Assignment No 7

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1 Aim

The aim of this assignment is to analyse circuits using the Laplace Transform of the impulse response, utilising the Symbolic algebra capabilities of Python.

2 Step Response of a Low pass filter

We aim to find the step response of the following circuit:

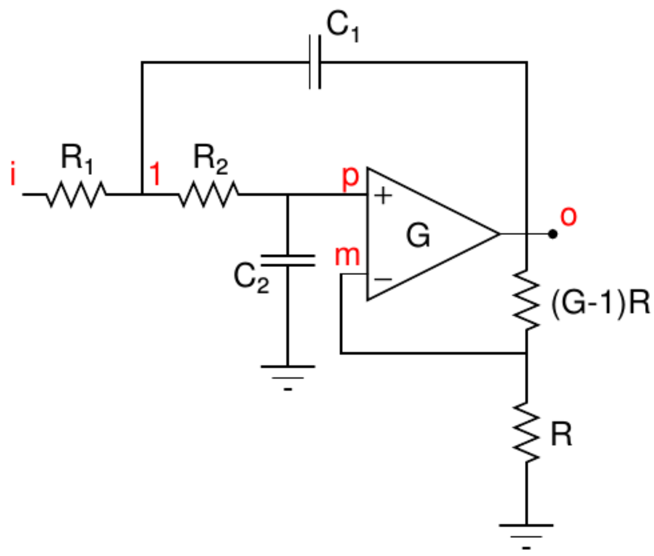

Figure 1: A Low-pass filter using an op-amp of gain G

The output voltage V_o can be obtained in the s-domain using the matrix capabilities of the `Sympy` module. The node voltages V_1, V_p,V_m,V_o can be obtained by solving the system equation in the matrix form. The following piece of code does the job :

```
def lowpass(R1,R2,C1,C2,G,Vi):
    s=symbols('s')
    A=Matrix([[0,0,1,-1/G],[-1/(1+s*R2*C2),1,0,0],[0,-G,G,1],
                         [-1/R1-1/R2-s*C1,1/R2,0,s*C1]])
    b=Matrix([0,0,0,-Vi/R1])
    V = A.inv()*b
    return (A,b,V)
```

Using the above `lowpass` function, we can obtain the impulse response in s-domain for the given values of components as follows :

```
A,b,V=lowpass(10000,10000,1e-9,1e-9,1.586,1)
Vo = V[3]
```

On obtaining the response in its symbolic representation, the following function is used to convert it into the polynomial representation compatible with the Signals toolbox of `scipy`:

```
def sympy_to_lti(xpr, s=symbols('s')):
    num, den = simplify(xpr).as_numer_denom()
    p_num_den = poly(num, s), poly(den, s)
    c_num_den = [expand(p).all_coeffs() for p in p_num_den]
    l_num, l_den = [lambdify((), c)() for c in c_num_den]
    return sp.lti(l_num, l_den)
```

The following graph is obtained :

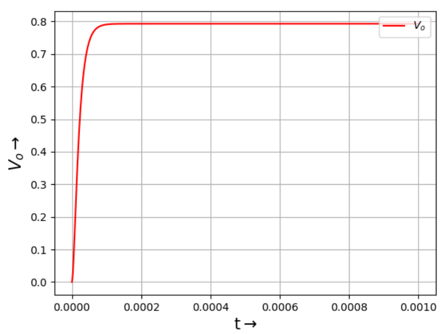

Figure 2: Step response of the lowpass filter

3 Response for a mixed frequency input

The input applied is

$$v_i = (sin(2000\pi t) + cos(2 * 10^6\pi t))u(t)$$

Ideally, since the cutoff frequency $1/2\pi$ MHz, the system is expected to allow the low frequency `sine` component while attenuating the high frequency `cosine` component. The following graph is obtained for $0 < t < 0.01s$ Using the given initial conditions, for we get the following graph :

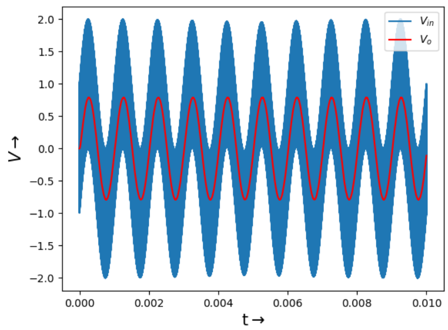

Figure 3: Output response to a mixed frequency input

One can notice that the output V_o oscillates at a frequency of 1000 Hz, thus verifying its low-pass characteristic.

