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Grade	**5.58** out of 15.00 (**37**%)

Question **1**

Incorrect

Mark 0.00 out of 1.00

Consider the vector space $\mathbb{R}_2[x]$ over $\mathbb{R}$ with the inner product $< f, g >= \int_{-1}^{1} f(x)g(x)dx$. The ordered basis $(x^2, x, 1)$ when converted to an orthogonal ordered basis, by taking elements in the given order, has the form

- a. $(x^2, x, 5 - 3x^2)$
- b. $(x^2, x, 1)$
- c. $(1, x, 3x^2 - 1)$ ✖
- d. $(x^2, x, 3 - 5x^2)$

Your answer is incorrect.

The correct answer is:
$(x^2, x, 3 - 5x^2)$

Question **2**

Incorrect

Mark 0.00 out of 1.00

Let $T : \mathbb{R}_3[x] \to \mathbb{R}_3[x]$ be a linear transformation over $\mathbb{R}$ given by $T(p(x)) = \frac{d}{dx}(p(x)) + p(x)$. What are the values of $\mathrm{null}(T)$ and $\mathrm{rank}(T)$?

- a. 3 and 1
- b. 2 and 2
- c. 4 and 0
- d. 0 and 4
- e. 1 and 3 ✖

Your answer is incorrect.

The correct answer is:
0 and 4

Question **3**

Incorrect

Mark 0.00 out of 1.00

Let $W = \{(1,1,-1)\}$ in the vector space $\mathbb{R}^3$ over $\mathbb{R}$. Inner product is the usual dot product. Which of the following are true?

- [ ] a. $(W^{\perp})^{\perp} = \{(1,1,-1)\}$
- [x] b. $W^{\perp} = \{(0,1,1)\}$ ✖
- [ ] c. $W^{\perp} = \{(1,0,1)\}$
- [x] d. $W^{\perp} = \{(1,0,1),(0,1,1)\}$ ✖
- [ ] e. $(W^{\perp})^{\perp} = \text{span}\{(-1,-1,1)\}$
- [ ] f. $W^{\perp} = \text{span}\{(1,1,2),(1,-1,0)\}$

Your answer is incorrect.

The correct answers are:

$W^{\perp} = \text{span}\{(1,1,2),(1,-1,0)\}$

,

$(W^{\perp})^{\perp} = \text{span}\{(-1,-1,1)\}$

Question **4**

Incorrect

Mark 0.00 out of 1.00

Consider the linear trasformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ over the field $\mathbb{R}$ given by $T((x,y,z)) = (x+y-z, x-y-z)$. A basis for $\ker(T)$ is

- [ ] a. $\{(1,1),(1,0)\}$
- [ ] b. $\{(1,0,1)\}$
- [x] c. $\{(1,1),(1,-1)\}$ ✖
- [ ] d. $\{(-1,0,-1)\}$

Your answer is incorrect.

The correct answers are:

$\{(1,0,1)\}$

,

$\{(-1,0,-1)\}$

Question 5
Partially correct
Mark 0.75 out of 1.00

Which of the following statements are true in an inner product space?

- [ ] a. linearly independent vectors are orthogonal
- [x] b. orthonormal vectors are orthogonal ✔
- [x] c. orthonormal vectors are linearly independent ✔
- [x] d. orthogonal vectors are linearly independent ✖
- [ ] e. linearly independent vectors are orthonormal
- [ ] f. orthogonal vectors are orthonormal

Your answer is partially correct.

You have selected too many options.
The correct answers are:
orthonormal vectors are orthogonal,

orthonormal vectors are linearly independent

Question 6
Partially correct
Mark 0.67 out of 1.00

What is always true abut the Hamel Base B of the vector space V over the field F?

- [x] a. Only finite linear combinations are taken from B ✔
- [ ] b. Only infinite linear combinations are taken from B
- [ ] c. B is finite
- [ ] d. B is uncountable
- [ ] e. $V = \text{span}(\text{span} B)$
- [ ] f. B is countable
- [x] g. B always exists ✔

Your answer is partially correct.

