Tutorial 5

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1 Free Particle in 3D

Consider a free particle in 3D (enclosed in a cube of side L, with periodic boundary conditions) whose wavefunction at time t=0 is given by:

$$\langle\vec{r}|\psi(0)\rangle \equiv \psi(r,0) = \frac{2}{\sqrt{L^3}}\cos\left(\frac{2\pi x}{L}\right)e^{i(3\pi y+5\pi z)/L} \tag{1}$$

(a) Momentum of the particle is measured at time t=0. What are the possible measurement outcome? State the probability of each.

(b) Write down the wavefunction of the particle at some later time $t > 0$.

(c) Suppose that the momentum of the particle is measured at time t=0 and has been found to be $\vec{p} = \frac{\pi\hbar}{L}(2\hat{i} + 3\hat{j} + 5\hat{k})$. What will be the wavefunction at some later time $t > 0$. **Recall, Measurement of an observable collapses the state of the particle into one of it's eigenvector.**

2 Spherical Harmonics

Consider the spherical harmonics given by:

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}} \qquad\qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta \tag{2}$$

(a) Show that these functions are normalised and orthogonal to each other.

(b) Show that both the functions are eigenfunctions of $\hat{L}^2$ and $\hat{L}_z$

The $\hat{L}^2$ and $\hat{L}_z$ operator are given by:

$$\hat{L}^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right) \qquad\qquad \hat{L}_z = -i\hbar\frac{\partial}{\partial\phi} \tag{3}$$

3 Spectrum for a 3D Problem

A system is described by the Hamiltonian

$$H = \frac{\mathbf{L}^2}{2I} + \alpha L_z. \tag{4}$$

What is the energy spectrum of the system?