PH3101 Classical Mechanics @2025

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Lecture 04

Newtonian Mechanics and coordinate systems

Cartesian Coordinate system

EOM : $\vec{F} = \dfrac{d\vec{P}}{dt}$

$$m\ddot{x} = F_x(x, y) \quad m\ddot{y} = F_y(x, y)$$

With out any force: $F_x = F_y = 0$

$$m\ddot{x} = 0 \quad m\ddot{y} = 0 \implies \begin{array}{l} P_x = const \\ P_y = const \end{array}$$

$$\vec{P} = P_x\hat{x} + P_y\hat{y}$$

Conservation of linear mometum!

Plane Polar Coordinate system

EOM : $\vec{F} = \dfrac{d\vec{P}}{dt}$

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = F_r(r, \theta)$$

Radial part of EOM

$$m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = F_\theta(r, \theta)$$

Angular part of EOM

With out any force: $F_r = F_\theta = 0$

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = 0$$

$$m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = 0$$

Newtonian Mechanics and coordinate systems

With out any force: $F_x = F_y = 0$

$$m\ddot{x} = 0 \qquad m\ddot{y} = 0 \implies P_x = const$$

$$P_y = const$$

$$\vec{P} = P_x\hat{x} + P_y\hat{y}$$

Conservation of linear mometum!

Plays an important role.

With out any force: $F_r = F_\theta = 0$

$$m\left(\ddot{r} - r\dot{\theta}^2\right) = 0 \longrightarrow 1$$

$$m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = 0 \longrightarrow 2$$

$$2 \implies r\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = 0$$

$$\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0$$

$mr^2\dot{\theta} = L_z$ $\quad L_z$ is constant $\longrightarrow$ 3

$$1 \implies \left(\ddot{r} - r\dot{\theta}^2\right) = 0$$

Substitue for $\dot{\theta} = \dfrac{L_z}{r^2}$ here, and simplify

Newtonian Mechanics and coordinate systems

With out any force: $F_x = F_y = 0$

$$m\ddot{x} = 0 \qquad m\ddot{y} = 0 \implies \begin{array}{l} P_x = const \\ P_y = const \end{array}$$

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Plays an important role.

$mr^2\dot{\theta} = L_z$ $\quad L_z$ is constant $\longrightarrow$ 3

$$1 \implies \dot{r}\left(\ddot{r} - r\dot{\theta}^2\right) = 0$$

Substitue for $\dot{\theta} = \dfrac{L_z}{r^2}$ here, and simplify

$$\frac{1}{2}\frac{d}{dt}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) = 0$$

$$\frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) = E$$ Is constant of motion. Linear momentum does not showup and angular momentum is conserved and plays an important role!

Angular Momentum

In polar coordinates, or in any system with a fixed point, angular momentum plays an important role. Angular momentum is defined as:

$$\vec{L} = \vec{r} \times \vec{P}$$

In the same style, we may define the torque for a force as :

$$\vec{N} = \vec{r} \times \vec{F}$$

Using Newtons law of motion, it is easy to show $\vec{N} = \dfrac{d\vec{L}}{dt}$

The equivalent conservation principle states that in the absence of torque, angular momentum is conserved.

Work Done by the Force

The effect of a force on a particle is quantified by the acceleration it produces, or equivalently, by the rate of change of momentum. (by the way it is called NLOM!)

But is this the only way to represent the effect of force on a physical system?

