Reference

The Art of Electronics → Horowitz and Hill

~~Electr~~

Grading

Endsem - 40 marks

Midsem - 20 marks.

Class Test (3) → best 2 out of 3 → 20 marks

Class Participation → 10 marks

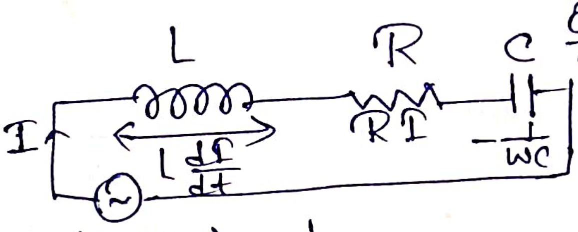
Assignments → 10 marks.

Prerequisites

1) Fourier Transform
2) Fourier Series
3) Linear Algebra → Hoffman kunze level.

* There is no tutorials. All 4 are lectures.

$$\frac{Q}{C} = \frac{\int I\,dt}{C}.$$



$V = V_0 \sin wt.$

$I = I_0 \sin(wt + \varphi)$ Let $\varphi = 0$

$\dot{I} = I_0 e^{jwt} \quad j = \sqrt{-1}.$

$V = Z I$ → impedence.

$$V = \left(jwL + R - \frac{j}{wC}\right)I = \left\{R + j\left(wL - \frac{1}{wC}\right)\right\}I$$

$$Z = R + j\left(wL - \frac{1}{wC}\right)$$

$$\Rightarrow z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

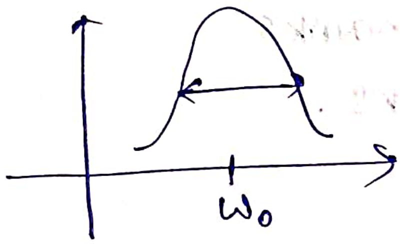
$$|z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\varphi = \tan^{-1}$$

Resonance Condition :-

$\cancel{|z| = \sqrt{\ }}$

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

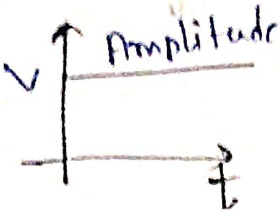


Lecture - 1

05/08/2025

Recap :

DC :-



Hydel Plant → AC generator.

Rectifier converts AC into DC.

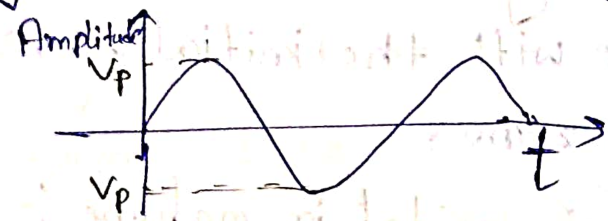
example :-

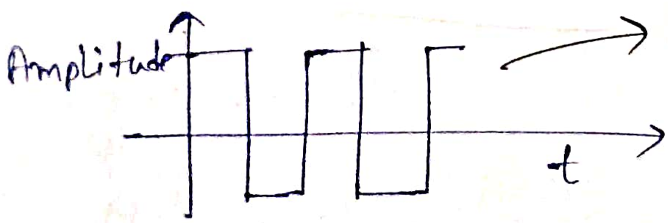
1) 1.5 V battery
2) Mobile Charger.

Alternating Current :-

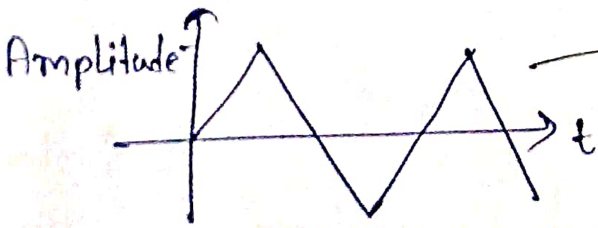
Flow of charge changes its direction periodically.

⇒ Voltage also reverses along with current.

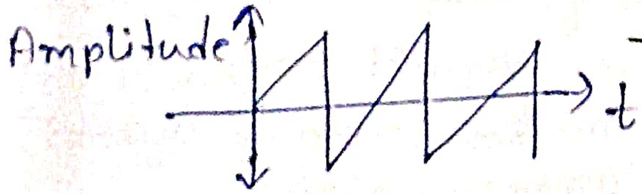




→ Square



→ Triangular.



→ Saw-tooth.

→ Continuous wave.

Function generator
or
AWG → Arbitary Wavefront Generator.

· Pulsed Signals.

Waveform of A.C. :-

AC can have various forms.
The most common one is sinusoidal.

$$v(t) = V_p \sin(\omega t + \phi)$$

V_p → Amplitude, ω → frequency, ϕ → Phase.

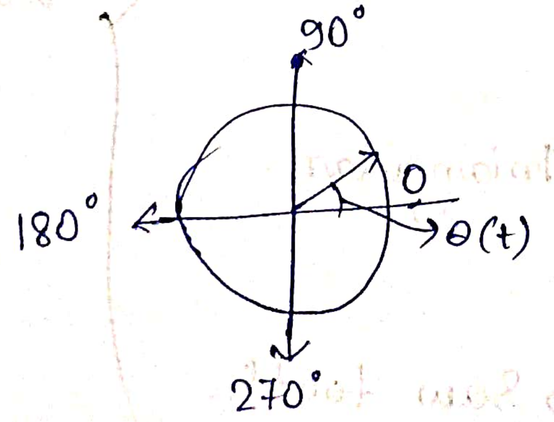
V_p :- Peak Voltage is the maximum Voltage it can reach in either the +ve side or the -ve side.

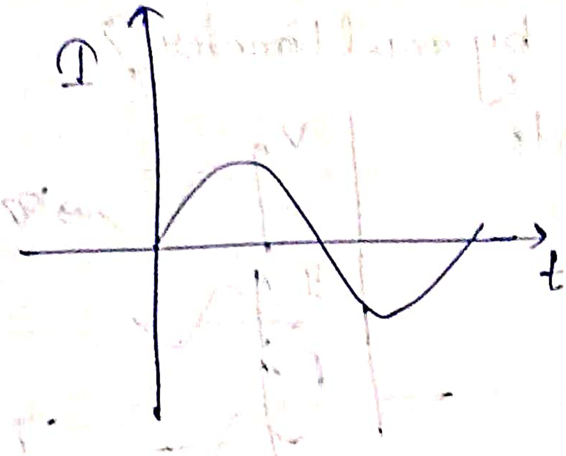
ω :- angular frequency, $\omega = 2\pi f$.

ϕ :- Phase difference with the initial wave
→ It can be zero.

Why is A.C. signal sinusoidal in nature :-

$$v(t) = V_p \sin(\omega t + \phi)$$





Mathematical expression :-

As the magnetic flux φ changes in a wire, the Faraday's law states that in E.M.F. is giving by the rate of change of magnetic flux.

$$\text{E.M.F.}: E = -\frac{d\varphi}{dt}.$$

If we have N turns in the coil,

$$E = -N\frac{d\varphi}{dt} \quad \ldots \text{(I)}$$

$$\varphi = \vec{B}.\vec{A} = BA\cos\theta \quad \ldots \text{(II)}$$

B = magnetic field

A = Area enclosed ; $\theta \rightarrow$ angle between the field lines and the normal of the coil area.

Using (I) and (II),

$$E = -N\frac{d\varphi}{dt} = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d(\cos\theta)}{dt} = BAN\sin\theta\,\underbrace{\dot{\theta}}_{\omega} \quad (\theta = \omega t)$$

$$= NBA\omega\sin\theta = NBA\omega\sin\omega t$$

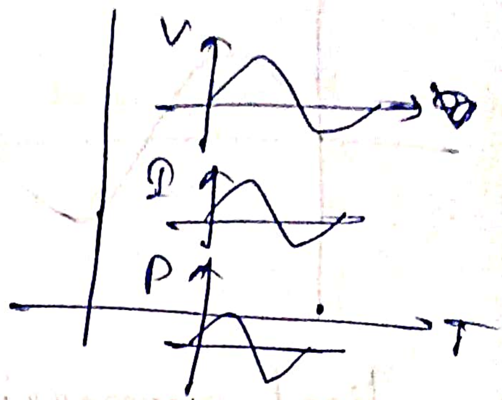
Note :-

(I) It is a continuous function of time.

(II) As it passes through an inductor or a capacitor (Energy storage material) its shape or frequency do not change.

Why rms is measured by multimeter?

$\Rightarrow$ Because rms represents the amount of heat dissipated in a resistor!



Square Wave :-

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin(2\pi(2k-1)ft)}{2k-1}$$

$$= \frac{4}{\pi}\left(\sin(2\pi ft) + \frac{1}{3}\sin(6\pi ft) + \frac{1}{5}\sin(10\pi ft) + \dots\right)$$

$1.2\pi ft$ $3.2\pi ft$ $5.2\pi ft$.

$\Rightarrow$ Square waves contains only odd integer harmonics.

Saw tooth wave :-

$$x(t) = \frac{A}{2} - \frac{A}{\pi} \sum_{k=1}^{\infty} (-1)^k \frac{\sin(2\pi k ft)}{k}.$$

fundamental frequency $= 2\pi f$.

Work out and find out the harmonics by expansion.

Triangular wave :-

$$x(t) = \frac{8}{\pi^2} \sum_{k=0}^{\infty} \frac{(-1)^k (\sin(2\pi(2k+1)ft))}{(2k+1)^2}$$

RMS

what we measure in a multimeter is the rms value of an A.C. signal.

RMS value is the value of the direct current that dissipates power in a resistor.

(i) Let us have n-values $(x_1, x_2, \ldots x_n)$

then $x_{rms} = \sqrt{\frac{1}{n}(x_1^2 + x_2^2 + \cdots + x_n^2)}$

(ii) If $f(t)$ is a continuous functions in an interval between T_1 and T_2, then

$$f_{rms} = \sqrt{\frac{1}{T_2 - T_1}\left(\int_{T_1}^{T_2} [f(t)]^2 dt\right)}.$$

for a large T, for a function over all time 'T'

$$f_{rms} = \lim_{T \to \infty} \sqrt{\frac{1}{T}\int_0^T [(f(t))^2] dt}.$$

For a periodic function, one period is enough to capture the entire duration.

$V(t) = V_p \sin \omega t$.

