2. Binomial Random Variable:-

↳ Its a Collection of Bernouli R.V

$$\boxed{P(X=i) = {}^{n}C_{i}\, p^{i} (1-p)^{n-i}}$$

$i \to$ no. of success
$n \to$ no. of trails
$p \to$ Probability of Success
$1-p \to$ probability of failure.

Ex:-

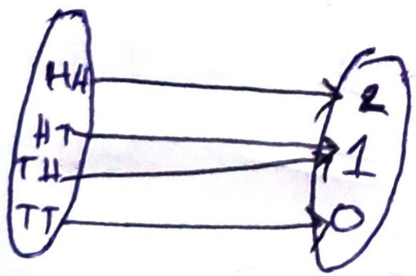
step-1:- R.E $\to$ Tossing 2 coins

step-2:- S.S $\to$ {HH, TH, HT, TT}

step-3:- R.V $\to$ X $\to$ Counting the no. of heads.

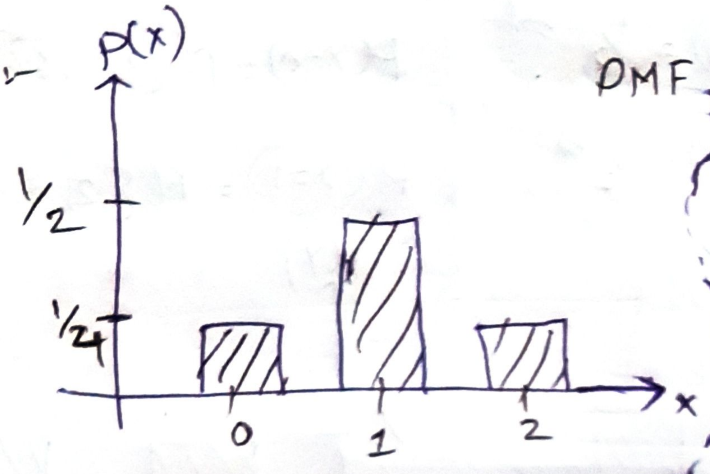
X: SS $\to$ {0, 1, 2}

0 heads, 1 head, 2 heads



Note:- It is Discrete Random Variable but not "Bernouli" because we are having 3 outcomes (0, 1, 2)
{but in bernouli, It has to be only 2 outcomes.

Step-4:-



PMF $P(x=0) = P(\{TT\}) = \frac{1}{4}$

$P(x=1) = P(\{HT\},\{TH\}) = \frac{1}{2}$

$P(x=2) = P(\{HH\}) = \frac{1}{4}$

↳ $x = \{0, 1, 2\}$.

Here we can't define 3 outcomes (success, failure)

so it is not Bernouli.

↳ But it is Binomial

$$\boxed{P(x=i) = {}^{n}C_{i}\, p^{i} (1-p)^{n-i}}$$

$n \rightarrow$ no. of trails $\quad$ $P \rightarrow$ Prob of Success

$i \rightarrow$ no. of success $\quad$ $1-P \rightarrow$ Prob of failure

$${}^{n}C_{i} = \frac{n!}{r!(n-r)!}$$

$(\frac{1}{2} \rightarrow$ success prob$)$

$$P(x=0) = {}^{2}C_{0} \left(\frac{1}{2}\right)^{0} \left(\frac{1}{2}\right)^{2-0} = \frac{2!}{(2-0)!\,0!} \left(\frac{1}{2}\right)^{2}$$

$$= \frac{1}{4}$$

$$P(x=1) = {}^{2}C_{1} \left(\frac{1}{2}\right)^{1} \left(\frac{1}{2}\right)^{2-1} = 2! \cdot \frac{1}{4} = \frac{1}{2}$$

$$P(x=2) = {}^{2}C_{2} \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{0} = \frac{1}{4}$$

Note:- Without doing all steps; we can directly compute the answer using binomial distribution.

Ex:-

Step-1 :- R.E $\longrightarrow$ Tossing 3 coins.

step-2 :- S.S $\longrightarrow$ { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

Step-3:- R.V $\longrightarrow$

x → Counting no. of heads

x: SS = { 0, 1, 2, 3 }

probability of 1 head out of 3 coins → $P(x=1)$ ⇒ case:-

1st: $\underset{1-p}{\underline{T}}$ $\underset{1-p}{\underline{T}}$ $\underset{p}{\underline{H}}$ ⇒ 3 coins

⇒ $(1-p)^2 p$

2nd case

$\underset{1-p}{\underline{T}}$ $\underset{p}{\underline{H}}$ $\underset{1-p}{\underline{T}}$ ⇒ $(1-p)\,p(1-p)$ $= (1-p)^2 p$

3rd case

$\underset{p}{\underline{H}}$ $\underset{1-p}{\underline{T}}$ $\underset{1-p}{\underline{T}}$ ⇒ $p(1-p)^2$

for $P(x=1) \Rightarrow$ we have 3 cases

$\Rightarrow P(x=1) = (1-p^2)p + p(1-p^2) + (1-p^2)p$

$$\boxed{P(x=1) = \underbrace{3}_{\text{no. of cases}}\underbrace{p}_{\text{Success probability}}\underbrace{(1-p^2)}_{\text{failure probability.}}}$$

no. of cases Success probability failure probability.

$$\boxed{P(x=i) = {}^{n}C_i * (p^i) * (1-p)^i}$$

Ex1-

R.E $\rightarrow$ Toss 100 Coins

S.S $\rightarrow$ { HHH.....T, HH.....T, ... }

2^{100}

R.V $\longrightarrow$ Counting the no. of heads.
$X \rightarrow$

X: SS $\rightarrow$ { 0, 1, 2, 3, 4, ..., 100 }

Probability of getting head/tail.

$P(x=1) = {}^{100}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{99} = 100\left(\frac{1}{2}\right)^{100}$

$P(x=0) = {}^{100}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{100} = \left(\frac{1}{2}\right)^{100}$

$P(x=100) = {}^{100}C_{100} \left(\frac{1}{2}\right)^{100} \left(\frac{1}{2}\right)^0 = \left(\frac{1}{2}\right)^{100}$