Quantum Approximate Optimization Algorithm (QAOA) as a Discretized and Optimized Adiabatic Path

A Theoretical and Numerical Investigation of QAOA as a Shortcut to Adiabaticity

The Fear State Vector

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Abstract

- This report explores QAOA as a discretized realization of Adiabatic Quantum Computation (AQC). We derive the formal mapping via Trotter-Suzuki decomposition.
- We demonstrate QAOA replaces a fixed, slow time schedule with optimized, non-linear time steps ($\vec{\gamma}$, $\vec{\beta}$).
- We analyze its behavior using a 3-qubit MaxCut problem and detail advanced heuristics like Recursive QAOA (RQAOA).
- We show QAOA provides a "shortcut to adiabaticity," enhancing performance and addressing the noise limitations of NISQ hardware.

Introduction & Fundamentals

The NISQ Era & Motivation

NISQ (Noisy Intermediate-Scale Quantum) hardware is limited by noise, gate errors, and short coherence times.

Deep circuits, like those required for full AQC, are impractical.

Goal: Show QAOA is a theoretically-grounded, shallow-circuit alternative that outperforms AQC's strict time constraints.

Qubit & Gates Fundamentals

Qubit: A quantum state in superposition. $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Hamiltonians: Encoded using Pauli operators, which form the basis for problem and mixer terms.

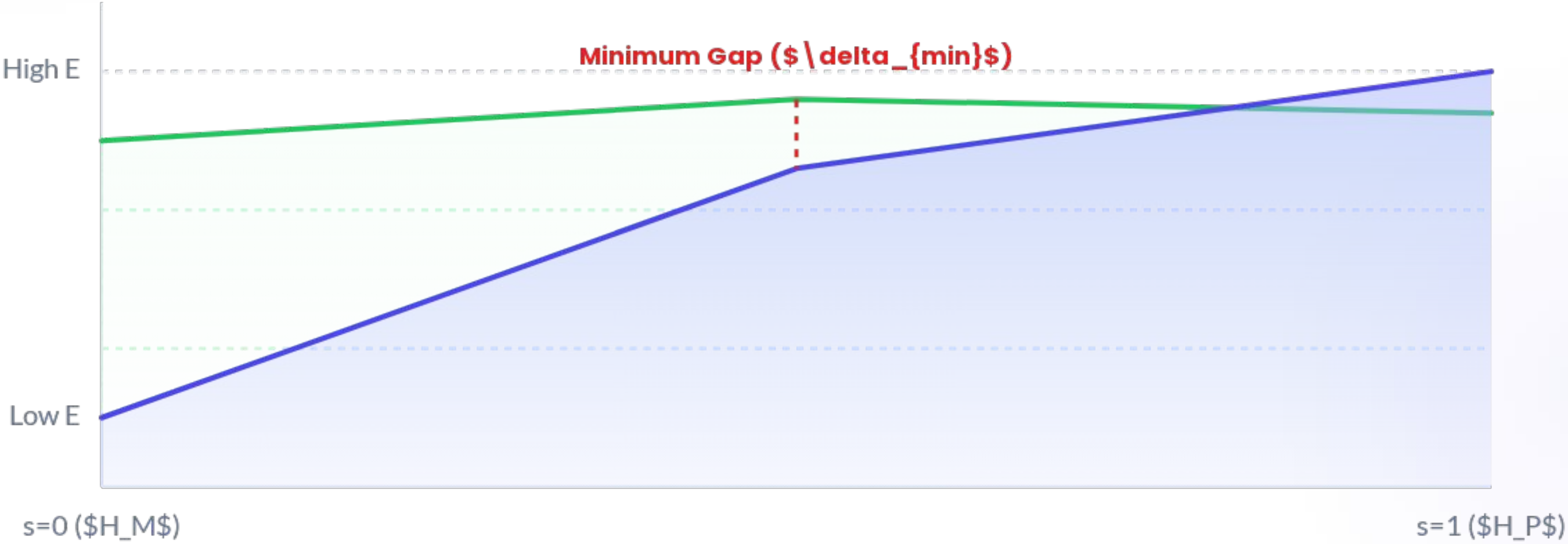
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Adiabatic Quantum Computation (AQC)

