Introduction to Biomedical Engineering

Section 1: Basic electronics

Lecture 1.3: Filters

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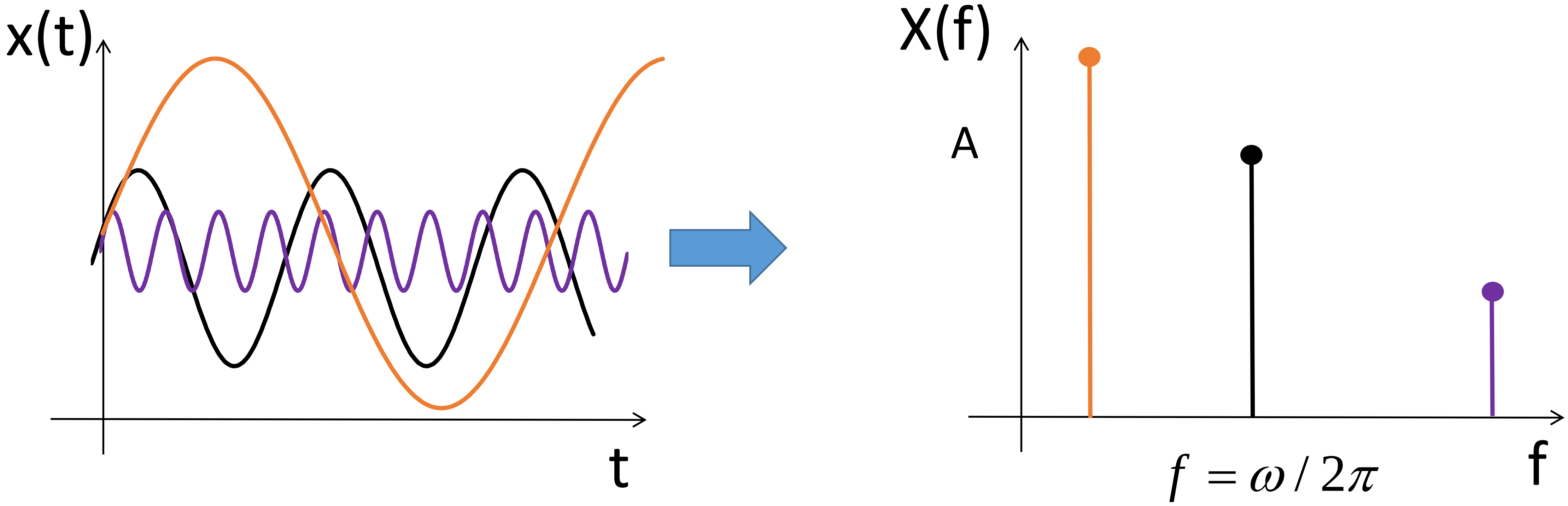
Frequency vs Time domains

The **Time Domain** refers to describing functions with respect to time, in other words the value of function *x* is known for all real values of *t*

$$x(t) = A\sin(\omega t + \phi)$$



Whereas the **Frequency Domain** is analysis of signals with respect to frequency, so a Frequency Domain graph shows the amplitude of the signal within each frequency band.

Bode Plots – Frequency Response

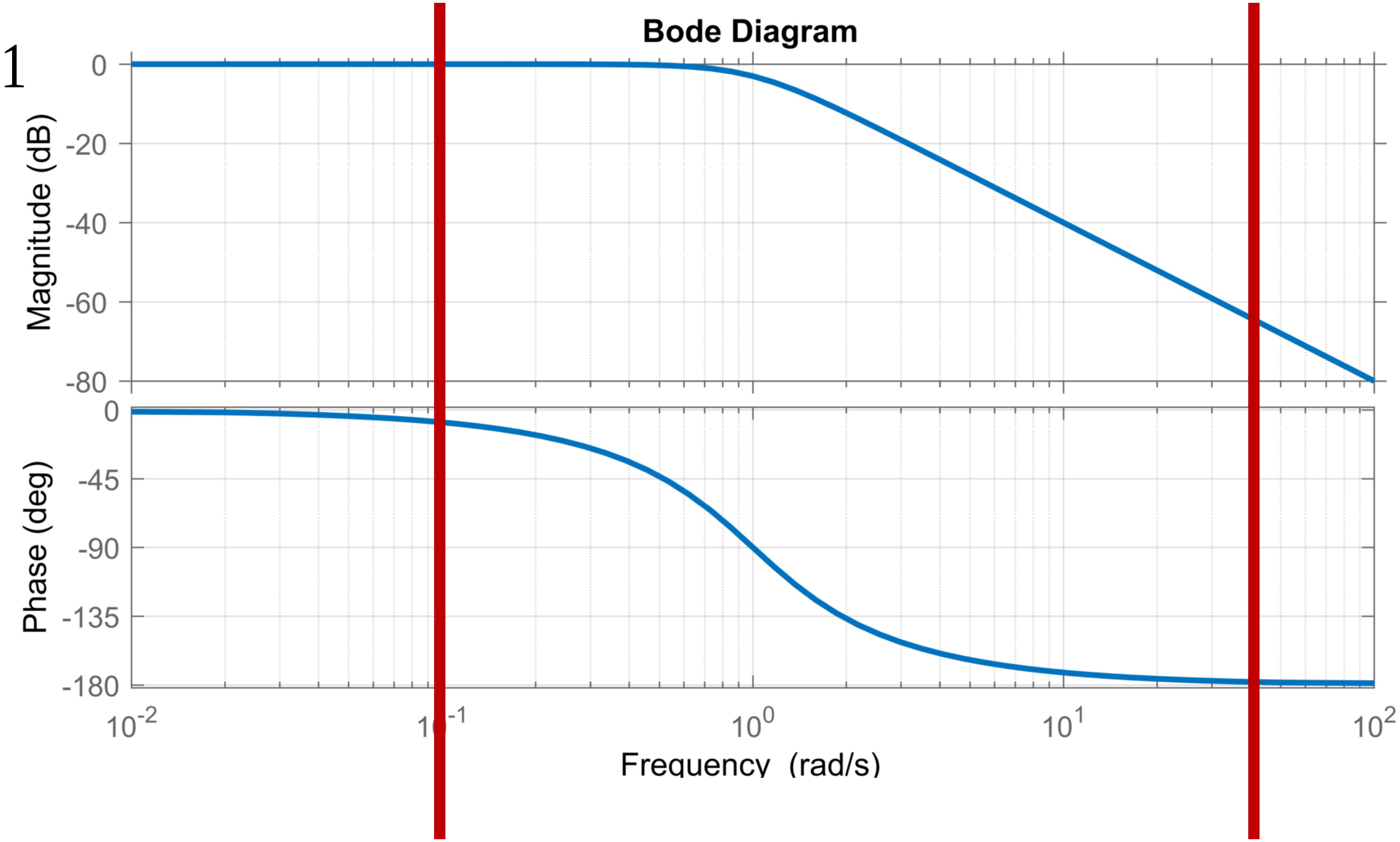
We typically view the magnitude and phase response on graphs with log frequency as x axis

$$0dB = 1$$

Change in magnitude or "Gain" expressed in dB, so this is LogLog plot

Change in phase in radians or degrees, so a LinLog plot

This is a *first order* low pass filter response, like the TIA example



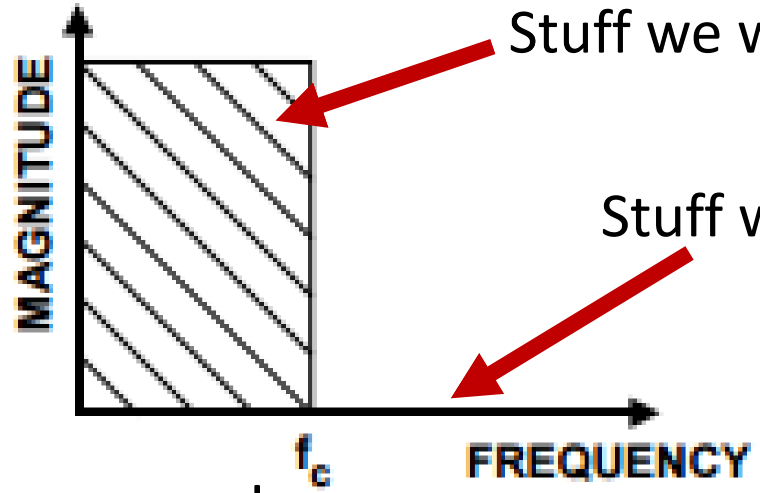
A sine wave here would have approx. same amplitude, and small phase delay

One here would be reduced by approx. 70dB (~3000) and be in anti-phase

Classes of ideal filter

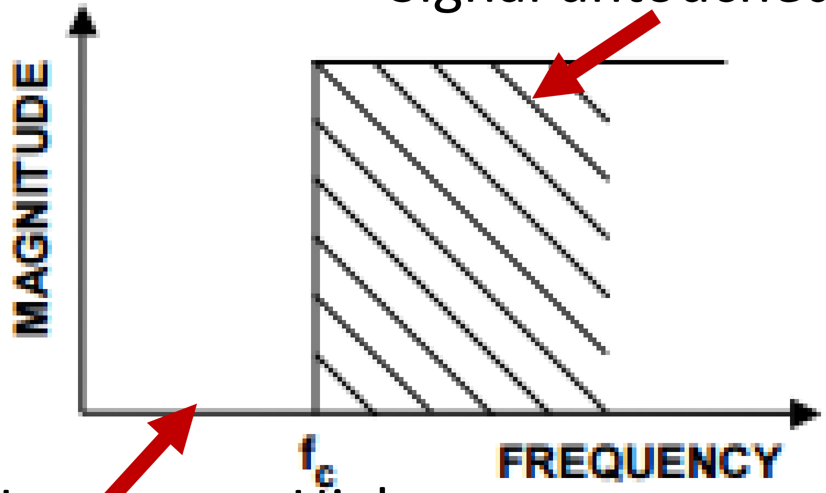
Stuff we want

Stuff we don’t



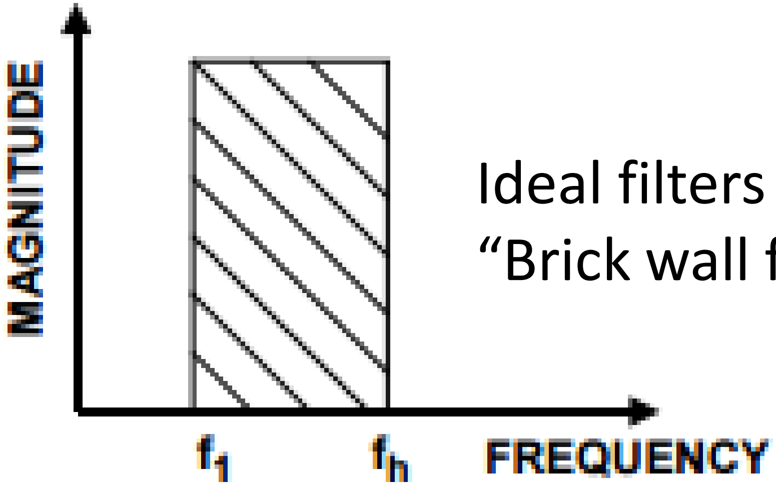
Lowpass

Signal untouched



Noise completely removed

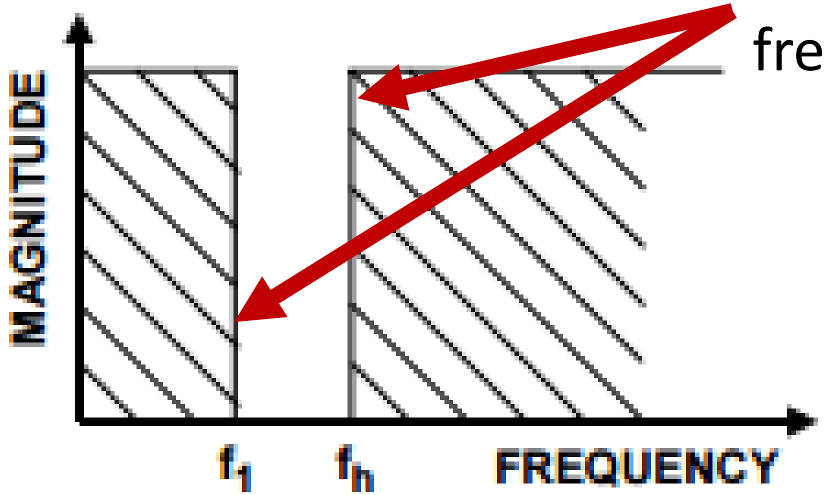
Highpass



Bandpass

Ideal filters are called “Brick wall filters”

