Assignment No 8

Sumanth R Hegde, EE17B032

April 2, 2019

1 Aim

This week's assignment is based on Discrete Fourier Transform (DFT) of signals. We examine the DFT of various functions using the `fft` library in `numpy`.

2 The Sinusoid

Here, we try to estimate the DFT of a sinusoidal function.

$$y = sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$$

Has the Continuous time Fourier Transform

$$Y(\omega) = \frac{[\delta(\omega - 1) - \delta(\omega + 1)]}{2j}$$

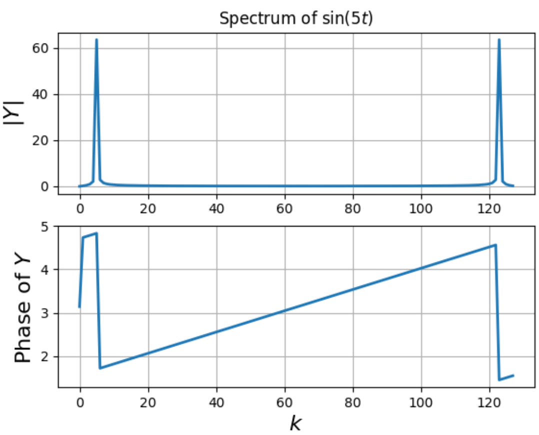

Figure 1: DFT of $sin(5t)$

There are multiple issues that need to be fixed here:

- The peaks are not where we expect them to be. This can be corrected using fft shift.
- The spikes have a height of 64, not 0.5. This should be rectified by dividing by the sample rate.
- The frequency axis is not in place. This is due to the duplicacy of 0 and 2π.
- The actual phase at the spikes is near but not exactly correct.

We end up with a much cleaner spectrum after making the modifications:

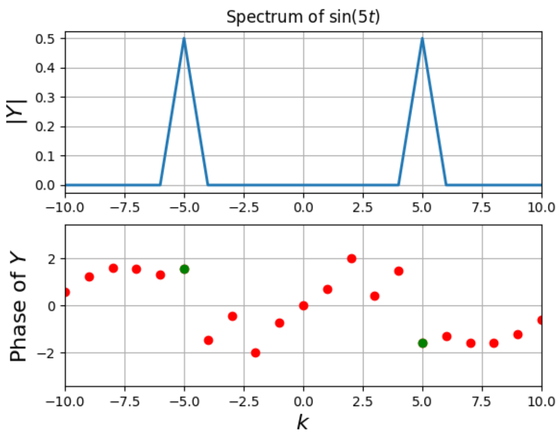

Figure 2: DFT of an amplitude modulated wave

In the phase response shown the point marked in green are those that have a magnitude $> 10^{-3}$, and thus all other points that are marked in red are merely spurious.

3 Amplitude modulated signal

Consider the amplitude modulated signal,

$$f(t) = (1 + 0.1cos(t))cos(10t)$$

We expect a shifted set of spikes, with a main impulse and two side impulses on each side. This is because,

$$0.1cos(10t)cos(t) = 0.05(cos11t + cos9t)$$
$$= 0.025(e^{11tj} + e^{9tj} + e^{11tj} + e^{9tj})$$

In order to see even the side peaks, the frequency resolution has to be improved. We can do so by keeping the number of samples constant and increasing the range in the time domain. The following spectrum is obtained :

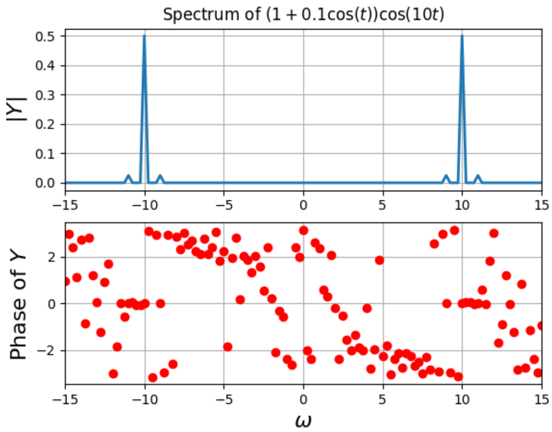

Figure 3: DFT of an amplitude modulated wave

4 The Assignment

4.1 The Sinusoids

Consider the sinusoids sin^3t and cos^3t :

$$sin^3t = \frac{3sin(t) - sin(3t)}{4}$$

$$cos^3t = \frac{cos(3t) + 3cos(t)}{4}$$

Thus, in the frequency spectrum, there will be 4 impulses, with on pair at thrice the frequency than the other. Taking the same precautionary measures as done previously, we obtain the following approximate spectrum :

