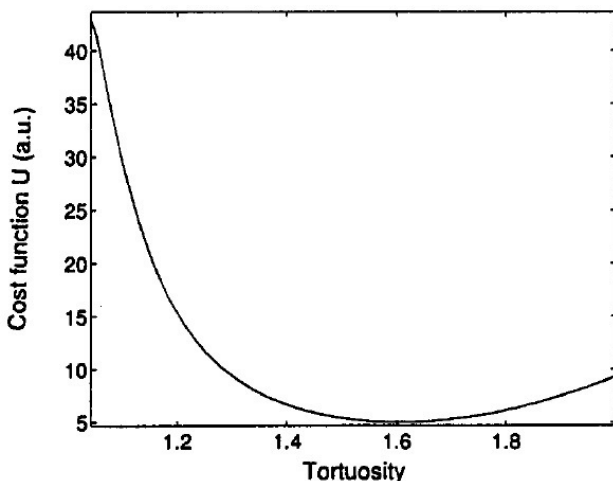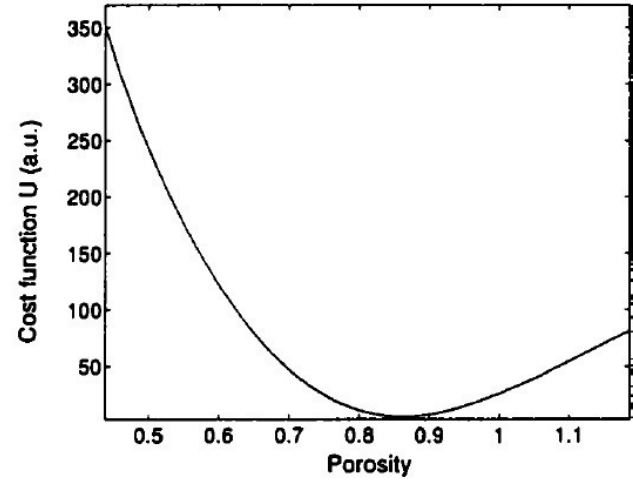FIG. 8. Variation of cost function U with the porosity for plastic foam M2.

When the incident angle is $\theta < \theta_c$, the reflection coefficient decreases slowly with the incident angle, and when it is $\theta > \theta_c$, the reflection coefficient increases quickly with the angle. It can also be seen from Figs. 2 and 3 that the sensitivity of porosity variation is more important than the sensitivity of tortuosity on the reflection coefficient at the first interface.

V. INVERSE PROBLEM

The propagation of acoustic waves in a slab of porous material in the high frequency asymptotic domain is characterized by four parameters: porosity ϕ, tortuosity α_∞, viscous characteristic length Λ, and thermal characteristic length Λ', the values of which are crucial to the behavior of sound waves in such materials. It is of some importance to work out new experimental methods and efficient tools for estimation of them. The basic inverse problem associated with the slab may be stated as follows: from measurement of the signals transmitted and/or reflected outside the slab, find the values of the medium's parameters. The inverse problem has been solved in transmission mode in Refs. 4–6 and an estimate of α_∞, Λ', and Λ' made therein that gives a good correlation between experience and theory. Porosity was not estimated in this mode because of its weak sensitivity. Porosity was estimated in reflected mode in our previous

FIG. 9. Variation of cost function U with the tortuosity for plastic foam M3.

FIG. 10. Variation of cost function U with the porosity for plastic foam M3.

studies[7,8,31,32] by solving the inverse problem at normal[8] and oblique incidence[31] for a fixed value of tortuosity. Porosity and tortuosity were measured[7,32] simultaneously for plastic foams and air-saturated random packings of beads by measuring reflected waves for each pair of incident angles.

In this article, we determine the porosity and tortuosity by solving the inverse problem for waves reflected by the first interface, and by taking into account experimental data for all measured incident angles.

The inverse problem is to find values of parameters ϕ and α_∞, which minimize the function

$$U'' = F(\phi, \alpha_\infty) \sum_{\theta_i} \sum_{t_i} [p^r(x, \theta_i, t_i)$$
$$- r(\theta_i, t_i) * p^i(x, \theta_i, t_i)]^2 \qquad (30)$$

where $p^r(x, \theta_i, t_i)$ represents the discrete set of values of the experimental reflected signal for different incident angles θ_i, $r(\theta_i, t_i)$ is the reflection coefficient at the first interface, and $p^i(x, \theta_i, t_i)$ is the experimental incident signal. The term $r(\theta_i, t_i) * p^i(x, \theta_i, t_i))$ represents the predicted reflected signal. The inverse problem is solved numerically by the least-square method.

Experiments were performed in air with two broadband Ultran NCT202 transducers with 190 kHz center frequency in air and 6 dB bandwith that extends from 150 to 230 kHz. An optical goniometer was used to position the transducers. Pulses of 400 V were provided by a 5052PR Panametrics puls receiver. The signals received were amplified to 90 dB and filtered above 1 MHz to avoid high frequency noise. Electronic interference was removed by 1000 acquisition averages. The experimental setup is shown in Fig. 4. The duration of the signal plays an important role: its spectrum must verify the condition of the high frequency approximation referred to in Sec. IV.

TABLE II. Reconstructed values of porosity and tortuosity by solving the inverse problem.

Material	M1	M2	M3
Tortuosity	1.12	1.1	1.6
Porosity	0.96	0.99	0.85