“Cut off” frequencies



Bandstop/Notch

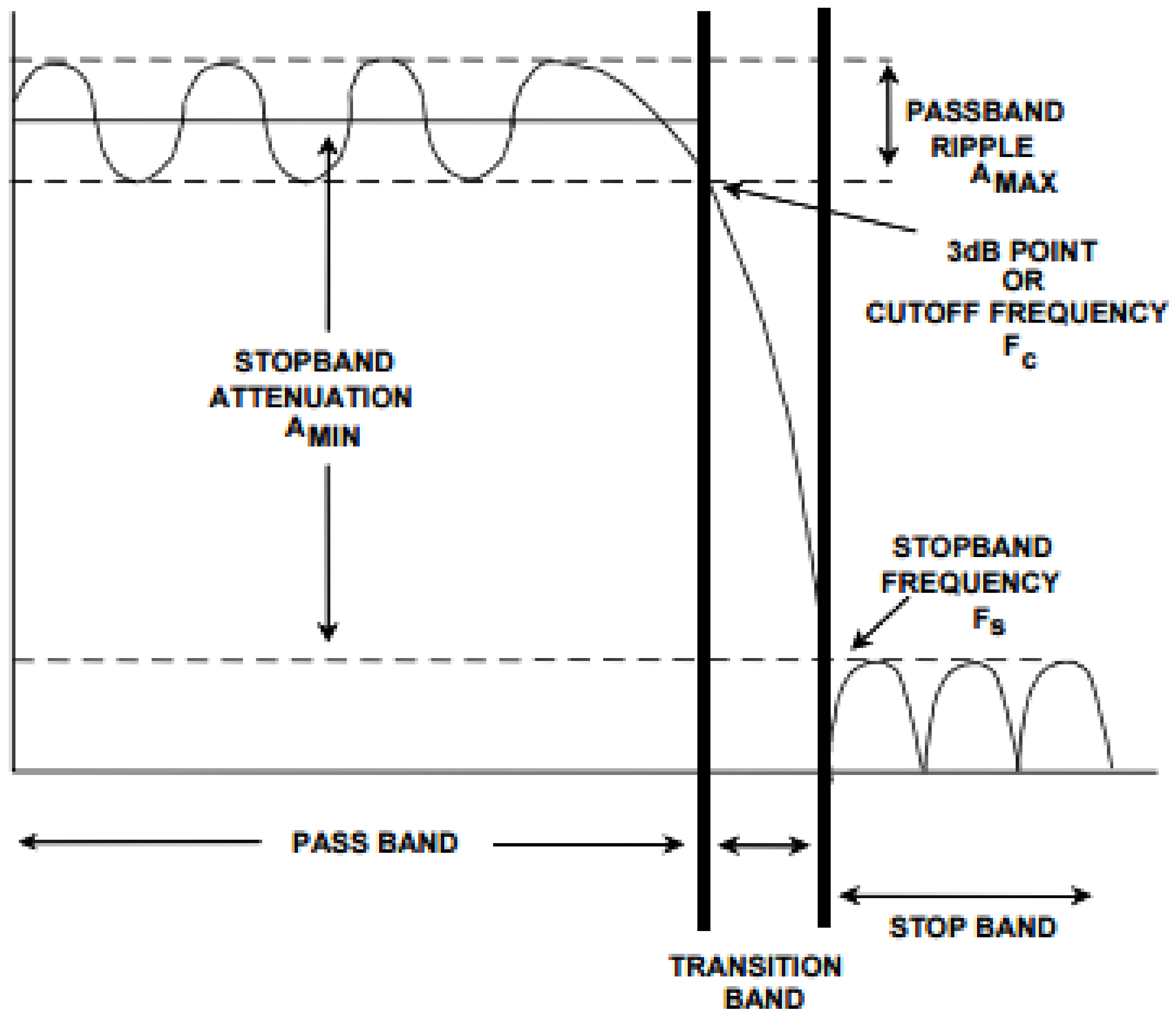
Filter response overview

In reality we cannot achieve a true "brick wall" filter! Not a limit of electronics or computation it's a physics/mathematical one – infinite sharp change in freq. requires infinite signal length

Ideal filter:

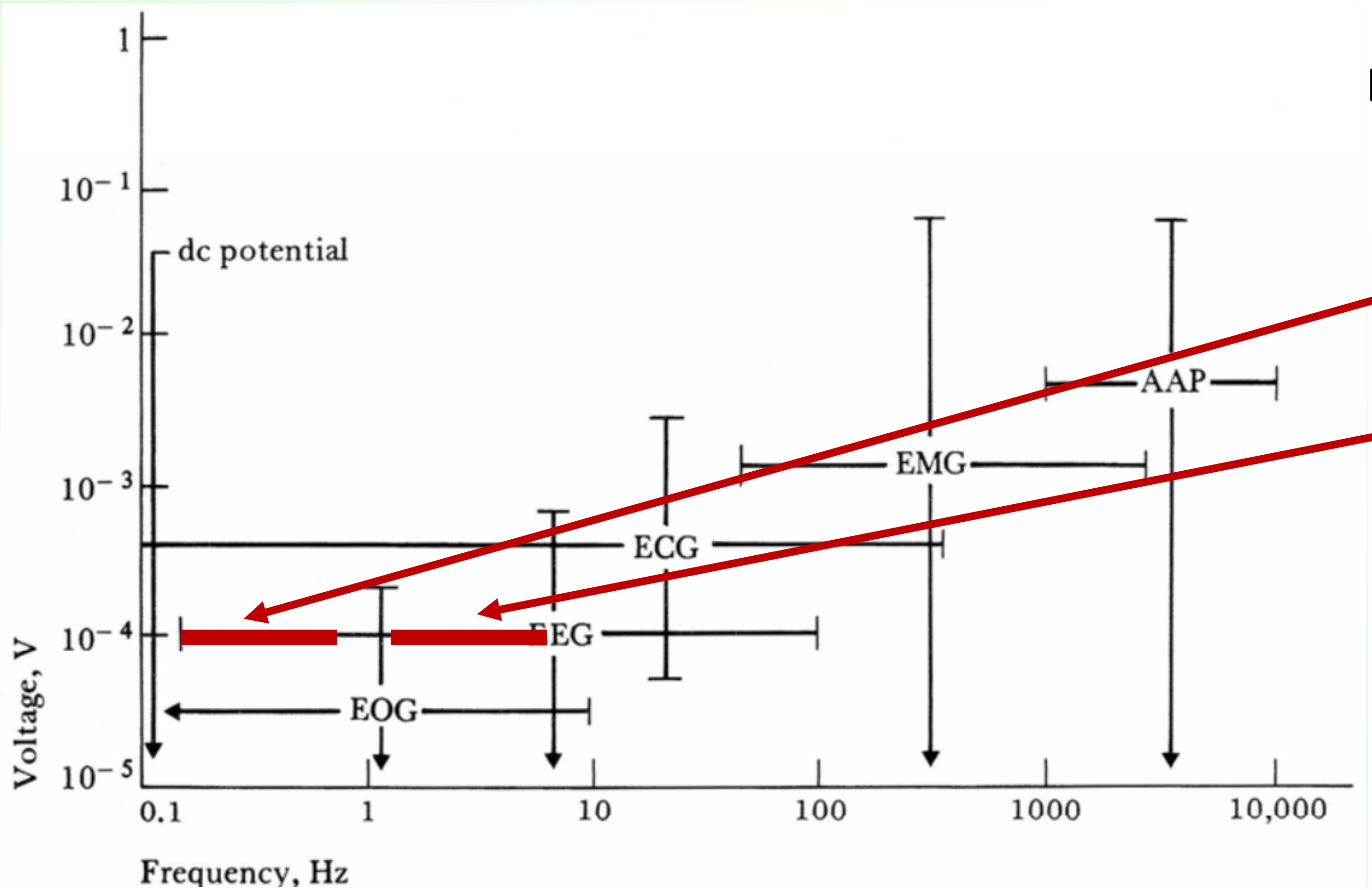
- Attenuation is ∞
- Ideal phase in pass band 0deg
- 0 Transition band
- 0 Ripple in pass or stop band

Depending on the application, some of these criteria are more important than others. So we can chose a filter to match our needs. There is no "best"

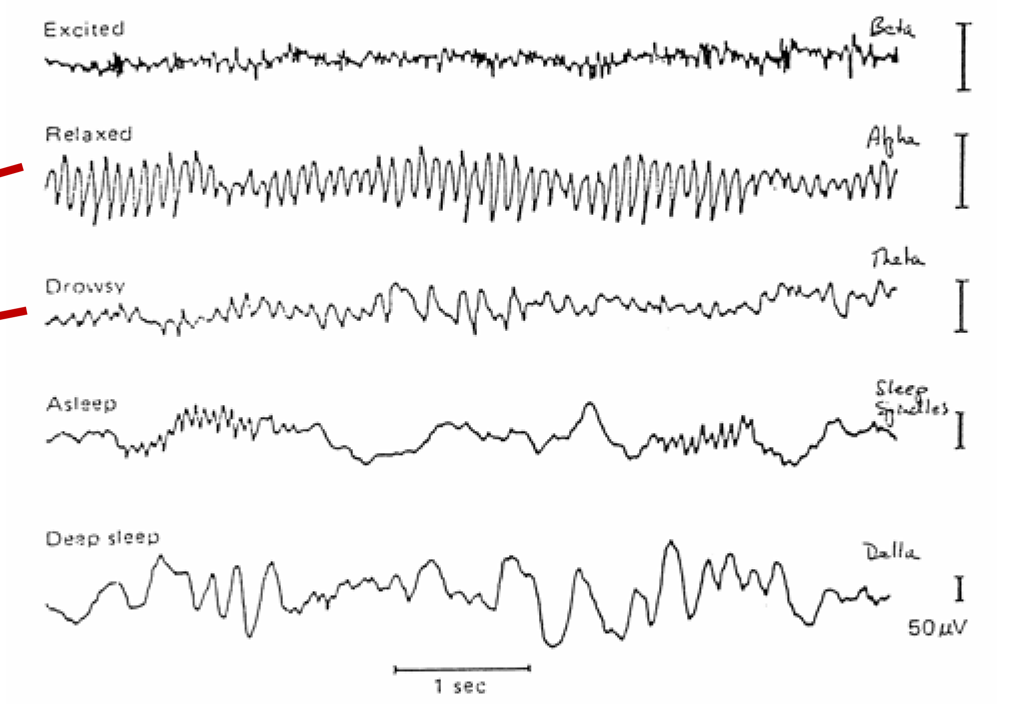


Why do we need filters?

Physiological signals are *bandlimited* i.e. they only exist at certain parts of the spectrum



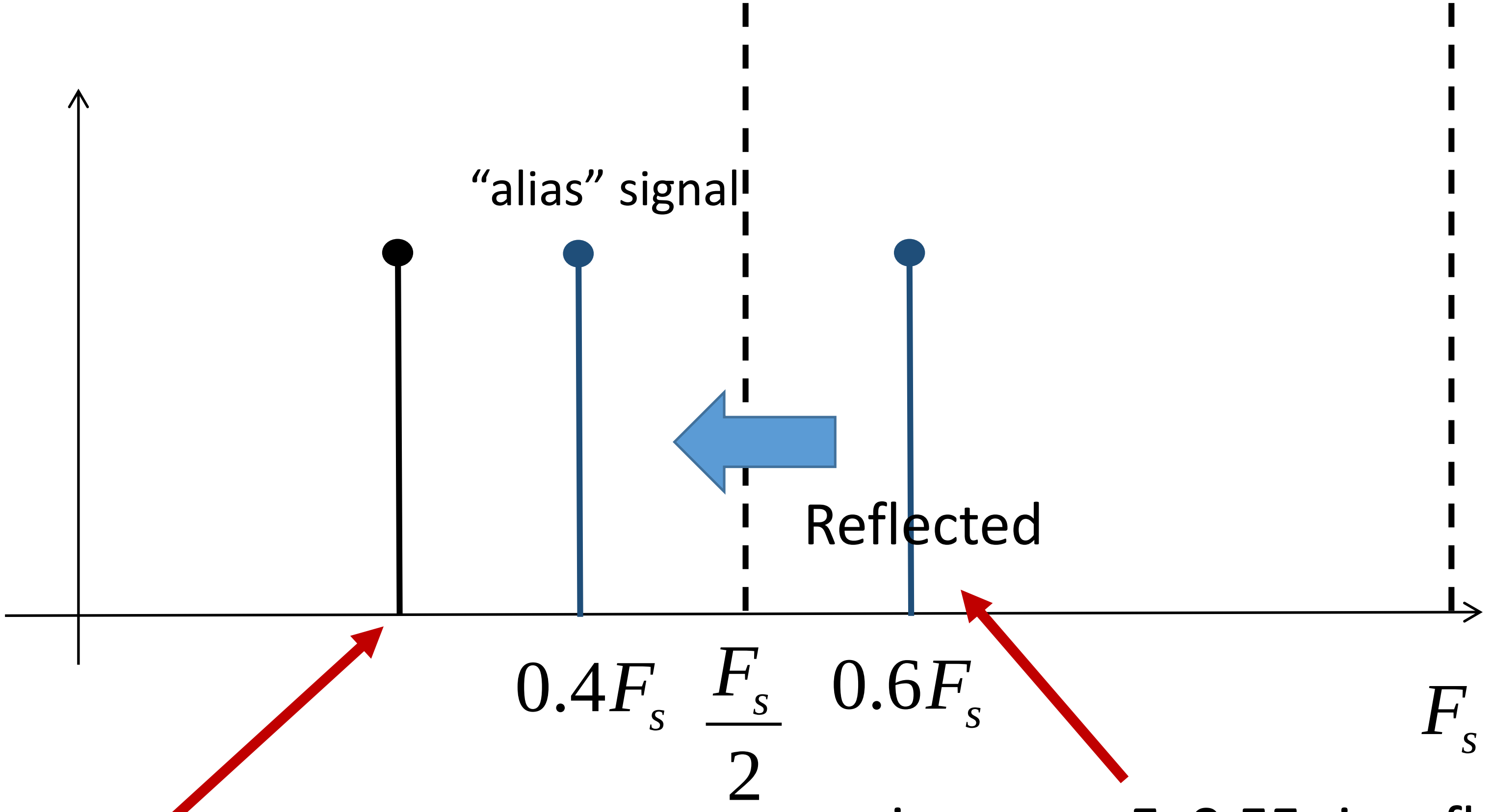
Different EEG “rhythms” occur at different bands



Filtering helps us remove interferences, reduce noise, emphasise certain parts of a signal. All of which aid further processing and analysis

Sampling – Nyquist Frequency

Another crucial application of filtering is to reduce artefacts when sampling an analogue signal. For a given sampling rate **Fs,** the maximum frequency which can be sampled correctly is **0.5Fs.** This is known as the *Nyquist frequency.*

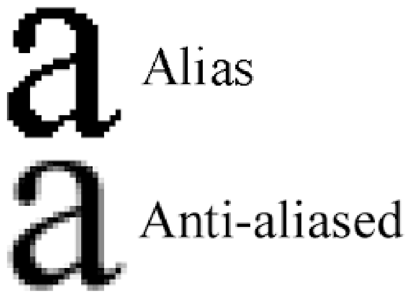


So your 0.6Fs signal will appear as a 0.4Fs signal when you sample it! We don’t want this!