You have correctly selected 2.
The correct answers are:
Only finite linear combinations are taken from B
,
$V = \text{span}(\text{span} B)$
,
B always exists

Question 7
Correct
Mark 1.00 out of 1.00

Let $T : \mathbb{R}_3[x] \to \mathbb{R}_3[x]$ be a linear transformation over $\mathbb{R}$ given by $T(p(x)) = \frac{d}{dx}(p(x))$. What are the values of $\text{null}(T)$ and $\text{rank}(T)$?

- a. 4 and 0
- b. 2 and 2
- c. 3 and 1
- d. 1 and 3 ✔
- e. 0 and 4

Your answer is correct.

The correct answer is:
1 and 3

Question 8
Incorrect
Mark 0.00 out of 1.00

Consider the linear Transformation $T : \mathbb{R}_2[x] \to \mathbb{R}_3[x]$ over $\mathbb{R}$ given by $T(p(x)) = \frac{d}{dx}(p(x)) + \int_0^x p(t)dt$. With the ordered base $(1, x, x^2)$ given for $\mathbb{R}_2[x]$ and the ordered base $(1, x, x^2, x^3)$ given for $\mathbb{R}_3[x]$, what is the matrix of T?

Here we use a notation where a matrix with 1st ROW (a, b, c) and 2nd ROW (e, f, g) is written as $((a, b, c), (d, e, f))$

- a. $((0, 1, 0), (1, 0, 2), (1, \frac{1}{2}, 0), (0, 0, \frac{1}{3}))$
- b. $((0, 1, 0, 0), (1, 0, \frac{1}{3}, 0), (0, 2, 0, \frac{1}{4}))$
- c. $((0, 1, 0), (1, 0, 2), (1, \frac{1}{3}, 0), (0, 0, \frac{1}{4}))$
- d. $((0, 1, 0, 0), (1, 0, \frac{1}{2}, 0), (0, 2, 0, \frac{1}{3}))$ ✖

Your answer is incorrect.

The correct answer is:
$((0, 1, 0), (1, 0, 2), (1, \frac{1}{2}, 0), (0, 0, \frac{1}{3}))$

Question 9

Incorrect

Mark 0.00 out of 1.00

Which of the following are examples for "subspace U of the vector space V over the field F"?
Here vector addition is usual addition and scalar multiplication is usual multiplication.

- [ ] a. $\mathbb{Q}$ of $\mathbb{R}$ over $\mathbb{Q}$
- [ ] b. $\mathbb{R}$ of $\mathbb{C}$ over $\mathbb{R}$
- [x] c. $\mathbb{Z}$ of $\mathbb{C}$ over $\mathbb{R}$ ✖
- [x] d. $\mathbb{Z}$ of $\mathbb{Q}$ over $\mathbb{Z}$ ✖
- [ ] e. $\mathbb{C}$ of $\mathbb{R}$ over $\mathbb{Q}$
- [x] f. $\mathbb{R}$ of $\mathbb{C}$ over $\mathbb{Q}$ ✔
- [ ] g. $\mathbb{Z}$ of $\mathbb{Q}$ over $\mathbb{Q}$

Your answer is incorrect.

The correct answers are:
$\mathbb{Q}$ of $\mathbb{R}$ over $\mathbb{Q}$
,
$\mathbb{R}$ of $\mathbb{C}$ over $\mathbb{R}$

,
$\mathbb{R}$ of $\mathbb{C}$ over $\mathbb{Q}$

Question 10

Partially correct

Mark 0.42 out of 1.00

Which of the following are examples for "vector space V over the field F"?
Here vector addition is usual addition and scalar multiplication is usual multiplication.

- [x] a. $\mathbb{R}$ over $\mathbb{Q}$ ✔
- [x] b. $\mathbb{Q}$ over $\mathbb{Z}$ ✘
- [ ] c. $\mathbb{Q}$ over $\mathbb{R}$
- [ ] d. $\mathbb{Z}$ over $\mathbb{Q}$
- [x] e. $\mathbb{Q}$ over $\mathbb{Q}$ ✔
- [ ] f. $\mathbb{R}$ over $\mathbb{R}$
- [ ] g. $\mathbb{Z}$ over $\mathbb{Z}$

Your answer is partially correct.