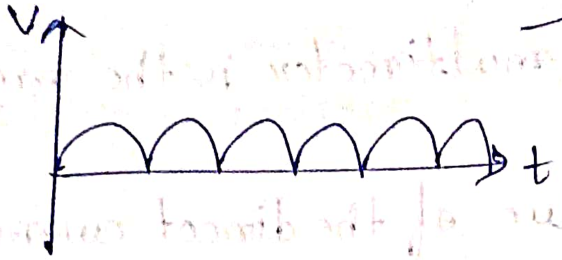
$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \sin^2 \omega t \, dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \frac{(1 - \cos 2\omega t)}{2} dt} \quad \left.\frac{\sin 2\omega t}{2\omega}\right|_0^{2\pi} = 0.$$

$$= \sqrt{\frac{1}{2\pi} \cdot \frac{2\pi}{2}} = \frac{1}{\sqrt{2}} V_p.$$

$$V_{rms} = \frac{V_p}{\sqrt{2}}.$$

Full wave rectifier :-

$\rightarrow V_{rms} = \frac{V_p}{\sqrt{2}}$

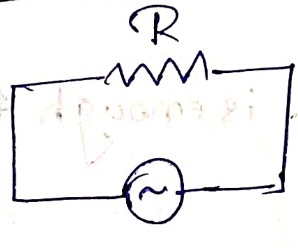


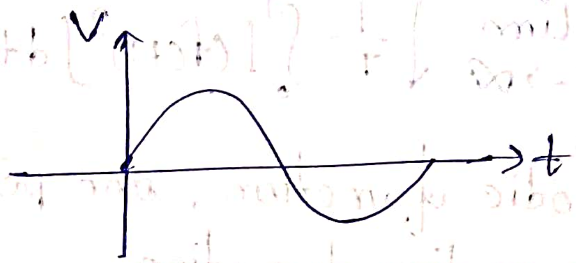
Half wave rectifier :- (HWR)

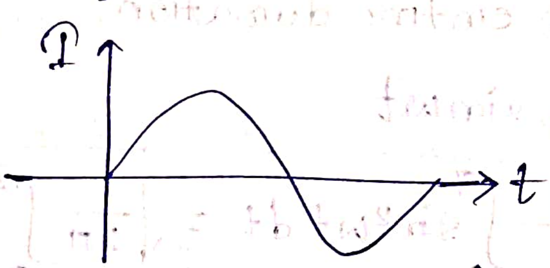
$\rightarrow V_{rms} = \frac{V_p}{2}$.

$$f(HWR) = \begin{cases} V \sin \omega t, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi \end{cases}$$

Single Phase :-

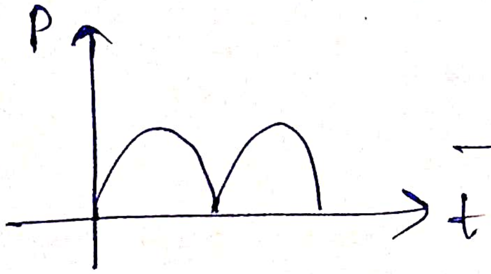






Power, $P(t) = (V_p \sin \omega t)(I_p \sin \omega t)$

$= V_p I_p \sin^2 \omega t = V_p I_p \left(\frac{1-\cos 2\omega t}{2}\right)$

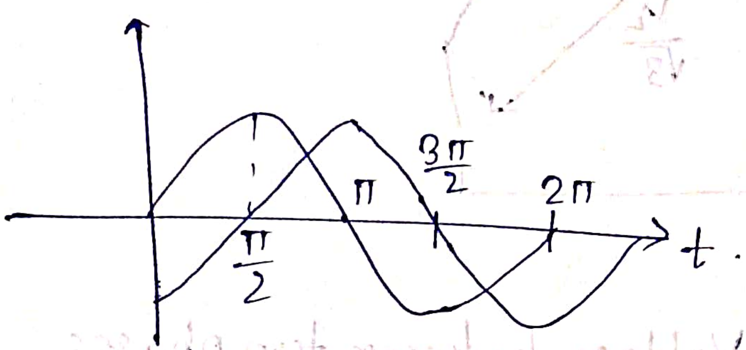


$\rightarrow$ Power that is varying with time

Two phase Line

$V_1(t) = V_p \sin\omega t$

$V_2(t) = V_p \{ \sin(\omega t - \frac{\pi}{2})$



Instantaneous power $P(t) = P_1(t) + P_2(t)$

$$= \frac{V_p I_p}{2}(1-\cos 2\omega t) + \frac{V_p I}{2}(1-\cos(2\omega t - \pi))$$

$\because \cos(x-\pi) = -\cos x$

$$\Rightarrow P = \frac{V_p I_p}{2} \{1-\cos 2\omega t + 1 + \cos 2\omega t\} = V_p I_p$$

Three Phase Signal / Line

All motors are running on three phase.

$V_1(t) = V_p \sin\omega t$

$V_2(t) = V_p \sin(\omega t - \frac{2\pi}{3})$

$V_3(t) = V_p \sin(\omega t - \frac{4\pi}{3})$.

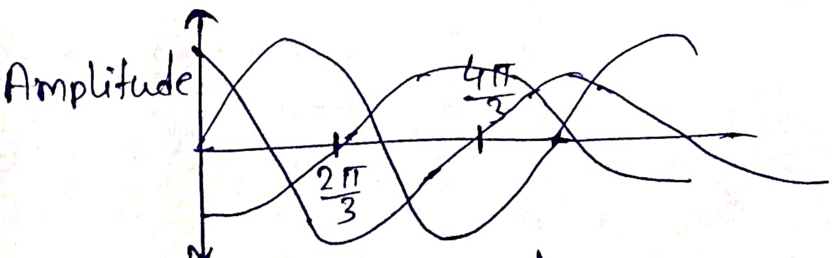
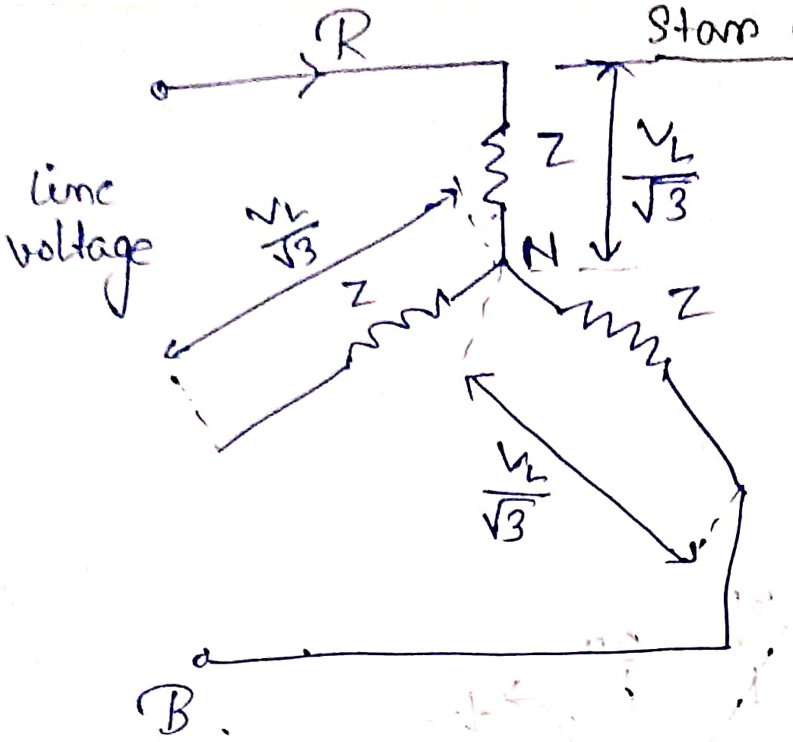


Fig:- 3 phase waveform with 120° phase shifts.

Sum of these 3 phases at any instant:

$V_1(t) + V_2(t) + V_3(t) = 0$.

Star Connection or Y



Line Voltage = Voltage between two phases.

Phase Voltage = Voltage between one phase and neutral point (N).

Line Voltage $V_L = \sqrt{3}\, V_P$.

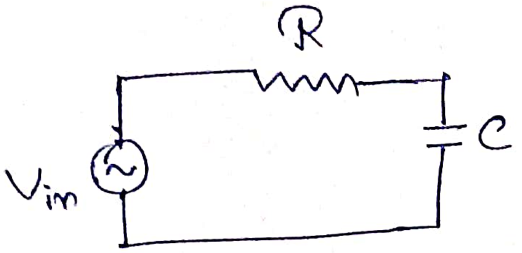
Line voltage = 415 V.

Phase voltage = 240 V.

Lecture - 3

07/08/2025

C-R Circuit

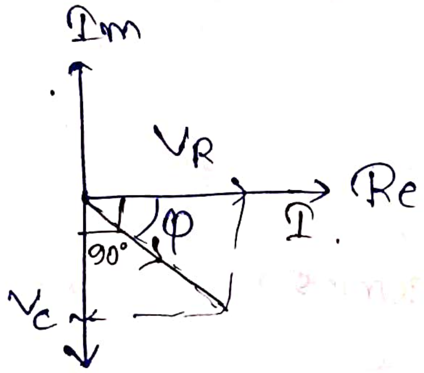


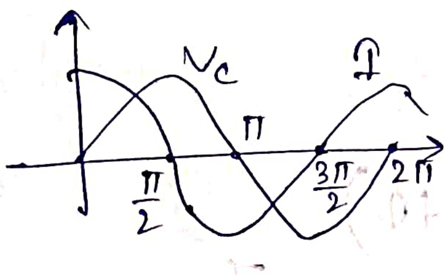
Phasor Diagram

$j = \sqrt{-1}$.

Voltage drop ~~accr~~ across 'R' = IR.

Voltage drop across 'c' = $V_c = \frac{I}{j\omega C} = -\frac{j}{\omega C} I$.





Current is leading V_c by 90°.

Net impedence in the R-C circuit is

$$Z = R - \frac{j}{\omega C} \quad \ldots \quad (I)$$

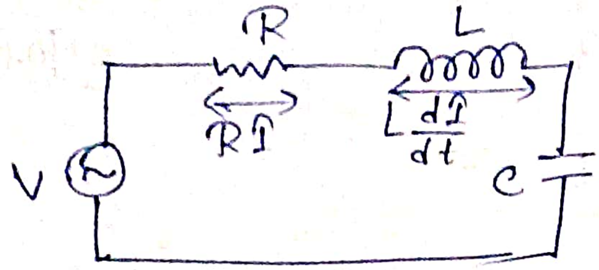
$$ZZ^* = |Z|^2$$

$$|Z| = \sqrt{\left(R - \frac{j}{\omega C}\right)\left(R + \frac{j}{\omega C}\right)} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad \ldots \quad (II).$$

$$\varphi = \tan^{-1}\left(\frac{1}{R\omega C}\right) \quad \ldots \quad (III).$$

$$\boxed{0 < \varphi < 90^\circ}$$

$$Z = |Z| e^{j\varphi} \quad \ldots \quad (IV).$$



~~a~~ $\frac{\int I\,dt}{C} = \frac{Q}{C}$. $j\omega L$

$I = I_0 \sin\omega t$. across $C: -\frac{j}{\omega C}$.
$= \mathrm{Im}(I_0 e^{j\omega t})$

$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

Impedence $X_C = -\frac{j}{\omega C}$.

$\Rightarrow V_C = X_C I$.

Power in R-C ~~cir~~ series

Let us assume,

$V = V_p \sin(\omega t)$

$I = I_p \sin(\omega t + \phi)$.