The High-pass filter

A high pass filter can be constructed by making a small modification to the low-pass circuit :

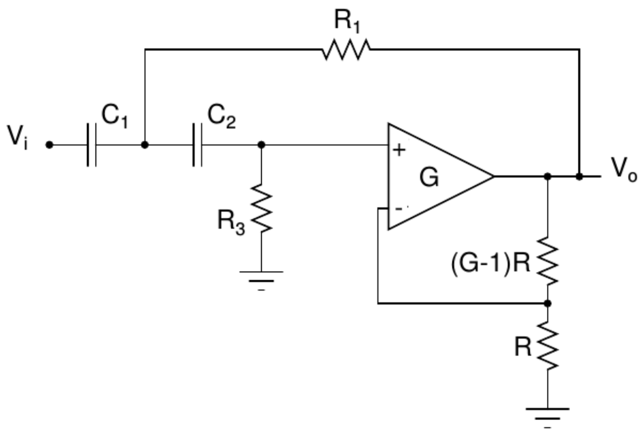

Figure 4: A high-pass filter using an op-amp of gain G

To obtain the value of the output V_o, we can use the above `lowpass` function itself, by noting that the capacitors and resistors have switched places. For the given values of the components, the following code block does the job:

```
def highpass(R1,R3,C1,C2,G,Vi):
    s=symbols('s')
    A=Matrix([[0,0,1,-1/G],[-1/(1+1/(s*R3*C2)),1,0,0],[0,-G,G,1],
                  [-s*C1-s*C2-1/R1,s*C2,0,1/R1]])
    b=Matrix([0,0,0,-Vi*s*C1])
    V = A.inv()*b
    return (A,b,V)
A,b,V = highpass(10000,10000,1e-9,1e-9,1.586,1)
Vo = V[3]
H = sympy_to_lti(Vo)
```

The magnitude response, as expected, is that of a high pass filter, with cut-off frequency at $1/2\pi$ MHz :

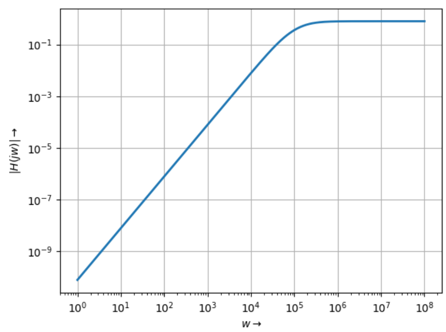

Figure 5: $|H(jw)|$ vs w in a loglog scale

4 Response to a damped sinusoid

Consider the following damped sinusoids :

$$v_{in} = e^{-0.5t}sin(2\pi t)$$

and

$$v_{in} = e^{-0.5t}sin(2\pi * 10^5 t)$$

The high pass filter is expected to attenuate the low frequency sinusoid. The system should allow frequencies such as 2x10^5 Hz as they are above the cut-off frequency.

The following outputs are obtained for the above mentioned frequencies:

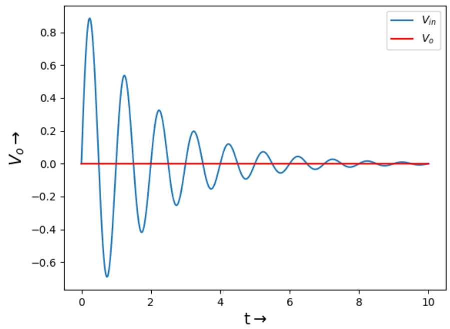

Figure 6: High-pass filter response for 1Hz sinusoid input

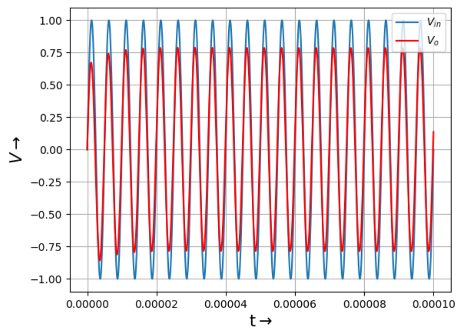

Figure 7: High-pass filter response for 2x10^5Hz sinusoid input

Thus, the high-pass behaviour of the circuit is verified. The change in the exponential would only affect the rate at which the sinusoid amplitude decays to zero.

5 Step Response

The response for a unit step can be found using the following code snippet :

```
t = np.linspace(0,10,1000)
Vo = sp.step(H,T=t)
```

The following plot is obtained for the output V_o :

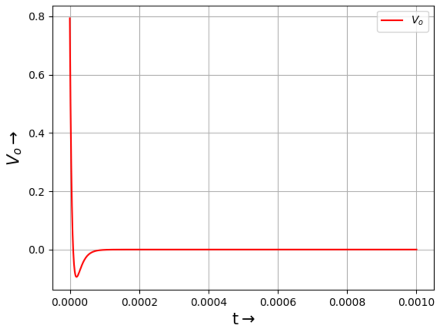

Figure 8: Step response

Initially, there is a sudden change in the voltage, due to which the capacitors act as a short, and we obtain non-zero voltage at the output. For the steady value of the input V_i , the capacitor C_1 in the circuit acts as an open switch and allows no current to pass through. Consequently, we have a zero voltage at the output node.

6 Conclusion

Sympy provides a convenient way to analyse LTI systems using their Laplace transforms. The toolbox was used to study the behaviour of a low pass filter, implemented using an opamp of gain G. For a mixed frequency sinusoid as input, it was found that the filter suppressed the high frequencies while allowing the low frequency components. Similarly, a high pass filter was implemented using an op-amp with the same gain. The magnitude response of the filter was plotted and its output was analysed for damped sinusoids. The step response of the filter was found to have a non-zero peak at $t = 0$, due to the sudden change in the input voltage.