You have correctly selected 2.
The correct answers are:
$\mathbb{R}$ over $\mathbb{R}$
,
$\mathbb{R}$ over $\mathbb{Q}$
,
$\mathbb{Q}$ over $\mathbb{Q}$

Question 11

Correct

Mark 1.00 out of 1.00

Consider the set of solutions V of the differential equation $\frac{dy}{dx} - \frac{1}{2y} = 0$ over the field $F = \mathbb{R}$. Which of the following are true?

- [x] a. $\{\sqrt{x}, -\sqrt{x}\} \subset V$ ✔
- [ ] b. $\{\sqrt{x}, -\sqrt{x}\}$ is a basis for V
- [ ] c. $\{\sqrt{x}, -\sqrt{x}\}$ is linearly independent
- [x] d. V is not a vector space ✔
- [ ] e. $V = \mathrm{span}\{\sqrt{x}, -\sqrt{x}\}$

Your answer is correct.

The correct answers are:
V is not a vector space
,
$\{\sqrt{x}, -\sqrt{x}\} \subset V$

Question 12
Partially correct
Mark 0.67 out of 1.00

Which of the following are inner products $< u, v >$ in the vector space V over the field F?
Here the vector addition is usual addition and the scalar multiplication is multiplying by a number.

- [x] a. $< x, y >= xy$ in $\mathbb{R}$ over $\mathbb{R}$ ✔
- [ ] b. $< A, B >= \det(AB)$ in $\mathbb{R}^{n\times n}$ over $\mathbb{R}$
- [x] c. $< f, g >= \int_0^\infty f(x)g(x)dx$ in $\mathcal{C}[0, \infty)$ over $\mathbb{R}$ ✘
- [ ] d. $< f, g >= \int_0^\infty e^{-x} f(x)g(x)dx$ in $\mathcal{C}[0, \infty)$ over $\mathbb{R}$

Your answer is partially correct.

You have selected too many options.
The correct answer is:
$< x, y >= xy$ in $\mathbb{R}$ over $\mathbb{R}$

Question 13
Partially correct
Mark 0.75 out of 1.00

Consider the inner product $< f, g >= \int_{-1}^{1} f(x)g(x)dx$ in the vector space $\mathbb{R}_3[x]$ over $\mathbb{R}$. The best approximation to $2 + 3x + 4x^2$ in $W = \text{span}\{1, x\}$ is

- [ ] a. $4x^2$
- [ ] b. 0
- [ ] c. $4 + 2x$
- [x] d. $2 + 3x$ ✘
- [x] e. $\frac{10}{3} + 3x$ ✔

Your answer is partially correct.

You have selected too many options.
The correct answer is:
$\frac{10}{3} + 3x$

Question 14
Partially correct
Mark 0.33 out of 1.00

Let $V = \mathbb{C}^{n\times n}$ and $F = \mathbb{C}$. The vector addition is the matrix addition and the scalar multiplication is multiplying the matrix by a number. Also define the function $< A, B >= \text{trace}(A^H B)$ where $A^H = \overline{A}^T$ is the conjugate transpose.

- a. V is a vector space over F if the vector addition is the matrix multiplication.
- b. $< A, A >= 0 \Leftrightarrow A = O$ is not true
- c. V is a vector space over F.
- d. Cauchy-Schwarz inequality looks like $|\text{trace}(A^H B)| \leq \|A\|_E \|B\|_E$ where $\|A\|_E = \sqrt{\sum_{i=1}^{n} |a_{ii}|^2}$ is the Frobeneous norm.
- e. $< A, B >$ is an inner product on V

Your answer is partially correct.

You have correctly selected 1.
The correct answers are:
V is a vector space over F.

,

$< A, B >$ is an inner product on V

,

Cauchy-Schwarz inequality looks like $|\text{trace}(A^H B)| \leq \|A\|_E \|B\|_E$ where $\|A\|_E = \sqrt{\sum_{i=1}^{n} |a_{ii}|^2}$ is the Frobeneous norm.

