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Programming Assignment 1 Report: Dynamic Modelling and State-Space Representation of the RRBot Robotic Arm

Defining all variables as symbolic:

```
%% Defining / Creating Symbols
syms l1 l2 r1 r2 h1 h2 theta_1(t) theta_2(t) theta_ddot1 theta_ddot2 T1 T2 g 'real'
syms m1 m2 v1 v2 w1 w2 I1 I2 'real'
```

Lagrangian Formulation:

Step 1: choose a set of independent coordinates $q \in \mathbb{R}^n$ that describes the system's configuration $\rightarrow$ generalized coordinates:

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} \in \mathbb{R}^n$$

This is done in the MATLAB code as follows:

```
%% Defining Generalised Co-ordinates
q = sym('q', [2,1]);
q(1) = theta_1;
q(2) = theta_2;
```

Step 2: define the generalized inputs $u \in \mathbb{R}^n$ acting on each q_i:

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \in \mathbb{R}^n$$

This is done in the MATLAB code as follows:

```
%% Defining Generalised Inputs
u = sym('u', [2,1]);
u(1) = T1;
u(2) = T2;
```

Step 3: define the Lagrangian function $L(q, \dot{q})$:

Lagrangian

$$L(q, \dot{q}) = K(q, \dot{q}) - P(q)$$

K: Kinetic energy $\qquad$ P: Potential energy

The KE and PE equations are formed in MATLAB as shown below and then they are substituted in the Lagrangian to get L.

```
%% Kinetic Energy Equations
KE = (m1*(v1.^2))/2 + (I1*w1.^2)/2 +  (m2*(v2.^2))/2 + (I2*w2.^2)/2;
KE = subs(KE,[v1,v2,w1,w2],[sqrt(x1_dot^2 + y1_dot^2), sqrt(x2_dot^2 + y2_dot^2), diff(theta_1,t), diff(theta_1 + theta_2,t)]);
KE = simplify(KE);
```

```
%% Potential Energy Equations
PE = (m1*g*h1) + (m2*g*h2);
PE = subs(PE,[h1,h2],[y1,y2]);
```

Step 4: derive the Euler-Lagrange equations:

Euler-Lagrange Equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = u_i, \qquad i = 1, \ldots, n$$

u_i: generalized force applied to q_i

Kinetic Energy :

$$\frac{I_1 \left(\frac{\partial}{\partial t}\,\theta_1(t)\right)^2}{2} + \frac{I_2 \left(\frac{\partial}{\partial t}\,\theta_1(t)\right)^2}{2} + \frac{I_2 \left(\frac{\partial}{\partial t}\,\theta_2(t)\right)^2}{2} + I_2 \frac{\partial}{\partial t}\,\theta_2(t) \frac{\partial}{\partial t}\,\theta_1(t) + \frac{{l_1}^2 m_2 \left(\frac{\partial}{\partial t}\,\theta_1(t)\right)^2}{2} + \frac{m_1 {r_1}^2 \left(\frac{\partial}{\partial t}\,\theta_1(t)\right)^2}{2} + \frac{m_2 {r_2}^2 \left(\frac{\partial}{\partial t}\,\theta_1(t)\right)^2}{2} + \frac{m_2 {r_2}^2 \left(\frac{\partial}{\partial t}\,\theta_2(t)\right)^2}{2} + m_2 {r_2}^2 \frac{\partial}{\partial t}\,\theta_2(t) \frac{\partial}{\partial t}\,\theta_1(t) + l_1 m_2 r_2 \cos(\theta_2(t)) \left(\frac{\partial}{\partial t}\,\theta_1(t)\right)^2 + l_1 m_2 r_2 \cos(\theta_2(t)) \frac{\partial}{\partial t}\,\theta_2(t) \frac{\partial}{\partial t}\,\theta_1(t)$$

Potential Energy:

$$(\cos(\theta_1(t) + \theta_2(t))\, r_2 + \cos(\theta_1(t))\, l_1)\, g\, m_2 + \cos(\theta_1(t))\, g\, m_1\, r_1$$

