 = 2 ← |
| $R_2$: -M | $A_1$ | 8 | 0 | 0 | -2 | -1 | 1 | 8/0 = 0 |
| | $z_j$ | -4-8M | -2 | -4 | -1+2M | M | -M | |
| | $z_j - c_j$ | | -8 ↑ | 0 | -1+2M | M | 0 | |

- $newR_1 = oldR_1 \times 2$
- $newR_2 = oldR_2$

| | $C_j \rightarrow$ | | 6 | -4 | 0 | 0 | -M |
|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $A_1$ |
| $R_1$: 6 | $x_1$ | 2 | 1 | 2 | 1/2 | 0 | 0 |
| $R_2$: -M | $A_1$ | 8 | 0 | 0 | -2 | -1 | 1 |
| | $z_j$ | 12-8M | 6 | 12 | 3+2M | M | -M |
| | $z_j - c_j$ | | 0 | 16 | 3+2M | M | 0 |

This solution is infeasible solution because there is a artificial variable even the optimal solution is obtained.

d. Max $Z = 12x_1 - 4x_2$

Subject to

$x_1 + 2x_2 \leq 2$

$2x_1 + 4x_2 \geq 8$

$x_1 = 6$ and $x_1, x_2 \geq 0$

Solution:

Formatting the problem in standard form of LPP

$$\text{Max } z = 12x_1 - 4x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

s.t.

$$x_1 + 2x_2 + S_1 + 0S_2 + 0A_1 + 0A_2 = 2$$

$$2x_1 + 4x_2 + 0S_1 - S_2 + A_1 + 0A_2 = 8$$

$$x_1 + 0x_2 + 0S_1 + 0S_2 + 0A_1 + A_2 = 6$$

Developing initial simplex table weget.

| | $C_j \rightarrow$ | | 12 | -4 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 0 | $S_1$ | 2 | 1 | 2 | 1 | 0 | 0 | 0 | $2/2 = 1 \leftarrow$ |
| $R_2$: -M | $A_1$ | 8 | 2 | 4 | 0 | -1 | 1 | 0 | $8/4 = 2$ |
| $R_3$: -M | $A_2$ | 6 | 1 | 0 | 0 | 0 | 0 | 1 | $6/0 = \infty$ |
| | $z_j$ | -14M | -3M | -4M | 0 | M | -M | -M | |
| | $z_j - c_j$ | | -3M-12 | -4M+4 ↑ | 0 | M | 0 | 0 | |

- $newR_1 = oldR_1 / 2$
- $newR_2 = oldR_2 - 4\,newR_1$
- $newR_3 = oldR_3$

| | $C_j \rightarrow$ | | 12 | -4 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: -4 | $x_2$ | 1 | 1/2 | 1 | 1/2 | 0 | 0 | 0 | $1/\tfrac{1}{2} = 2 \leftarrow$ |
| $R_2$: -M | $A_1$ | 4 | 0 | 0 | -2 | -1 | 1 | 0 | $4/0 = \infty$ |
| $R_3$: -M | $A_2$ | 6 | 1 | 0 | 0 | 0 | 0 | 1 | $6/1 = 6$ |
| | $z_j$ | -4-10M | -2-M | -4 | -2+2M | M | -M | -M | |
| | $z_j - c_j$ | | -14-M ↑ | 0 | -2+2M | M | 0 | 0 | |

- $newR_1 = oldR_1 \times 2$
- $newR_2 = oldR_2$
- $newR_3 = oldR_3 - newR_1$

| | | $C_j \rightarrow$ | 12 | -4 | 0 | 0 | -M | -M | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $A_1$ | $A_2$ | |
| $R_1$: 12 | $x_1$ | 2 | 1 | 2 | 1 | 0 | 0 | 0 | |
| $R_2$: -M | $A_1$ | 4 | 0 | 0 | -2 | -1 | 1 | 0 | |
| $R_3$: -M | $A_2$ | 4 | 0 | -2 | -1 | 0 | 0 | 1 | |
| | $z_j$ | 24-8M | 12 | 24+2M | 12+M | M | -M | -M | |
| | $z_j - c_j$ | | 0 | 28+2M | -M+12 | M | 0 | 0 | |

Since, there is an artificial variable even if the optimal solution is obtained so the solution is infeasible solution.

e. Min $z = 4A + 5B$

Subject to

$A + B \leq 2$

$2A + B \geq 20$

$A, B \geq 0$

Solution:

Formatting the problem in standard form of LPP, we get,

$\text{Min } z = 4A + 5B + 0S_1 + 0S_2 - MA_1$

s.t

$A + B + S_1 + 0S_2 + 0A_1 = 2$

$2A + B + 0S_1 - S_2 + A_1 = 20$

Developing initial simplex table,

| | | $C_j \rightarrow$ | 4 | 5 | 0 | 0 | -M | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | $S_1$ | $S_2$ | $A_1$ | |
| $R_1$: 0 | $S_1$ | 2 | 1 | 1 | 1 | 0 | 0 | 2/1 = 2 ← |
| $R_2$: -M | $A_1$ | 20 | 2 | 1 | 0 | -1 | 1 | 20/2 = 10 |
| | $z_j$ | -20M | -2M | -M | 0 | M | -M | |
| | $z_j - c_j$ | | -2M-4 ↑ | -M-5 | 0 | M | 0 | |

- $newR_1 = oldR_1$
- $newR_2 = oldR_2 - 2newR_1$

| | $C_j \rightarrow$ | | 4 | 5 | 0 | 0 | $-M$ | Ratio |
|---|---|---|---|---|---|---|---|---|
| CB | BV | $x_B$ | A | B | $S_1$ | $S_2$ | $A_1$ | |
| $R_1$: 4 | A | 2 | 1 | 1 | 1 | 0 | 0 | |
| $R_2$: $-M$ | $A_1$ | 16 | 0 | $-1$ | $-2$ | $-1$ | 1 | |
| | $z_j$ | $8-16M$ | 4 | $4+M$ | $4+2M$ | $M$ | $-M$ | |
| | $z_j - c_j$ | | 0 | $M-1$ | $2M+4$ | $M$ | 0 | |

Since, even optimal solution is obtained there exists an artificial variable so the given solution is infeasible solution.