$P = VI = V_p \sin\omega t \, I_p \sin(\omega t + \phi)$

$= \frac{V_p I_p}{2}\left[2\sin\omega t \sin(\omega t + \phi)\right]$

$= \frac{V_p I_p}{2}\left[\cos(\omega t - \omega t - \phi) - \cos(2\omega t + \phi)\right]$

$= \frac{V_p}{\sqrt{2}}\frac{I_p}{\sqrt{2}}\cos\phi - \frac{V_p}{\sqrt{2}}\frac{I_p}{\sqrt{2}}\cos(2\omega t + \phi)$.

$P_{av} = \frac{1}{T}\int_0^T P\,dt = V_{rms} I_{rms} \cos\phi$.

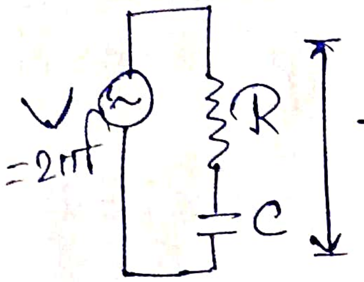
$\hookrightarrow$ Power factor.

$$\cos\varphi = \frac{R}{|z|}$$

$$\Rightarrow P_{av} = V_{rms} I_{rms} \frac{R}{|z|} = I_{rms} |z| I_{rms} \frac{R}{|z|}$$

$$= I_{rms}^2 R.$$

Variation of z and phase angle with frequency



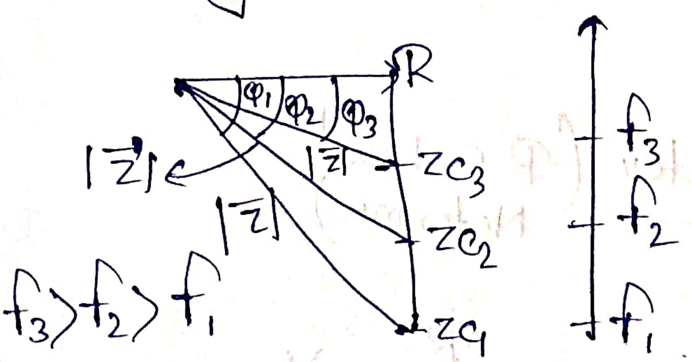
For a series R-C circuit -

(I) R remains constant.

(II) $|z_c| = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ → decreases as frequency increases.

(III) Total z or $|z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$: decreases as "

(IV) Phase angle decreases as "

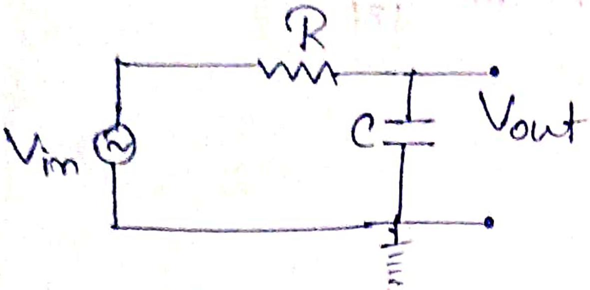


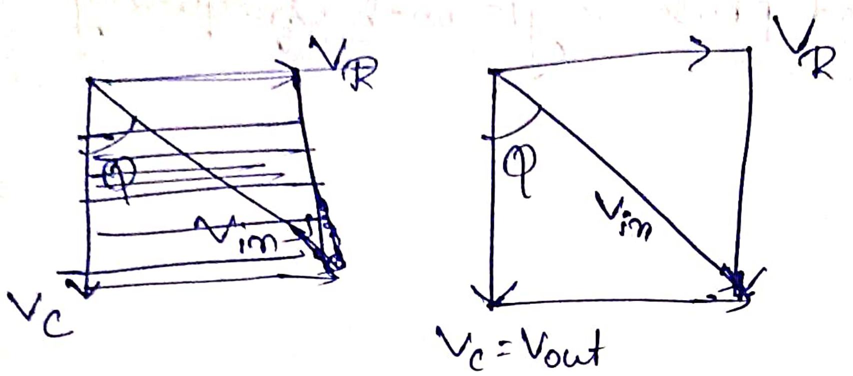
Homework :-

Write a small program to ~~ca~~ plot P_{av} as a function of φ.

from φ : 0 to 4π.

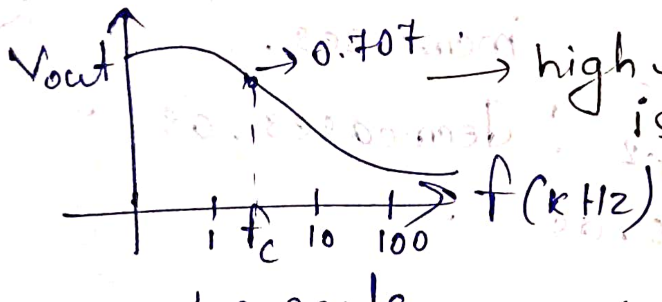
R.C. Lag Network (Low-Pass filter).





$$V_{out} = I Z_C = Z_C \cdot \frac{V_{in}}{\sqrt{R^2 + Z_C^2}}$$

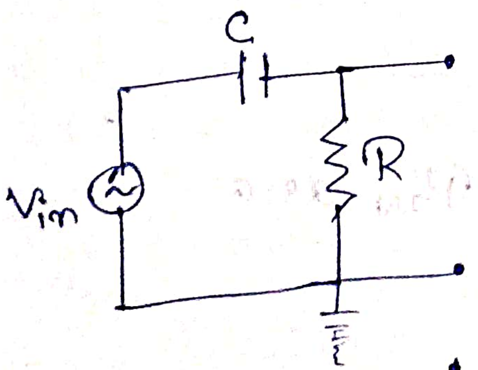
$$\varphi = 90^\circ - \tan^{-1}\left(\frac{Z_C}{R}\right).$$



→ high frequency is rejected.

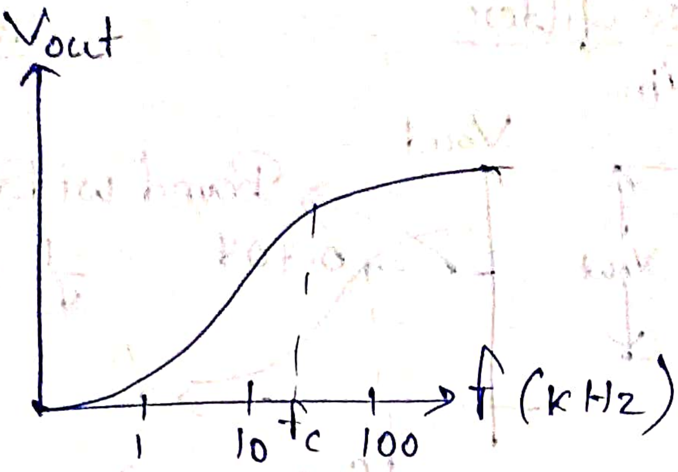
log scale.

High Pass Filter (R.C lead Network).



$$V_{out} = I.R = \frac{R}{\sqrt{R^2 + Z_C^2}} V_{in}$$

The phase lead $\varphi = \tan^{-1}\left(\frac{Z_C}{R}\right)$



Cut off frequency :-

Frequency at which the resistor load = capacitor load.

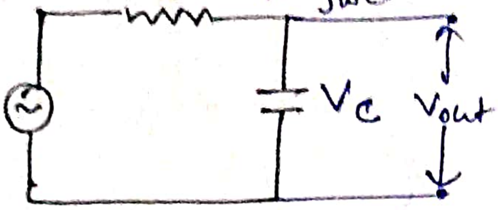
$$R = |Z_c| \Rightarrow R = \frac{1}{\underset{\text{cutoff}}{\underbrace{w_c}} C} = \frac{1}{2\pi f_c C}$$

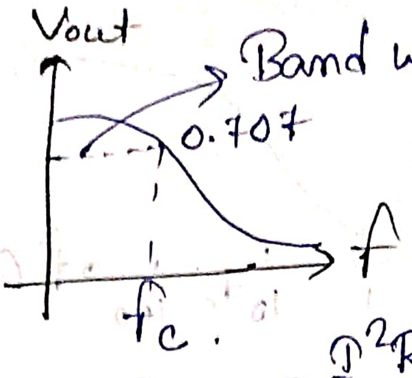
$$f_c = \frac{1}{2\pi RC}$$

12/08/2025

Low Pass filter

$V_R = \frac{R}{R + \frac{1}{j\omega C}} V_{in}$





$\frac{1}{\sqrt{2}} = 0.707.$

$V_c = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{in}.$

Power $= I^2R = VI = \frac{V^2}{R}.$

$V = IR.$

Half power $= \frac{V_{out}}{\sqrt{2}}.$

$Z = R - \frac{j}{\omega C}.$

$Z_R = Z_C$

$\parallel$

$R = \frac{1}{\omega C}.$

Total $|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}.$

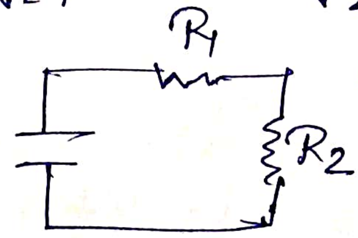
$\boxed{|V_c| = \frac{1}{\omega C}}$

$R = \frac{1}{\omega C} = X_C.$

$V_R = V_C.$

$|V_{out}| = \frac{\frac{1}{\omega C}}{\sqrt{R^2 + X_C^2}} V_{in} \longrightarrow$ at $R = X_C$

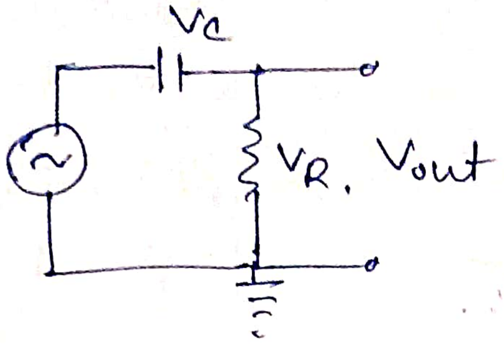
$= \frac{\frac{1}{\omega C}}{\sqrt{R^2 + R^2}} V_{in} = \frac{R}{\sqrt{2}R} V_{in} = \frac{V_{in}}{\sqrt{2}}.$