Lagrangian Equation :

$$\frac{I_1 \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2}{2} + \frac{I_2 \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2}{2} + \frac{I_2 \left(\frac{\partial}{\partial t}\, \theta_2(t)\right)^2}{2} + I_2 \frac{\partial}{\partial t}\, \theta_2(t) \frac{\partial}{\partial t}\, \theta_1(t) + \frac{l_1^2\, m_2 \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2}{2} + \frac{m_1\, r_1^2 \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2}{2} + \frac{m_2\, r_2^2 \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2}{2} + \frac{m_2\, r_2^2 \left(\frac{\partial}{\partial t}\, \theta_2(t)\right)^2}{2} + m_2\, r_2^2 \frac{\partial}{\partial t}\, \theta_2(t) \frac{\partial}{\partial t}\, \theta_1(t) - g\, m_2\, r_2 \cos(\theta_1(t) + \theta_2(t)) - g\, l_1\, m_2 \cos(\theta_1(t)) - g\, m_1\, r_1 \cos(\theta_1(t)) + l_1\, m_2\, r_2 \cos(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2 + l_1\, m_2\, r_2 \cos(\theta_2(t)) \frac{\partial}{\partial t}\, \theta_2(t) \frac{\partial}{\partial t}\, \theta_1(t)$$

Eq1 :

$$I_1\, \theta_{\text{ddot1}} - T_1 + I_2\, \theta_{\text{ddot1}} + I_2\, \theta_{\text{ddot2}} + l_1^2\, m_2\, \theta_{\text{ddot1}} + m_1\, r_1^2\, \theta_{\text{ddot1}} + m_2\, r_2^2\, \theta_{\text{ddot1}} + m_2\, r_2^2\, \theta_{\text{ddot2}} - g\, m_1\, r_1 \sin(\theta_1(t)) - g\, m_2\, r_2 \sin(\theta_1(t) + \theta_2(t)) - g\, l_1\, m_2 \sin(\theta_1(t)) - l_1\, m_2\, r_2 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_2(t)\right)^2 + 2\, l_1\, m_2\, r_2\, \theta_{\text{ddot1}} \cos(\theta_2(t)) + l_1\, m_2\, r_2\, \theta_{\text{ddot2}} \cos(\theta_2(t)) - 2\, l_1\, m_2\, r_2 \sin(\theta_2(t)) \frac{\partial}{\partial t}\, \theta_2(t) \frac{\partial}{\partial t}\, \theta_1(t) = 0$$

Eq2:

$$-T_2 - \sin(\theta_1(t) + \theta_2(t))\, g\, m_2\, r_2 + I_2\, \theta_{\text{ddot2}} + m_2\, r_2^2\, \theta_{\text{ddot2}} + I_2\, \theta_{\text{ddot1}} + m_2\, r_2^2\, \theta_{\text{ddot1}} + \cos(\theta_2(t))\, l_1\, m_2\, r_2\, \theta_{\text{ddot1}} + \sin(\theta_2(t))\, l_1\, m_2\, r_2 \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2$$

b) State Space Representation:

$$X = \begin{bmatrix} \theta_1(t) \\ \frac{\partial}{\partial t}\, \theta_1(t) \\ \theta_2(t) \\ \frac{\partial}{\partial t}\, \theta_2(t) \end{bmatrix}$$

