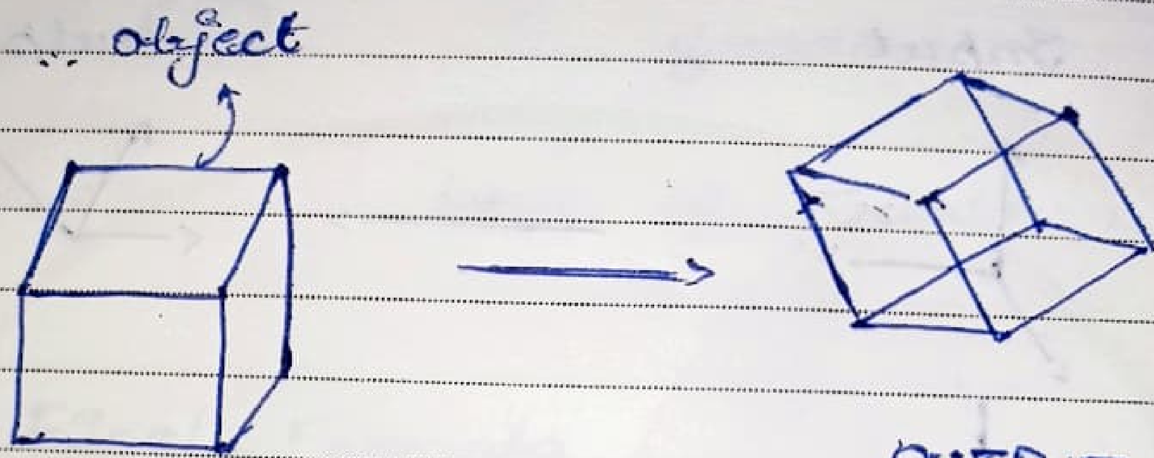
Task 3

a)

Euler's Angle



INPUT → OUTPUT

INPUT

↓

chose 3 axis

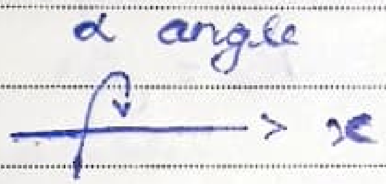
↓

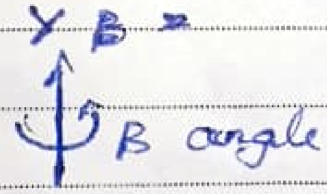
Rotate α along axis 1 → Rotate β along axis 2 → Rotate γ along axis 3 → OUTPUT

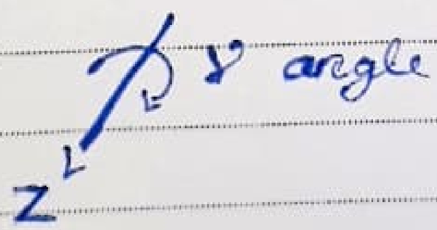
6 Types based on Order of Rotation

1) XYZ
2) XZY
3) YXZ
4) YZX
5) ZYX
6) ZXY

⇒ For ex: XYZ means
X 1st
Y 2nd
Z 3rd

①


②


③


Date :

Notes

Quaternion

A 4-D Quantity which is used to represent 3-D rotation

General Form

$$H = \underset{\text{Scalar}}{a} + \underbrace{bi + cj + dK}_{\text{vector part}}$$

where

★ $i^2 = j^2 = K^2 = -1$

★ $ij = K \qquad iK = -j \qquad jK = i$

$ji = -K \qquad Ki = j \qquad Kj = -i$

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Advantages

1) Avoids changing Axes
2) Avoids ~~Gym~~ Gimbal Lock

in Euler angle

For ex : after an XYZ rotation, since rotation changed axis orientation further rotation along the same axis is not ~~causes~~ ~~not~~ possible

Sometime if two axis after rotation lines up with one another, then there would be a loss of DOF causing Gymbal Lock.