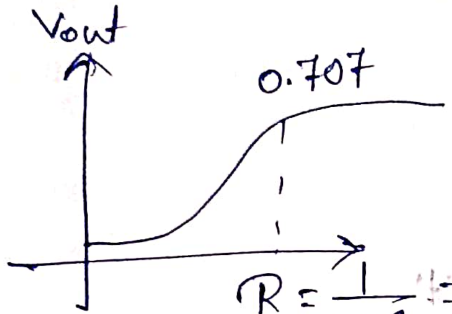


$V_{R_1} = \frac{R_1}{R_1 + R_2} V_{in}$

$V_{R_2} = \frac{R_2}{R_1 + R_2} V_{in}$

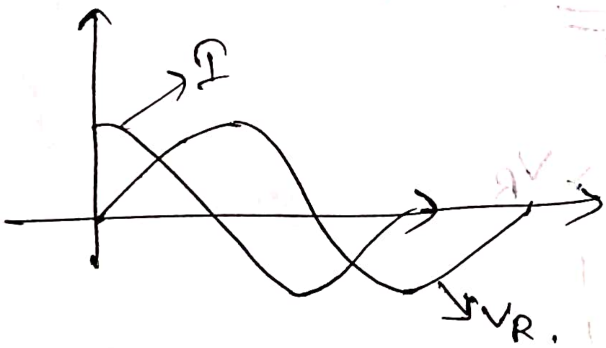
High Pass Filter





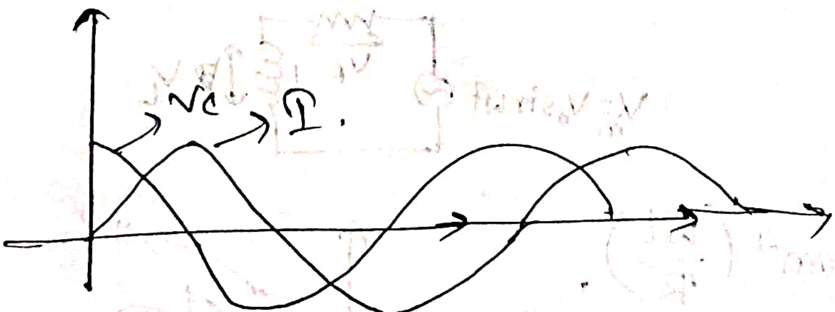
$R = \frac{1}{WC}$ = cut off frequency

Voltage Lag :-

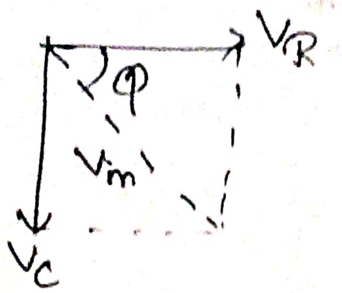


I and V_R are in phase

Voltage Leading :-

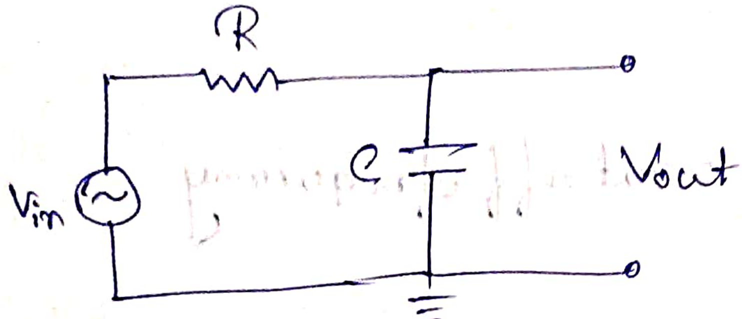


Exact phase difference is not 90'. there is a factor $\phi = \tan^{-1}\left(\frac{1}{RWC}\right)$.

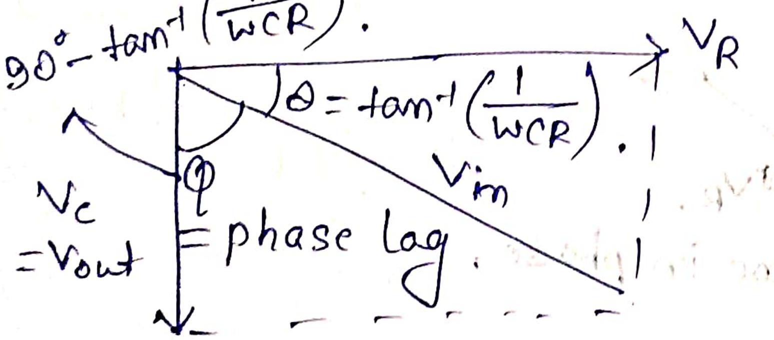


$$\varphi = \tan^{-1}\frac{Z_C}{R} = \tan^{-1}\frac{1}{\omega RC}.$$

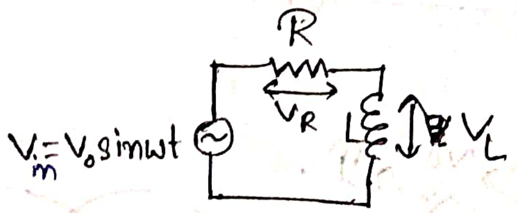
RC Lag Network



$$V_{out} = I Z_C = \frac{Z_C V_{in}}{\sqrt{R^2 + Z_C^2}}$$



R-L Circuit



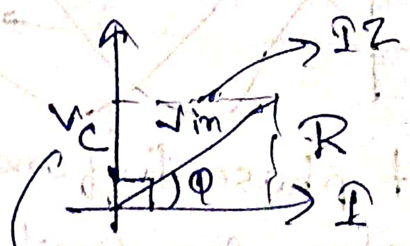
$V_R = IR.$

$V_L = j I \omega L.$

$$\varphi = \tan^{-1}\frac{V_C}{V_R} = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

$Z = R + j\omega L.$

$|Z| = \sqrt{R^2 + \omega^2 L^2}.$



Leading the current by 90°.

Power,

$V = V_0 \sin\omega t$

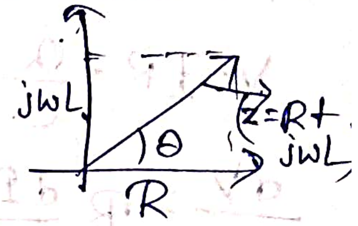
$I = I_0 \sin(\omega t - \varphi)$

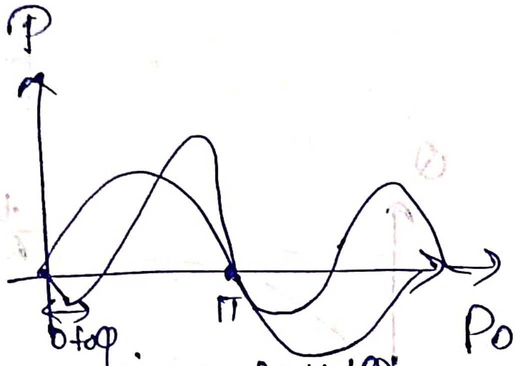
$P = VI = V_0 \sin\omega t \; I_0 \sin(\omega t - \varphi)$

$\Rightarrow P = V_{Rms} I_{rms} \{\cos\varphi - \cos(2\omega t - \varphi)\}$.

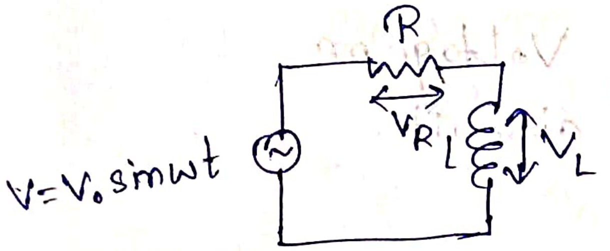
$P_{av} = V_{Rms} I_{rms} \cos\varphi$., $\cos\varphi = \frac{R}{|Z|}$.

$P_{av} = \frac{VIR}{Z} = I^2 R \quad (\because V = IZ)$.





Power is -Ve. (Physically it means backflow of energy into the source).

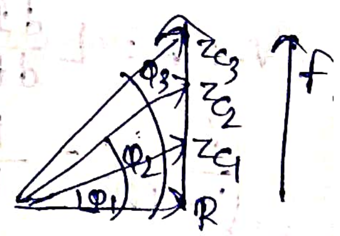


Variation of 'z' and 'φ' with frequencies.

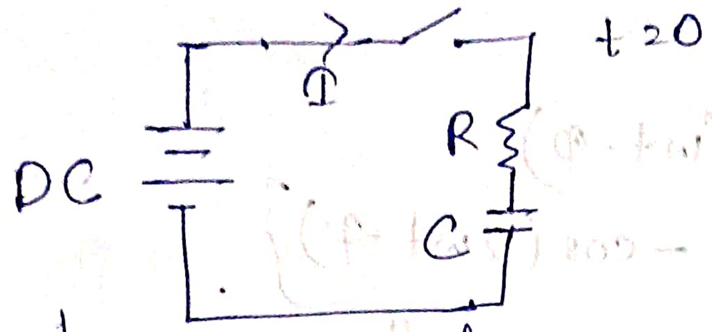
$|Z_L| = \omega L = 2\pi f L$.

$|Z| = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + 4\pi^2 f^2 L^2}$

$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{2\pi f L}{R}\right)$.



Charging of a Capacitor

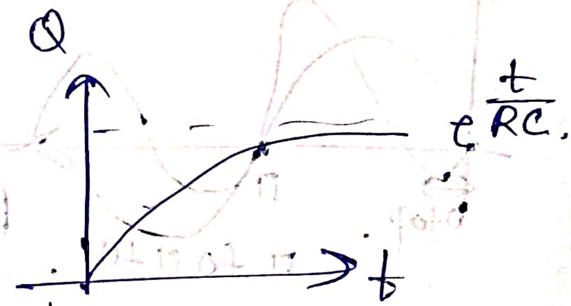


Transient current

As soon as circuit will connected –

$$V = IR + \frac{Q}{C}$$

$$\frac{dV}{dt} = R\frac{dI}{dt} + \frac{I}{C}.$$



Homework :-

(i) Evaluate the charging of a capacitor in a RC circuit.

(ii) Evaluate the build up of Voltage or current $(I = \frac{V}{R})$ in an R-L circuit.

For RL circuit

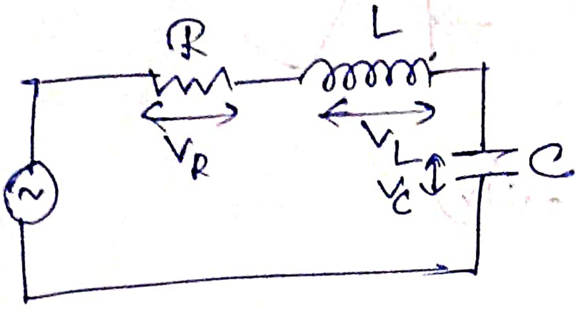
$$V = IR + L\frac{dI}{dt}.$$

$$V - IR = L\frac{dI}{dt}.$$

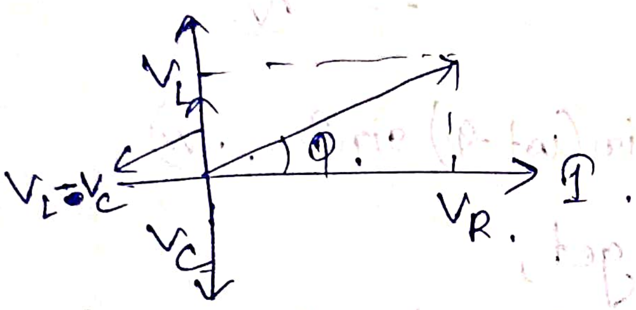
$$\frac{dI}{(V-IR)} = \frac{1}{L}dt.$$

time constant.