AQC evolves a system from an initial mixer Hamiltonian (H_m) to a final problem Hamiltonian(H_p) :

$H(s)=(1-s)H_m+sH_p$

The Constraint: Evolution time T is dominated by the minimum energy gap (δmin). An exponentially small gap forces an exponentially long time: T ∝ 1 / (δ min)^2



The Bridge: From AQC to QAOA

1. Continuous AQC

A smooth, continuous evolution over time

$T.$ $U(t) = e^{-i \int_{0}^{T} H(t')dt'}$

2. Trotterization

Discretize the evolution into p steps.

$U(0,T) \approx \left[\prod_{k=1}^{p} e^{-i(1-8_{k})\Delta t H_m} e^{(-i s_k \Delta t H_p)}\right]$

3. Parameter Mapping

Re-label the discrete time segments as trainable parameters.

Problem Angle: $\gamma_k = s_k \Delta t$

Mixer Angle: $\beta_k = (1 - s_k)\Delta t$

4. QAOA Ansatz

The result is the variational QAOA circuit.

$|\psi(\gamma,\beta)\rangle = \prod_{k=1}^{p} e^{-i\beta_k H_m} e^{-i\gamma_k H_p} |+\rangle^{\otimes n}$

QAOA Circuit Layers

1. Problem Operator $U_P(\gamma)$

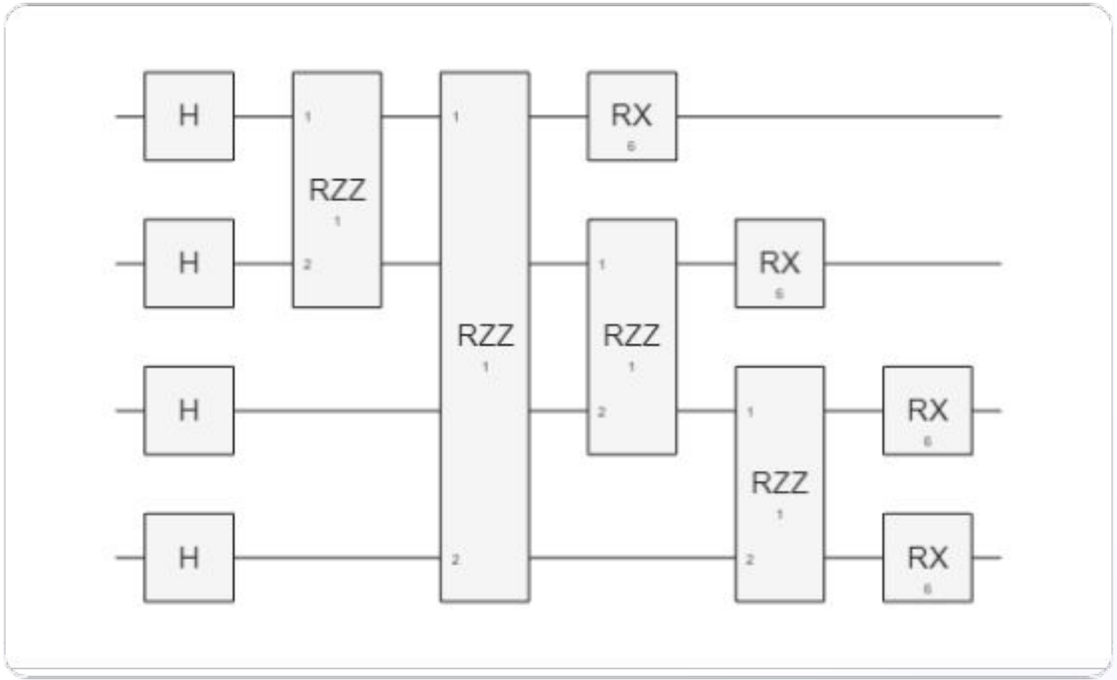
Encodes the cost function into the phase. For MaxCut, this involves R_{ZZ} gates for each edge in the graph.

$$U_P(\gamma) = e^{-i\gamma H_P} = \prod_{(i,j)} e^{-i\gamma Z_i Z_j}$$

2. Mixer Operator $U_M(\beta)$

Explores the solution space by allowing transitions between states. This is a layer of parallel R_X rotations.