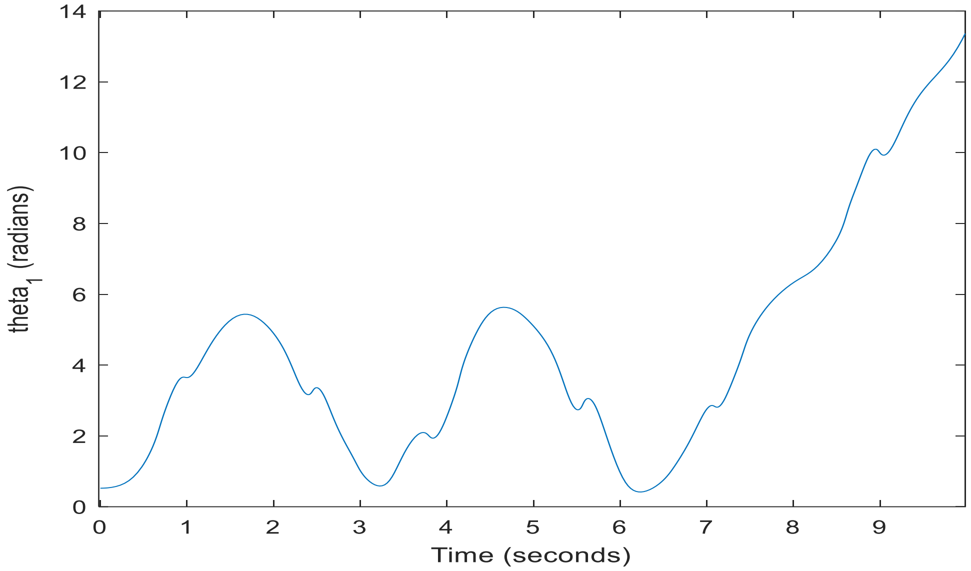
Theta_dot1 =

$$\frac{I_2\, T_1 - I_2\, T_2 + T_1\, m_2\, r_2^2 - T_2\, m_2\, r_2^2 + g\, l_1\, m_2^2\, r_2^2 \sin(\theta_1(t)) + I_2\, g\, l_1\, m_2 \sin(\theta_1(t)) + I_2\, g\, m_1\, r_1 \sin(\theta_1(t)) - T_2\, l_1\, m_2\, r_2 \cos(\theta_2(t)) + l_1\, m_2^2\, r_2^3 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2 + l_1\, m_2^2\, r_2^3 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_2(t)\right)^2 + g\, m_1\, m_2\, r_1\, r_2^2 \sin(\theta_1(t)) + 2\, l_1\, m_2^2\, r_2^3 \sin(\theta_2(t)) \frac{\partial}{\partial t}\, \theta_2(t) \frac{\partial}{\partial t}\, \theta_1(t) + I_2\, l_1\, m_2\, r_2 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2 + I_2\, l_1\, m_2\, r_2 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_2(t)\right)^2 + l_1^2\, m_2^2\, r_2^2 \cos(\theta_2(t)) \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2 - g\, l_1}{I_1\, I_2 + l_1^2\, m_2^2\, r_2^2 + I_2\, l_1^2\, m_2 + I_2\, m_1\, r_1^2 + I_1\, m_2\, r_2^2 - l_1^2\, m_2^2\, r_2^2 \cos(\theta_2(t))^2 + m_1\, m_2\, r_1^2\, r_2^2}$$