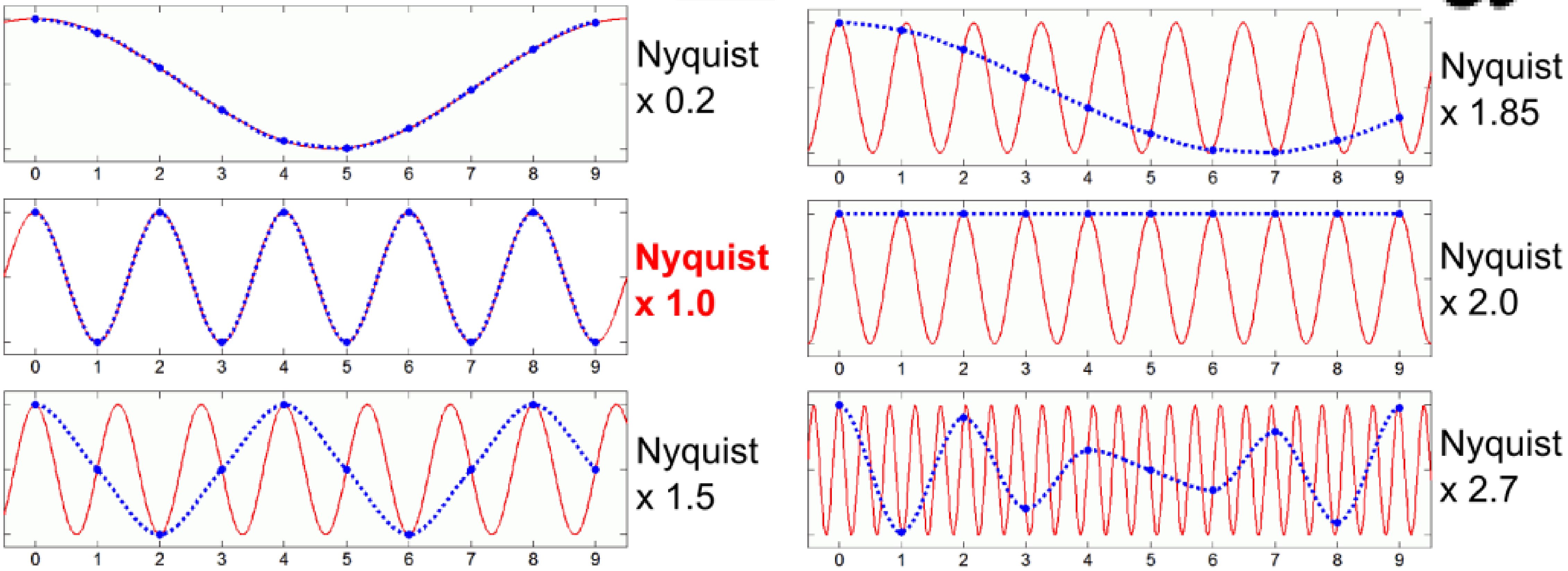
sine wave **F<0.5Fs** sampled ok

sine wave **F>0.5Fs** is reflected in the line **0.5Fs** known as *folding*

Sampling – Nyquist Frequency



Aliasing like this can cause some very weird results!

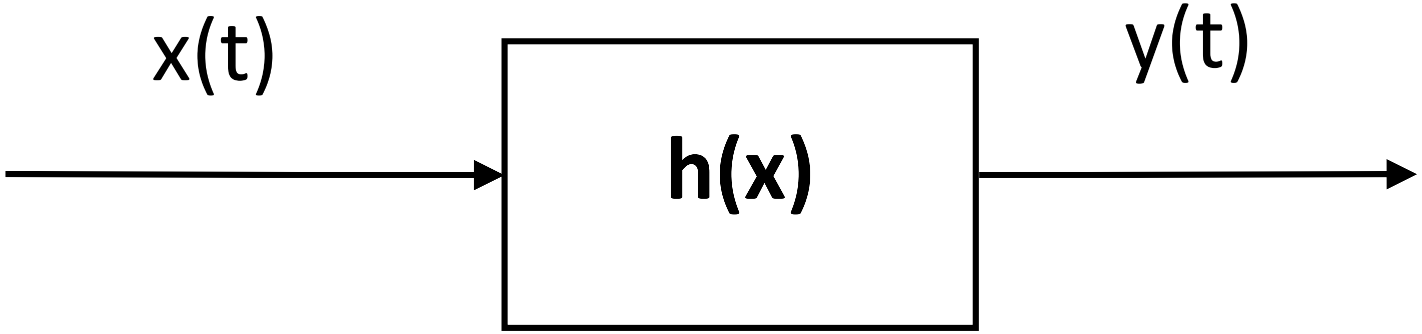


Best to avoid these frequencies being sampled at all. So use a *low pass* filter. This is known as an *anti-aliasing filter* as it is designed to prevent these aliased signals.

E.g. Digital audio Fs/2 = 22.1 kHz, with an anti-alias cut off at ~20kHz

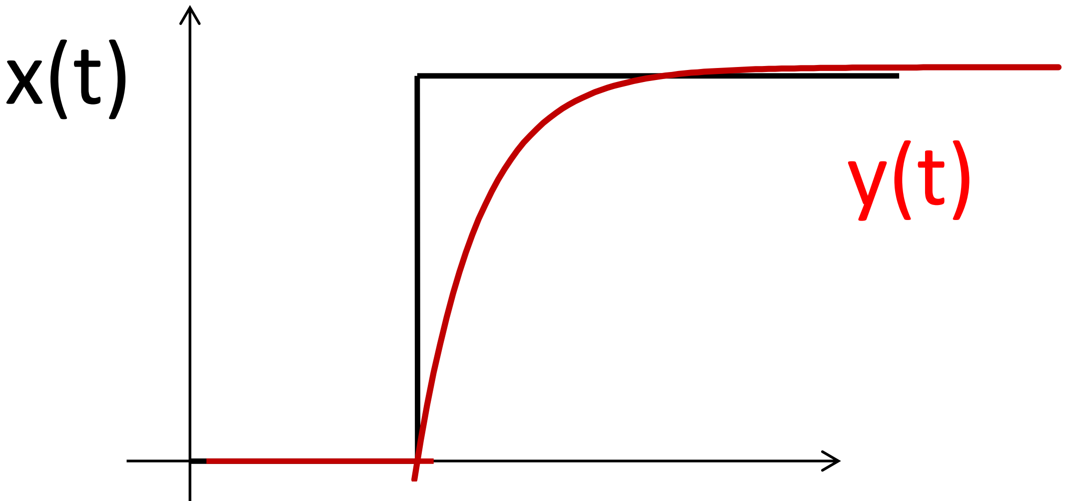
Transfer functions

As we are interested in describing something that *changes* with time, it is useful to express the function block of the system F(t) as an ordinary differential equation (ODE)



$$y = h * x$$

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_o = bx$$



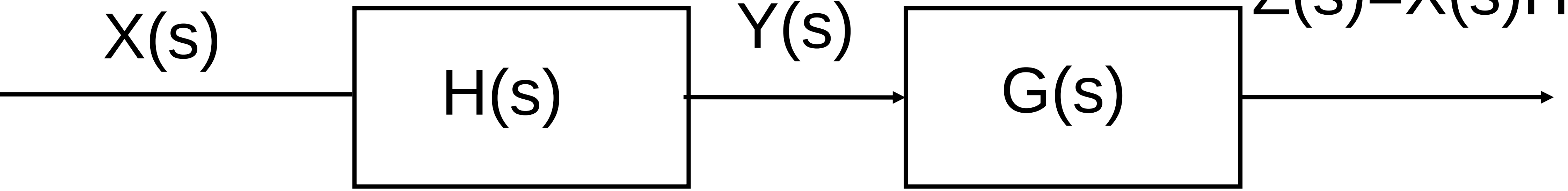
X is input function

Y is output

N is *order* of the ODE

A0… are coefficients

Transfer functions – Laplace



$Z(s)=X(s)H(s)G(s)$

The Laplace transform is an extension of the Fourier transform which allows us to consider things *changing over time*. Rather that just sine waves.

Exponential Decays

$$s = \sigma + j\omega$$

Sine Waves

The Laplace transform of *x(t)* is defined by:

$$L\{x(t)\} = X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

$$\boxed{s \equiv \frac{d}{dt}}$$

$$\boxed{\frac{1}{s} \equiv \int_{0^-}^{\infty} dt}$$

The Laplace variable, s, can be considered to represent the differential operator

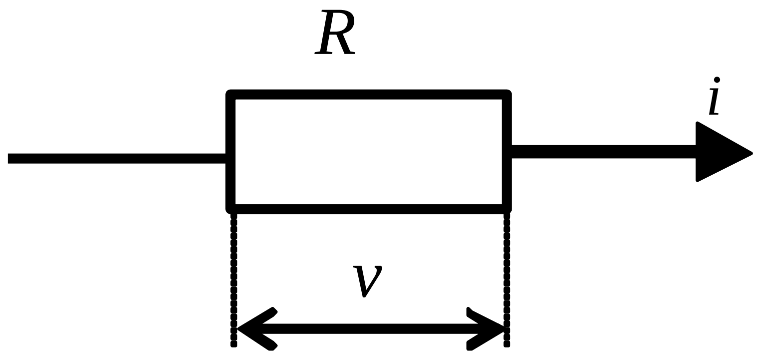
Transfer functions of electronic components

$$L\left\{\frac{d^n x}{dt^n}\right\} = s^n X(s)$$

Convention is input voltage, output current. Balance **Voltages**

Resistor

Ohm's law V=IR:

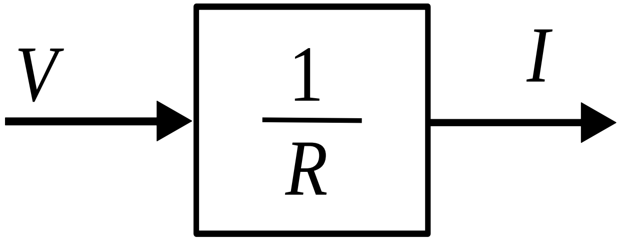


Time domain $v(t) = i(t)R$

Laplace domain $V(s) = I(s)R$

Transfer function:

$$G(s) = \frac{I(s)}{V(s)} = \frac{1}{R}$$

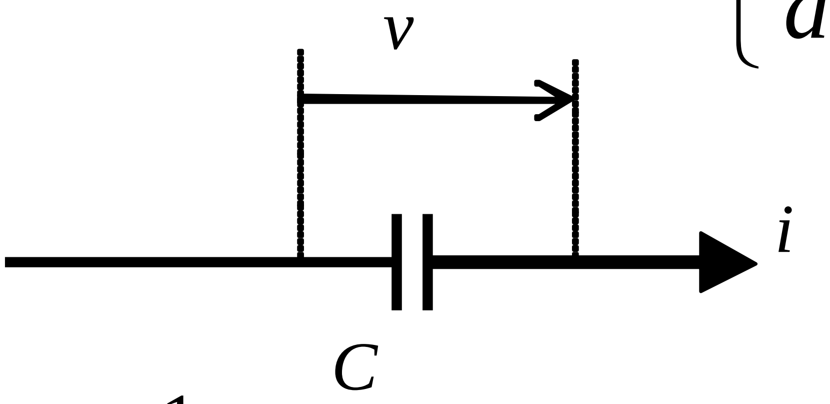


Transfer functions of electronic components

$$L\left\{\frac{d^n x}{dt^n}\right\} = s^n X(s)$$

Capacitor

Either definition of current/voltage relationship gives same result

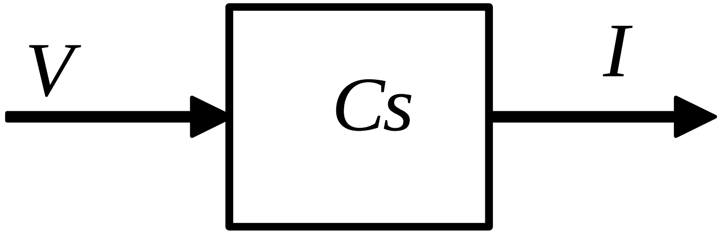


Time domain $\quad i(t) = C\dfrac{dv}{dt} \qquad v(t) = \dfrac{1}{C}\displaystyle\int i(t)dt$

Laplace domain $\quad I(s) = CsV(s) \qquad V(s) = \dfrac{1}{C}\dfrac{1}{s}I(s)$

Transfer function:

$$G(s) = \frac{I(s)}{V(s)} = Cs$$

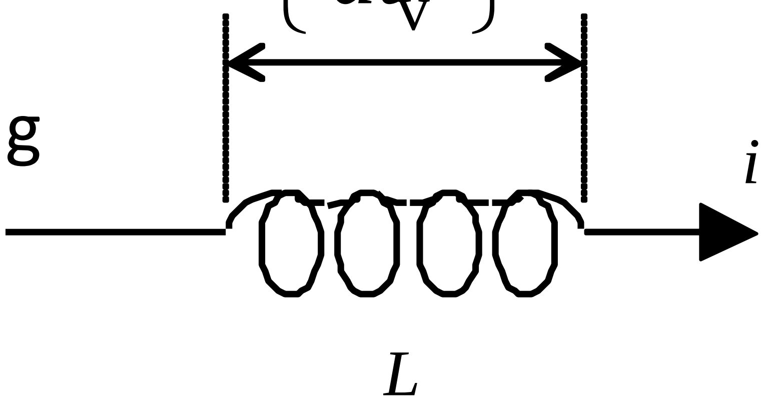


Transfer functions of electronic components

$$L\left\{\frac{d^n x}{dt^n}\right\} = s^n X(s)$$

Inductor

An inductor resists changes of current by generating a voltage in opposition via magnetic induction



