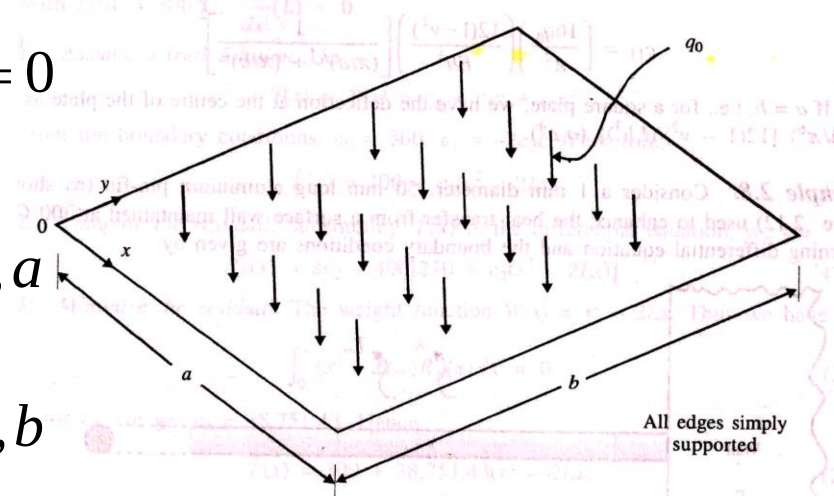
EXAMPLE 1: Consider a simply supported rectangular plate subjected to uniform load as shown in the figure. The governing differential equation is given by:

$$\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-q_0=0$$



the boundary conditions are:

$$w(0,y)=0=w(a,y) \qquad \frac{\partial^2 w}{\partial x^2}=0, \; for....x=0,a$$

$$w(x,0)=0=w(x,b) \qquad \frac{\partial^2 w}{\partial y^2}=0, \; for....y=0,b$$

- As the problem is a two dimensional problem and the governing differential equation may be seen in terms of both x and y, hence the trial solution to be assumed must contain both the functions of x and y.
- Let us choose a one term trigonometric trail solution to the above differential equation:

$$w(x,y)\approx \hat{w}(x,y)=C_{11}\sin(\pi x/a)\sin(\pi y/b)$$

- It may be observed that the assumed trial solution is satisfying all given boundary conditions.
- The assumed trial solution may be written in terms of the general form:

$$u\approx\hat{u}=\phi+\sum_{i=1}^{n}C_i N_i$$

$$w(x,y)\approx \hat{w}(x,y)=\underset{\phi}{\underset{\downarrow}{0}}+\underset{C_1}{\underset{\downarrow}{C_{11}}}\underbrace{\sin(\pi x/a)\sin(\pi y/b)}_{N_1}$$

- Substituting the Trial solution into the governing differential equation will lead domain residual indicating the amount of error in the assumed Trial solution.

$$R_d = \frac{Eh^3}{12(1-\nu^2)} C_{11}[(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2] \sin(\pi x/a)\sin(\pi y/b) - q_0$$

- To minimize the domain residual in an overall sense, the weighted integral of the Residual is equated to zero. The weighting function will be:

$$W_1 = N_1 = \sin(\pi x/a)\sin(\pi y/b)$$

$$\int_0^a \int_0^b W_1 R_d (dx dy) = 0$$

$$\int_0^a \int_0^b \{\sin(\pi x/a)\sin(\pi y/b)\}\{\frac{Eh^3}{12(1-\nu^2)} C_{11}[(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2] \sin(\pi x/a)\sin(\pi y/b) - q_0\} dx dy = 0$$

- Solving for C_{11} we get: $C_{11} = (\frac{16q_0}{\pi^2})(\frac{12(1-\nu^2)}{Eh^3})[\frac{1}{(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2}]^2$

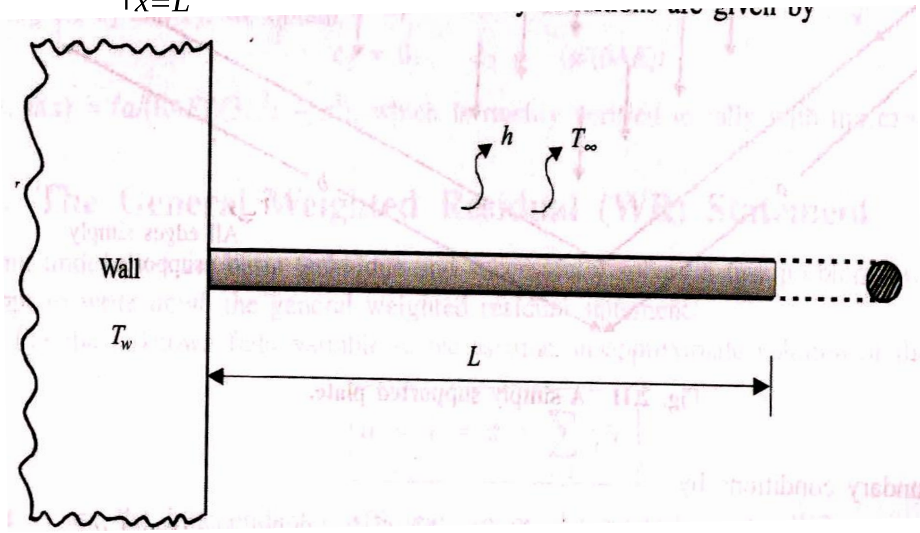
- The trial solution becomes:

$$w(x,y) \approx \widehat{w}(x,y) = (\frac{16q_0}{\pi^2})(\frac{12(1-\nu^2)}{Eh^3})[\frac{1}{(\frac{\pi}{a})^2 + (\frac{\pi}{b})^2}]^2 \sin(\pi x/a)\sin(\pi y/b)$$

ASSIGNMENT-2: Consider a 1mm diameter, 50 mm long aluminium pin-fin used to enhance the heat transfer from a surface wall maintained at 300^0C. The governing differential equation and the boundary conditions are given by:

$$k\frac{d^2T}{dx^2} = \frac{Ph}{A_c}(T - T_0) \qquad T(0) = T_w = 300^0 C \qquad \left.\frac{dT}{dx}\right|_{x=L} = 0$$

where: k = coefficient of thermal conductivity, P = Perimeter, A_c= cross sectional area, h = convective heat transfer coefficient, T_w= wall temperature, T_0= Ambient temperature.



Considering, k = 200 W/m/^{0}C for aluminium, h = 20 W/m^2 ^{0}C, T_0 = 30^0C, estimate the temperature distribution in the fin using the Galerkin Weighted Residual Method through use of QUADRATIC, CUBIC, and QUARTIC Trial Solutions.

$$T(x) = T_\infty + \left(\frac{T_w - T_\infty}{\cosh(mL)}\right)\cosh[m(L-x)], where...m^2 = \frac{hP}{kA_c}$$ ← **Exact Solution**

$$T(x) \approx \hat{T}(x) = C_0 + C_1 x + C_2 x^2$$ ← **Quadratic Trial Solution**

$$T(x) \approx \hat{T}(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3$$ ← **Cubic Trial Solution**

$$T(x) \approx \hat{T}(x) = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4$$ ← **Quartic**