



governing equation

for the deflection :  $EI \frac{\partial^4 v}{\partial x^4} - q_0 = 0$

Boundary conditions - (i)  $v(0) = 0$

(ii)  $\frac{\partial v}{\partial x} \Big|_{x=0} = 0$

(iii)  $\frac{\partial^2 v}{\partial x^2} \Big|_{x=L} = 0$

(iv)  $\frac{\partial^3 v}{\partial x^3} \Big|_{x=L} = 0$

Let the deflection be  $\hat{v}(x)$ , it is not the exact solution.

assume  $v(x) \approx \hat{v}(x)$   
 $\uparrow$   
 exact

we assume that deflection be a polynomial of degree four.

$$\hat{v}(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$$

①

As Now by applying boundary cond<sup>n</sup> we can find  $c_0, c_1, c_2, c_3, c_4$ .

$$v(0) \approx \hat{v}(0) = c_0 = 0$$

$$\therefore c_0 = 0$$

$$\left. \frac{\partial \hat{v}}{\partial x} \right|_{x=0} = \left. \frac{\partial \hat{v}}{\partial x} \right|_{x=0} = 0$$

$$\frac{\partial \hat{v}(x)}{\partial x} = C_1 + 2C_2x + 3C_3x^2 + 4C_4x^3$$

$$\left. \frac{\partial \hat{v}}{\partial x} \right|_{x=0} = C_1 = 0$$

$$\boxed{C_1 = 0}$$

$$\left. \frac{\partial^2 \hat{v}}{\partial x^2} \right|_{x=L} = \left. \frac{\partial^2 \hat{v}}{\partial x^2} \right|_{x=L} = 0$$

$$\left. \frac{\partial^2 \hat{v}}{\partial x^2} \right|_{x=L} = 2C_2 + 6C_3L + 12C_4L^2 = 0$$

— (2)

$$\left. \frac{\partial^3 \hat{v}}{\partial x^3} \right|_{x=L} = 6C_3 + 12C_4L = 0$$

$$C_3 = -4C_4L \quad \text{— (3)}$$

put (3) in (2) we get,

$$\boxed{C_2 = 6C_4L^2}$$

putting all the coefficient value in eq (1) we get



$$\hat{v}(x) = 6C_4 L^2 x^2 - 4C_4 L x^3 + C_4 x^4$$

$$\boxed{\hat{v}(x) = C_4 x^2 (6L^2 - 4Lx + x^2)}$$

— (4)

Now this must satisfy the governing eq<sup>n</sup>. But as this is not the exact solution it must incur some residual error

$$EI \frac{\partial^4 \hat{v}(x)}{\partial x^4} - q_0 = R_e$$

↑  
Residual error

$$EI (C_4) - q_0 = R_e$$

We want our solution close to exact sol<sup>n</sup> in order to that  $R_e \rightarrow 0$

Here  $R_e$  can be absolute '0'

$$\therefore EI (C_4) - q_0 = 0$$

$$C_4 = \frac{q_0}{EI} \quad \text{— (5)}$$

putting this in eq (4)

we get

$$\boxed{\hat{v}(x) = \frac{q_0}{EI} x^2 (6L^2 - 4Lx + x^2) = v(x)}$$