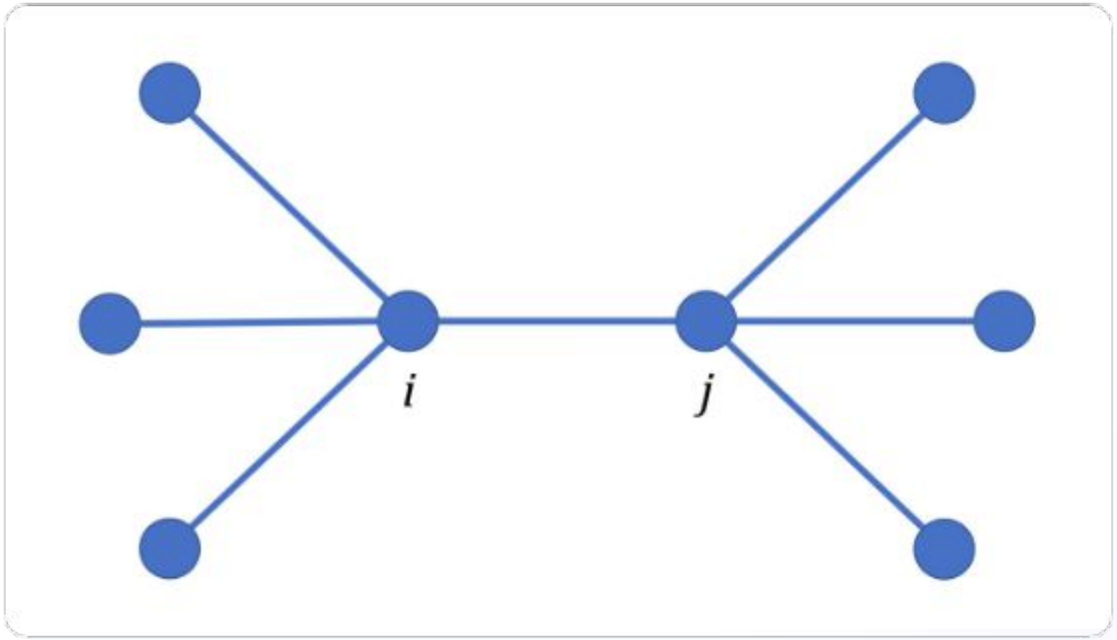
$$U_M(\beta) = e^{-i\beta H_M} = \prod_i e^{-i\beta X_i}$$



Case Study: 3-Qubit MaxCut

Problem: Triangle Graph

We analyze a simple 3-vertex graph to find the partition that maximizes the edges cut (C_{max}=2).



Problem Hamiltonian (H_P)

Encodes the cost of each edge.

$$H_P = \frac{1}{2}\left[(1 - Z_0 Z_1) + (1 - Z_1 Z_2) + (1 - Z_0 Z_2) \right]$$

Mixer Hamiltonian (H_M)

Enables transitions between all states.