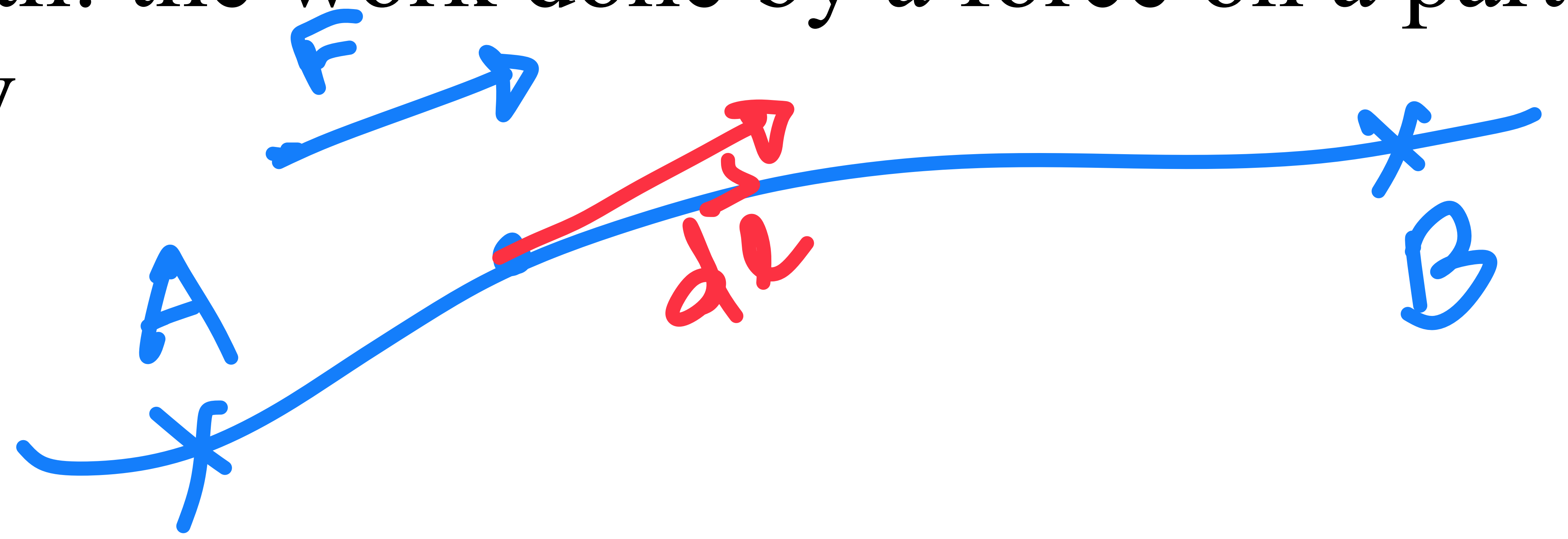
The answer is: No!

Let's define the work done by a force on a particle that is displaced by a certain amount, say $\vec{dl}$

$$dW = \vec{F} \cdot \vec{dl}$$

This can be extended to a finite path: the work done by a force on a particle moving from point A to point B is given by

