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AI24BTEC11027 - R Sumanth

1) Let $f(x+iy) = u(x,y) + iv(x,y)$ be an analytic defined on the complex plane satisfying $2u^2 + 3v^2 = 1$. then
   a) f is a constant
   b) $f(z) = kz$ some nonzero real number k
   c) $u(x,y) = \frac{\cos(x+y)}{\sqrt{2}}$
   d) $v(x,y) = \frac{\sin(x-y)}{\sqrt{3}}$

2) The value of $\oint_c (xy^2 + 2x)dx + (x^2y + 4x)dy$ along the circle $C : x^2 + y^2 = 4$ in the anticlockwise direction is
   a) -16π
   b) -4π
   c) 4π
   d) 16π

3) The volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 2$ and whose top lies in the plane $z = 5 - x - y$ is
   a) 2
   b) 4
   c) 6
   d) 10

4) The general solution of $x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2)$ is
   a) $F(x^2 + y^2 + z^2, xyz) = 0$
   b) $F(x^2 + y^2 - Z^2, xyz) = 0$
   c) $F(x^2 - y^2 + Z^2, xyz) = 0$
   d) $F(-x^2 + y^2 + Z^2, xyz) = 0$

5) Choose a point uniformly distributed at random on the disc $x^2 + y^2 \leq 1$. Let the random variable X denote the distance of this point from the center of the disc. Then the variance of X is
   a) $\frac{1}{16}$
   b) $\frac{1}{17}$
   c) $\frac{1}{18}$
   d) $\frac{1}{19}$

6) If Runge-kutta method of order 4 is used to solve the differential equation $\frac{dy}{dx} = f(x), y(0) = 0$ in the interval $[0.h]$ with step size h, then
   a) $y(h) = \frac{h}{6}[f(0) + 4f(h/2) + f(h)]$
   b) $y(h) = \frac{h}{6}[f(0) + 4f(h)]$
   c) $y(h) = \frac{h}{2}[f(0) + 4f(h)]$
   d) $y(h) = \frac{h}{6}[f(0) + 2f(h/2) + f(h)]$

7) If a polynomial of degree three interpolates a function $f(x)$ at the points $(0,3),(1,13),(3,99)$ and $(4,187)$, then $f(2)$ is
   a) 20
   b) 36
   c) 43
   d) 58

8) Let $f : R \to R$ be defined by $f(x) = x^2$ for $-\pi \le x \le \pi$ and $f(x+2\pi) = f(x)$. The Fourier series of f in $[-\pi, \pi]$ is
   a) $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$
   b) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$
   c) $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$
   d) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$

9) Let $f : R \to R$ be defined by $f(x) = x^2$ for $-\pi \le x \le \pi$ and $f(x+2\pi) = f(x)$. The sum of the absolute value of the Fourier coefficients of f is
   a) $\frac{\pi^2}{6}$
   b) $\frac{\pi^2}{3}$
   c) $\frac{2\pi^2}{3}$
   d) π^2

10) Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$ be a solution of the differential equation $\frac{d^2y}{dx^2} + xy = 0$. The value of a_{11} is
   a) 0
   b) 1
   c) 2
   d) 3

11) Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$ be a solution of the differential equation $\frac{d^2y}{dx^2} + xy = 0$. The value of $a_1 1$ is The solution of the differential equation given above satisfying $y(0) = 1$ and $y'(0) = 0$ is
   a) $y(x) = 1 + \frac{1}{2.3}x^2 - \frac{1}{2.3.5.6}x^4 + \frac{1}{2.3.5.6.8.9}x^6 - \ldots.$
   b) $y(x) = 1 - \frac{1}{2.3}x^2 + \frac{1}{2.3.5.6}x^4 - \frac{1}{2.3.5.6.8.9}x^6 + \ldots.$
   c) $y(x) = 1 + \frac{1}{2.3}x^3 - \frac{1}{2.3.5.6}x^6 + \frac{1}{2.3.5.6.8.9}x^9 - \ldots.$
   d) $y(x) = 1 - \frac{1}{2.3}x^3 - \frac{1}{2.3.5.6}x^6 - \frac{1}{2.3.5.6.8.9}x^9 + \ldots.$

12) The potential $u(x,y)$ satisfies the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square $0 \le x \le \pi, 0 \le y \le \pi$. Three of the edges $x = 0$ and $y = 0$ of the square are kept at zero potential and the edge $y = \pi$ is kept at nonzero potential. the potential $u(x,y)$ is given by
   a) $u(x,y) = \sum_{n=1}^{\infty} A_n \cos hnx \sin ny$
   b) $u(x,y) = \sum_{n=1}^{\infty} A_n \sin nx \cosh ny$
   c) $u(x,y) = \sum_{n=1}^{\infty} A_n \sin hnx \sin ny$
   d) $u(x,y) = \sum_{n=1}^{\infty} A_n \sin nx \sin hny$

13) The potential $u(x,y)$ satisfies the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square $0 \le x \le \pi, 0 \le y \le \pi$. Three of the edges $x = 0$ and $y = 0$ of the square are kept at zero potential and the edge $y = \pi$ is kept at nonzero potential. If the edge $y = \pi$ is kept at the potential $\sin x$, then the potential $u(x,y)$ is given by
   a) $u(x,y) = \sum_{n=1}^{\infty} \frac{\sin nx \sin hny}{\sin hn\pi}$

b) $u(x,y) = \frac{\sin x \sin hy}{\sin h\pi}$

c) $u(x,y) = \frac{\sin x \cos hy}{\cos h\pi}$

d) $u(x,y) = \sum_{n=1}^{\infty} \frac{\cos hnx \sin ny}{\cos hn\pi}$