

# Floating Bodies

## Floataction of Body

A body is said to be floating if the entire mass of the body is not submerged in a fluid. A body floats or sinks based on the relative density of the body w.r.t. the fluid. Body floats if the relative density is less than 1, else it sinks.

## Density

Density is the characteristic property of a substance. It is defined as the ratio of mass of a substance to its volume.

$$\rho = \frac{m}{V}$$

## Definition

Relationship between volume submerged, density of fluid and density of object

The fraction submerged is the ratio of the volume submerged to the volume of the object,

$$\text{Fraction submerged} = \frac{V_{\text{submerged}}}{V_{\text{obj}}} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}$$

where,  $\rho_{\text{obj}}$  is the average density of the object and  $\rho_{\text{fl}}$  is the density of the fluid.

## Definition

### Relative Density definition

Relative density is defined as the ratio of density of a substance to the density of the standard substance. Usually water at 4°C is used as a standard for a liquid or solid and air for a gas. It is a unitless quantity.

## Relative Density

Density of water at 4°C is given by  $\rho_w = 1 \text{ g/cm}^3$

Relative density, R.D. =  $\frac{\rho}{\rho_w}$

=  $\rho$  (in CGS units)

## Uses of relative density

Following are the uses of relative density:

- 1) It is used for determining the density of unknown substance from the known density of another substance.
- 2) It is used by geologist to find out the mineral content in the rock.
- 3) Testing purity of a substance (example gold).

## Lactometer

A lactometer is an application of hydrometer and is used to check purity of cow's milk. It works on the principle of Archimedes' principle. The instrument is graduated into a hundred parts. Milk is poured in and allowed to stand until the cream has formed, then the depth of the cream deposit in degrees determines the quality of the milk. If the milk sample is pure, then the lactometer floats on it and if it is adulterated or impure, then the lactometer sinks.

Note: The relative density of milk does not give a conclusive indication of its composition since milk contains a variety of substances that are either heavier or lighter than water.

## Principle of Floatation

According to the Principle of Floatation, a body floats in a liquid if the weight of the body is equal to the weight of the liquid displaced by it.

Note: The floating of a body occurs if and only if the density of the body is less than or equal to the density of fluid.

### Definition

#### Experimental verification of principle of floatation

Consider a wood of mass  $m$  with known density which is less than the density of water, uniform cross section  $A$  and length  $L$ . Mark equally spaced points on the wood and label them to facilitate the measuring of length of wood immersed in water. When slowly immersed in the liquid, it floats at the surface of water. Measure the length immersed in the fluid  $l$ .

Since the wood is at rest, it means that there is no net force acting on the wood. Hence, liquid is applying an upward force equal to the weight of the wood. This is the buoyant force ( $F_b$ ).

$$F_b = mg = \rho_{\text{wood}} L A \rho_{\text{wood}} g$$

Calculate the weight of displaced liquid,  $F_d = \rho_{\text{water}} l A \rho_{\text{wood}} g$

Find the ratio,  $F_b / F_d = \rho_{\text{wood}} L / \rho_{\text{water}} l$

The ratio comes out to be 1 hence proving principle of floatation.

### Result

#### Natural effects of atmospheric pressure

Some natural effects of atmospheric pressure are:

1. As the temperature in a region is increased, it creates a region of low pressure. This causes air to flow from high pressure regions to low pressure regions. Hence, winds are caused due to a difference in atmospheric pressure.
2. Atmospheric pressure decreases with increase in height. This creates harsher environments in higher altitudes. The combined effect of low temperatures and pressure at high altitudes is reflected by a reduction in the density of forests.

#### Experimental demonstration of atmospheric pressure

Fill a cup one-third with water. Cover the entire mouth with an index card. Holding the card in place, turn the cup upside down. Remove your hand from underneath.

Atmospheric pressure holds up the card and the water on it.

## Pressure Gauge

Pressure measurement is the analysis of an applied force by a fluid on a surface. Pressure is typically measured in units of force per unit of surface area. Many techniques have been developed for the measurement of pressure and vacuum. Instruments used to measure and display pressure in an integral unit are called pressure gauges or vacuum gauges

Variation of pressure with height in a fluid

For an infinitesimally small change in height, change in pressure is given by

$$dP = \rho g dh$$

Note:

This equation is useful in situations where density and acceleration due to gravity are functions of height.

Buoyant Force

Buoyant force is an upward force exerted by a fluid that opposes the weight of an immersed object. In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid. This causes an upward force on the immersed object because of the difference in pressure.

Definition

Apparent weight of floating body

Apparent weight is given by subtracting the upthrust on the body from the weight of the body.

Case 1- Floating body:  $W_{\text{apparent}} = 0$  because weight equals upthrust

Case 2- Sinking body:  $W_{\text{apparent}} = (\rho - \rho_0)Vg$

where  $\rho$ : density of the body

$\rho_0$ : density of the fluid

$V$ : volume of the body

Example

Buoyant Force on an immersed object in terms of specific gravity

Example: When a body lighter than water is completely submerged in water, the buoyant force acting on it is found to be  $n$  times its weight. Find the specific gravity of the material of the body.

Solution:

Let volume of the body be  $V$  and its density be  $\rho_s$

Let density of fluid be  $\rho$

Weight of the body is  $V\rho_s g$

Weight of the fluid displaced = Buoyant Force =  $V\rho g$

$n(\text{weight of body}) = \text{Buoyant force acting}$

$$\square n(V\rho_s g) = (V\rho g)$$

$$\rho_s = 1/n$$

Definition

Archimedes' Principle

Archimedes' principle states that when a body is totally or partially immersed in a fluid, there is an apparent loss in its weight due to upthrust and this apparent loss in weight is equal to the weight of the fluid displaced by the immersed part of the body.

## Applications of Archimedes' Principle

Some of the applications of Archimedes' Principle are:

Finding the volume of irregular shaped solids (knowing the density of water).

Finding the density of irregularly shaped solids (knowing the density of water).

Finding the relative density of a liquid.

Finding the density of liquids (knowing the density of water).

## Definition

### Pascal's Law

Pascal's law or the principle of transmission of fluid-pressure (also Pascal's Principle) is a principle in fluid mechanics, that states that pressure exerted anywhere in a confined fluid is transmitted equally in all directions throughout the fluid.

Note:

Pressure acts normally on the walls of the container in which it is present.

Result

Hydraulic jack

The hydraulic jack works on Pascal's law. An effort is applied on the piston of the jack with smaller area. This results in a greater force on the other end of the jack and hence results in a multiplication of force.

In the given diagram,

$$F_1 A_1 = F_2 A_2$$

Since,  $A_2 > A_1$

$$F_2 > F_1$$