Question 15
Incorrect
Mark 0.00 out of 1.00

Consider the linear trasformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ over the field $\mathbb{R}$ given by $T((x, y, z)) = (x + y - z, x - y - z)$. A basis for $\text{ran}(T)$ is

- a. $\{(1, 0, 1)\}$
- b. $\{(1, 1), (1, 0)\}$
- c. $\{(-1, 0, -1)\}$
- d. $\{(1, 1), (1, -1)\}$

Your answer is incorrect.

The correct answers are:
$\{(1, 1), (1, -1)\}$

,

$\{(1, 1), (1, 0)\}$

Previous activity

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Assignment 1-Due Sunday 5th June ►

Stay in touch

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No Date

1.) $\langle f,g \rangle = \int_{-1}^{1} f(x)g(x)dx$ $(x^2, x, 1)$

$y_1 = x^2$ $y_2 = x - \frac{\int_{-1}^{1} x \times x^2 dx}{\int_{-1}^{1} x^2 \times x^2 dx} \times x^2 = x - \frac{\left[\frac{x^4}{4}\right]_{-1}^{1}}{\left[\frac{x^5}{5}\right]_{-1}^{1}} x^2 = x$

$y_3 = 1 - \frac{\int_{-1}^{1} 1 \times x^2}{\int_{-1}^{1} x^4 dx} x^2 - \frac{\int_{-1}^{1} 1 \times x}{\int_{-1}^{1} x^2 da} \times x = 1 - \frac{\left(\frac{x^3}{3}\right)_{-1}^{1}}{\left(\frac{x^5}{5}\right)_{-1}^{1}} x^2 = 1 - \frac{5}{3}\frac{2x^2}{2}$
$= \frac{3-5x^2}{3}$

New basis.
$= (x^2, x, \frac{3-5x^2}{3})$ since ~~nor~~ n or m dosent matter,
$-(x^2, x, 3-5x^2)$

a ✗ b ✗ c ✗ ~~d✗~~ d ✓

2.) $T(P(x)) = P'(x) + P(x)$
Consider $ax^3 + bx^2 + cx + d = p(x)$

$T(p(x)) = 3ax^2 + 2bx + c + ax^3 + bx^2 + cx + d = 0$ to find $\ker(T)$
$= ax^3 + (3a+b)x^2 + (2b+c)x + (c+d) = 0$

$a=0$ $3a+b=0$ $2b+c=0$ $c+d=0$
$b=0$ $c=0$ $d=0$ $\ker(T) = \{\underline{0}\}$

$\therefore$ null $T = \underline{\underline{0}} \Rightarrow \text{rank}(T) = 4 - 0 = 4$.

3.) a) $(W^\perp)^\perp$ is also a span. Cannot be a single vector.
so (a) is wrong.

b.) Cannot be a single vector. (b) is wrong.

c.) (c) is wrong. $W^\perp$ is all vectors perpendicular to $(1,1,-1)$
which is clearly a vector space with dimension 2
$W^\perp$ can't be a single vector.

d.) Can't be just two vectors. should be a span. (d) wrong

e.) correct. $(W^\perp)^\perp = \text{span}\{(1,1,-1)\} = \text{span}\{(-1,-1,1)\}$ because
$(-1,-1,1) = -1(1,1,-1) =$ they are L.D.

f) Correct $(1,1,2)\cdot(1,-1,0)=0 \Rightarrow$ L.I
$(1,1,-1)\cdot(1,1,2)=0 \Rightarrow$ they are perpendicular
$(1,1,-1)\cdot(1,-1,0)=0 \Rightarrow$,, ,, ,,

$\therefore w^{\perp} = \text{span}\{(1,1,2),(1,-1,0)\}$

4.) $T(x,y,z)=(x+y-z,\ x-y-z)$ to find ker (T),

$x+y-z=0$ — ①
$x-y-z=0$ — ②

① − ② $2y=0$ $y=0$
$x=z$

$\begin{pmatrix} x \\ 0 \\ x \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$\therefore$ ker(T) = span$\{(1,0,1)\}$

a) X b) ✓ c) X d) ✓ $(-1,0,-1) = -1(1,0,1)$

5) a) X. (0,1) and (1,1) are L.I. but not orthogonal

b.) ~~corr~~ ✓ orthonormal means orthogonal and normal.

c.) ✓ Orthonormal vectors are L.I. It is a therom.

d.) X $\underline{0}$ is orthogonal to any vector. but $\underline{0}$ is L.D to any vector. ∴ not all orthogonal vectors are L.I.

e.) X (0,1), (1,1) are L.I. but not orthonormal.

f.) X (0,1) and (2,0) are orthogonal but not orthonormal.

