POLYTECH
Peter the Great
St.Petersburg Polytechnic
University

Course

« Introduction to Biomedical Engineering»

Dr. Kirill Aristovich

Section 2: Control theory

Lecture 2.2: 1st and 2nd order systems analysis

1st and 2nd order systems analysis

Hello, in this lecture will continue to analyze transfer functions and information we can get from them.

Let's start by remembering the second order system behavior, and the fact that it was controlled by the roots of the denominator. We will formalize this for any arbitrary transfer function, denoting that any polynome of n-th order can be factorized by its roots. Since s is a complex variable, we always can find n complex roots.

Now, the polynomic numerator of the transfer function will produce so-called zeros, the points where transfer function is equal to zero. The denominator will give us so-called poles. Both are important, and knowing all zeros, poles, and constant gain would completely define the transfer function, and, hence, the system behavior. Servo, for example would have no zeros and 2 complex poles.

A system is characterized by its poles and zeros as they allow the reconstruction of the input/output differential equation. It is possible to get a sense of the system dynamics from plotting the poles and zeros graphically on the s –plane. A pole is commonly represented by a cross (x) and a zero by a circle (○).

By plotting the locations of the poles and zeros on the complex s-plane, we can obtain a considerable amount of information about the response of the system without having to take the Inverse Laplace transform. Each pole corresponds to a component of the time domain response, so from the plot it is possible to determine: What components exist, their relative importance (and possible simplifications), and how they change with gain.

First, let's consider poles that have real components only. They will be located on real axis. They will transform to exponents in time domain, and will be decaying if negative, and increasing if positive. The relative position will control the speed of exponential increase or decay.

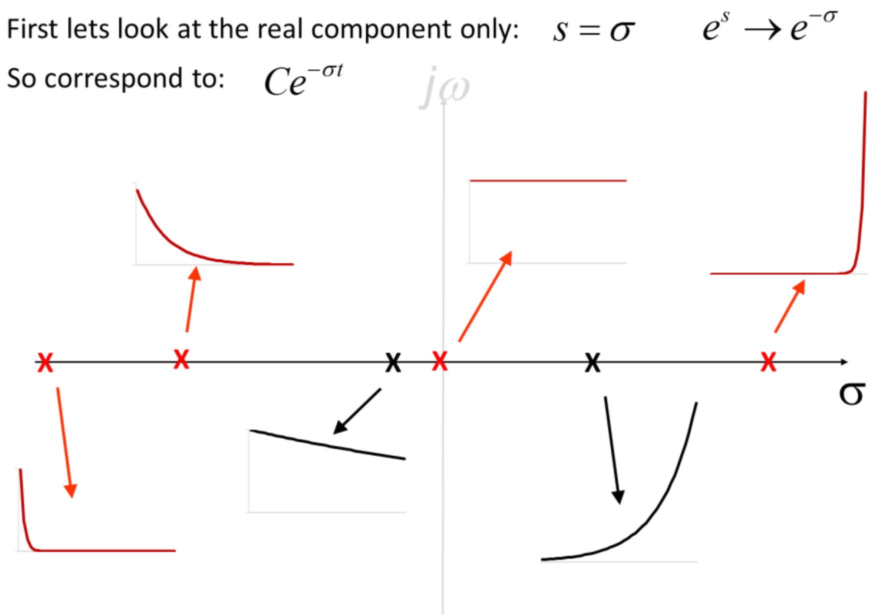

Figure 1- Real Poles – Exponentials

If we have purely imaginary poles, they will transfer to pure sinusoids in time domain, according to Euler formula. So they generate an oscillatory component with a constant amplitude. Poles closer to the origin have a low frequency, which increases the further poles are from the origin.

If we have complex poles, it is easy to see that they generate oscillatory component bound by exponential amplitude. Poles located in the left hand side decay to zero, whereas poles in the right hand increase to infinity, thus making the system unstable. The relative location and proximity to real or imaginary axis controls the speed of amplitude change and the frequency of oscillations respectively.

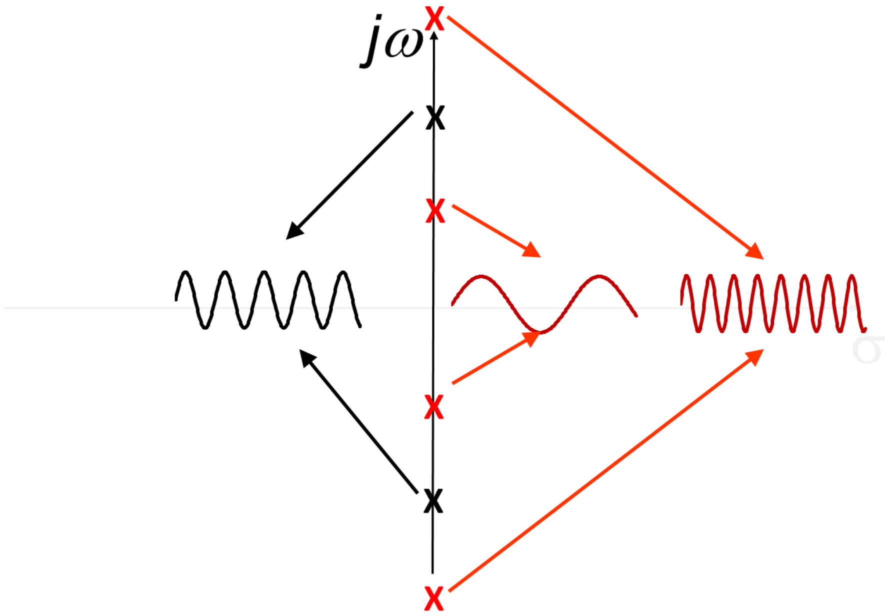

Figure 2 - Imaginary Poles – Sinusoids

Now, we can easily see what to do when designing a controller, we can start with plotting original open-loop poles, and decide what to do and how to move them around. For example, to stabilize the unstable system, we need to move all poles across imaginary axis to the left side. Now the rest is to decide what is there to add in the feedback to achieve that.

