A Comparative Study of Asymptotically-Optimal Sampling-Based Path Planning Methods

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Abstract: In this report, we propose a comparative study between three asymptotically-optimal sampling-based path planning algorithms: Fast-marching trees (FMT*), batch informed trees (BIT*), and neural rapidly random-exploring trees (NRRT*). The individual algorithms will be implemented, and their simulated performances in terms of execution time and path costs will be observed on environments of various types and complexities. The performance with varying sample counts will also be observed, and a qualitative comparison of the optimal paths achieved from each algorithm for a subset of the environments will be provided to demonstrate how the planners handle obstacles.

Keywords: Sampling-based planning, asymptotically optimal planning

1 Introduction

Path planning is a search problem where the goal is to find a sequence of actions that will lead to an obstacle-free optimal path from a start state to a target state. Grid-based heuristic search algorithms [1] have been proposed in the past that guarantee finding optimal paths that exists. However, these techniques do not scale well to higher dimensional problems as they require discretization of the state space. On the other hand, sampling-based path planning algorithms [2] can solve high-dimensional path planning problems more efficiently. However, these algorithms only converge to optimal paths asymptotically, and in practice may compute sub-optimal paths. Many attempts have been made to make improvements to sampling-based methods [3][4][5], using techniques such as heuristic guided searches, or the reduction of the sampling space. The goal of this project is to implement three sampling-based path planning algorithms, and compare their performances through simulations.

2 Problem

We can formulate the path planning problem by defining the state space and the cost function similar to [2]. Let $\mathcal{X} \in \mathbb{R}^d$, $\mathcal{X}_{free}$, and $\mathcal{X}_{obs}$ be the state space, free space, and the obstacle space respectively where $\mathcal{X}_{obs} \subset \mathcal{X}$ and $\mathcal{X}_{free} = \mathcal{X} \setminus \mathcal{X}_{obs}$. Let $x_{init} \in \mathcal{X}_{free}$, $x_{goal} \in \mathcal{X}_{free}$ be the start and end states respectively. The goal of the path planning problem is to determine a path σ : [0, 1] $\mapsto \mathcal{X}_{free}$ such that $\sigma(0) = x_{init}$, $\sigma(1) = \mathcal{G}(x_{goal})$, where the acceptable goal region is defined as $\mathcal{G}(x_{goal}) = \{x \in \mathcal{X} \mid x - x_{goal} < r\}$. Then we can define the path planning problem as minimizing the following cost function

$$\sigma^* = \underset{\sigma \in \Sigma}{\operatorname{argmin}}\, c(\sigma) \tag{1}$$
$$s.t.\ \ \sigma(0) = x_{init},\ \sigma(1) = \mathcal{G}(x_{goal}),\ \sigma(t)\ \mathcal{X}_{free},\ \forall t \in [0, 1]$$

where Σ is the set of feasible paths and $c(\sigma)$ is the cost of a single path σ.

3 Related Work

Rapidly random-exploring tree (RRT) [6] is a well studied algorithm, and many variants have been proposed to improve its performance. RRT* [2] optimizes the tree in each iteration, allowing the planner to converge to an optimal solution asymptotically. Informed RRT* [3] reduces the number of iterations to convergence from RRT* by reducing the sampling space once a sub-optimal path is found. Similar work has been done to extended Probabilistic roadmap algorithm (PRM) as PRM* [2] that converges to an asymptotically optimal solution. However, these approaches do not utilize the ordered search available to grid-based planners. RA* [5] and SBA* [4] directly extend A* to the continuous domain by sampling near heuristically selected vertices of the graph. While this is a good approach for cases with no obstacles, it must address the problem of local minima for cases with obstacles.

4 Proposed Method

In this project, we propose a comparison study of three asymptotically-optimal sampling-based path planning algorithms: Fast-marching trees (FMT*), batch informed trees (BIT*), and neural RRT (NRRT*) [7]. The following section will briefly describe each of these algorithms.

4.1 Fast-Marching Trees

FMT* incorporates the advantages from both single and multiple query sampling-based algorithms in that it eliminates the greediness in RRT* by creating connections nearly optimally instead of trying to find the exactly optimal steering direction. However, in finding the optimal path, it grows a tree (as in the case of RRT*) instead of creating a bi-directional graph employed in the PRM* algorithm. The tree grows outwards from the source and performs "lazy" collision-checks which is where the algorithm trades convergence rate with optimality. It is however shown that as the number of sampling points increases ($N \to \infty$), the number of sub-optimal nodes in the path become vanishingly small ($x_{subopt} \in Path(\mathcal{X}_{goal}, T = (V_{open} \bigcup V_{closed}, E)) \to 0$). The performance benefits of FMT* are observed in higher dimensions (e.g. 6DoF Robotic Manipulators) where performing collision-checks is costly.