6.) a.) ✓ definition of Hamel base

b.) X.

c.) X Hamel basis can be infinite

d.) X Hamel basis can be countable

e.) ✓ definition of Hamel basis.

f.) X B can be uncountable.

g.) ✓ Theorem.

7.) $T(p(x)) = p'(x)$ consider $ax^3+bx^2+cx+d = p(x)$
$T(p(x)) = 3ax^2+2bx+c=0$ ∴ we need ker(T)

$a=0$ $b=0$ $c=0$ d is a parameter.
ker(T) = span$\{(0,0,0,1)\}$ null T = 1 and rank T = 4 − 1 = 3

8) $T(p(x)) = p'(x) + \int_0^x p(t)\,dt$ $\quad (1, x, x^2)$ $\quad (1, x, x^2, x^3)$

$T(1) = 0 + \int_0^x dx = x = (0,1,0,0)^T$

$T(x) = 1 + \int_0^x t\,dt = 1 + \left[\frac{t^2}{2}\right]_0^x = 1 + \frac{x^2}{2} = (1,0,\frac{1}{2},0)^T$

$T(x^2) = 2x + \int_0^x t^2\,dt = 2x + \frac{x^3}{3} = (0,2,0,\frac{1}{3})^T$

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$ a) ✓ b) X c) X d) X

9) c) X $\mathbb{Z}$ is not a vector space over $\mathbb{R}$ not closed under scalar multiplication.
$2 \in \mathbb{Z}$, $0.25 \in \mathbb{R}$, $2 \times 0.25 = 0.5 \notin \mathbb{Z}$

d) X $\mathbb{Z}$ is not a Field

e) X $\mathbb{C}$ is not a subset of $\mathbb{R}$

g) X $\mathbb{Z}$ over $\mathbb{Q}$ is not a vector space. not closed under scalar multiplication.
$2 \in \mathbb{Z}$, $0.25 \in \mathbb{Q}$, $2 \cdot 0.25 = 0.5 \in \mathbb{Z}$

10) b) X $\mathbb{Z}$ is not a field.

c) X Not closed under scalar multiplication $2 \in \mathbb{Q}$ is a vector, $\pi \in \mathbb{R}$ is a scalar. $2\pi \in \mathbb{Q}$.

d) X not closed under scalar mutiplication. $2 \in \mathbb{Z}$ is a vector, $1.2 \in \mathbb{Q}$ is a scalar.
$2 \times 1.2 = 2.4 \notin \mathbb{Z}$

g) X $\mathbb{Z}$ is not a field. No multiplicative inverse
$2^{-1} \notin \mathbb{Z}$.

11) $\frac{dy}{dx} - \frac{1}{2y} = 0$ $\quad \int 2y\, dy = \int dx$.

$\frac{2y^2}{2} = x + C \quad y = \pm\sqrt{x+C}$ //

a) ✓ $\sqrt{x}$ and $-\sqrt{x}$ are solns when $c=0$

~~b) X $\sqrt{x+2}$~~ b) since $\sqrt{x}$ and $-\sqrt{x}$ are L.D,

$\{\sqrt{x}, -\sqrt{x}\}$ can't be a basis.

c) X $-\sqrt{x} = -1\sqrt{x} \Rightarrow$ they are L.D.

d) X $(\sqrt{x+3} + \sqrt{x+2}) \in V \Rightarrow$ not closed under addition.

e) X $\sqrt{x+3} \notin \text{span}\{\sqrt{x}, -\sqrt{x}\}$.