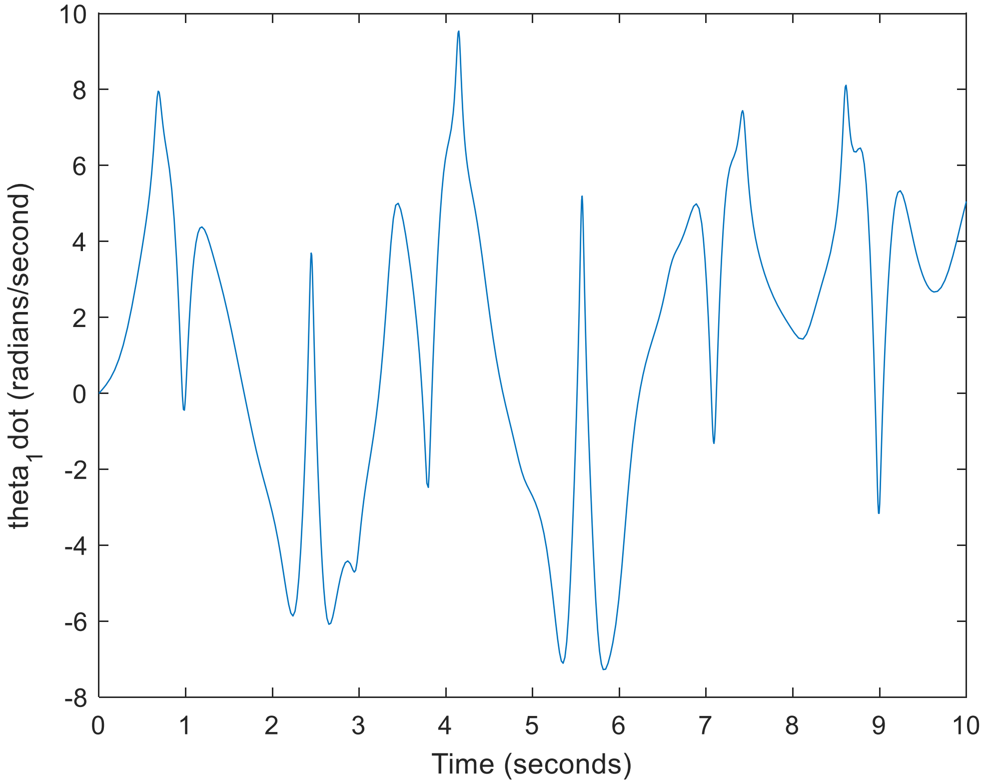
Theta_dot2 =

$$\frac{I_2\, T_1 - I_1\, T_2 - I_2\, T_2 - T_2\, l_1^2\, m_2 - T_2\, m_1\, r_1^2 + T_1\, m_2\, r_2^2 - T_2\, m_2\, r_2^2 + g\, l_1\, m_2^2\, r_2^2 \sin(\theta_1(t)) + I_2\, g\, l_1\, m_2 \sin(\theta_1(t)) + I_2\, g\, m_1\, r_1 \sin(\theta_1(t)) + T_1\, l_1\, m_2\, r_2 \cos(\theta_2(t)) - 2\, T_2\, l_1\, m_2\, r_2 \cos(\theta_2(t)) - g\, l_1^2\, m_2^2\, r_2 \sin(\theta_1(t) + \theta_2(t)) - I_1\, g\, m_2\, r_2 \sin(\theta_1(t) + \theta_2(t)) + I_1\, m_2^2\, r_2^3 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2 + l_1^3\, m_2^2\, r_2 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_1(t)\right)^2 + I_1\, m_2^2\, r_2^3 \sin(\theta_2(t)) \left(\frac{\partial}{\partial t}\, \theta_2(t)\right)^2 + g\, m_1\, m_2\, r_1\, r_2^2 \sin(\theta_1(t)) + 2\, I_1\, m_2^2}{}$$