LCR Circuit



Phasor Diagram



$$\varphi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right).$$

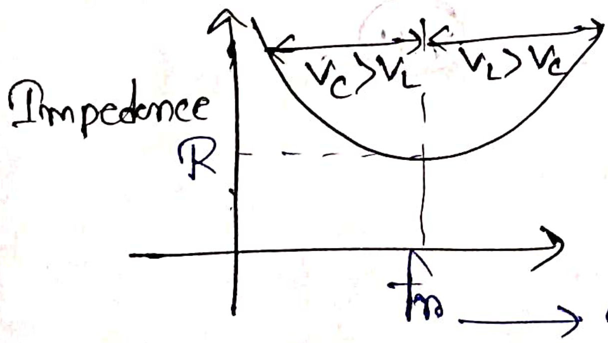
Assume: $V_L > V_C$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

$\downarrow$

Resonance condition :- when $\omega L = \frac{1}{\omega C}$.

$$|Z| = R.$$



$f_0 \longrightarrow$ resonance frequence.

Solve for the LCR circuit :-

$$V(t) = V_R + V_L + V_C$$

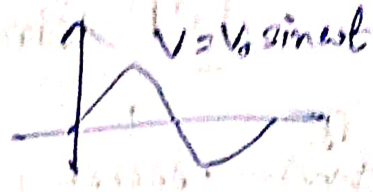
$$\Rightarrow V(t) = RI(t) + L\frac{dI}{dt} + \frac{Q(t)}{C} \qquad I = \frac{dQ}{dt},\ \frac{dI}{dt} = \frac{d^2Q}{dt^2}.$$

Derivative on both sides,

$$V'(t) = RI'(t) + LI''(t) + \frac{I(t)}{C} \quad \ldots (1).$$

at $t=0$, $V(0)=0$

At $t=t$, $V(t)=V_0 \sin \omega t$



$V'(t) = V_0 \omega \cos \omega t$

both take $I = A \sin(\omega t - \varphi)$ ... (iv)

Substituting in equation (1),

$-L\omega^2 A \sin(\omega t - \varphi) + RA\omega \cos(\omega t - \varphi) + \frac{1}{C} A \sin(\omega t - \varphi)$
$= \omega V_0 \cos \omega t$ ... (V)

$\cos \omega t = \cos(\omega t - \varphi + \varphi)$.
$= \cos(\omega t - \varphi)\cos\varphi - \sin(\omega t - \varphi)\sin\varphi$ ... (VI)

Substituting (VI) in (V) we get,

$-L\omega^2 A \sin(\omega t - \varphi) + RA\omega \cos(\omega t - \varphi) + \frac{1}{C} A \sin(\omega t - \varphi)$
$= \omega V_0 \left[\cos(\omega t - \varphi)\cos\varphi - \sin(\omega t - \varphi)\sin\varphi\right]$

Equating the coefficient of $\sin(\omega t - \varphi)$ and $\cos(\omega t - \varphi)$, we get -

$R\omega A = \omega V_0 \cancel{\cos \omega t} \cos\varphi$ ... (VII)

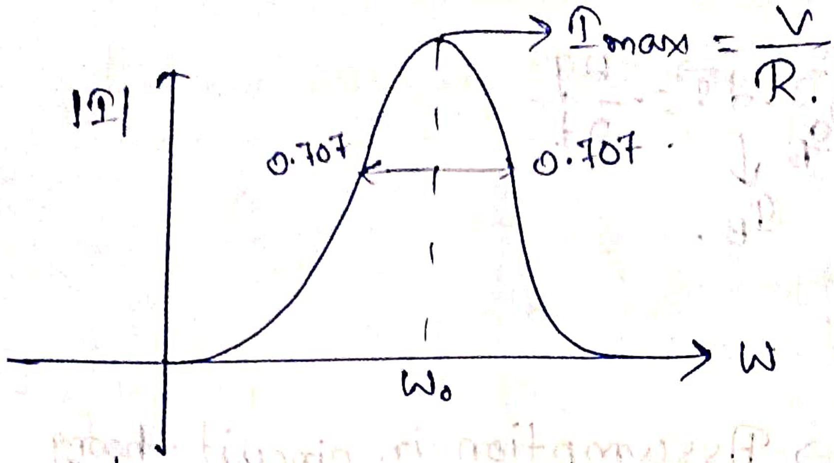
$\left(L\omega^2 - \frac{1}{C}\right) A = \omega V_0 \sin\varphi$ ... (VIII)

Divide (VIII) by (VII) —

$$\tan\varphi = \frac{L\omega^2 - \frac{1}{C}}{R\omega} = \frac{X_L - X_C}{R}.$$

$$A = \frac{\omega V_0}{\sqrt{(R\omega)^2 + \left(L\omega^2 - \frac{1}{C}\right)^2}}$$ [Calculate Explicitely]

$$V_s = ZI, \quad |I| = \frac{|V|}{|Z|} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



$$W_0 = \frac{1}{\sqrt{LC}}$$

Quality Factor

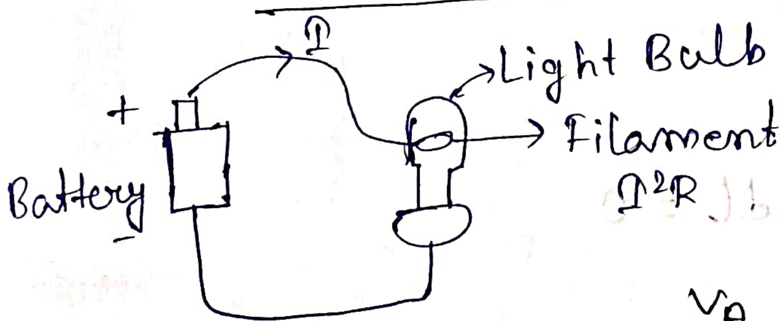
I is maximum at $X_L = X_C \Rightarrow wL = \frac{1}{wC}$.

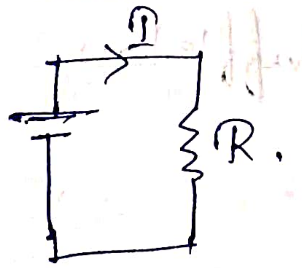
$$|I| = \frac{I_0}{\sqrt{1 + Q^2\left(\frac{W}{W_0} - \frac{W_0}{W}\right)^2}}.$$

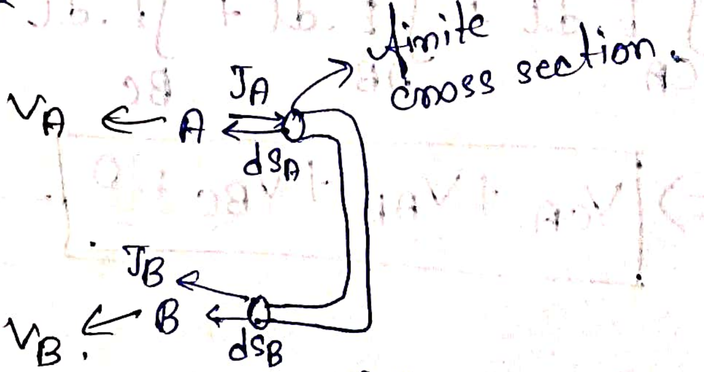
$Q = \frac{W_0}{\Delta W}$, ΔW corresponds to $I = \frac{I_0}{\sqrt{2}}$.

$$Q^2 = \frac{L}{R^2 C}.$$

Lumped Circuit Approximation







$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}.$$

$$\oint \vec{J} \cdot d\vec{s} = -\frac{\partial q}{\partial t}.$$

$$\underbrace{\int_{S_A} \vec{J}\cdot d\vec{s}}_{I_A} - \underbrace{\int_{S_B} \vec{J}\cdot d\vec{s}}_{I_B} = -\frac{\partial q}{\partial t}$$

$$I_A = I_B.$$

$\Rightarrow \frac{\partial q}{\partial t} = 0 \rightarrow$ Assumption in circuit theory.

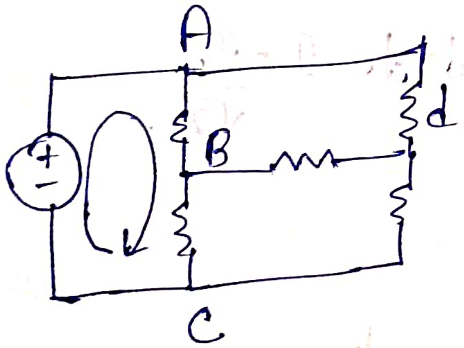
$\hookrightarrow$ This is kirchaff's current Law.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}.$$

$$V_{AB} = \oint E.dl = -\frac{\partial \phi_B}{\partial t} = 0.$$

$\hookrightarrow$ second assumption.



CAB

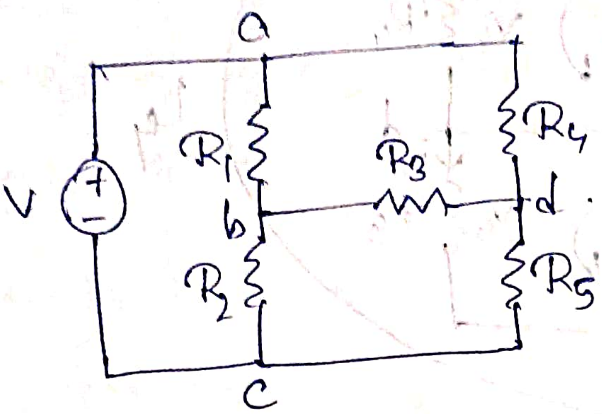
$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t} = 0.$$

$$\int_{CA} E.dl + \int_{AB} E.dl + \int_{BC} E.dl = 0$$

$\Rightarrow \boxed{V_{CA} + V_{AB} + V_{BC} = 0} \rightarrow$ kirchaff's Law.

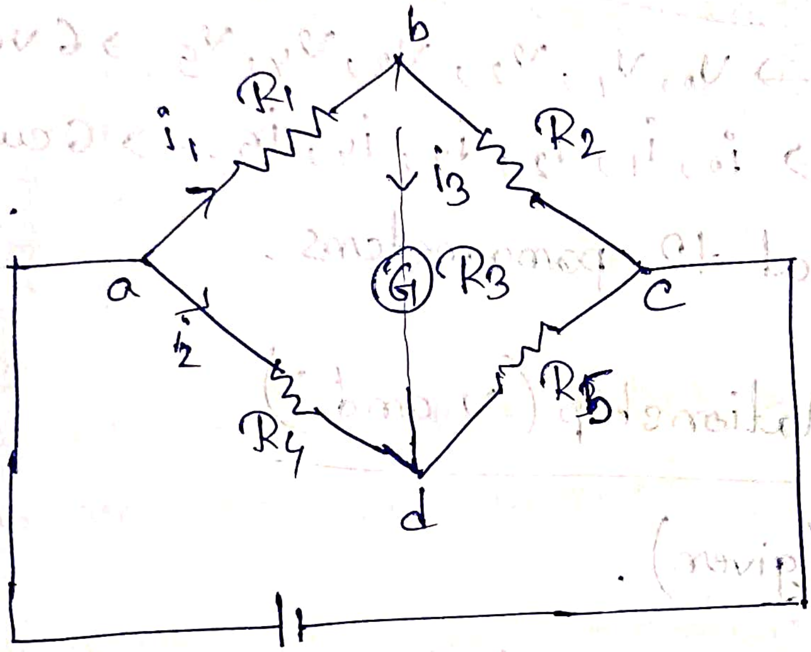
14/08/2025

Lumped Circuit Approximation (LCA)



Inside elements:

$$\frac{\partial q}{\partial t} = 0$$



$i_3 = 0$.

