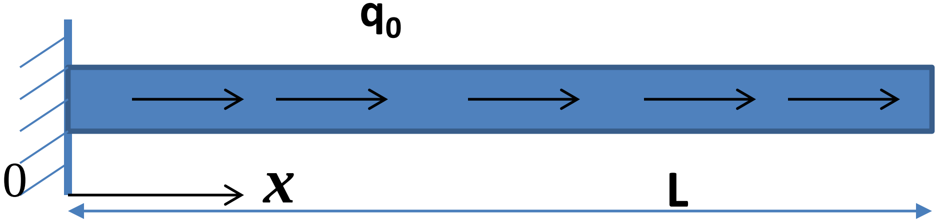
RESIDUAL MINIMIZATION METHOD

EXAMPLE 1: A uniform rod has been subjected to uniform axial load as shown in Figure. Find out an appropriate solution to this problem by RMM. The deformation of the bar (u) is governed by the governing differential equation:

$$AE\frac{d^2u}{dx^2}+q_o=0$$



With the boundary conditions: $u(0)=0, \left.\frac{du}{dx}\right|_{x=L}=0$

Step-1: (Assume the Trial Solution satisfying the boundary conditions)

- Let the trial or guess solution to the above differential equation be:

$$u(x)\cong\hat{u}(x)=C_0+C_1x+C_2x^2$$

Where the constants C_0, C_1, C_2 are yet to be determined in the trial solution so that the trial solution satisfies the given boundary conditions.

- Considering the first boundary condition: $u(0)=0$

$$u(0)\cong\hat{u}(0)\Rightarrow C_0+C_1x+C_2x^2=0\Rightarrow C_0=0$$

- Considering the second boundary condition: $\left.\frac{du}{dx}\right|_{x=L}=0$

$$\left.\frac{du(x)}{dx}\right|_{x=L}\cong\left.\frac{d\hat{u}(x)}{dx}\right|_{x=L}=0\Rightarrow C_1+2C_2L=0\Rightarrow C_1=-2C_2L$$

- So the final Trial solution satisfying the boundary conditions is: $\hat{u}(x)=C_2\left(x^2-2Lx\right)$

Since the trial solution contains only one free parameter C_2, it is often referred as a One parameter solution

Step-2: (To find the Domain Residual)

- Substituting the Trial solution into the governing differential equation will lead domain residual indicating the amount of error in the assumed Trial solution.

$$AE\frac{d^2u(x)}{dx^2}+q_o=0 \qquad u(x)\cong\widehat{u}(x)=C_2\left(x^2-2Lx\right)$$

$$AE\frac{d^2\widehat{u}(x)}{dx^2}+q_o\neq 0$$ [As $\widehat{u}(x)$ is not the exact solution of the D.E]

$$R_d=AE\frac{d^2\widehat{u}(x)}{dx^2}+q_o=AE(2C_2)+q_o$$

Step-3: (To minimize the Domain Residual)

- The Domain Residual indicates the error through out the domain (Ω) of the given differential equation. So in order to approximate the Trial Solution with the Exact Solution, the Domain Residual is tried to be minimized.

$$R_d=AE\frac{d^2\widehat{u}(x)}{dx^2}+q_o=AE(2C_2)+q_o=0 \quad \Rightarrow C_2=\frac{-q_o}{2AE}$$

- So the final Trail Solution to the Governing Differential Equation becomes:

$$u(x)\cong\widehat{u}(x)=\frac{q_0}{2AE}\left(2Lx-x^2\right)$$

- **For this simple example, as we could make the Domain Residual identically zero every where in the domain (Ω) , our final Trial Solution matches with the Exact Solution.**