PH3103 Mathematical Methods of Physics
Autumn Semester - 2025
Indian Institute of Science Education and Research, Kolkata
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Homework: 4 Submission Date: 1/09/2025

The hand written solutions must be submitted at the start of the class

1. Find the order of the pole and residue for the following functions: (a) f(z) = cosech z (b) tan z .
2. Obtain the Laurent series expansion of the following functions, valid in the region indicated in each case: (a) $(z+z^{-1})^{100}$ $(0<|z|<\infty)$ (b) $(z-1)^{-1}(z-2^{-2})$ $(1<|z|<2)$ (c) $e^{1/z}(1-z)^{-1}$ $(0<|z|<1)$.
3. Find the location, order and residue of each pole for finite z. Also analyse each function at the point of infinity. (a) e^{-z^2}, (b) $(2^z-1)/z$, (c) $1/(e^z+1)$.
4. Let a and b are positive constants, and let $n=0,1,....$ Use contour integration and the residue theorem to show these real integration that

$$(a)\quad \int_{-\infty}^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}=\frac{\pi}{ab(a+b)} \tag{1}$$

$$(b)\quad \int_{-0}^{\infty}\frac{dx\ x^2}{(x^2+a^2)^3}=\frac{\pi}{16a^3} \tag{2}$$

$$(c)\quad \int_{0}^{2\pi}d\theta e^{a\ cos\theta}cos(a\ sin\theta-n\theta)=\frac{2\pi a^n}{n} \tag{3}$$

$$(d)\quad \int_{-\infty}^{\infty}\frac{cosx\ dx}{1+x^2}=\pi/e \tag{4}$$

Give it a try to do these integration without using complex analysis, i.e residue theorem!