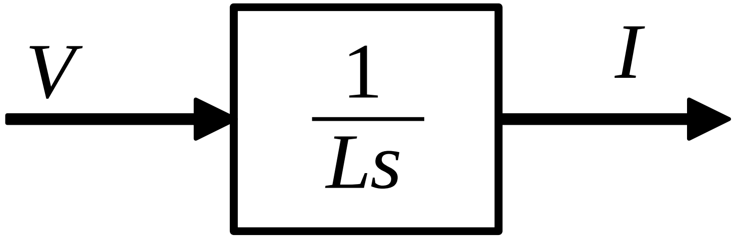
From Faraday's Law:

Time domain
$$v(t) = L\frac{di}{dt}$$

Laplace domain
$$V(s) = LsI(s)$$

Transfer function

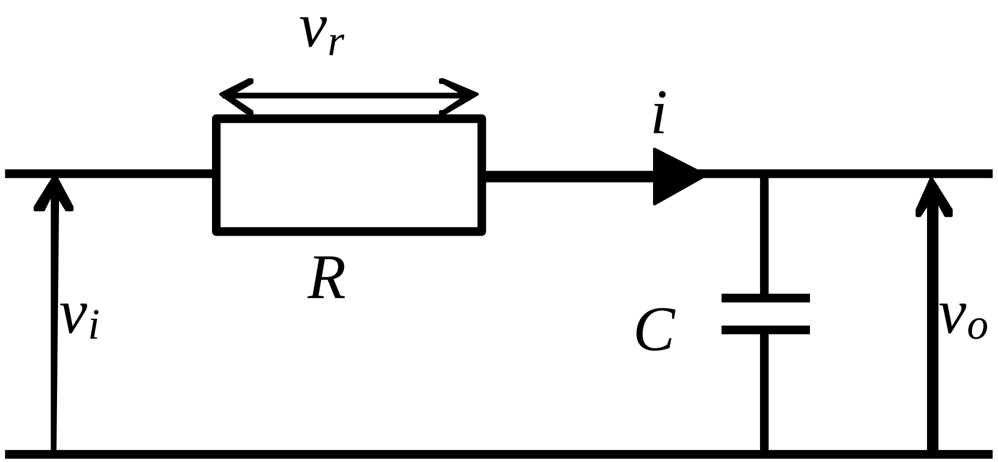
$$G(s) = \frac{I(s)}{V(s)} = \frac{1}{Ls}$$



Transfer function RC filter example

The goal is the transfer function, with
v_o the output of interest:

$$H(s) = \frac{V_0(s)}{V_i(s)}$$



Input voltage is sum of voltage drops in circuit

$$V_i = V_r + V_o$$

Kirchhoff's Voltage Law

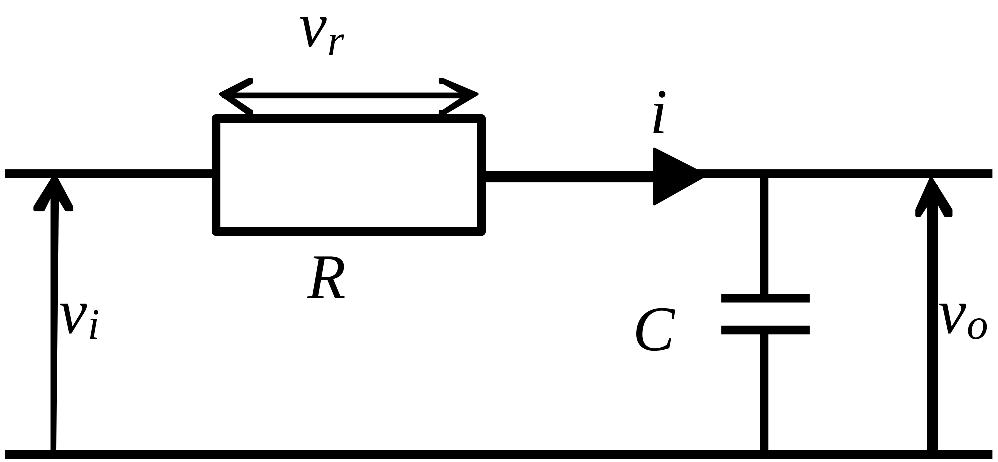
Starting with Time domain equations:

$$v_i(t) = v_r(t) + v_o(t) \qquad v_r(t) = i(t)R \qquad i(t) = C\frac{dv_o}{dt}$$

We can write v_i in terms of v_o

$$v_i(t) = RC\frac{dv_o}{dt} + v_o(t)$$

Transfer function RC filter example



$$v_i(t) = RC\frac{dv_o}{dt} + v_o(t) \qquad s \equiv \frac{d}{dt}$$

$$V_i(s) = RCsV_o(s) + V_o(s)$$

$$V_i(s) = V_o(s)(RCs + 1)$$

Transfer function w.r.t. voltage:

$$H(s) = \frac{V_0(s)}{V_i(s)} = \frac{1}{RCs+1} = \frac{\gamma}{1+\tau s} \qquad \gamma = 1 \qquad \tau = RC$$

OK, so now what!?

Transfer function – Freq Respor

Obtain the **frequency domain** response by ignoring the time component of *s* σ=0

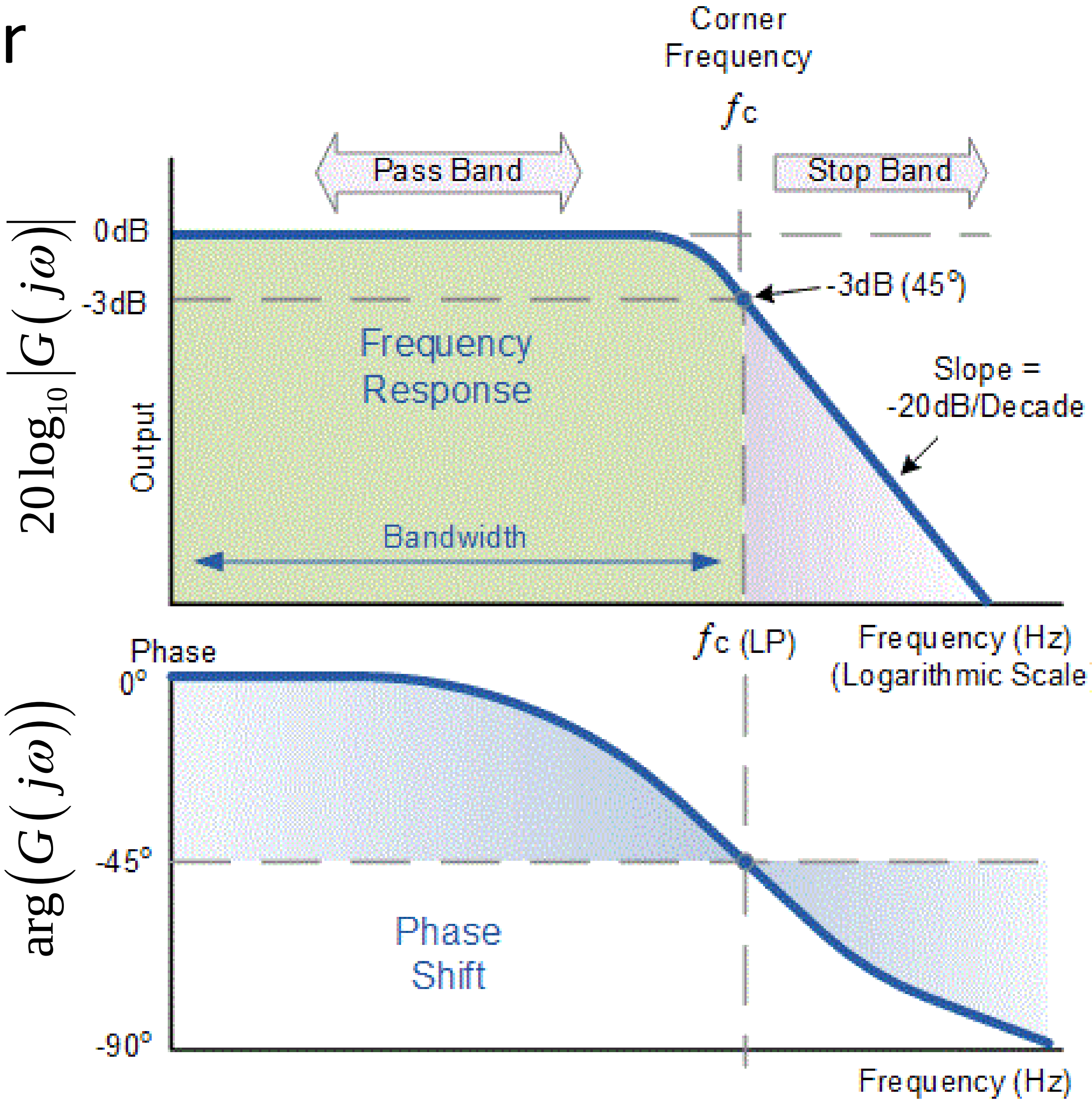
$$s = \cancel{\sigma} + j\omega = j\omega$$

$$G(s) = \frac{V_0(s)}{V_i(s)} = \frac{1}{1+RCs}$$

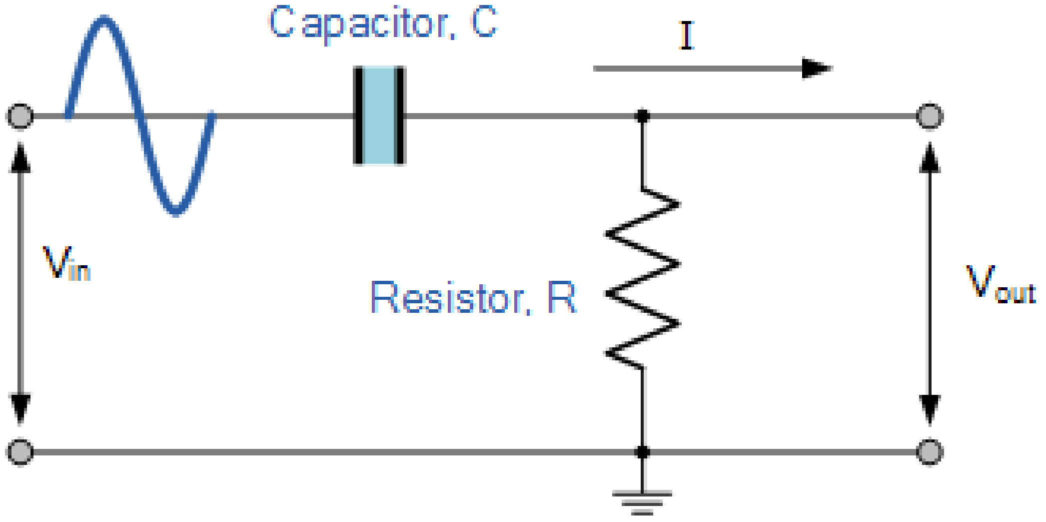
$$G(j\omega) = \frac{1}{1+j\omega RC}$$

$$\omega_c = \frac{1}{RC} \qquad f_c = \frac{1}{2\pi RC}$$

"At low freq, no current goes into capacitor. At high freq *all* current flows into capacitor"