$V_{ad} = V_{ab}$.

$i = i_1 + i_2$

outside $\rightarrow \frac{\partial \Phi_B}{\partial t} \rightarrow 0$ In resistor or other elements.

$\hookrightarrow$ Voltage source of current.

$$i_1 R_1 + i_1 R_2 = i_2 R_4 + i_2 R_5$$

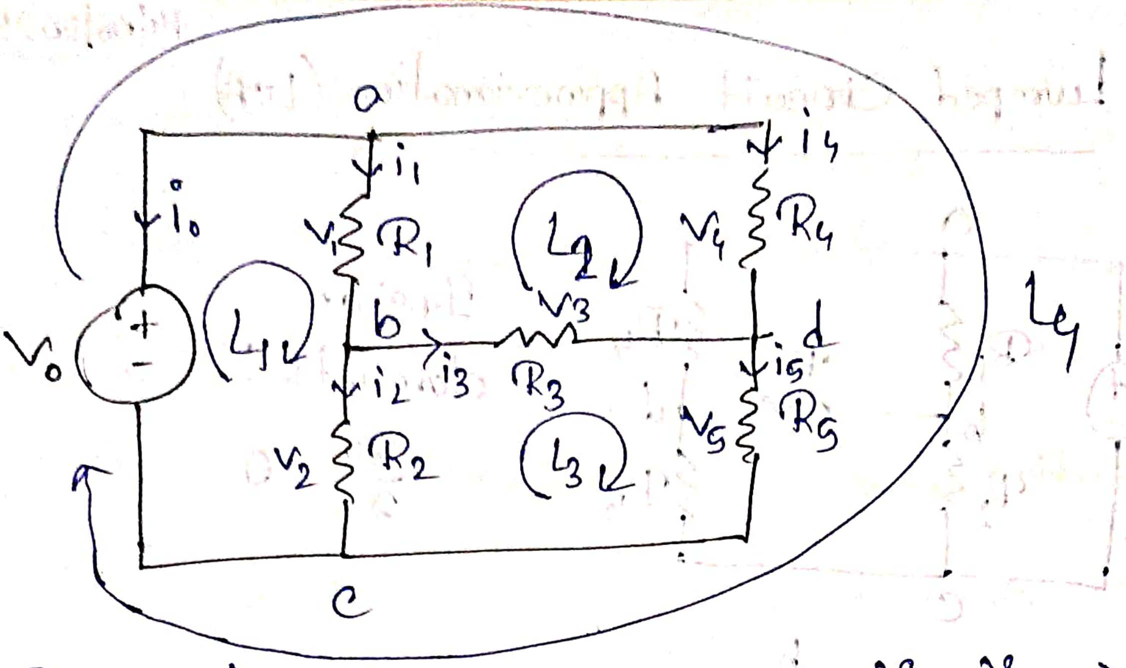
$$\frac{R_1}{R_4} = \frac{R_2}{R_5}$$

~~Inside~~ $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$.

$\oint \vec{J} \cdot d\vec{s} = 0$

$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t} = 0$;

$V_{ca} + V_{ab} + V_{bc} = 0 \longrightarrow$ Recap of last class.



Parameters $\rightarrow v_0, v_1, v_2, v_3, v_4, v_5 \rightarrow$ 6 voltages

$\rightarrow i_0, i_1, i_2, i_3, i_4, i_5 \rightarrow$ 6 currents

$\rightarrow$ Total 12 parameters.

1. Element Relationship (v and i)

$v_0 = V_0$ (given).

$v_1 = i_1 R_1$ $v_3 = i_3 R_3$ $v_5 = i_5 R_5$.

$v_2 = i_2 R_2$ $v_4 = i_4 R_4$

2. KCL at the Nodes :-

Convention :- Current leaving is +ve.

a : $i_0 + i_1 + i_2 = 0$ ... (i)

b : $-i_1 + i_2 + i_3 = 0$ ... (ii)

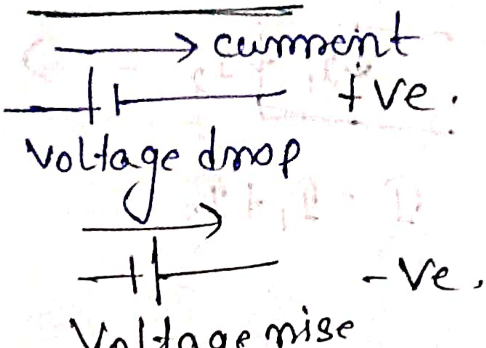
d : $-i_4 - i_3 + i_5 = 0$ ... (iii).

c : $-i_2 - i_5 - i_0 = 0$ $\longrightarrow$ not independent as

c :→ (i) + (ii) + (iii).

3. Loop Equations :- (KVL equations for loops).

Convention :-



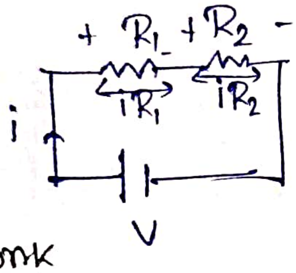
L1 : $-V_0 + V_1 + V_2 = 0$ .. (IV)

L2 : $-V_1 + V_4 - V_3 = 0$ ..... (V)

$\Rightarrow V_1 + V_3 - V_4 = 0$ .... (V)

L3 : $-V_2 + V_3 + V_5 = 0$ .. (VI)

L4 : $-V_0 + V_4 + V_5 = 0$ → not independent as L4 : (IV) + (V) + (VI).



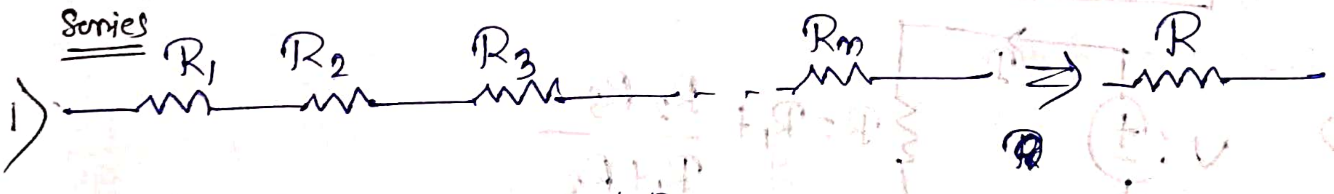
$V = iR_1 + iR_2 = i(R_1 + R_2)$.

Homework

We have 12 equations. Solve for them to find voltages and current.

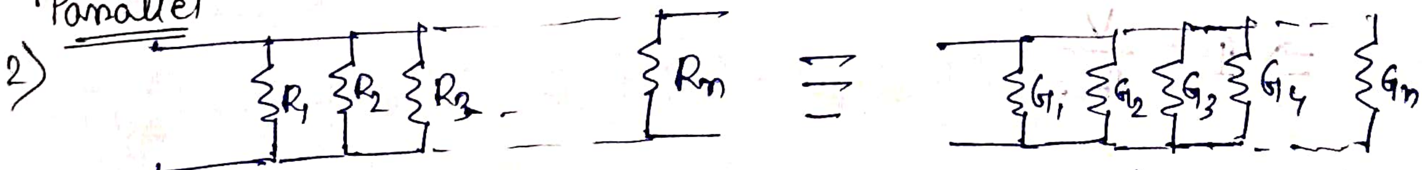
Intuitive Methods for circuit combination rules

Series

1) 

$R = R_1 + R_2 + \dots + R_n$.

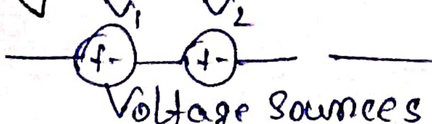
Parallel

2) 

G = conductance = $\frac{1}{R}$.

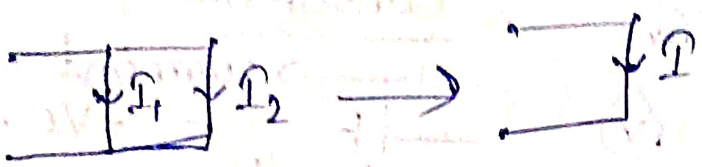
$G = G_1 + G_2 + \dots + G_n$

3) Voltage Sources :-



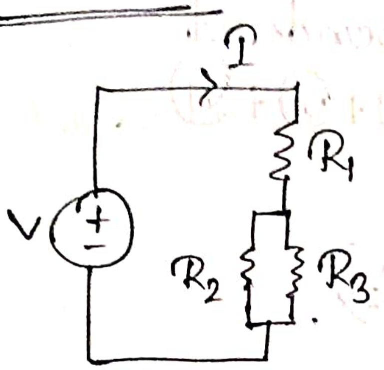
$\longrightarrow V = V_1 + V_2$.

4) Current Sources.



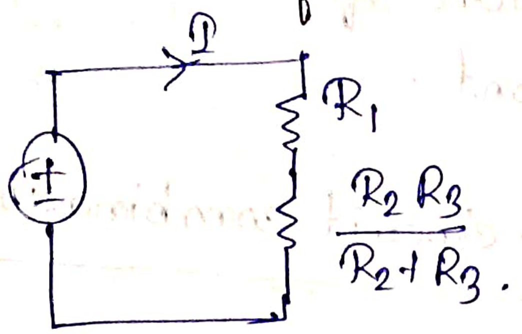
$I = I_1 + I_2$

Question

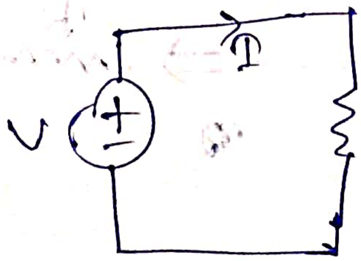


What is I for this circuit?

$\Rightarrow$



$\Rightarrow$



$R = R_1 + \frac{R_2 R_3}{R_1 + R_3}$

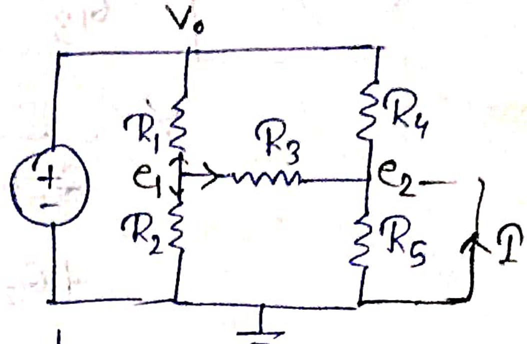
$\Rightarrow I = \frac{V}{R}$.

Node Analysis Method

18/08/2025

Step 1:- Select a reference node (ground inside) from which voltages will be measured.

Step 2:- Label the voltages of the remaining nodes ("e_i") w.r.t. the ground.