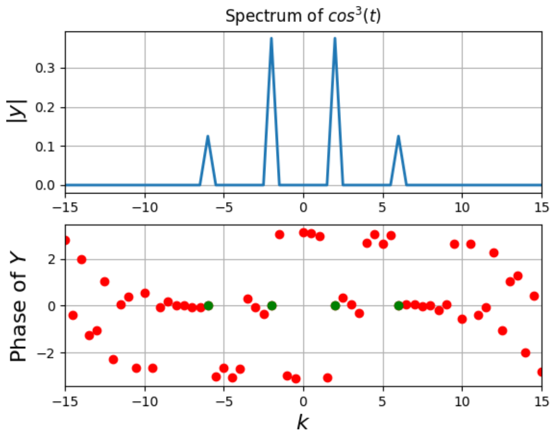

Figure 4: DFT of cos^3t

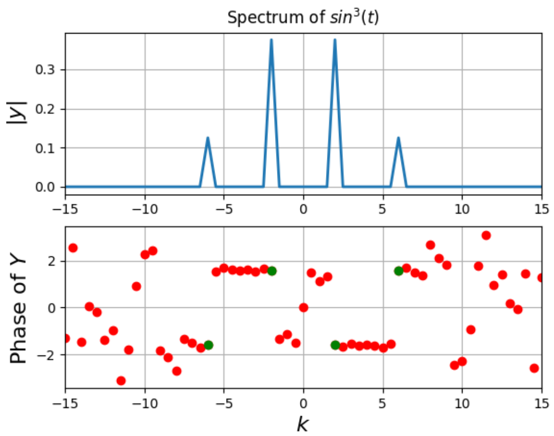

Figure 5: DFT of sin^3t

4.2 Frequency Modulation

Here, we consider the frequency modulated function $cos(20t + 5cos(t))$

Because the argument of the cosine function itself consists of a cosine term, the wave can be expressed using Bessel functions. Reference used : https://www.dsprelated.com/freebooks/mdft/Sinusoidal_Frequency_Modulation_FM.html Thus, the frequency spectrum consists of impulses at the carrier frequency ($w_c = 20$ here) and impulses at multiple side band frequencies, that are close to the carrier frequency.

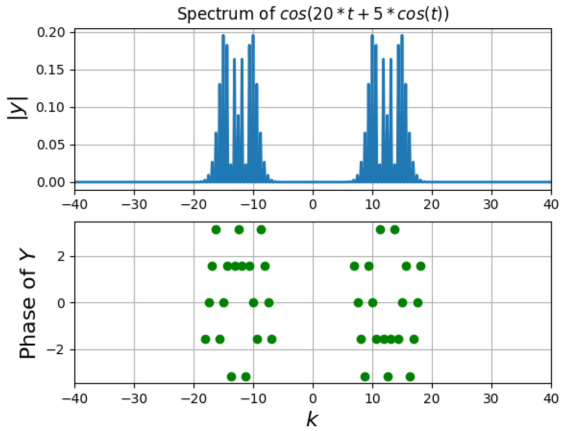

Figure 6: DFT of $cos(20t + 5cos(t))$

4.3 The Gaussian

The Gaussian function $f(x) = e^{-x^2/2}$ is not band-limited, in the sense that the frequency spectrum has non zero values even for very large frequencies.

The Continuous Time Fourier Transform for the Gaussian is given by

$$F(w) = \sqrt{2\pi}e^{-w^2/2}$$

Thus, the phase is zero for all w while the magnitude is a Gaussian function. After appropriate normalization, the DFT can be used to obtain the error. For different time ranges(t_{lim}) , the error in the estimate for the DFT obtained using the `fftshift` function was found to vary. The value of error was also found to vary with the resolution in frequency domain (w_{lim}), as well as the sampling rate. For a choice of

$$N = 512, w_{lim} = 32\ rad\ s^{-1}, t_{lim} = 8\pi\ s$$

the error was of the order 10^{-15} ,which is well within the allowed error. For different sampling rates, we observe that the peak of the gaussian broadens.