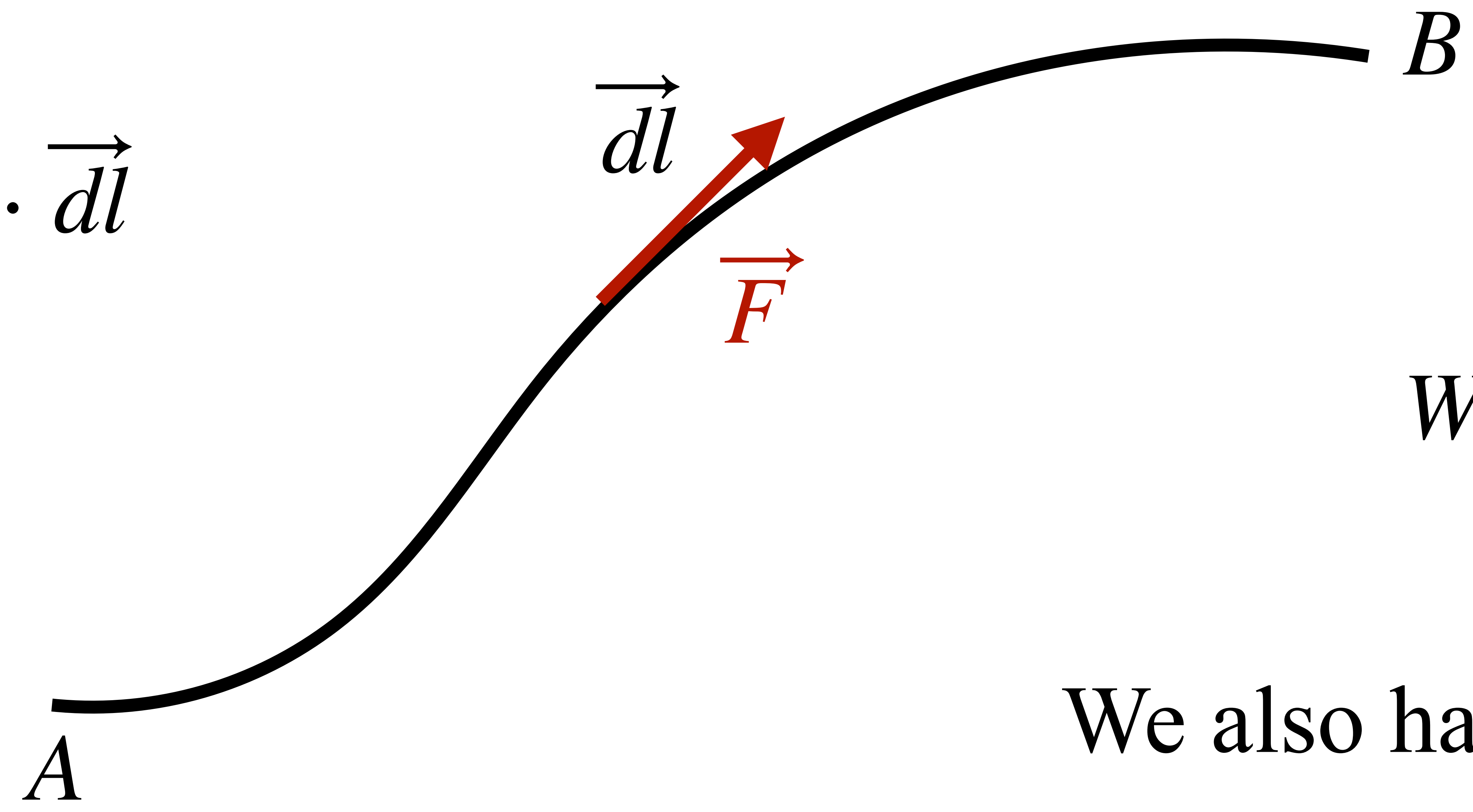
$$W = \int_A^B \vec{F} \cdot \vec{dl}$$



Work Force

Let's consider the workdone for moving particle from A → B

$$W = \int_A^B \vec{F} \cdot \vec{dl}$$



Let's get more from this

$$W = \int_A^B \vec{F} \cdot \vec{dl} = \int_A^B \vec{F} \cdot \frac{\vec{dl}}{dt} dt$$

We also have $\vec{v} = \dfrac{\vec{dl}}{dt}$ and $\vec{F} = m\dfrac{d\vec{v}}{dt}$

$$W = \int_A^B m\frac{d\vec{v}}{dt} \cdot \vec{v}\ dt = \int_A^B d\left(\frac{1}{2}m\,\vec{v}\cdot\vec{v}\right) = \int_A^B d\left(\frac{1}{2}m\,v^2\right)$$

Where, the Kinetic energy, $T = \dfrac{1}{2}m\,v^2$

$$W = \frac{1}{2}m\,v_B^2 - \frac{1}{2}m\,v_A^2 = T_B - T_A$$

Work and Energy

The work done by a force in moving a particle from point A to point B equals the difference in kinetic energy between the two points and is independent of the path followed.

Conservative Systems

An interesting type of vector field is one that can be written as the gradient of a scalar potential,