Passive Filters – High Pass



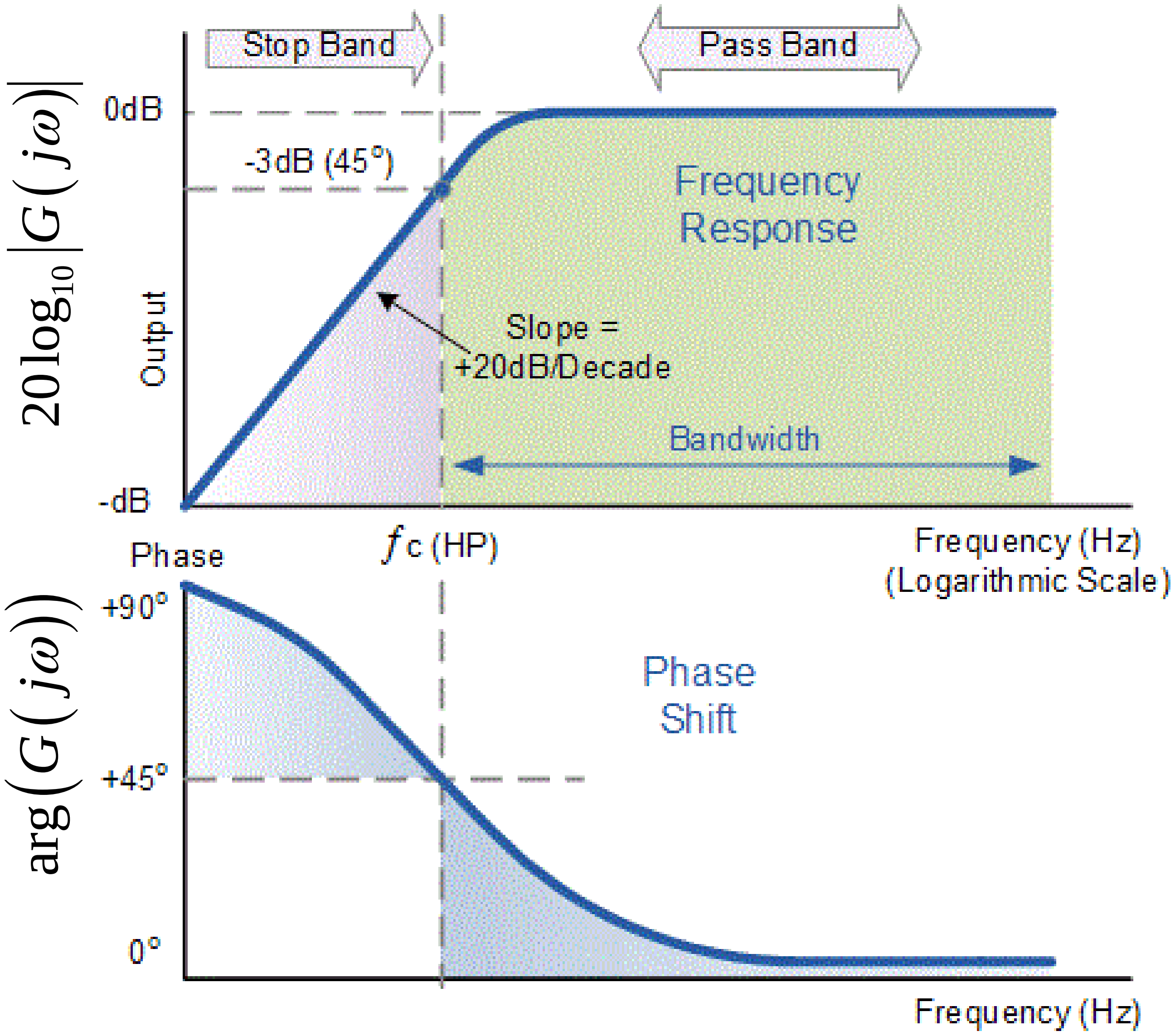
$$G(s) = \frac{RCs}{1+RCs}$$

$$f_c = \frac{1}{2\pi RC}$$

$$\left(G(j\omega) = \frac{RCj\omega}{1+RCj\omega} \right)$$

$$\omega_c = \frac{1}{RC}$$

"At low freq, no current can pass capacitor. At high freq *all* current flows through capacitor"

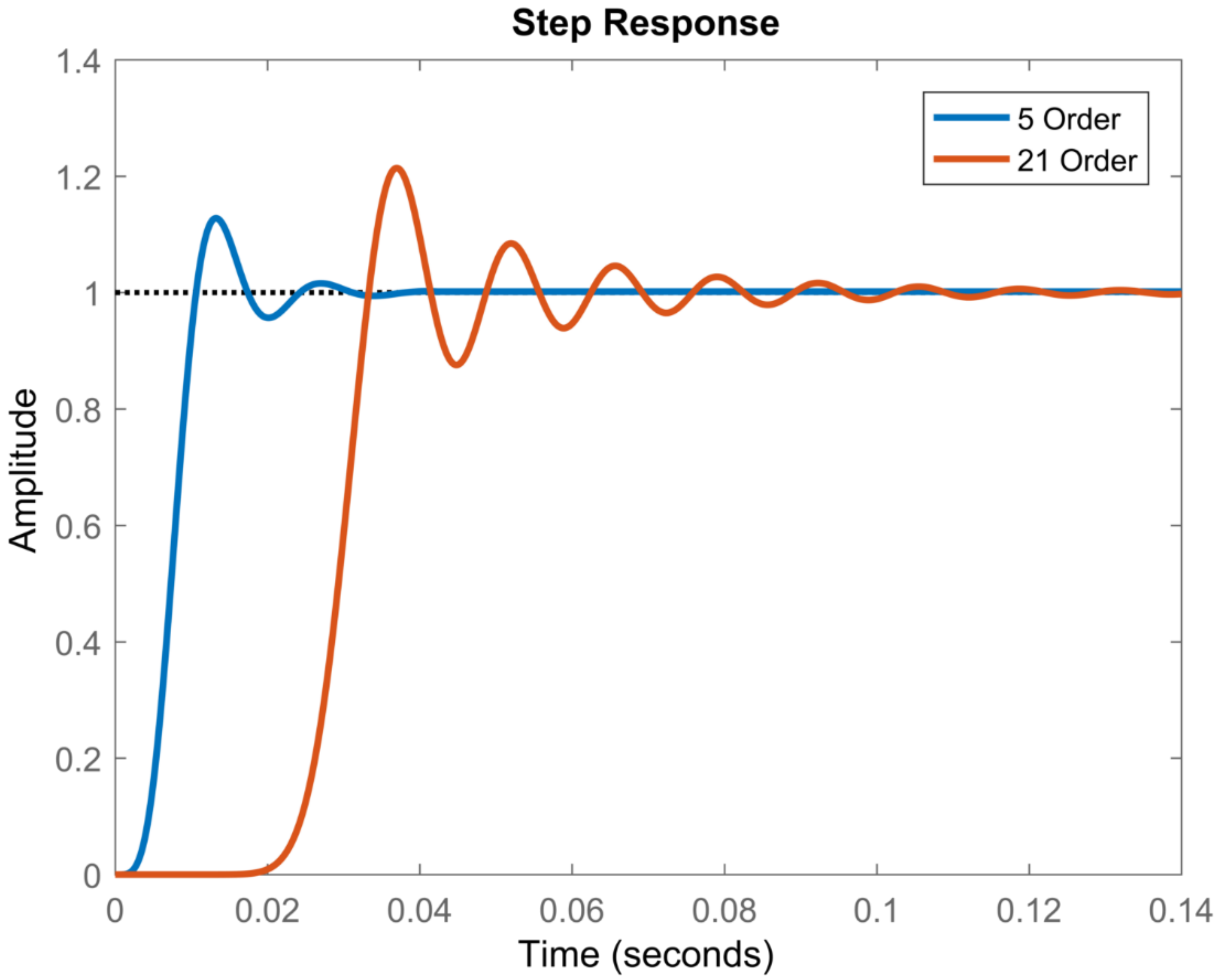


Transfer functions – Step response

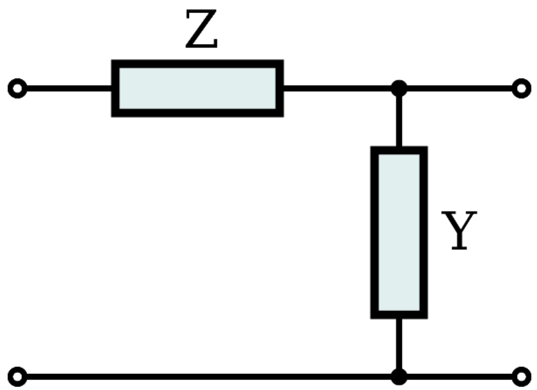
The transfer function also shows us the response of the system in time to a “step”. This is why we have the σ bit!

Increasing the order of the system increases the delay at the start also increases the time taken to settle to a steady output.

Application dependent if this is ok or not!

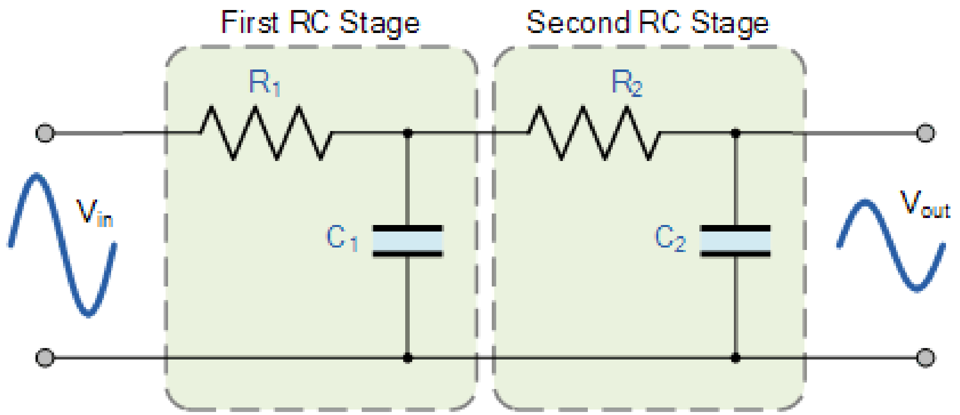


Passive filters – Cauer Topology

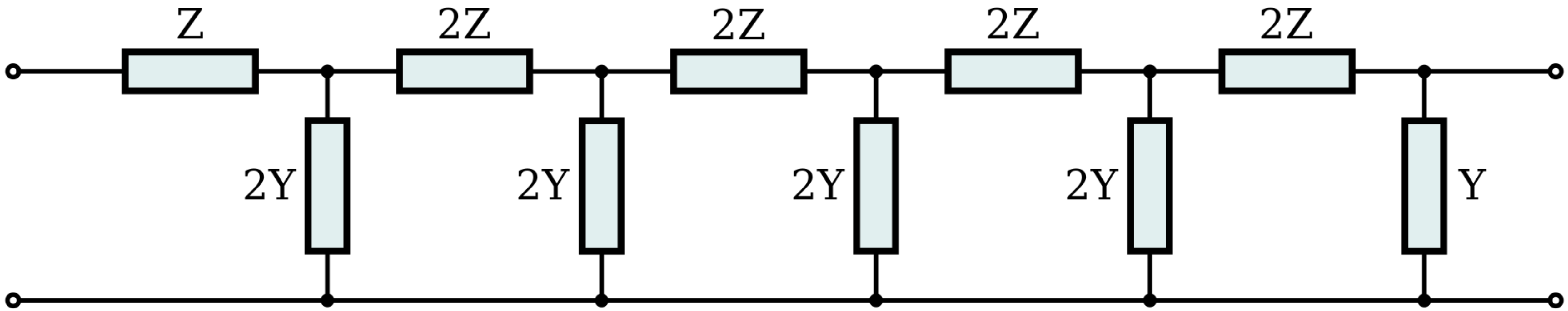


Both the High and Low pass filters are a simple of example of a type of circuit layout known as a “ladder” or “Cauer” topology.

Connecting multiple sections of passive components – known as “cascading” or “daisy chaining” – allows for variety of transfer functions

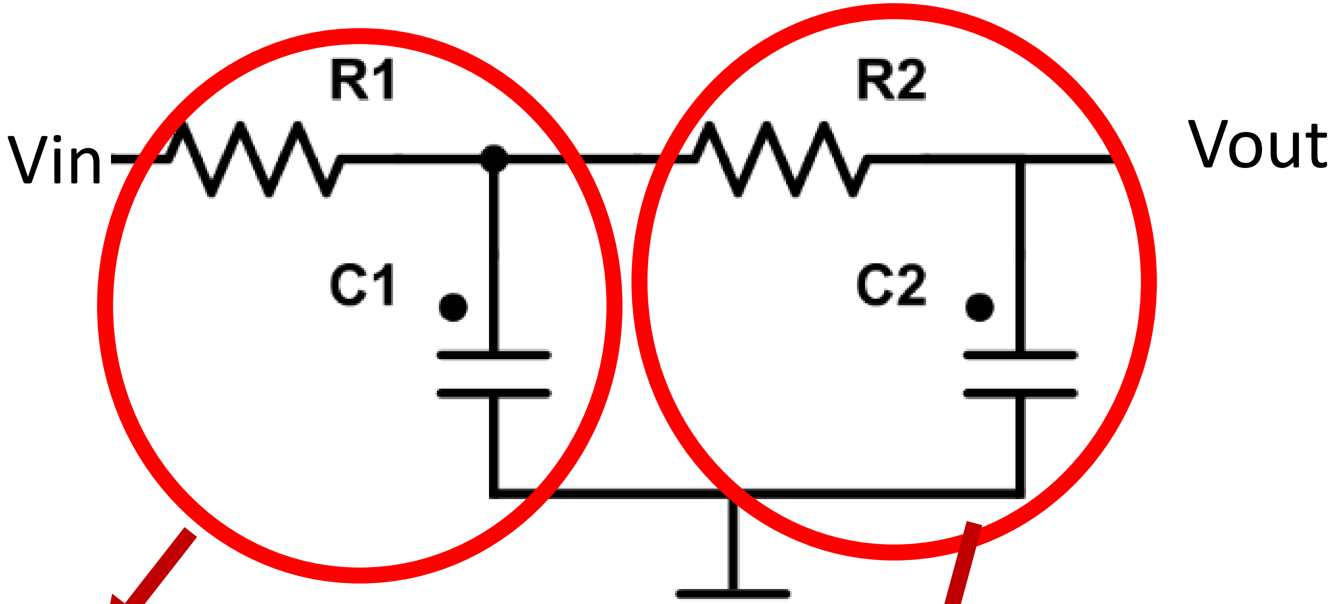


These have maximum unity gain, and higher orders require inductors – heavy and expensive

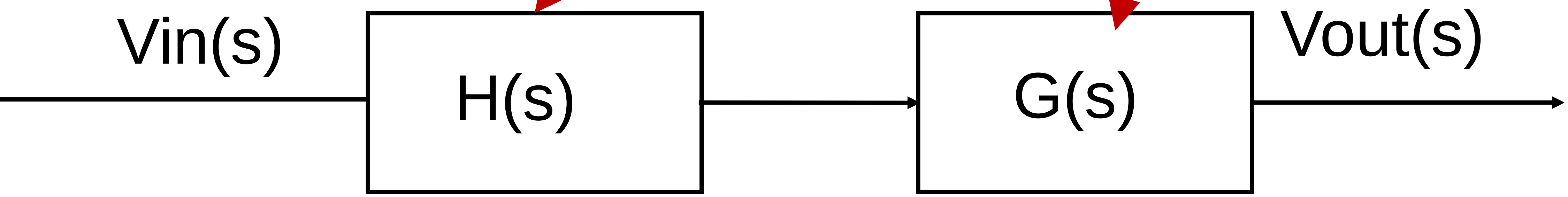


Making 2nd Order filter – Naïve attempt

We want to make a second order RC filter



Can we combine the two circuits together and get the same cutoff freq?



