Tutorial 4

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1 Trace Invariance, Commutators and Dynamics

The trace of an operator is defined as sum of the diagonal elements of the matrix representation of the operator in any orthonormal basis. Show that:

1. Trace of product of operators is cyclic, $\mathrm{Tr}(\hat{A}\hat{B}\hat{C}) = \mathrm{Tr}(\hat{C}\hat{B}\hat{A})$
2. Trace of an operator is independent of the basis chosen.
3. $\mathrm{Tr}(\hat{A}^\dagger)$=$\mathrm{Tr}(\hat{A})^*$

Using the operator in appropriate representation, show that the following commutation relation hold:

$$[\hat{x}, F(\hat{p})] = i\hbar\frac{\partial F}{\partial p} \qquad\qquad [\hat{p}, G(\hat{x})] = -i\hbar\frac{\partial G}{\partial x} \tag{1}$$

Using the Heisenberg equation of motion, show that for a particle described by the usual Hamiltonian, $\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + V(\hat{x})$,the following relation holds:

$$\frac{\mathrm{d}\hat{x}_H}{\mathrm{d}t} = \frac{\hat{p}_H}{m} \qquad\qquad \frac{\mathrm{d}\hat{p}_H}{\mathrm{d}t} = -\frac{\partial V(\hat{x}_H)}{\partial \hat{x}} \tag{2}$$

Combining the above results, we get:

$$m\frac{\mathrm{d}^2\hat{x}_H}{\mathrm{d}t^2} = -\frac{\partial V(\hat{x}_H)}{\partial \hat{x}} \tag{3}$$

2 An electron inside a dielectric

A large dielectric cube with edge length L is uniformly charged throughout its volume so that it carries a total charge Q. It fills the space between condenser plates, which have a potential difference Φ_0 across them. An electron is free to move in a small canal drilled in the dielectric normal to the plates. The Hamiltonian for the electron is

$$\mathcal{H} = \frac{\hat{p}^2}{2m} + \frac{K\hat{x}^2}{2} + \frac{e\phi_0\hat{x}}{L}. \tag{4}$$

Find the ground state wave function and energy of the Hamiltonian $\mathcal{H}$

3 Uncertainty principle revisited

Let $|0\rangle$ and $|1\rangle$ denote the normalized eigenstates corresponding to the ground state and first excited states of a one dimensional harmonic oscillator . Find out the uncertainty in position and momentum, Δx and Δp respectively for the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. Verify Heisenberg uncertainty relation for this state.