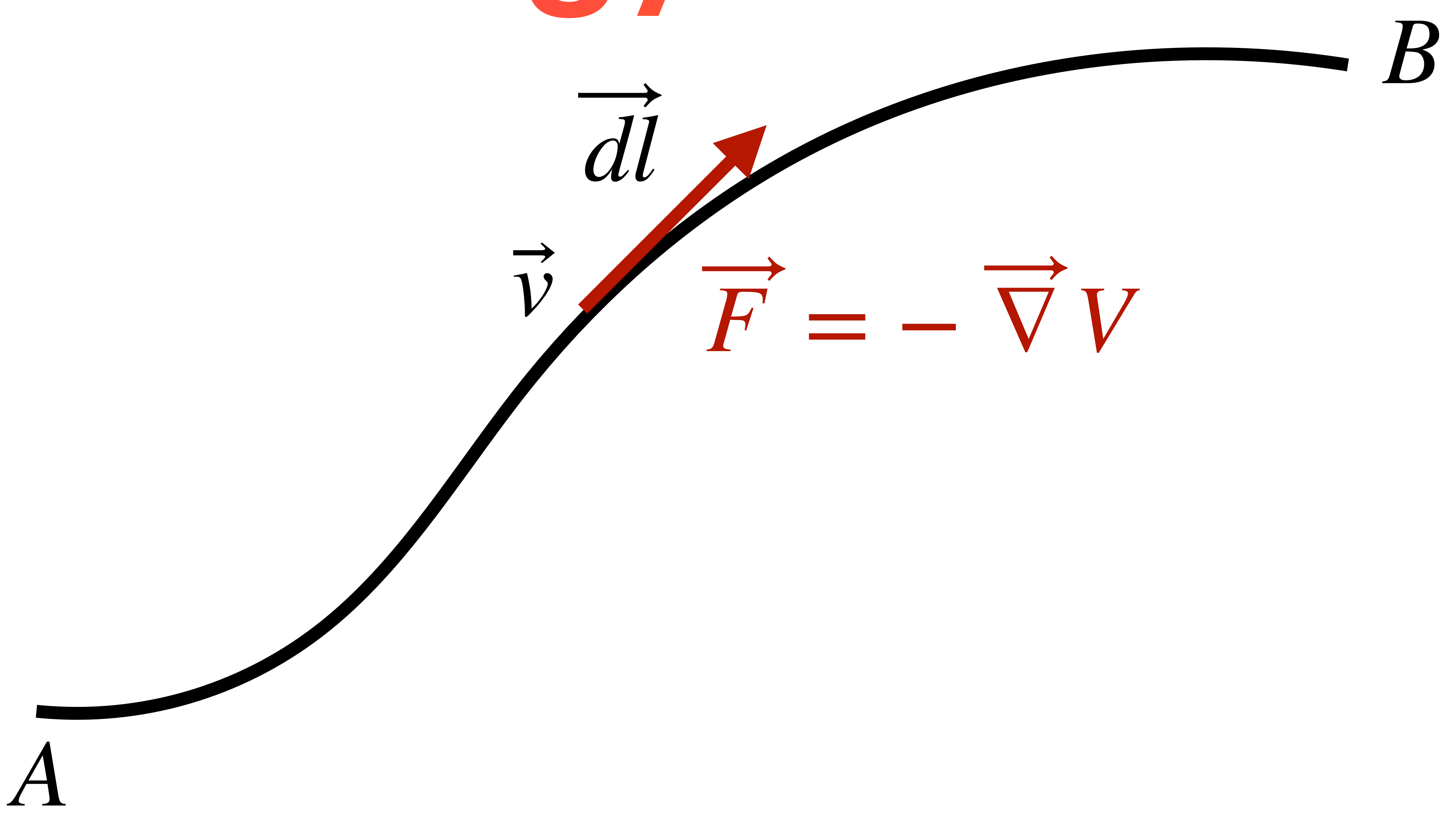
$$\vec{F} = -\vec{\nabla} V, \text{ where } V(\vec{X}) \text{ is potential function or potential energy}$$

This is interesting, because workdone takes a simpler form

$$W = \int_A^B \vec{F} \cdot \vec{dl} \quad = -\int_A^B \vec{\nabla} V \cdot \vec{dl} \quad = -\int_A^B dV \quad = V(A) - V(B)$$

Here as well, the work done depends solely on the potential difference between the initial and final points, and is independent of the path taken.

Conservation of total energy



Let $\vec{F}$ be a conservative force, then the workdone by the force in moving a particle A to B is given by

$$W = \int_A^B \vec{F} \cdot \overrightarrow{dl}$$

Let's start with NLOM

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$\vec{v}$ is the velocity of the particle

Because force is conservative,

$$-\vec{\nabla} V = m\frac{d\vec{v}}{dt}$$

take a dot product with $\overrightarrow{dl}$ on both sides and intergrating from A to B

$$-\int_A^B \vec{\nabla} V \cdot \overrightarrow{dl} = m\int_A^B \frac{d\vec{v}}{dt} \cdot \overrightarrow{dl}$$

Conservation of total energy

Let's start with NLOM

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$$-\int_A^B \vec{\nabla} V \cdot \vec{dl} = m\int_A^B \frac{d\vec{v}}{dt} \cdot \vec{dl} \implies -\int_A^B d(V) = \int_A^B d\left(\frac{1}{2}mv^2\right)$$

$$V(A) - V(B) = T_B - T_A \implies T_A + V_A = T_B + V_B$$

Total energy remains constant

For a particle moving under a conservative force, the total energy, $E = T + V$, remains constant along its path.

Conservation of total energy

Conservation of energy can be used to derive the equation of motion, but this approach is valid only for conservative systems.

We have used NLOM while showing conversations of energy

$$\boxed{\vec{F} = \frac{d\vec{P}}{dt}} \quad \vec{F} = -\vec{\nabla} V \quad \boxed{E = T + V}$$