$$\frac{V_{out}}{V_{in}} = H(s)G(S) = \left(\frac{1}{sC_1R_1+1}\right)\left(\frac{1}{sC_2R_2+1}\right)$$

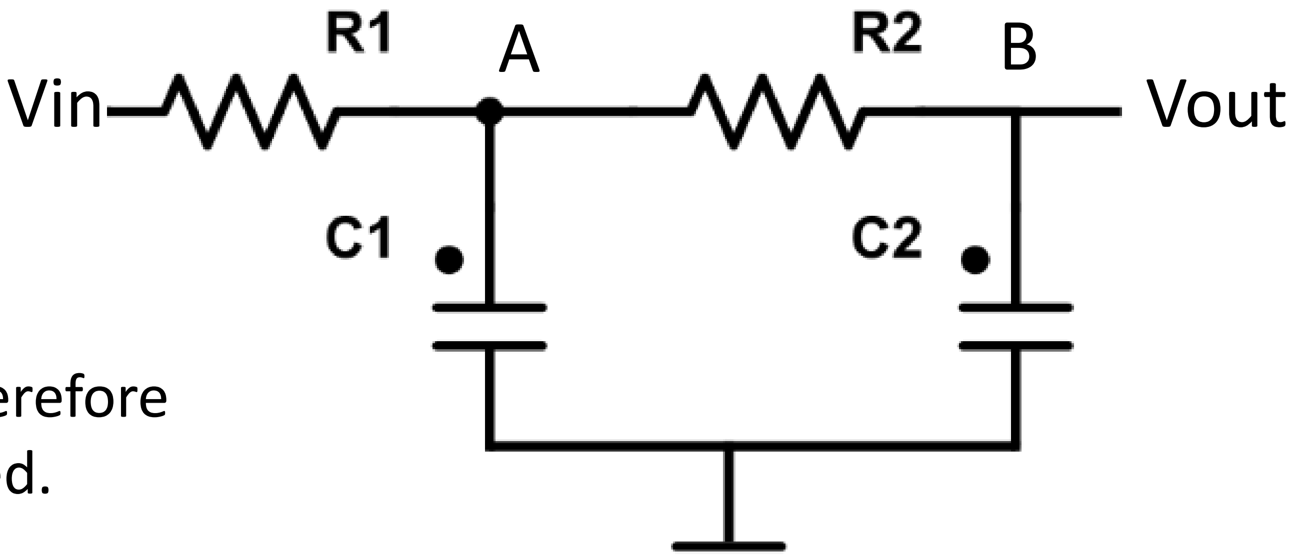
Is this the transfer function we get?

Higher order systems

However…

What about the voltage at point A?

It is dependent upon *all four components*. Therefore the two transfer functions cannot be separated.



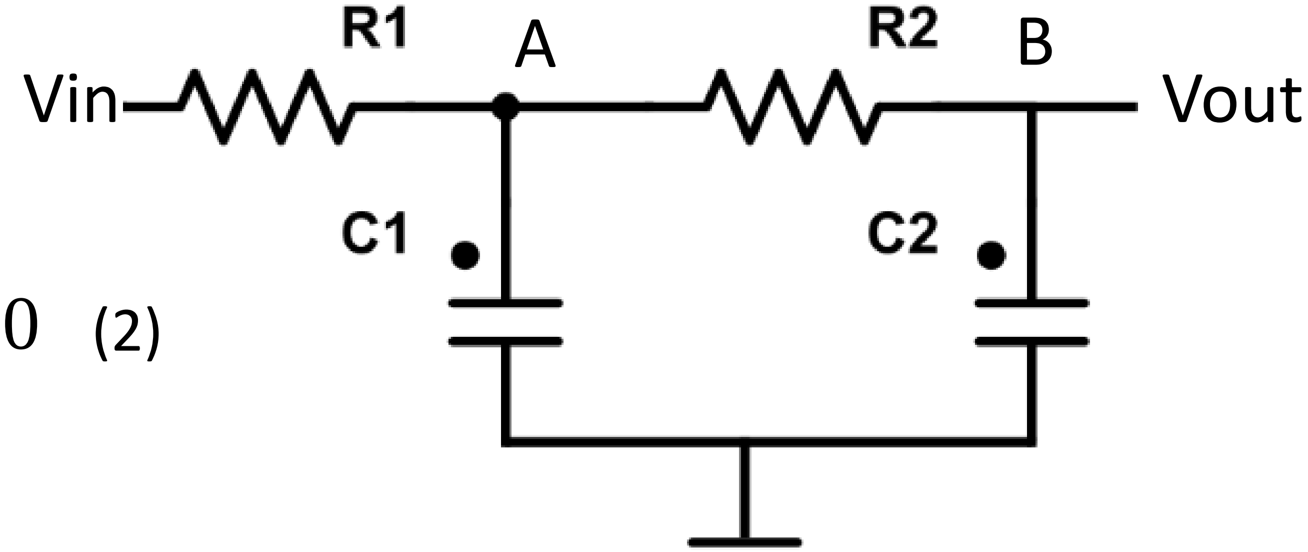
From KCL at point B:

$$\frac{V_{out} - V_A}{R_2} + sC_2V_{out} = 0 \longrightarrow V_A = V_{out}(1 + sC_2R_2) \quad (1)$$

From KCL at point A:

$$\frac{V_A - V_{in}}{R_1} + \frac{V_A - V_{out}}{R_2} + sC_1V_A = 0 \longrightarrow V_A\left(R_1 + R_2 + sC_1R_1R_2\right) - R_2V_{in} - R_1V_{out} = 0 \quad (2)$$

Higher order systems



$$V_A = V_{out}(1 + sC_2R_2) \quad (1)$$

$$V_A\left(R_1 + R_2 + sC_1R_1R_2\right) - R_2V_{in} - R_1V_{out} = 0 \quad (2)$$

Subbing (1) in to (2) yields:

$$V_{out}(1 + sC_2R_2)\left(R_1 + R_2 + sC_1R_1R_2\right) - R_2V_{in} - R_1V_{out} = 0$$

Rearranging into transfer function:

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2 + sC_1R_1R_2 + sC_2R_1R_2 + sC_2R_2{}^2 + s^2C_1C_2R_1R_2{}^2 - R_1}$$

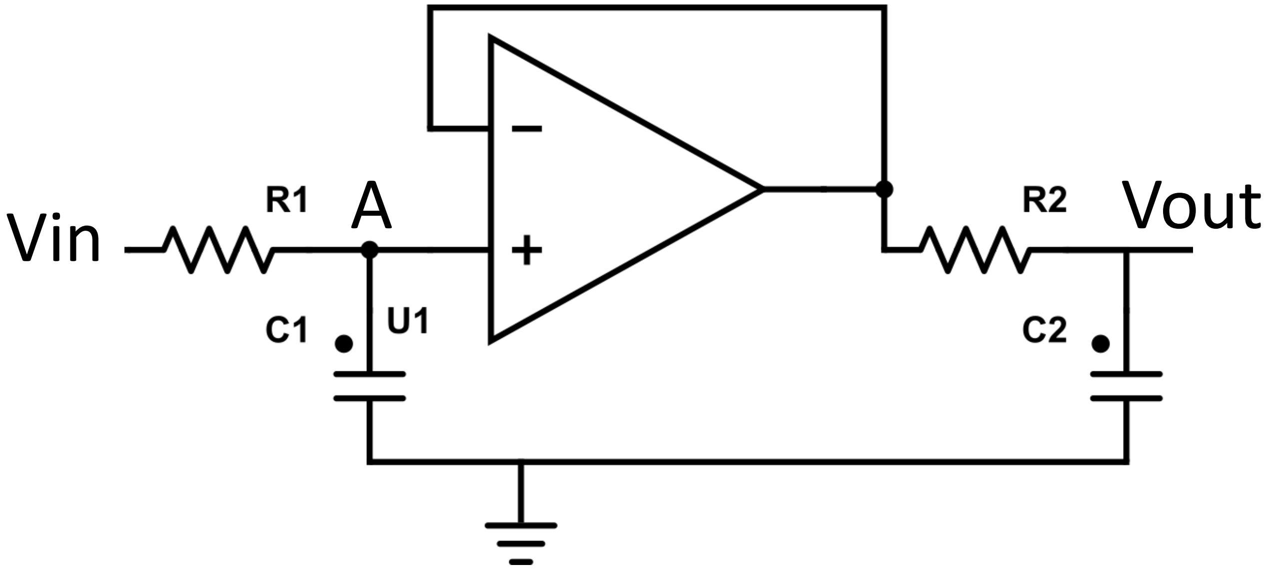
Tidying up:

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2C_1C_2R_1R_2 + s\left(C_1R_1 + C_2R_1 + C_2R_2\right) + 1}$$

Making 2nd Order filter – V 2.0

We can make the two transfer functions independent if we **buffer** the two circuits.

The infinite input impedance and zero output impedance isolates the two circuits from each other.



Now we can combine the two transfer functions as desired:

$$\frac{V_{out}}{V_{in}} = \left(\frac{1}{sC_1R_1+1}\right)\left(\frac{1}{sC_2R_2+1}\right) \rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{s^2C_1C_2R_1R_2+sC_1C_2+sC_2R_2+1}$$

Rearranging:

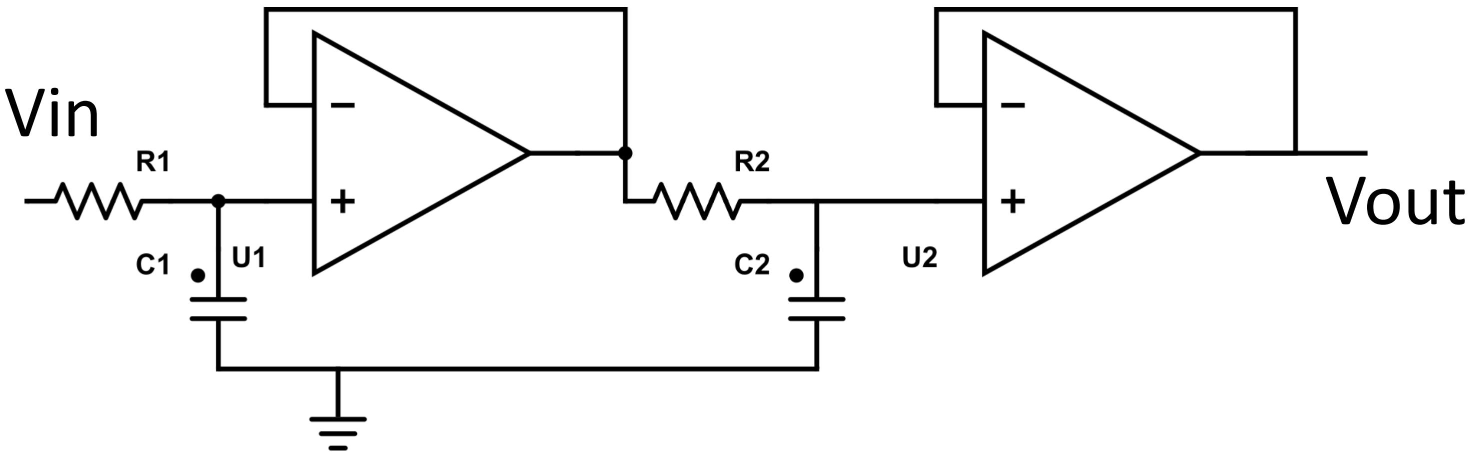
$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2\left(C_1C_2R_1R_2\right)+s\left(C_1C_2+C_2R_2\right)+1}$$

This is the answer we want!

Making 2nd Order filter – V 2.0

However…

The circuit now has poor output impedance, as it is determined by the impedance of the second RC filter



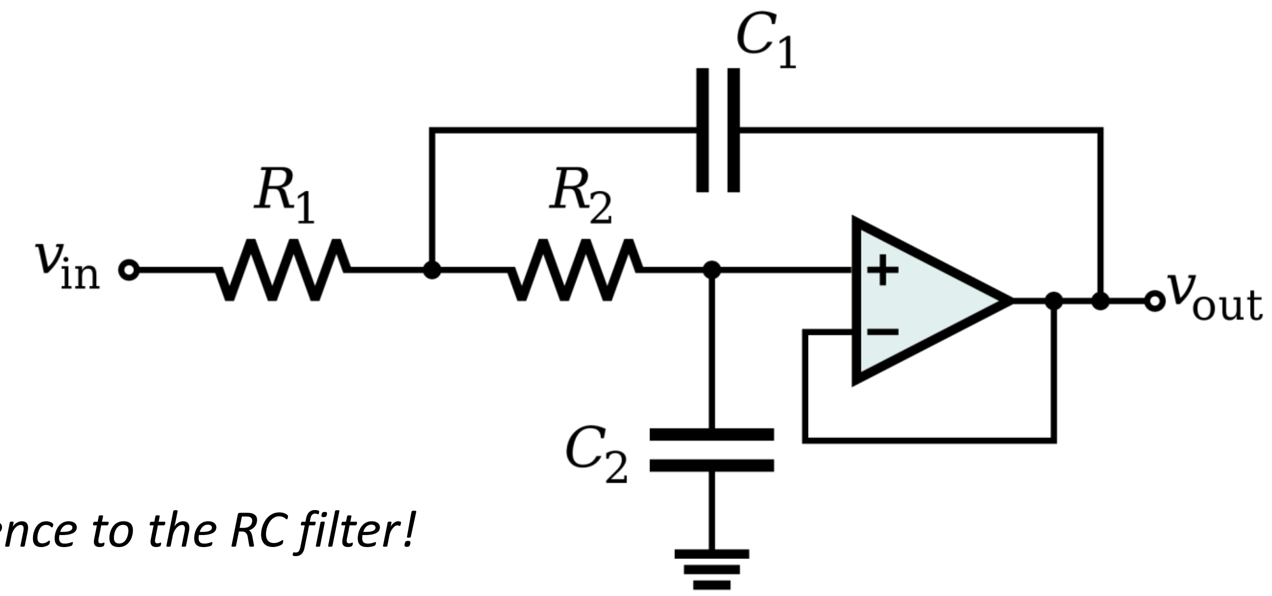
We can improve this by adding a second buffer at the end. Now the output impedance is determined by the op-amp and is theoretically zero.