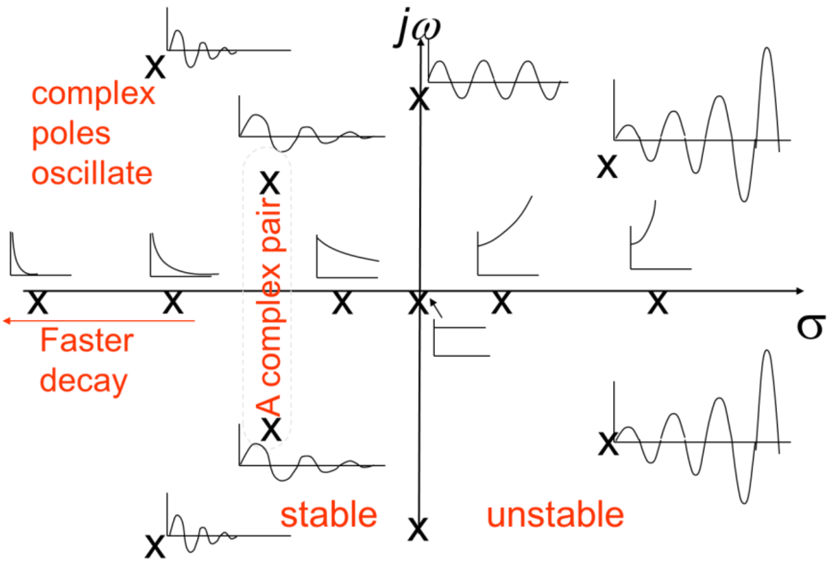

Figure 3 - Pole Location and Impulse response summary

Speaking of controlling different systems, it is important to know how accurately the control system tracks the demand (our desired input) once it has settled down, in other words the steady state behavior. For example, in case of step input response, does the system settles to a desired location, or there is a gap? What is the system accuracy in tracing the changing demand (this can be evaluated by supplying the ramp input)?

It turns out both parameters can be evaluated using the finite value theorem which takes advantage of some handy properties of the Laplace transform. The theorem says that the limit of the output when time reaches infinity, will be equal to the limit of s times Laplace of output, when s reaches zero. We are normally more interested in the final value of the ERROR rather than the output, as the goal of the controller is to drive the error as close to zero.

Further for some inputs such as a ramp or sinusoidal input, the final output isn’t really meaningful. Normally, we consider the system we need to control in the forward loop, and stuff we are adding to the system, or controller, in the feedback loop. It is not that important, but drawing it like that allows us to compute the error quite easily as Input minus Output times transfer function of the feedback. From the other hand, output is equal to the error (after summation element) times transfer function of the system. Substituting and rearranging, we get the expression for the error through transfer functions and the input. Using the final value theorem, it is possible to find the steady state error of the control system and thus judge the success of the controller without having to calculate the full dynamic response at all! Remembering how complicated it is using inverse tables combined with boundary conditions, we can appreciate the usefulness of these.

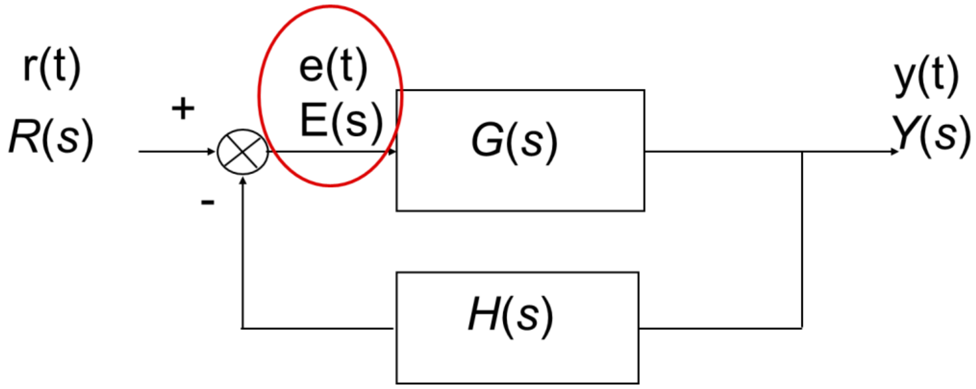

Figure 4 - Steady State Error

For the arbitrary system, which has a feedback controller, we can deliver generalized expressions for step steady state error, which depends on so-called position error constant, and ramp steady state error, which similarly depends on velocity error constant.

Finally, to illustrate the point we can compute the steady-state error of the simple feedback-controlled servo, and see that it is zero. I will leave it for homework for you to evaluate the ramp response error, knowing that the ramp input in Laplace domain is A over s square.

This course was developed with the support of
the "Open Polytech" educational project

OPEN
POLYTECH