$$H_M = X_0 + X_1 + X_2$$

Optimization Landscape

The Classical Challenge

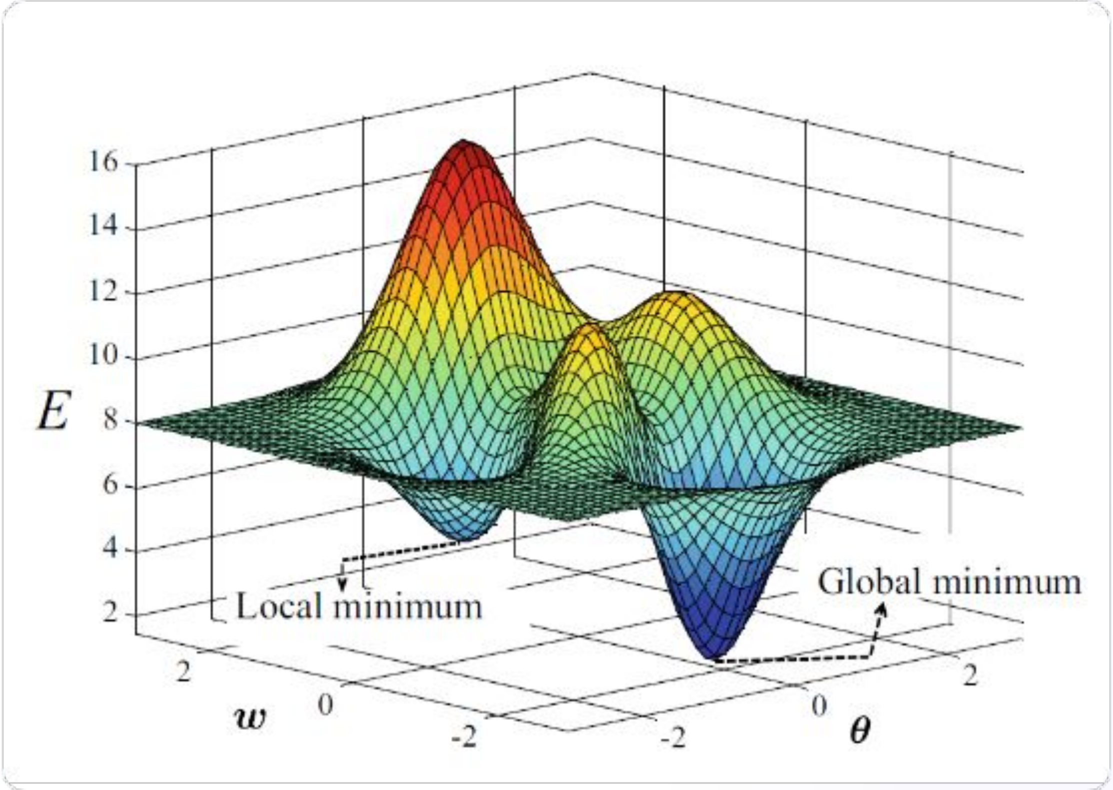
The classical optimizer's job is to find the optimal angles (γ^*, β^*) that maximize the measured cost $\langle H_p \rangle$

Barren Plateaus

A key challenge for VQAs. As the problem size or depth increases, the landscape can become "flat" (gradients vanish), making optimization exponentially hard.

Our Approach: Derivative-Free

We use optimizers like **COBYLA** or **Nelder-Mead**. They are robust to the statistical noise from quantum measurements and do not require calculating gradients.



Advanced Heuristics: Recursive QAOA (RQAOA)

1. Shallow QAOA

Run a low-depth QAOA ($p<=2$) on the N-qubit graph. This is fast and noise-resilient.

2. Measure Correlations

Calculate the correlators $\langle Z_i Z_j \rangle$. Find the edge with the strongest correlation.

3. Classical Reduction

Enforce the correlation ($(Z_i = +\text{-}Z_j)$ and classically simplify the graph to N-1 qubits.

4. Repeat

Loop on the smaller graph. This avoids deep circuits and barren plateaus entirely.