This circuit has several drawbacks:

- An op amp for each order
- Cannot have a gain higher than 1
- Cannot implement Chebyshev, Bessel, Elliptical filters

i.e. Always critically damped as $\zeta = 1$ (if R1= R2 C1=C2)

2nd Order filter – Sallen Key



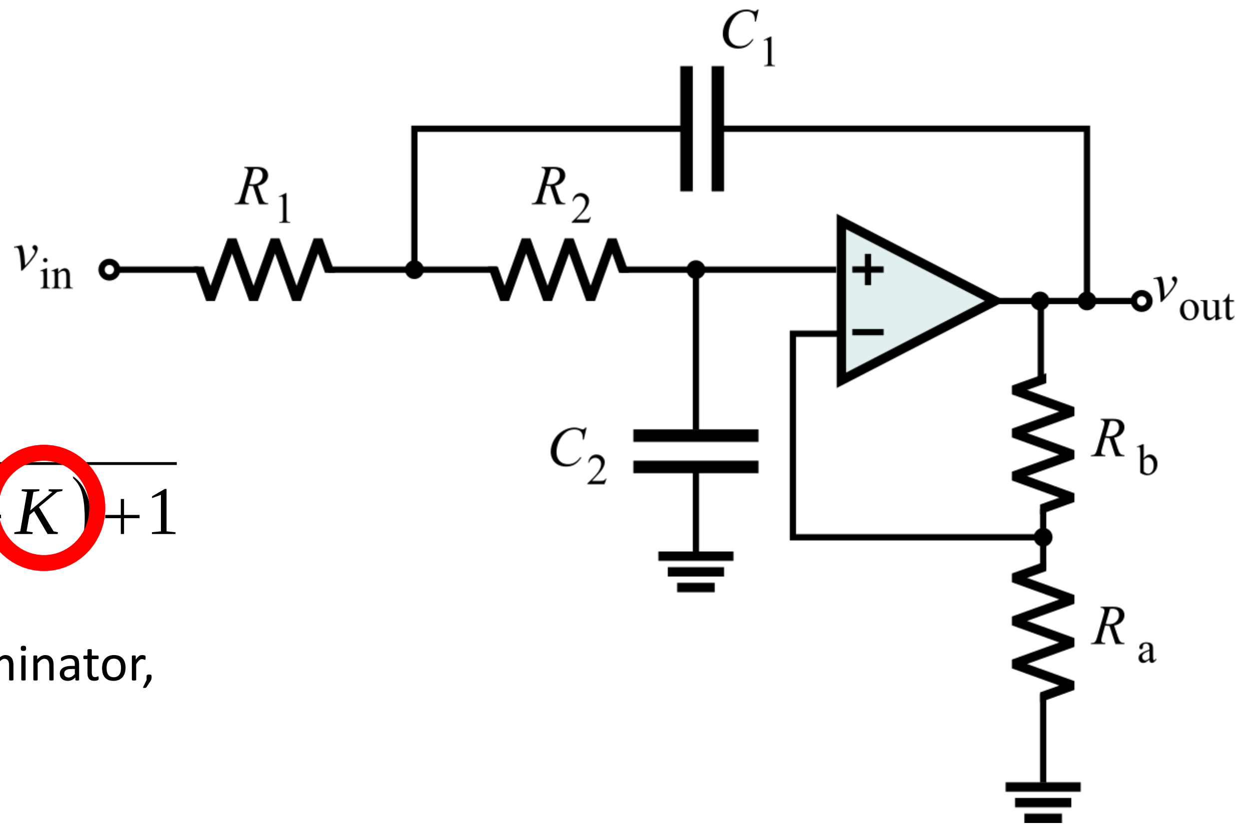
Sallen Key topology advantages:

- 1 Op Amp
- Zero output impedance

Notice the difference to the RC filter!

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2\left(C_1C_2R_1R_2\right) + s\left(C_2R_1 + C_2R_2\right) + 1}$$

Can be implemented with higher gain than 1:



$$\frac{V_{out}}{V_{in}} = \frac{K}{s^2\left(C_1C_2R_1R_2\right) + s\left(C_2R_1 + C_2R_2 + R_1C_1(1-K\right) + 1}$$

$$K = 1 + \frac{R_b}{R_a}$$

Gain also alters the roots of the denominator, so alters frequency response

Active Filters – Sallen Keys

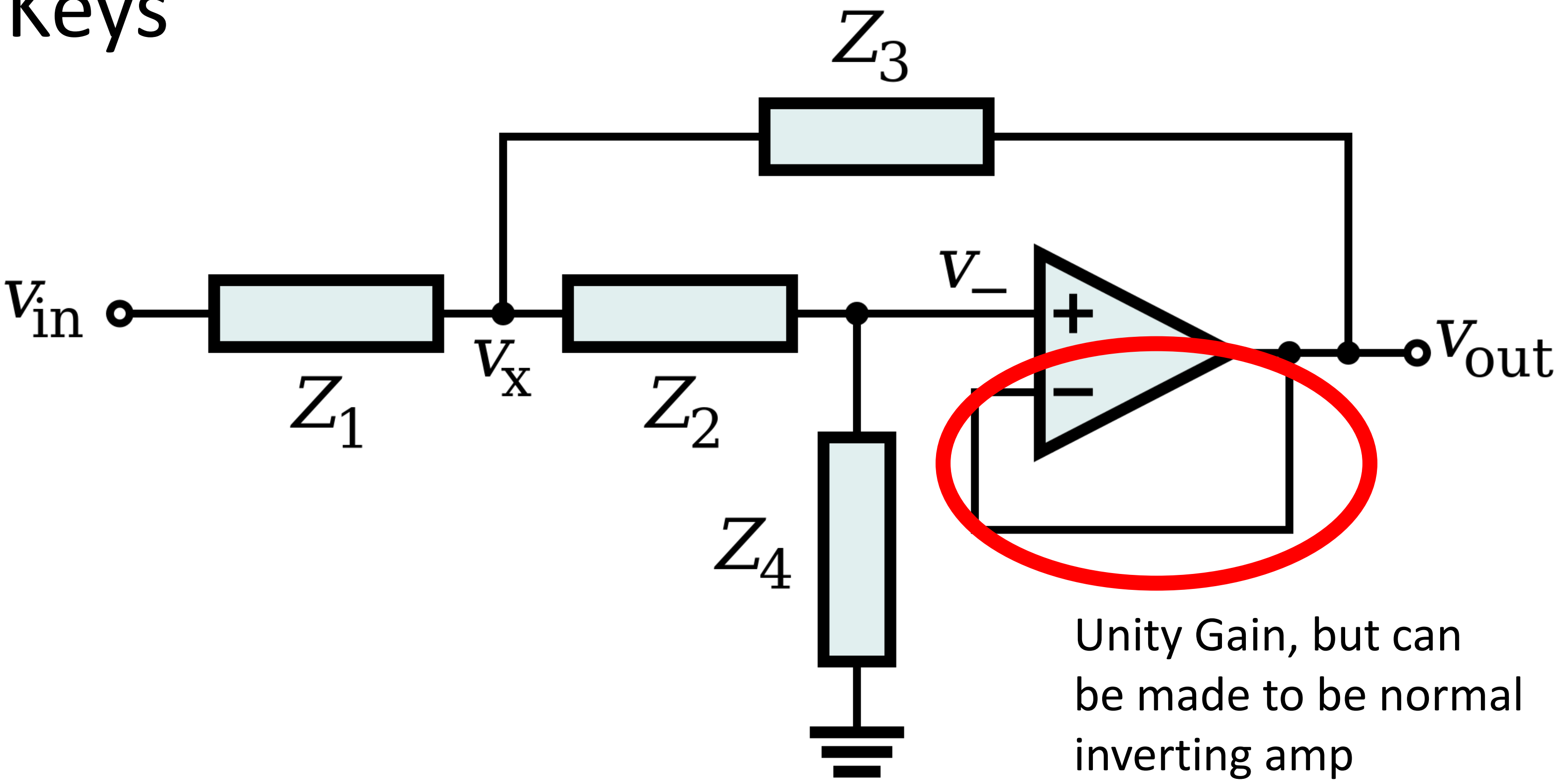
Versatile Op-amp topology

Can implement 2nd order filters low/high/band pass/notch etc.

The Z blocks can either be a resistor or a capacitor

Butterworth/Chebyshev etc. are determined by the *ratio* of these impedances

Gain Bandwidth Product of op amp limits the filters which can be realised



Unity Gain, but can be made to be normal inverting amp

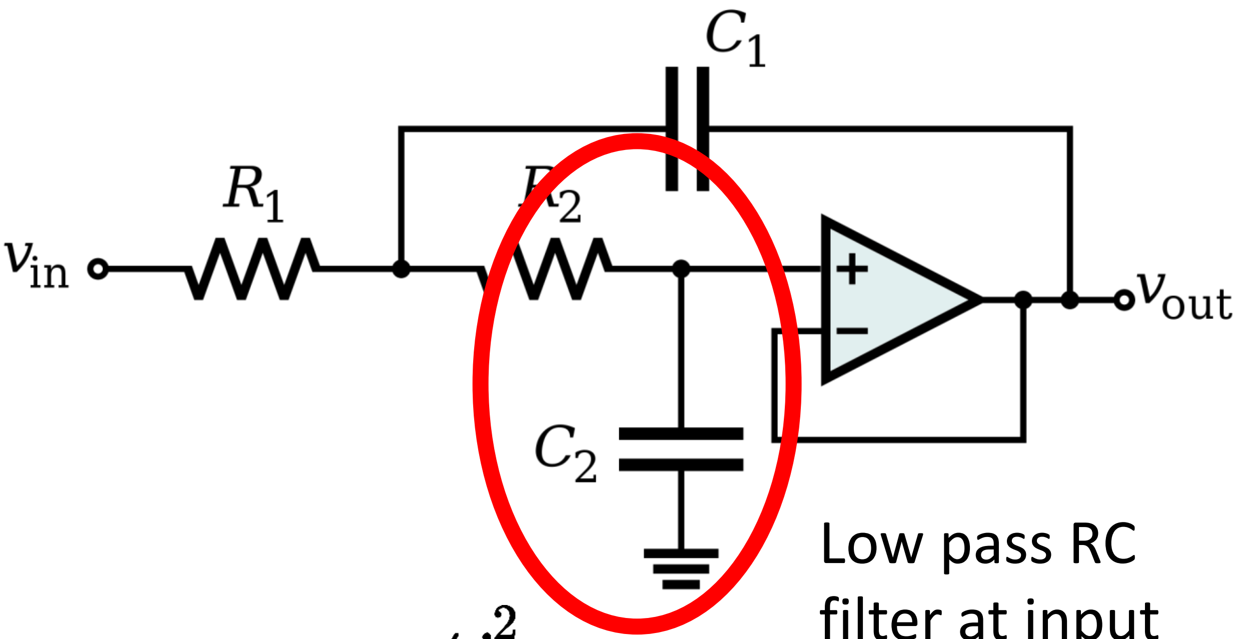
$$\frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4},$$

Same trade offs as we have already seen

Active Filters – Sallen Keys

These circuits are more complicated now, but we don't need to analyse them to work out what their job is if we come across them:

Low Pass Filter

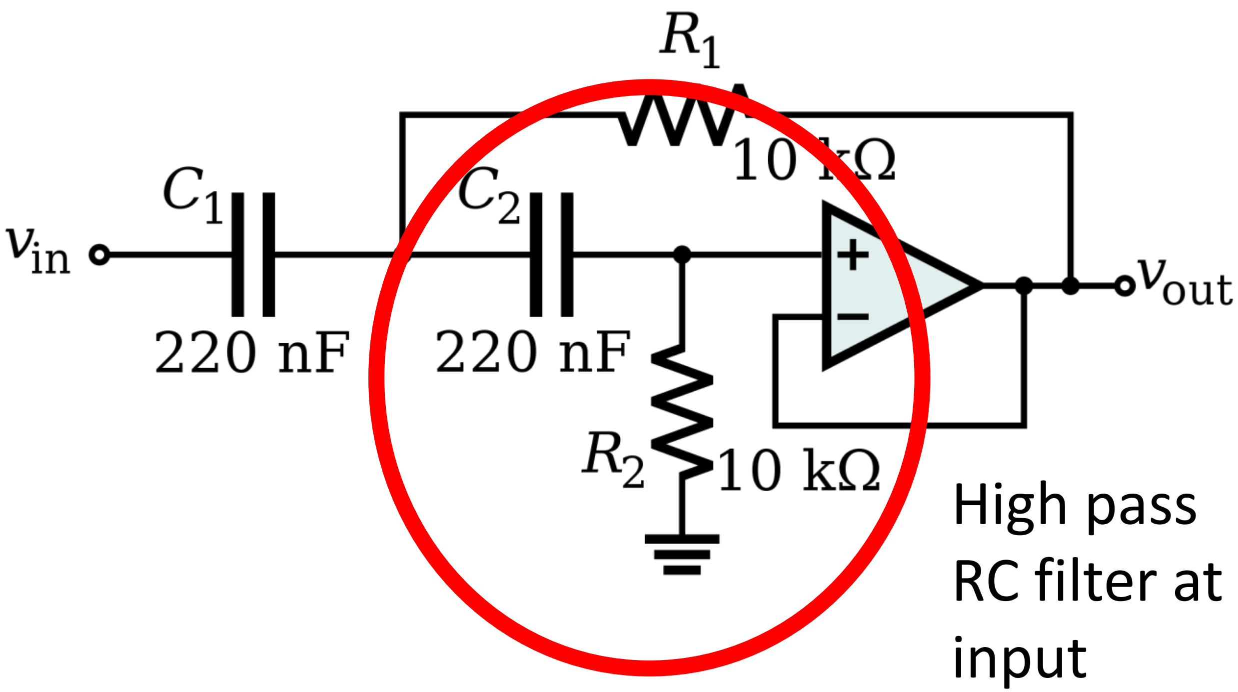


Low pass RC filter at input

$$H(s) = \frac{\omega_0^2}{s^2 + 2\alpha s + \omega_0^2},$$

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

High Pass Filter

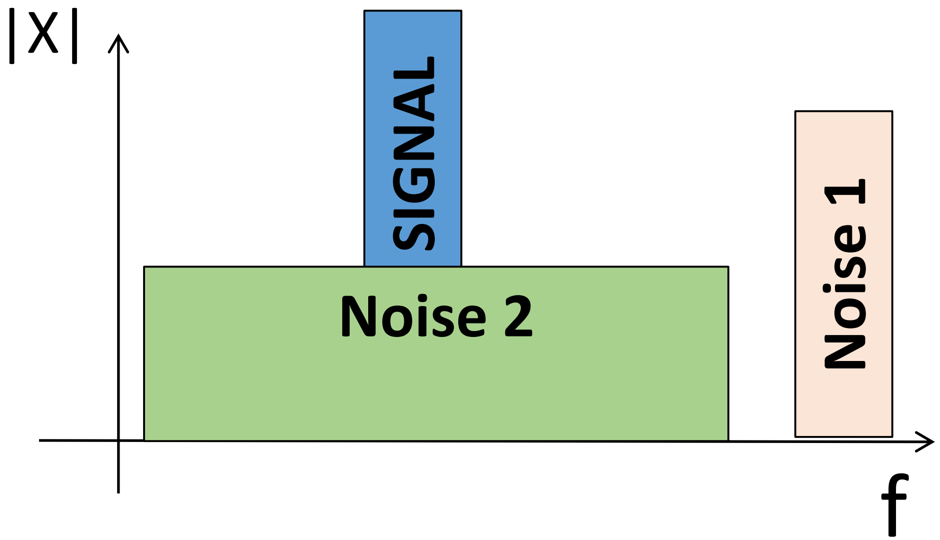


High pass RC filter at input

Exactly *what* values we need are specified by standard equations (Butterworth, Chebyshev etc.) So we can just look them up!

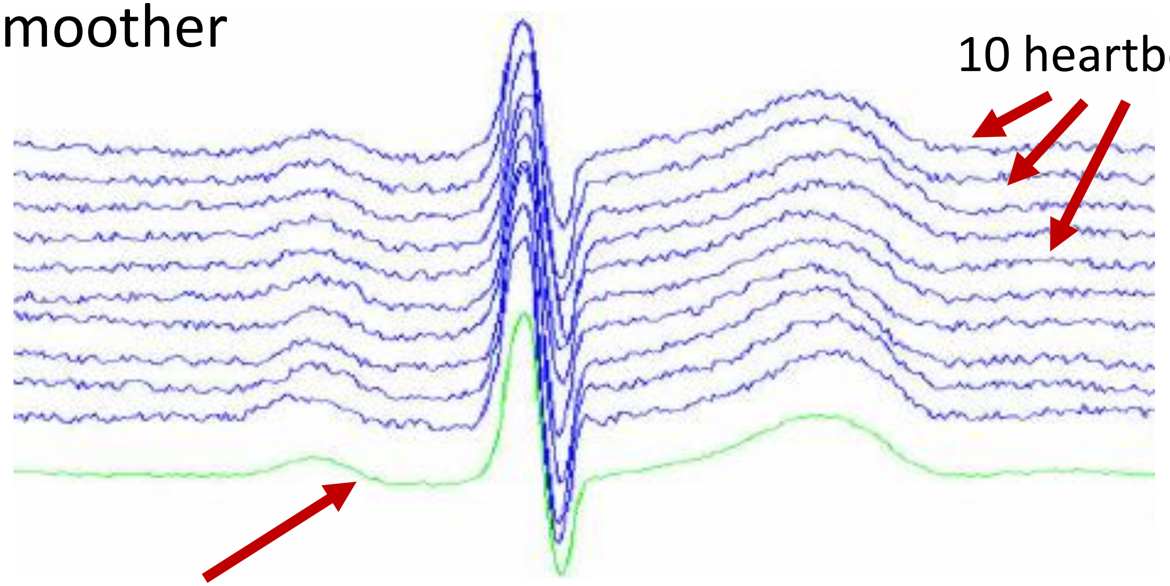
When filtering isn't enough – Coherent averaging

So far assumed the noise and signal are *separable* in frequency (noise 1).
But often it is *in band* (noise 2) i.e. shares some of the same frequencies



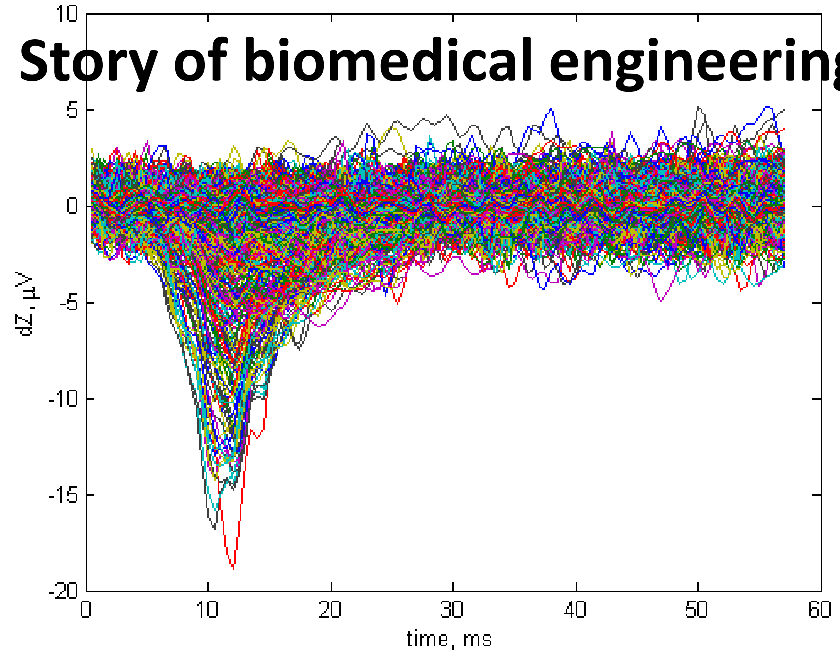
If signal is repeated, align and average across repeats. As noise is random with respect to signal i.e. *uncorrelated*, averaged signal will be smoother

10 heartbeats



Averaged (smoother) ECG signal

Story of biomedical engineering!



EIT signals *after* averaging

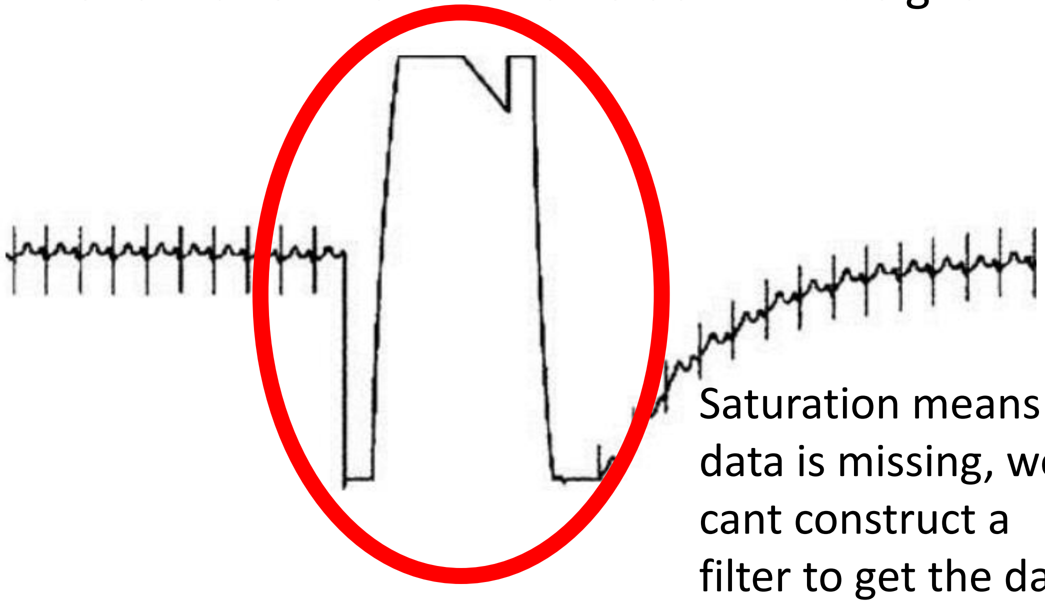
Filters ≠ Magic

Defibrillation transient artefact in ECG signal

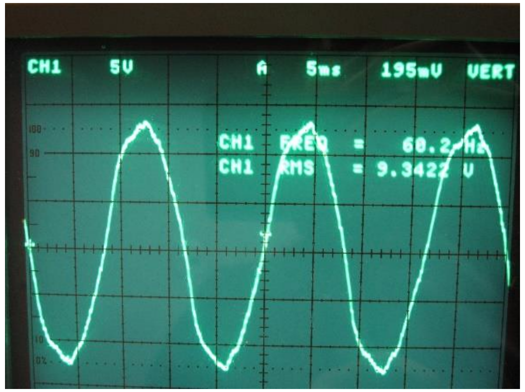
In general, filters cannot add data which is missing:

They can only reduce a signal by so much

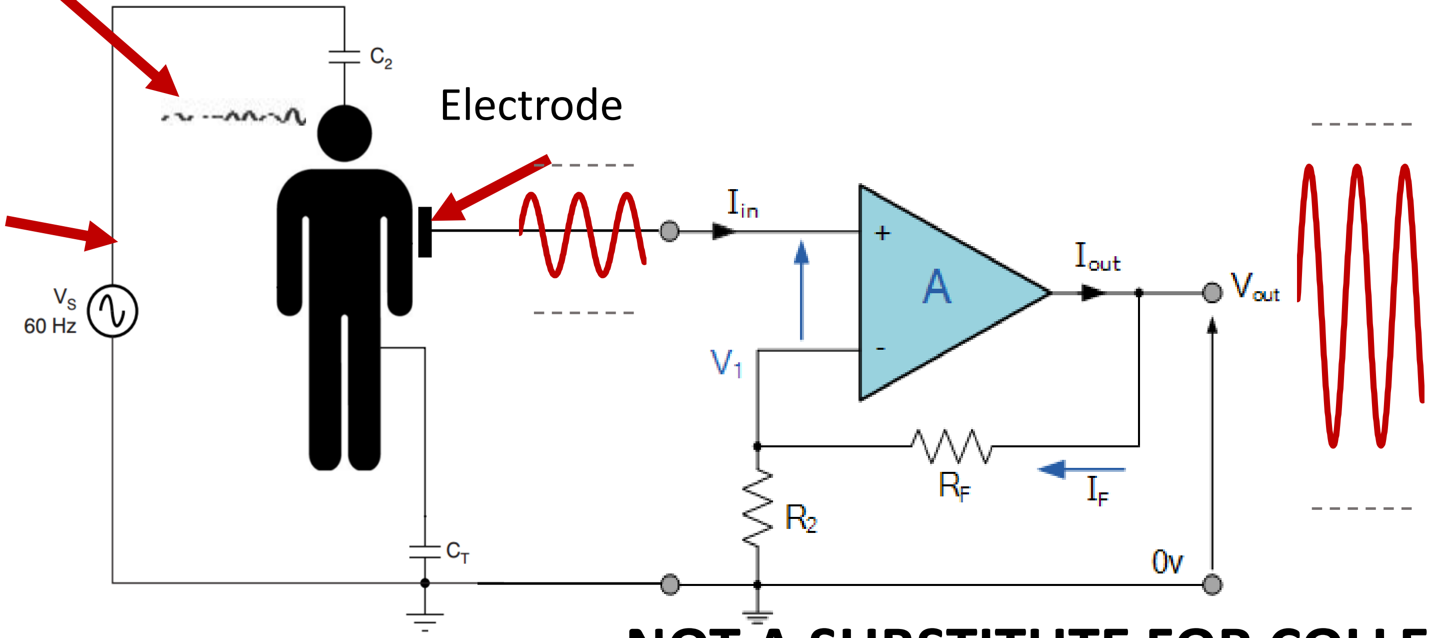
Single ended EEG recording
EEG signal ~100uV



Saturation means data is missing, we cant construct a filter to get the data back



Mains noise in room



Noise ~2V or ~20,000 larger.
So a reduction of 80 dB (10,000) still not enough!

NOT A SUBSTITUTE FOR COLLECTING GOOD DATA!

Thank you for your attention!

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