Tutorial 2

Ranbir Barman, Arnabesh Samadder

1 Solving Schrödinger equation in momentum space

You must have solved the Schrödinger equation in position-space representation for a free particle. A particle is considered to be free if the net force acting on it is zero. Let us now consider that the particle is under a constant force F.

(a) Write down the potential energy corresponding to this force. Recall that for a conservative force, the force and potential energy V are related by $F = -\nabla V$, which in 1D reads $F = -\frac{\mathrm{d}V}{\mathrm{d}x}$

Hint for next question:

The state of a particle is represented by a vector $|\Psi\rangle$, which is postulated to contain all the information about the particle that we are allowed to ask. The state evolves in time according to the relation:

$$\hat{\mathcal{H}}\,|\Psi\rangle = i\hbar\frac{\partial\,|\psi\rangle}{\partial t} \tag{1}$$

In quantum mechanics, the Hamiltonian of a single particle is written as:

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + V(\hat{x}) \tag{2}$$

The position and momentum operator in position space representation is:

$$\hat{x} = x\delta(x-x') \qquad\qquad \hat{p} = -i\hbar\frac{\partial}{\partial x}\delta(x-x') \tag{3}$$

The position and momentum operator in momentum space representation is:

$$\hat{x} = i\hbar\frac{\partial}{\partial p}\delta(p-p') \qquad\qquad \hat{p} = p\delta(p-p') \tag{4}$$

(b) Write down the Schrödinger equation for this potential in position space representation as well as momentum space representation.

(c) Solve the Schrödinger equation in the momentum space representation to obtain the wave function in momentum space. You should get:

$$\psi_E(p) = Ne^{\frac{i}{\hbar F}\left(Ep-\frac{p^3}{6m}\right)} \tag{5}$$

(d) Solve for the normalisation constant so that:

$$\int \psi^*_{E'}(p)\psi_E(p)dp = \delta(E-E') \tag{6}$$

Use the property:

$$\int_{-\infty}^{\infty} e^{ikx}dk = 2\pi\delta(x) \qquad\qquad \delta(kx) = \frac{\delta(x)}{|k|} \tag{7}$$

2 Question 2

Show that any operator $\hat{A}$ may be expressed as the linear combination of a Hermitian and an anti-Hermitian $\left(B^\dagger = -\hat{B}\right)$ operator.