Step 3:- Write KCL for all the nodes (Solve the ground node).

Step 4:- Solve for the node voltages 'e_i'.

Step 5:- Solve the V's or I's.

KCL at node e_1:-

$$\frac{e_1 - V_0}{R_1} + \frac{e_1 - 0}{R_2} + \frac{e_1 - e_2}{R_3} = 0 \quad \ldots (1) \Rightarrow (e_1 - V_0)G_1 + e_1 G_2 + (e_1 - e_2)G_3 = 0 \quad \ldots (1a)$$

e_2:-

$$\frac{e_2 - e_1}{R_3} + \frac{e_2 - V_0}{R_4} + \frac{e_2 - 0}{R_5} - I = 0 \Rightarrow (e_2 - e_1)G_3 + (e_2 - V_0)G_4 + e_2 G_5 = I \quad \ldots (11a)$$

$$\frac{1}{R_i} = G_i$$

$$\Rightarrow e_1(G_1 + G_2 + G_3) + e_2(-G_3) = V_0 G_1 \quad \ldots (1b)$$

$$e_1(-G_3) + e_2(G_3 + G_4 + G_5) = V_0 G_4 + I \quad \ldots (11b)$$

$$\underbrace{\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 + G_4 + G_5 \end{bmatrix}}_{M} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{bmatrix} G_1 V_0 \\ G_4 V_0 + I \end{bmatrix}$$

$$M^{-1} = \frac{\text{Adj } M}{|M|}.$$

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{\begin{bmatrix} G_3 + G_4 + G_5 & G_3 \\ G_3 & G_1 + G_2 + G_3 \end{bmatrix} \begin{pmatrix} G_1 V_0 \\ G_4 V_0 + I \end{pmatrix}}{(G_1 G_3 + G_1 G_4 + G_1 G_5 + G_2 G_4 + G_2 G_5 + G_2 G_3 + G_3 G_4 + G_3 G_5)}.$$

Homework

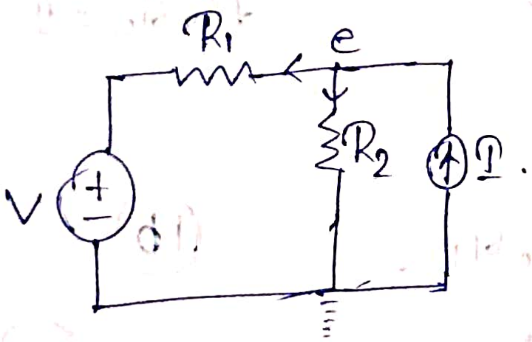
Find e_1, e_2.

$G_1 = G_5 = \frac{1}{8.2\,k}$.

$G_2 = G_4 = \frac{1}{3.9\,k}$. $I = 0, V_0 = 3V$.

$G_3 = \frac{1}{1.5\,k}$.

Linearity and Superposition



Node Equation at e :-

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0.$$

$$\Rightarrow \left[\frac{1}{R_1} + \frac{1}{R_2}\right] e = \frac{V}{R_1} + I$$

source terms.

→ Linear in Voltage and current sources.

Conduction Matrix.

node voltage

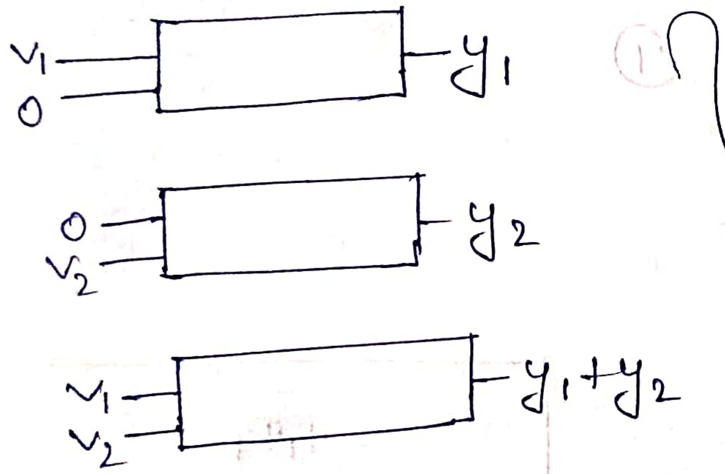
Solve :- $\boxed{e = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I}$ --- (iii).

In general in the presence of multiple voltage sources and current sources

$$\boxed{e = a_1 v_1 + a_2 v_2 + \ldots + b_1 I_1 + b_2 I_2 + \ldots} \rightarrow \text{Linear}$$

a_i's are dimensionless and b_i's has dimension of resistance (ohm).

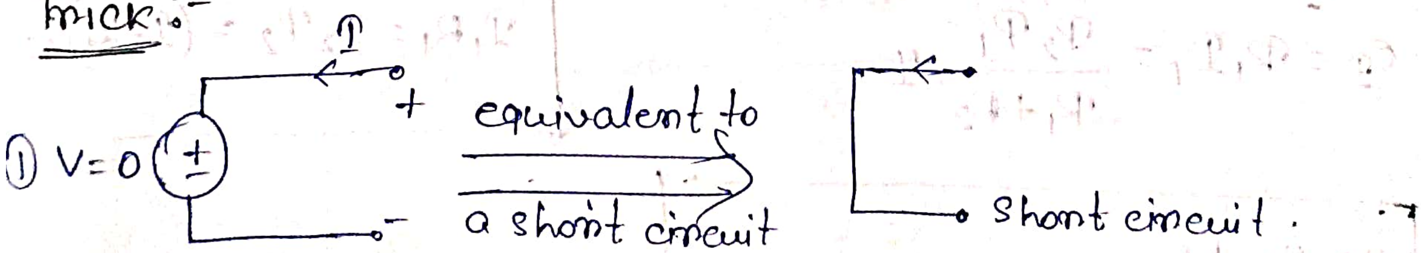
SuperPosition

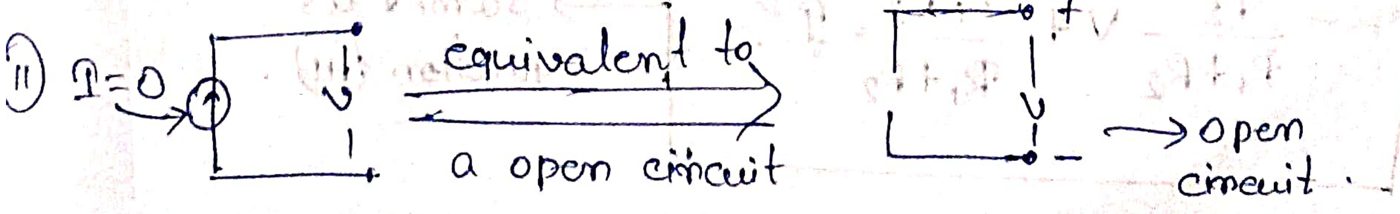


→ Linearity of a circuit.

Output of a circuit is determined by summing the responses to each sources acting alone.

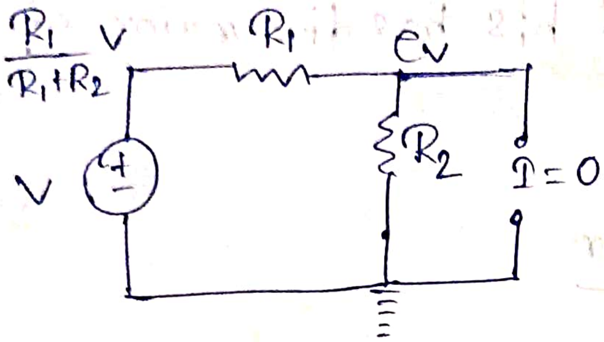
Trick :-





Output of the linear circuit $\xrightarrow[\text{by}]{\text{determined}}$ summing up the responses of each source acting alone.

a) V acting alone ($I=0$) :-

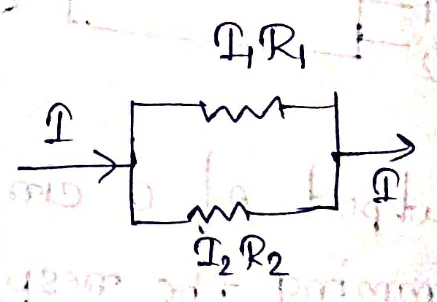


$$e_V = \frac{R_2}{R_1+R_2} V \quad \ldots \text{(I)}$$

b) I acting alone ($V=0$) :-

$$I_1 = \frac{R_2}{R_1+R_2} I$$

$$e_I = R_1 I_1 = \frac{R_2 R_1}{R_1+R_2} I$$



$I = I_1 + I_2$

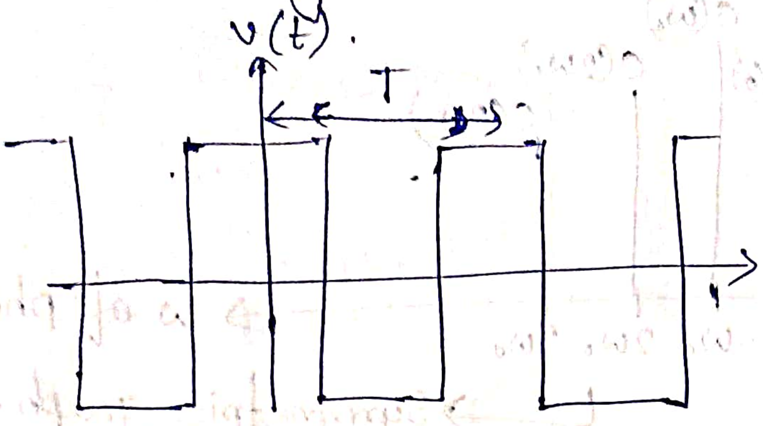
$I_1R_1 = I_2R_2 = (I - I_1)R_2$

$$\boxed{\begin{aligned} e &= e_I + e_V \\ &= \frac{R_2}{R_1+R_2} V + \frac{R_1R_2}{R_1+R_2} I \end{aligned}}$$

$\rightarrow$ Same as equation (III).

Fourier Series 21/08/2025

Periodic Signal.