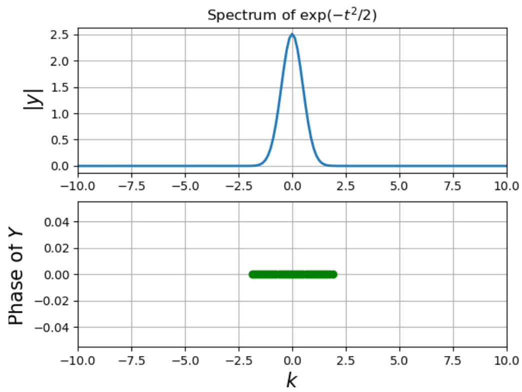

Figure 7: DFT of the Gaussian for $t_{lim} = 4\pi$

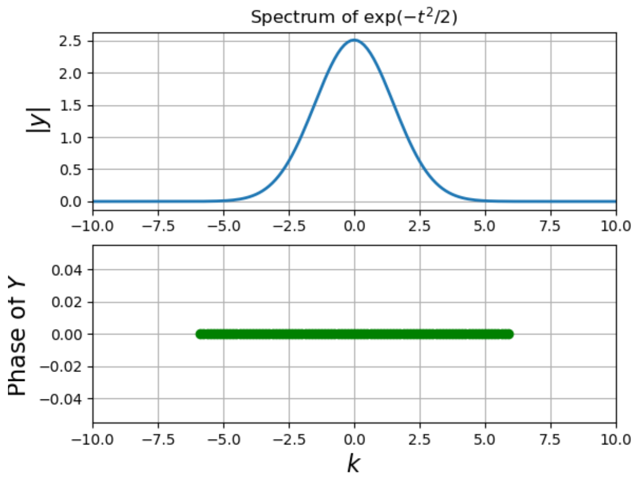

Figure 8: DFT of the Gaussian for $t_{lim} = 12\pi$

For different sampling rates, the we observe that the peak sharpens with an increase in the sampling rate.

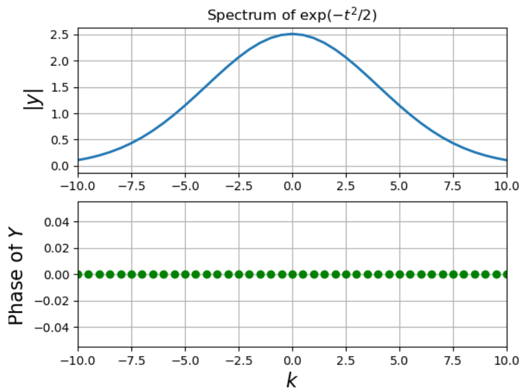

Figure 9: DFT of the Gaussian for $N = 256$

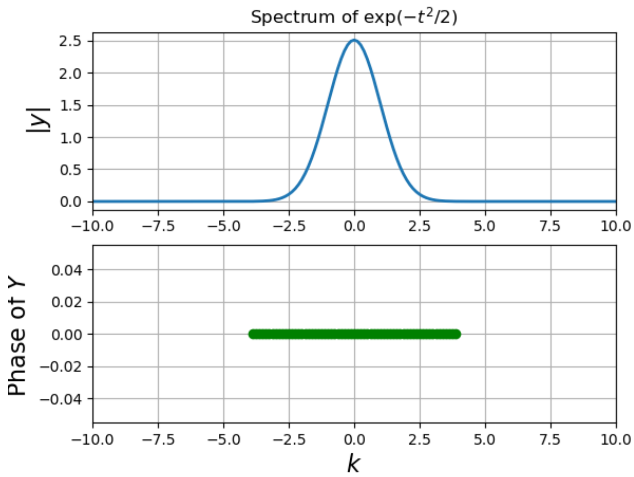

Figure 10: DFT of the Gaussian for $N = 1024$

5 Conclusion

The `fft` library in python provides a useful toolkit for analysis of DFT of signals. The Discrete Fourier Transforms of sinusoids, amplitude modulate signals, frequency modulated signals were analysed. In the case of pure sinusoids, the DFT contained impulses at the sinusoid frequencies. The amplitude modulated wave had a frequency spectrum with impulses at the carrier and the side band frequencies. The frequency moduated wave, having an infinite number of side band frequencies, gave rise a DFT with non zero values for a broader range of frequencies. The DFT of a gaussian is also a gaussian and the spectrum was found to sharpen for higher sampling rates, while broaden for greater time ranges.