Probability

1.) Random Experiment:-

The process of observing something uncertain is known as Random experiment.

Ex:- (1) Tossing a fair coin

(2) Rolling a fair dice.

2.) Sample space:-

All possible out comes of a random Experiment

Ex:- (1) Tossing a fair coin → { H, T }

(2) Rolling a fair dice → { 1, 2, 3, 4, 5, 6 }

3.) Event:-

Event is a set of outcomes of a Random Experiment

↳ Event is a Subset of "Sample Space"

Ex:- getting head when a coin is tossed

Event — Random Experiment

Axioms of probability:-

(1). $0 \leq P(E) \leq 1 \Rightarrow$ probability of an event always lies b/w 0 & 1

(2.) $P(S) = 1 \Longrightarrow$ Sum of probability of all outcomes is "1"

$P(S)$ ↓ Sample space

(3.) $P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)$

when they are Mutually exclusive (Independent)

$$P(E_1 \cup E_2 \cup E_3 \cdots \cup E_n) = P(E_1) + P(E_2) + \cdots + P(E_n)$$

Ex:- R.E → Tossing two coins

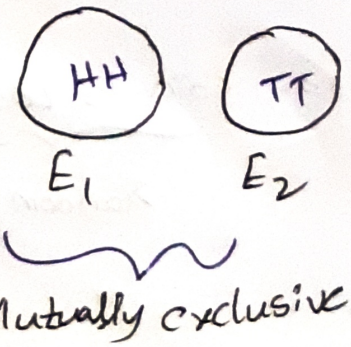
S.S → {HH, HT, TH, TT}

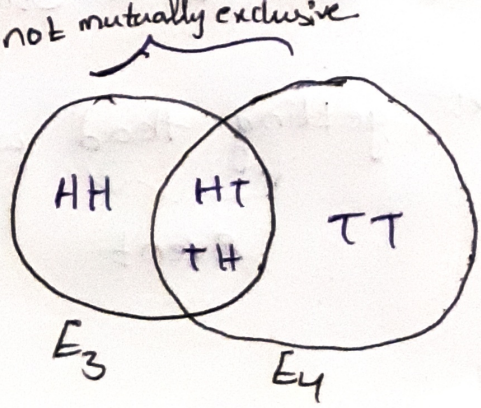
E_1 → Getting both heads → {HH}

E_2 → Getting both Tails → {TT}

E_3 → Getting atleast 1 Head → {HH, HT, TH}

E_4 → Getting atleast 1 Tail → {TT, TH, HT}





$\Rightarrow E_1 \cap E_2 = \phi \Rightarrow$ Mutually exclusive (nothing in blw E_1 & E_2)

$\Rightarrow E_3 \cap E_4 \neq \phi \Rightarrow$ Not Mutually exclusive (HT, TH are in blw E_3 & E_4)

$E_5 \rightarrow$ Getting both heads (or) tails $\rightarrow \{HH, TT\}$

$P(E_5) = P(E_1 \cup E_2) = P(E_1) + P(E_2) \Rightarrow$ as they are Mutually exclusive

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

When they are not mutually exclusive.

Conditional probability:-

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$$

Ex:- probability of getting a prime number (Event) when given that number is even number. (when a dice is rolled) ← R.E

Sol:- Method-1:-

Sample space = $\{1,2,3,4,5,6\}$

Prime numbers :- $\{2,3,5\}$

given number is even number $(2,4,6) =$

$\Rightarrow$ sample space = $\{2,4,6\}$

Sample Space = {2, 4, 6}

Prime numbers

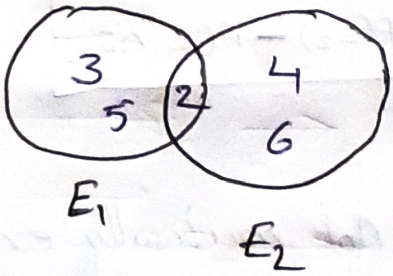
2 → Prime

∴ Probability of getting prime no:- $\frac{1}{3}$ ↗ 3 → 3 → Sample Space

Method 2.

$E_1 \Rightarrow$ getting prime number $\{2, 3, 5\} \Rightarrow P(E_1) = 1/2$

$E_2 \Rightarrow$ number is even $\{2, 4, 6\} \Rightarrow P(E_2) = 1/2$



$P(E_1 \cap E_2) = \frac{1}{6}$

(or)

$P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$

$= \frac{1}{2} + \frac{1}{2} - \frac{5}{6} = \frac{1}{6}$

($\frac{1}{2} = \frac{3}{6}$, $\frac{1}{2} = \frac{3}{6}$)

$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

$= \frac{1/6}{1/2} = \frac{2}{6} = \frac{1}{3}$

∴ $\boxed{P(E_1/E_2) = 1/3}$