12.) a.) $\langle x,y \rangle = xy$ $\quad \mathbb{R}$ over $\mathbb{R}$

$\langle y,x \rangle = yx = xy = \langle x,y \rangle$

$\langle x+y, z \rangle = (x+y)z = xz + yz = \langle x,z \rangle + \langle y,z \rangle$

$\langle ax, y \rangle = axy = a(x,y) = a\langle x,y \rangle$

$\langle x,x \rangle = x^2 \geq 0$

$\langle x,x \rangle = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$.

a) is correct //.

13) $\langle f,g \rangle = \int_{-1}^{1} f(x)g(x)\,dx \quad u = 2 + 3x + 4x^2 \quad$ ~~wE~~ $W = [1, x\}$

First orthonormalize the basis :

$y_1 = 1, \; z_1 = \frac{y_1}{\|y_1\|} = \frac{1}{\sqrt{\int_{-1}^{1} dx}} = \frac{1}{\sqrt{[x]_{-1}^{1}}} = \frac{1}{\sqrt{2}} = z_1,$

$y_2 = x - \int_{-1}^{1} \frac{1}{\sqrt{2}} x\, dx \frac{1}{\sqrt{2}} = x - \frac{1}{2}\left[\frac{x^2}{2}\right]_{-1}^{1} = x$ //

$z_2 = \frac{x}{\|x\|} = \frac{x}{\sqrt{\int_{-1}^{1} x^2 dx}} = \frac{x}{\sqrt{\left[\frac{x^3}{3}\right]_{-1}^{1}}} = \sqrt{\frac{3}{2}}\, x$ //

$V = \left\{\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}\, x\right\} \leftarrow$ orthogonal basis .

$u = Pu + Qu$
$\uparrow$ closest approx.

$Pu = \int_{-1}^{1} (2+3x+4x^2)\frac{1}{\sqrt{2}}\,dx\,\frac{1}{\sqrt{2}} + \int_{-1}^{1}(2+3x+4x^2)\sqrt{\frac{3}{2}}x\,dx\,\frac{\sqrt{3}}{\sqrt{2}}x$

$= \frac{1}{2}\left(2[x]_{-1}^{1} + \frac{3}{2}[x^2]_{-1}^{1} + \frac{4}{3}[x^3]_{-1}^{1}\right) + \frac{3}{2}x\int_{-1}^{1}(2x+3x^2+4x^3)\,dx$

$= \frac{1}{2}\left(4+0+\frac{8}{3}\right) + \frac{3x}{2}\left([x^2]_{-1}^{1} + [x^3]_{-1}^{1} + [x^4]_{-1}^{1}\right)$

$= \frac{20}{6} + \frac{3x}{2}(0+2+0) = \frac{10}{3} + 3x$ //

(e) is correct.

14.) a) X matrix multiplication is not commutadive.

~~b)~~

e.) correct.

c.) showing V is a vector space.

1) V, + is obelian. $A, B \in \mathbb{C}^{n\times n} = V$.

- $V \neq \emptyset$.
- $A + B = \mathbb{C}^{n\times n}$
- $A + (B+C) - (A+B) + C$
- $A + 0 = 0 + A = A$
- $A + (-A) = (-A) + A = 0$
- $A + B = B + A$.

2.) $(\mathbb{C}, +, \cdot)$ is a field ✓

3.) $aA \in \mathbb{C}^{n\times n}$. $a \in \mathbb{C}$; $A \in \mathbb{C}^{n\times n}$.

4) $a(A+B) = aA + aB \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}^{n\times n}$.

5.) $(a+b)A = aA + bA \in \mathbb{C}^{n\times n}$ $b \in \mathbb{C}$

6.) $(ab)A = a(bA)$

7.) $IA = A$ $I \in \mathbb{C}^{n\times n}$.

15.) $T(x,y,z) = (x+y-z, x-y-z)$

let's consider the standard basis

$T((1,0,0)) = (1,1)$

$T((0,1,0)) = (1,-1)$

the right side of the L.T is $\mathbb{R}^2$. And we already have 2. L.I. vectors. That means $\forall v \in \mathbb{R}^2$ is rank(T).
2D vectors
so the basis can be any 2 L.I.

a) X b) ✓ c) X d) ✓