Results & The "Shortcut" Mechanism

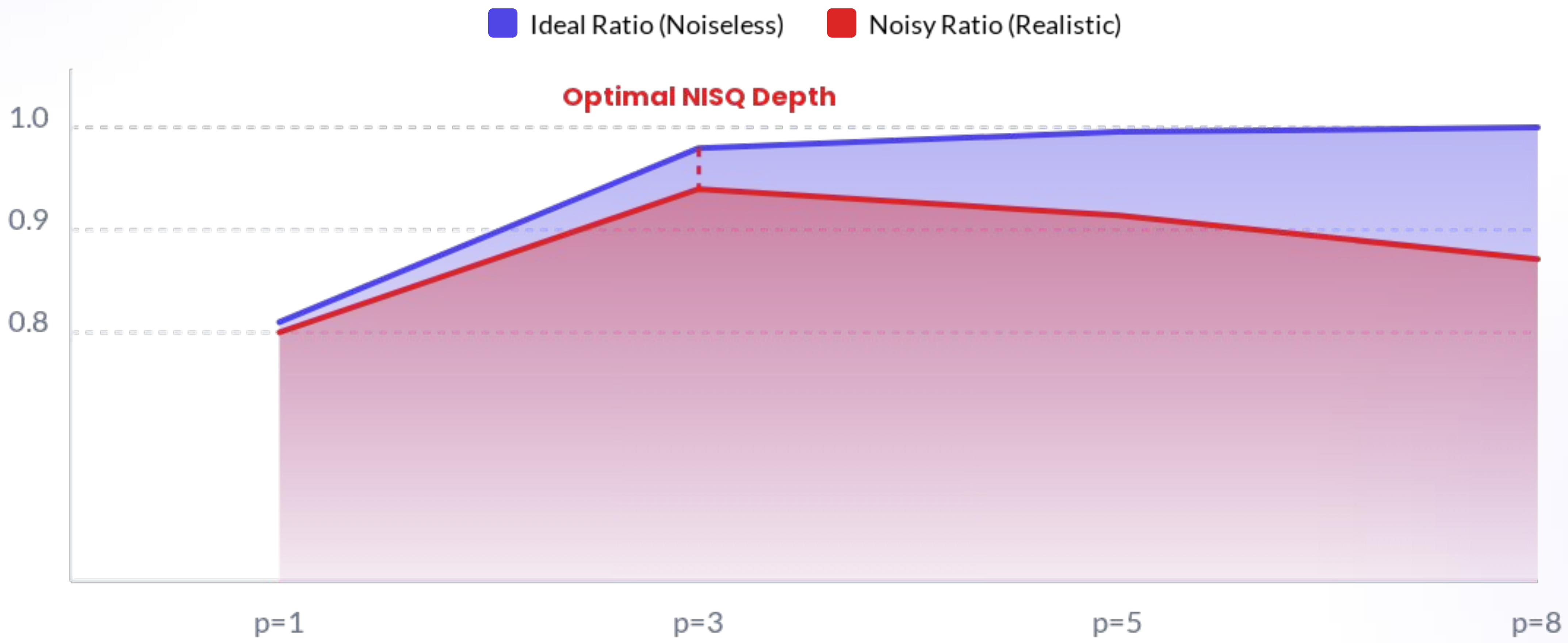
Data for the 3-Qubit MaxCut problem shows the approximation ratio (quality of solution) rapidly improves with just a few layers.

The Shortcut: *The optimized* (γ^*, β^*) *parameters are *not* linear like AQC's schedule. They define a non-linear, high-speed path that acts as a **"counterdiabatic" drive**, suppressing errors and finding the solution much faster than AQC's time limit.*

Noise & Realistic Performance

There is a trade-off: higher p gives better theoretical accuracy but also accumulates more noise.

Result: Real performance *peaks* at a shallow depth and then *degrades*.



Discussion & Conclusion

Discussion

QAOA is the practical, variational implementation of AQC, perfectly suited for the NISQ era.

Its flexibility and optimized "shortcut" mechanism (via counterdiabatic driving) allow it to overcome the rigid time constraints and noise limitations that make traditional AQC unfeasible.

Conclusion

QAOA is not just a heuristic; it is a **theoretically grounded** algorithm that achieves near-adiabatic performance with shallow, fixed-depth circuits.

It is confirmed as a powerful and viable pathway toward achieving quantum advantage for optimization problems.

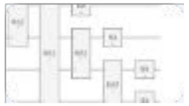
Questions?

Thank You | The Fear State Vector

Key References

- Farhi, E. et al. *Quantum Approximate Optimization Algorithm.* arXiv:1411.4028 (2014).
- Zhou, L. et al. *QAOA: Performance, Mechanism, and Implementation.* Nature (2020).
- Zhu, D. et al. *Shortcuts to Adiabaticity in QAOA.* Phys. Rev. Research (2022).

Image Sources



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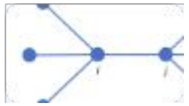
Source: www.mathworks.com



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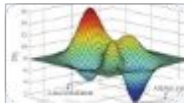

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