Assignment No 9

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1 Aim

In this assignment, we explore the nature of DFTs of non periodic signals, and the use of DFT in parameter estimation.

2 Spectrum of an aperiodic signal

Using the method followed for obtaining the DFT for periodic signals, the following spectrum is obtained for $sin(\sqrt{2}t)$:

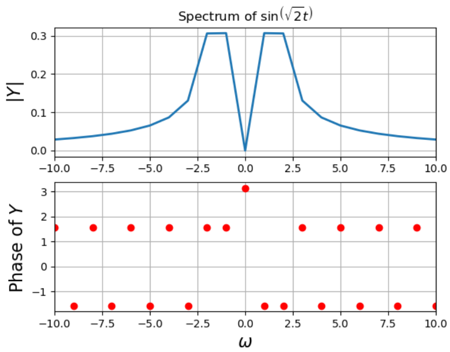

Figure 1: Spectrum of $sin(\sqrt{2}t)$

This is because, the DFT is trying to analyse the 2π periodic extension of the function in the interval $[-\pi, \pi]$. This function, having discontinuities, results in a slowly decaying frequency response and hence we do not obtain shark peaks at the expected frequencies.

3 Windowing

The fix for the above problem is windowing, where we multiple the function in the time domain with the Hamming window function :

$$x[n] = \begin{cases} 0.54 + 0.46 \cos \frac{2\pi n}{N-1} & |n| N1 \\ 0 & else \end{cases}$$

The DFT after windowing shows significant improvement :

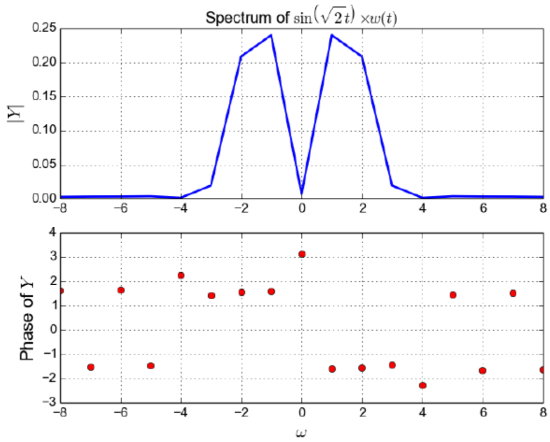

Figure 2: Spectrum of $sin(\sqrt{2}t)$ after windowing

4 The DFT for $cos^3(w_o t)$

The Spectrum of $cos^3(w_o t)$ for $w_o = 0.86$ without windowing is as follows :

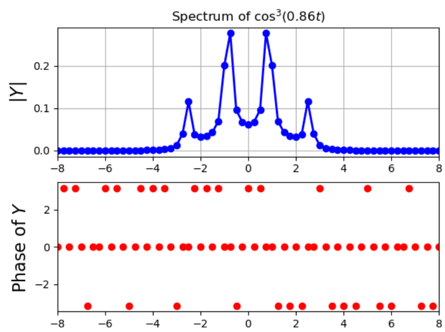

Figure 3: Spectrum of $cos^3(0.86t)$ without windowing

The Spectrum after windowing has a narrower peak :

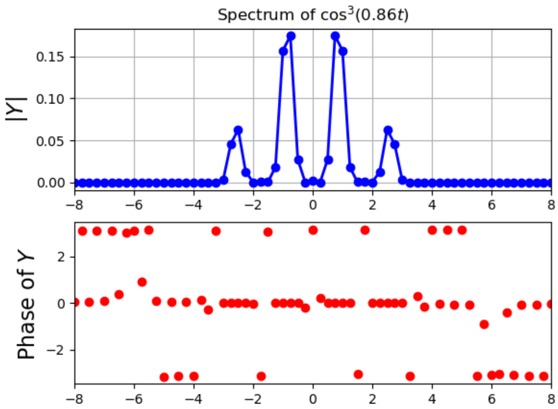

Figure 4: Spectrum of $cos^3(0.86t)$ after windowing

5 Parameter estimation using the DFT

Given a 128 element vector known to contain $cos(w_o t + \delta)$ for arbitrary δ and $0.5 < w_o < 1.5$, we try to estimate the value of w_o and δ using the DFT of the vector.

5.1 Frequency estimation

On first glance, notice that due to the low sampling rate of the function, the resolution in the frequency domain is low. The following plot is obtained for $cos(0.5t + \pi)$:

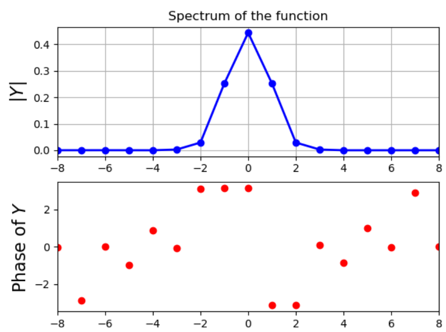

Figure 5: Spectrum of $cos(0.5t + \pi)$

The peaks overlap and thus it is challenging to obtain a good estimate. Since we cannot rely on the location of the peak for estimating the frequency, an alternative method should be considered. By simply noticing that expectation value of w_o over $|Y|$ should produce results close to the true value, we experiment with our estimation method as:

$$w_{0,est} = \frac{\Sigma|Y|^p w}{\Sigma|Y|^p}$$

On experimenting with different values of p, it was found that a value of $p = 1.7$ gave good results in the given range of w_o