Date :

Notes

Rotational Matrix

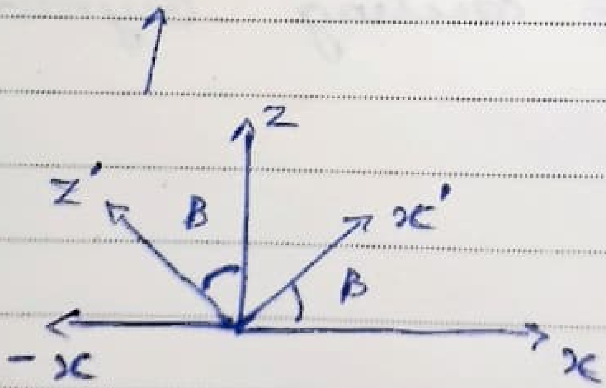
Is a Matrix which encodes co-ordinates of ~~a~~ rotated frame wrt to a reference frame.

They are always orthogonal Matrices

Can be ~~of~~ multiplied with a vector to find how the vector would rotate if the frame rotates.

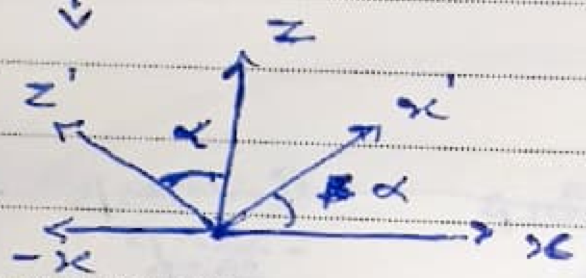
Ex

$$R_1 = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

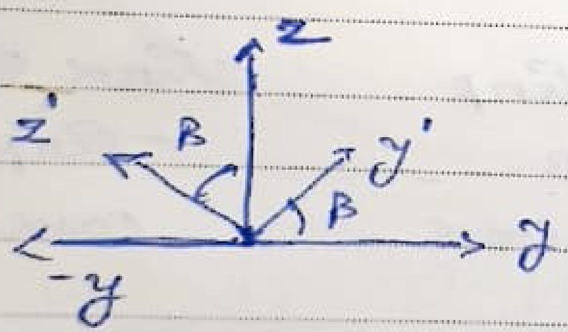


$R_{20} =$

b) $R_1 = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$



$R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix}$



$R_{net} = \cancel{R_0} \; R_1^0 \times R_2^1$

$\cancel{R_{20}} = R_{10}$

$\boxed{R_{net}^{-1} P = R_0}$

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$$R_{net} = \begin{bmatrix} \cos\alpha & -\sin\alpha\sin\beta & -\sin\alpha\cos\beta \\ 0 & \cos\beta & -\sin\beta \\ \sin\alpha & \sin\beta\cos\alpha & \cos\alpha\cos\beta \end{bmatrix}$$

$R_{net}^{-1} =$

$$\begin{array}{ccccc} \cos\alpha & -\sin\alpha\sin\beta & -\sin\alpha\cos\beta & \cos\alpha & -\sin\alpha\sin\beta \\ 0 & \cos\beta & -\sin\beta \ \cancel{\cos\beta} & 0 & \cos\beta \\ \sin\alpha & \sin\beta\cos\alpha & \cos\alpha\cos\beta \ \cancel{\cos\beta\cos\alpha} & \sin\alpha & \sin\beta\cos\alpha \\ \cos\alpha & -\sin\alpha\sin\beta & -\sin\alpha\cos\beta & \cos\alpha & -\sin\alpha\sin\beta \\ 0 & \cos\beta & -\sin\beta \ \cancel{\cos\beta} & 0 & \cos\beta \end{array}$$

$$\begin{bmatrix} \cos\alpha & -\sin\alpha\sin\beta & -\sin\alpha\cos\beta \\ 0 & \cos\beta & -\sin\beta \\ \sin\alpha & \sin\beta\cos\alpha & \cos\alpha\cos\beta \end{bmatrix}^T$$

$$R^{-1} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ -\sin\alpha\sin\beta & \cos\beta & \sin\beta\cos\alpha \\ -\sin\alpha\cos\beta & -\sin\beta & \cos\alpha\cos\beta \end{bmatrix}$$

Notes