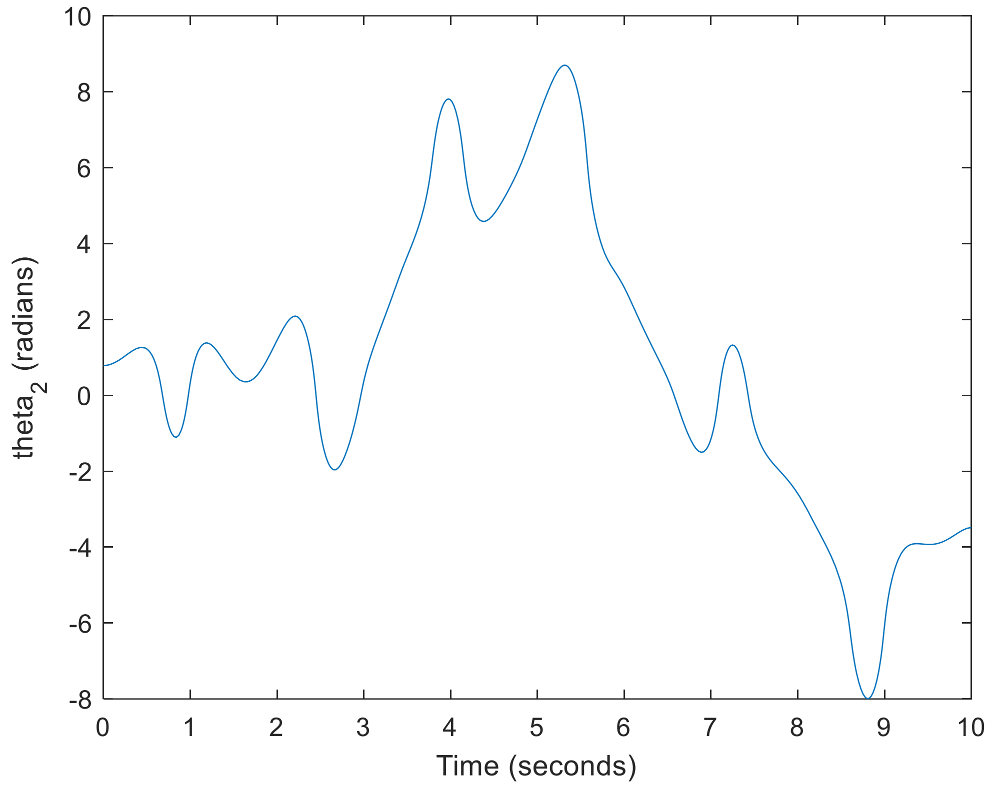
Graphs : -

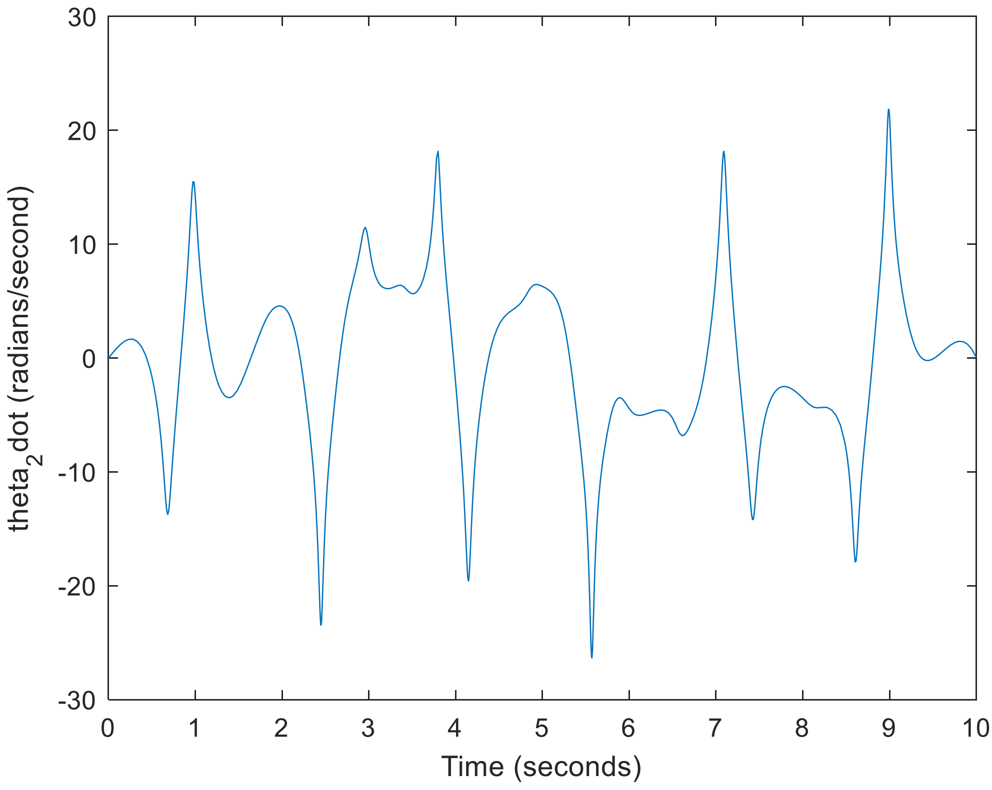


14
12
10
8
6
4
2
0
theta1 (radians)
0
1
2
3
4
5
6
7
8
9
Time (seconds)



10
8
6
4
2
0
-2
-4
-6
-8
theta1 dot (radians/second)
0
1
2
3
4
5
6
7
8
9
10
Time (seconds)


10
8
6
4
2
0
-2
-4
-6
-8
0 1 2 3 4 5 6 7 8 9 10
Time (seconds)
theta$_2$ (radians)


30
20
10
0
-10
-20
-30
0 1 2 3 4 5 6 7 8 9 10
Time (seconds)
theta$_2$dot (radians/second)

Discussion : -

The goals of the assignment were to find the Kinetic Energy, Potential Energy and then finally find the Lagrangian Euler equation to find the equations of motion which was done as explained in the report above stepwise and also then finding the values in terms of theta_ddot. We also wanted to represent these in the state space representation form. The MATLAB model does not account for Friction and energy losses and this results in an energy constant system which causes it to keep on moving perpetually. From the results obtained above in the graphs, we can see that all graphs are pretty unstable in MATLAB, especially the graph 1 of theta1 vs time which tends to overshoot. My future work for this assignment would be to make the code even more automated so that the manual inputs are as less as possible.