4.2 Batch Informed Trees

BIT* is a sampling based approach to path planning that utilizes the ordered nature of grid based approaches with the goal of creating a path planning algorithm that is both scalable and efficient. To do this, BIT* samples in batches and performs an ordered search on the batches [8]. In other words, the algorithm samples an entire batch randomly but uses a heuristic to determine which sample in the batch to process next [8]. Furthermore, similar to [3], once a sub-optimal solution is found, it reduces the region that is sampled to focus on the sub-problem that contains a better solution [8]. In this way, it maintains anytime resolution: finding a sub-optimal solution quickly, and using this sub-optimal solution to converge asymptotically towards the optimal solution [8].

4.3 Neural RRT*

Neural RRT* is a non-uniform sampling-based path planning algorithm that aims to take advantage of the optimal path costs of the A* algorithm as well as the asymptotic optimality of the RRT* algorithm. Instead of sampling from the state space using just a uniform distribution, a non-uniform distribution learned by a convolutional neural network (CNN) trained on optimal paths generated from the A* algorithm is used to achieve non-uniform sampling. Unlike many of the RRT variant algorithms, the NRRT* algorithm does not depend on any human-designed heuristics to improve its performance. It also maintains the probabilistic completeness and the asymptotic optimality of the

RRT* algorithm. Through simulations, it has been shown to perform superior compared to RRT* and IRRT* [7].

5 Proposed Evaluation

We will implement the individual algorithms in python as per the discussions and directives mentioned in the individual papers. In order to make direct comparisons between optimal paths returned by the different algorithms, they will be evaluated on a common set of cluttered environments of various types. We will measure 2 metrics as part of running the simulation: Optimal Path Cost ($\mathcal{J}$) and Execution Time ($\mathcal{T}$), where ET is calculated based on the convergence criteria for each algorithm. We evaluate the cost ($\mathcal{J}$) as follows: For a set of N nodes in the free space $\mathcal{X}_{free}$, let $\mathcal{P} \subset \mathcal{X}_{free}$ be the set of nodes along the optimal path as computed by the path planning algorithm $\{\mathcal{X}_{free}, x_{init}, \mathcal{G}(x_{goal})\}$; Then the cost $\mathcal{J}$ is defined as:

$$\mathcal{J} = \sum_{i=1}^{N} ||x_i - Parent(x_i)||_2, \text{ where } x_i \in \mathcal{P} \tag{2}$$

An additional comparison of the variation of cost ($\mathcal{J}$) and ET ($\mathcal{T}$) with sample count will be completed, with 10 different sample counts in the range $[10, 10000]$. Finally, we will also provide a qualitative comparison of the optimal paths resulting from each algorithm for the same set of environments highlighting their behaviour around corners and near obstacles. If time permits, we will also be comparing costs for obstacles in higher dimensions using simulations in OMPL.

References

[1] P. E. Hart, N. J. Nilsson, and B. Raphael. A formal basis for the heuristic determination of minimum cost paths. *IEEE Transactions on Systems Science and Cybernetics*, 4(2):100–107, 1968. doi:10.1109/TSSC.1968.300136.

[2] S. Karaman and E. Frazzoli. Sampling-based algorithms for optimal motion planning. *CoRR*, abs/1105.1186, 2011.

[3] J. D. Gammell, S. S. Srinivasa, and T. D. Barfoot. Informed rrt: Optimal sampling-based path planning focused via direct sampling of an admissible ellipsoidal heuristic. In *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 2997–3004, 2014. doi:10.1109/IROS.2014.6942976.

[4] S. M. Persson and I. Sharf. Sampling-based a* algorithm for robot path-planning. *The International Journal of Robotics Research*, 33(13):1683–1708, 2014.

[5] R. Diankov and J. Kuffner. Randomized statistical path planning. In *2007 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pages 1–6, 10 2007. doi:10.1109/IROS.2007.4399557.

[6] S. M. LaValle. Rapidly-exploring random trees : a new tool for path planning. *The annual research report*, 1998.

[7] J. Wang, W. Chi, C. Li, C. Wang, and M. Q.-H. Meng. Neural rrt*: Learning-based optimal path planning. *IEEE Transactions on Automation Science and Engineering*, 17(4):1748–1758, 2020.

[8] J. D. Gammell, S. S. Srinivasa, and T. D. Barfoot. Bit*: Batch informed trees for optimal sampling-based planning via dynamic programming on implicit random geometric graphs. *CoRR*, abs/1405.5848, 2014.