PH3103 Mathematical Methods of Physics
Autumn Semester - 2025
Indian Institute of Science Education and Research, Kolkata
Instructor: Koushik Dutta

Homework: 3 Submission Date: 25/08/2025

The hand written solutions must be submitted at the start of the class

1. Identify the functions that are analytic functions of z in some region, and find their domain of analyticity
$$(a) \quad x \quad (b) \quad r \quad (c) \quad iy + x^2 + y^2 \qquad (d)(x - iy)/(x^2 + y^2) \tag{1}$$

2. Given u (or v) as indicated below, find the corresponding v (or u) by integrating CR equations, and hence find out the analytic function $f(z)$ up to an additive constant. Also find out the domain of analyticity of $f(z)$ in each case
$$(a) \quad v = 2xy \quad (b) \quad cos\ x\ cosh\ y \quad . \tag{2}$$

3. Show that $\nabla^2(|f(z)|^2) = 4|f'(z)|^2$.

4. Consider the following complex function $f(z) = (x^2 - y^2) + 2xyi$. Evaluate the integral $\int f(z)dz$ from the origin to the point $z = 1 + i$ along the paths (a) $x = t^2,\ y = t^3$ (b) $x = 4t^2,\ y = 8t^3$. In both the cases, show the path clearly. Also evaluate the integral in cartesian coordinates by first moving along the x-axis, and then moving parallel to the y-axis.

   Does the result remind something about what you have learned already for conservative forces? Think!

5. Suppose $f(z)$ is analytic on the closed disk of radius r centered at z_0. Then, show the following identity
$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + re^{i\theta}) d\theta \ . \tag{3}$$