CS6790 : Assignment 2

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1 Aim

The aim of the assignment is to obtain the camera calibration matrix K by obtaining the Image of the Absolute Conic (IAC) in four different methods. The implementation can be found here.

2 Principle

For any projective camera, the camera induces a transformation from the world to the camera sensor plane. For a point $\mathbf{X}$ ($\in P^3$), the corresponding point $\mathbf{x}$ ($\in P^2$) is given by

$$\mathbf{x} = P\mathbf{X} \tag{1}$$

where P is the camera projection matrix. This matrix, in turn, can be expressed as

$$P = KR\ [\ I \mid \text{-}\tilde{C}\ ] \tag{2}$$

where K is the *camera calibration matrix*, R is a rotation matrix and $\tilde{C}$ is the coordinates of the camera centre in the world coordinate system.

The camera calibration matrix K can be found if one can identify the transformation induced on the absolute conic Ω_∞ . The equation for the Image of the Absolute Conic (IAC) is given by :

$$w = (KK^T)^{-1} \tag{3}$$

Once w is identified, we can find K by first performing Cholesky decomposition of w and later computing inverse of the result . For validation, an auxillary method was also developed. Here, the parameters of K were used to compute the general expression for the inverse of w and the computed w was used to directly obtain K, bypassing the Cholesky decomposition step. Both have been used interchangeably in the implementation.

We now look at estimating K under (a) no assumptions (b) zero skew and square pixels assumption.

3 Estimating full K matrix

3.1 From five pairs of orthogonal vanishing points

The angle b/w two lines l_1 and l_2, as given by the corresponding vanishing points v_1 and v_2 is :

$$\cos\theta = \frac{v_1^T \Omega_\infty v_2}{\sqrt{v_1^T \Omega_\infty v_1}\sqrt{v_2^T \Omega_\infty v_2}} \tag{4}$$

It can be shown that this equation is invariant under a homography. Now, consider the action of a projective camera on the plane at infinity (π_∞). The camera induces a homography from π_∞ to the sensor plane. Let v_1 and v_2 be the transformed vanishing points of two originally perpendicular lines. Let w be the IAC. Now, the lines being perpendicular (originally), we have

$$\cos\theta = 0$$
$$\implies v_1^T w v_2 = 0$$

The above is an equation in the parameters of w. With w being a symmetric matrix, five such equations would completely determine w (upto scale). One can then proceed to find K by Cholesky decompostion.

Results

The camera calibration matrix obtained from the five images of the checkerboard pattern is :

$$K = \begin{bmatrix} 6791.65 & -392.12 & 185.02 \\ 0.0000 & 2978.96 & -572.98 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

3.2 From Three Homography Relations

The projective camera induces a homography b/w each world plane and the camera sensor plane. In this method, the goal is to first establish the transformation on the circular points I and J of the plane, from which we obtain constraints on the IAC. The following steps are followed :

- Identify a world plane of interest and fix a metric coordinate system to the plane.
- Find the homography H b/w the world plane and the sensor plane by identifying the transformation induced on four points of the plane. Say $H = [\mathbf{h}_1 \ \ \mathbf{h}_2 \ \ \mathbf{h}_3]$
- The images of the circular points can be found as :

$$I' = H \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} ; J' = H \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

- Now, Since the circular points lie on Ω_∞, we have

$$I'^T wI' = 0 \ \ ; \ \ J'^T wJ' = 0$$

  Which yields

$$h_1^T w \, h_2 = 0 \quad ; \quad h_1^T w \, h_1 = h_2^T w \, h_2$$

- One plane yields 2 equations in the five parameters of w. Thus, three homographies would completely determine w. K can now be found as before.

Results

The following are the camera calibration matrices obtained for the checkerboard pattern and `image2_2.png` respectively :

$$K_1 = \begin{bmatrix} 7876.5 & -1035.55 & 858.89 \\ 0. & 8809.45 & 7755.87 \\ 0. & 0. & 1. \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1126.77 & -13.95 & 456.61 \\ 0.00 & 1128.99 & 347.13 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

4 Estimating K with zero skew and square pixels

The K matrix is now of the form :

$$K = \begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix}$$

For w of the form

$$w = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{12} & w_{22} & w_{23} \\ w_{13} & w_{23} & w_{33} \end{bmatrix}$$

The zero skew condition and the square pixels constraint introduce additional constraints on w :

$$w_{12} = 0 \quad ; \quad w_{11} = w_{22}$$

4.1 From three pairs of orthogonal vanishing points

Following section 3.1, each pair of orthogonal vanishing point gives one equation on w. Now, due to the zero skew and square pixel constraint, w has only three parameters. Thus, we only need three equations *ie.* three pairs of orthogonal vanishing points to determine w. K can then be found as before.

Results

The camera calibration matrices obtained for the checkerboard pattern, `image2_2.png`, `img1.jpg` and `img2.jpg` respectively are :

$$K_1 = \begin{bmatrix} 7687.76 & 0.00 & 2723.01 \\ 0.00 & 7687.77 & 8592.86 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1132.22 & 0. & 440.04 \\ 0. & 1132.22 & 382.98 \\ 0. & 0. & 1. \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 8317.79 & 0. & 2794.05 \\ 0. & 8317.79 & 4898.21 \\ 0. & 0. & 1. \end{bmatrix}$$

$$K_4 = \begin{bmatrix} 11784.30 & 0.00 & 4628.91 \\ 0.00 & 11784.30 & 2573.61 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}$$

4.2 From Two Homography Relations

Following section 3.2, each homography relation yields two equations in the three parameters of w. Thus, we need two homography relations (albeit not completely, as any three equations among the four obtained would do) for determining w.

Results

The camera calibration matrices obtained for the checkerboard pattern and `image2_2.png` :

$$K_1 = \begin{bmatrix} 13531.35 & 0. & 2995.79 \\ 0. & 13531.35 & 1141.62 \\ 0. & 0. & 1. \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 1127.54 & 0. & 440.84 \\ 0. & 1127.54 & 335.16 \\ 0. & 0. & 1. \end{bmatrix}$$

5 Interpreting the results

Clearly, one can notice significant difference in the K matrix obtained using methods in section 3.1 and 3.2 . One of the reasons is the inherent errors in the pixel coordinates. This was also noticed in the fact that the choice of the world plane could drastically affect the results, and in many cases yielded a non-positive definite w. One can also notice that the results of section 4.1 and 4.2 differ for the checkerboard pattern, but are in agreement for `image2_2.png`. This can be explained if we go back to the results of section 3.2, and observe that the camera used for the checkerboard pattern has a significant skew factor, while the camera matrix for `image2_2.png` has a negligible skew factor. Thus, when the zero skew condition is enforced, inconsistent results are obtained for the checkerboard pattern. This was also noticed in the fact that different homographies ie. different planes gave different results for section 4.2 (for the checkerboard pattern). For `image2_2.png`, both methods yield the same results, as the zero skew and the square pixels conditions are truly satisfied by the camera used.