5.2 Phase estimation

The phase of the function is obtained using the phase plot of the DFT. Using the fact that

$$cos(w_o t + \phi) \xrightarrow{\mathscr{F}} \frac{1}{2}(e^{j\phi}\delta(w - w_o) + e^{-j\phi}\delta(w + w_o))$$

The phase is estimated by observing the phase at w_o ,which, in this case can be taken at the peak value in the magnitude spectrum.

5.3 Estimation in the case of noisy data

In the case of added gaussian noise, the parameter p used in frequency estimation has to be changed, while the phase estimation algorithm would still work, considering the fact that the DFT of a gaussian is a gaussian in frequency, and for the given range of w_o, the phase remains almost unaffected. Through an exhaustive grid search, the parameter $p = 2.4$ gave good results for this case.

6 DFT of a chirped signal

The chirped signal is a signal of varying frequency, with frequency increasing from 16 to 32 radians per second as we move from $-\pi$ to π .

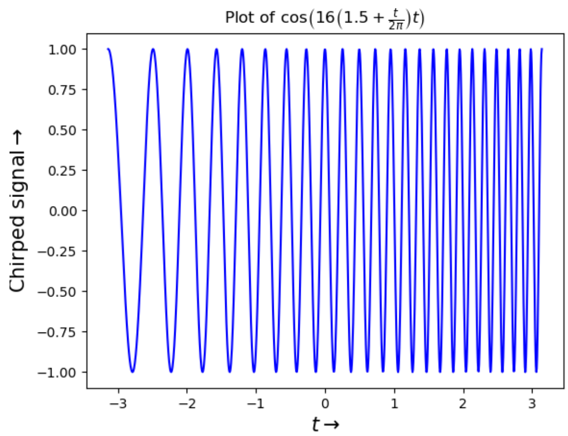

Figure 6: Plot of $cos(16(1.5 + \frac{t}{2\pi})t)$

The DFT of the chirped signal is as follows :

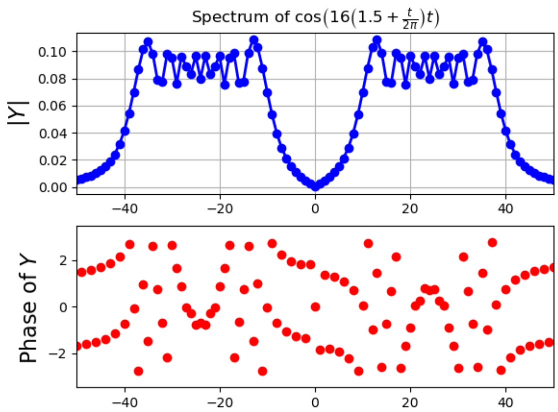

Figure 7: Spectrum of $cos(16(1.5 + \frac{t}{2\pi})t)$

On breaking the 1024 length vector into 16 pieces, each 64 samples wide, we can analyse the DFT in how it evolves over time.

The following surface plot is obtained for the time-frequency variation of the DFT

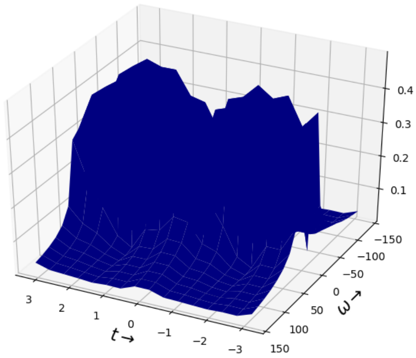

Figure 8: Time frequency plot of the DFT

We observe that the peak frequency of the signal increases with time, which is made clear with the below plot :

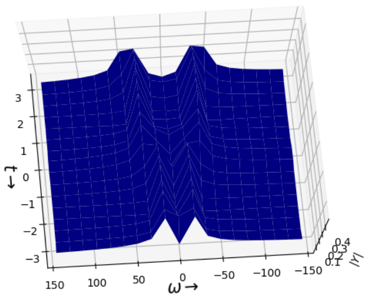

Figure 9: Variation of frequency contained with time

7 Conclusion

The DFT was obtained using a 2π periodic extension of the signal, and thus the spectrum was found to be erroneous for a non periodic function. The spectrum was rectified by the using a windowing technique, by employing the Hamming window. Given a vector of cosine values in the a time interval, the frequency and phase were estimated from the DFT spectrum, by using the expectation value of the frequency and a parameter grid search for optimum values. The DFT of a chirped signal was analysed and its time-frequency plot showed the gradual variation of peak frequency of the spectrum with time.