$\omega_0 = \frac{2\pi}{T}$

$$v(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

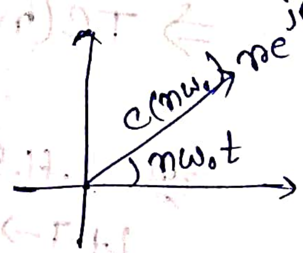
$$= a_0 + \sum_{n=1}^{\infty} \left[a(n\omega_0)\cos(n\omega_0 t) + b(n\omega_0)\sin(n\omega_0 t) \right].$$

$$= \sum_{n=-\infty}^{\infty} \underbrace{c(n\omega_0)\, e^{jn\omega_0 t}}_{\text{phasor}}.$$

→ phasor. (discrete infinite sum of phasor).

$$\cos(n\omega_0 t) = \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}$$

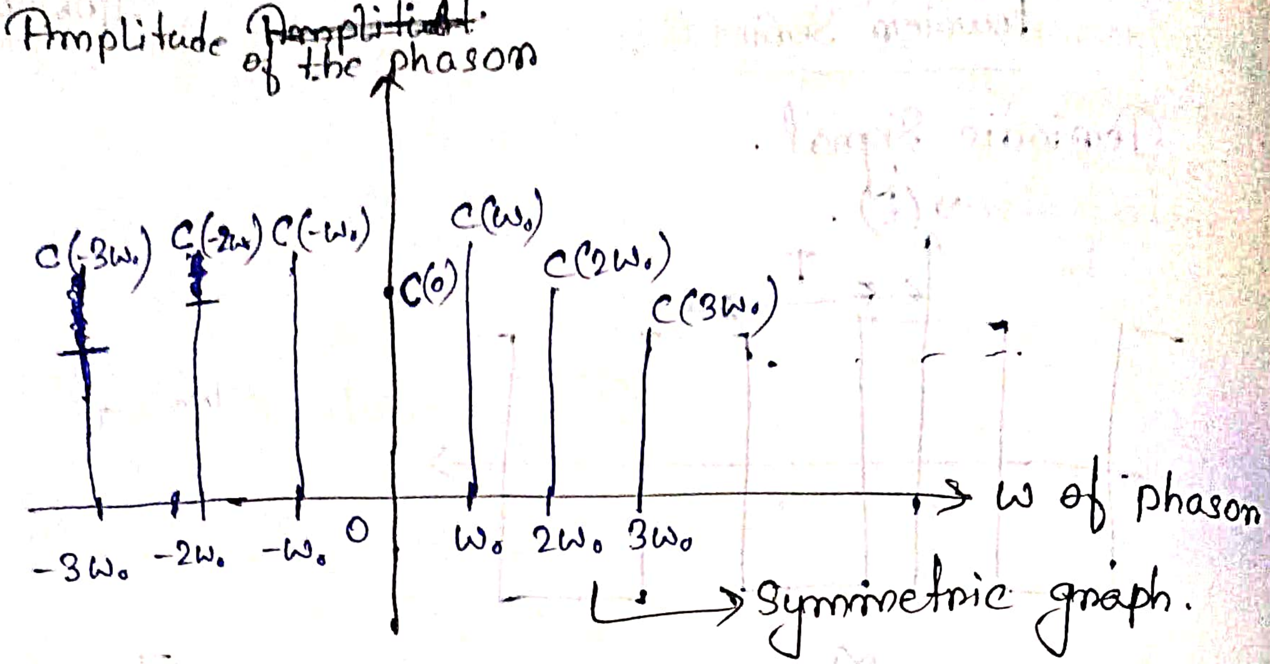
$$\sin(n\omega_0 t) = \frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}$$



$$v(t) = a(0) + \sum_{n=1}^{\infty} \left[a(n\omega_0) \left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right] + b(n\omega_0) \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right] \right]$$

$$= \underbrace{a(0)}_{c(0)} + \sum_{n=1}^{\infty} \left[e^{jn\omega_0 t} \underbrace{\left[\frac{a(n\omega_0)}{2} + \frac{b(n\omega_0)}{2j} \right]}_{c(n\omega_0 t)} + e^{-jn\omega_0 t} \underbrace{\left[\frac{a(n\omega_0)}{2} - \frac{b(n\omega_0)}{2j} \right]}_{c(-n\omega_0 t)} \right].$$

$$= \sum_{n=-\infty}^{\infty} c(n\omega_0)\, e^{jn\omega_0 t}.$$



Fourier Transform

Consider $T \to \infty$ in fourier series.

$\omega_0 \to 0$ as $T \to \infty$

$\Rightarrow \Delta\omega \to 0 \Rightarrow$ Graph will be continuous.

$$c(n\omega_0) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} v(t)\, e^{-jn\omega_0 t}\, dt.$$

$$\Rightarrow T c(n\omega_0) = \int_{-\frac{T}{2}}^{\frac{T}{2}} v(t)\, e^{-jn\omega_0 t}\, dt.$$

$$\Rightarrow \lim_{\substack{T\to\infty \\ \omega_0 \to 0}} \text{L.H.S.} = \lim_{\substack{T\to\infty \\ \omega_0\to 0}} T\{c(n\omega_0)\}$$

$$= \lim_{\omega_0 \to 0} \left(\frac{2\pi}{\omega_0}\right) c(n\omega_0)$$

$$= \lim_{\substack{\Delta\omega \to 0 \\ \omega_0 \to 0}} \frac{2\pi}{\Delta\omega}\, c(n\omega_0) \quad \omega = n\omega_0$$

$$= \lim_{\omega_0 \to 0} V(n\omega_0)$$

$$= V(\omega)$$

density of frequency content near $n\omega_0$

$$\text{R.H.S.} = \lim_{\substack{T\to\infty \\ \omega_0 \to 0}} \int_{-\frac{T}{2}}^{\frac{T}{2}} v(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \int_{-\infty}^{\infty} v(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt \qquad \omega = n\omega_0$$

$$\Rightarrow V(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt$$

$$\text{Spectrum} = V(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt.$$

$|V(\omega)| \rightarrow$ amplitude spectrum.

phase of $V(\omega) \rightarrow$ phase spectrum.

Inverse Fourier Transform

$$v(t) = \sum_{n=-\infty}^{\infty} \frac{c(n\omega_0)\, e^{jn\omega_0 t}}{T} \times T.$$

$$\Rightarrow v(t) = \sum_{n=-\infty}^{\infty} Tc(n\omega_0) \times \frac{\omega_0}{2\pi}\, e^{jn\omega_0 t}$$

$$\Rightarrow v(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} [Tc(n\omega_0)] \times \Delta\omega\, e^{jn\omega_0 t}.$$

$$\Rightarrow v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega)\, e^{j\omega t}\, d\omega.$$

$\rightarrow$ can write any function (whether periodic or not) as continuous infinite sum of phasors.

22/08/2025

Recap:-

↳ For periodic signal $v(t)$.

Fourier series $\rightarrow v(t) = \sum_{n=-\infty}^{\infty} c(n\omega_0) e^{jn\omega_0 t}$.

↓

discrete infinite sum of phasors.

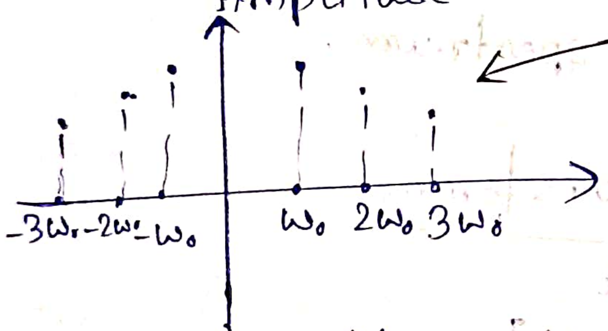
↓

Fourier transform.

$$V(\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t} dt.$$

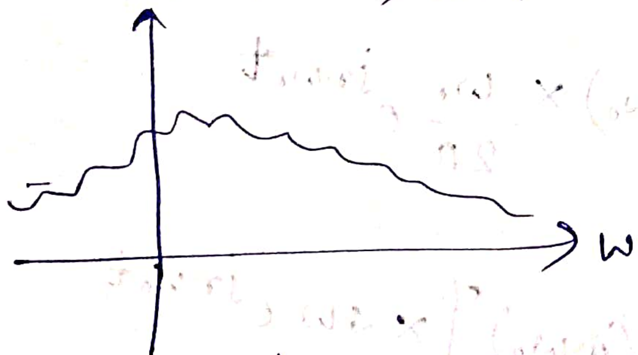
↑

density of frequency content / spectral content at ω.

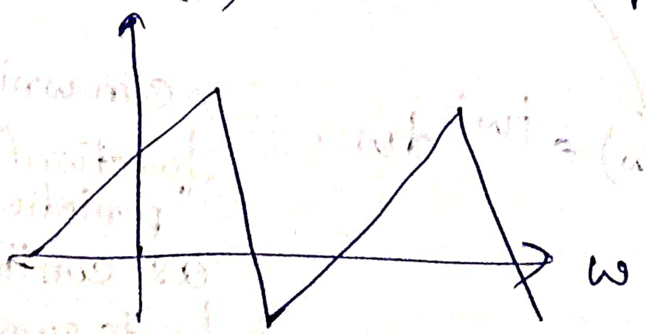


Analogous to probability mass function.

$|V(\omega)| \longrightarrow$ amplitude spectrum :



$V(\omega) \longrightarrow$ Phase spectrum



$$V(\omega) = \underbrace{r(\omega)}_{\text{Amplitude}} e^{j\underbrace{\theta(\omega)}_{\text{Phase}}}$$

Inverse Fourier Transform

$$v(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} v(\omega)\, e^{j\omega t}\, d\omega .$$

continuously infinite sum of phasors.

Example :-

1) $v(t) = e^{j\omega_0 t}$

$$v(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{j\omega_0 t}\, e^{-j\omega t}\, dt = \int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t}\, dt$$

$\omega = \omega_0$, $v(\omega) \to \infty$.

$\omega \neq \omega_0$, $v(\omega) = -\frac{1}{j(\omega-\omega_0)}\, e^{-j(\omega-\omega_0)t}\Big|_{-\infty}^{\infty}$

can be solved by complex integration and then by the distribution of dirac delta.

$$= -\frac{1}{j(\omega-\omega_0)}\left[\lim_{t\to\infty} e^{-j(\omega-\omega_0)t} - \lim_{t\to-\infty} e^{-j(\omega-\omega_0)t}\right]$$

Select some $v(\omega)$ so that it chooses a particular ω_0 from all the ω's.

Let $v(\omega) = \delta(\omega-\omega_0) \cdot \times 2\pi$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t}\, \cancel{dt}\, d\omega \cdot \times 2\pi$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega_0 t}\, \delta(\omega-\omega_0)\, d\omega \cdot \times 2\pi$$

$$= e^{j\omega_0 t}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, d\omega = e^{j\omega_0 t}$$

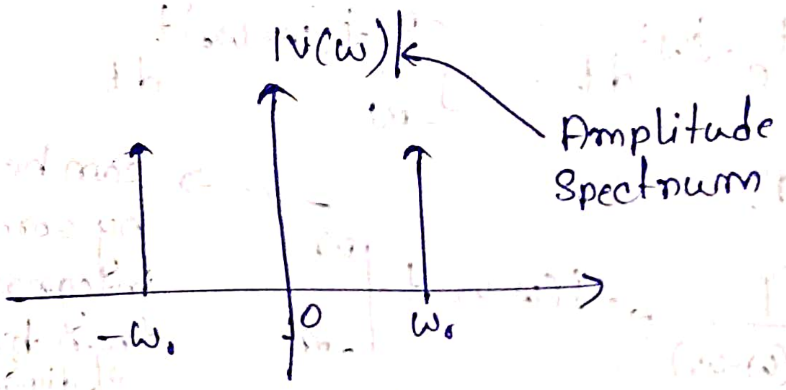
$$\Rightarrow v(t) = e^{j\omega_0 t} \longleftrightarrow v(\omega) = 2\pi\, \delta(\omega-\omega_0).$$

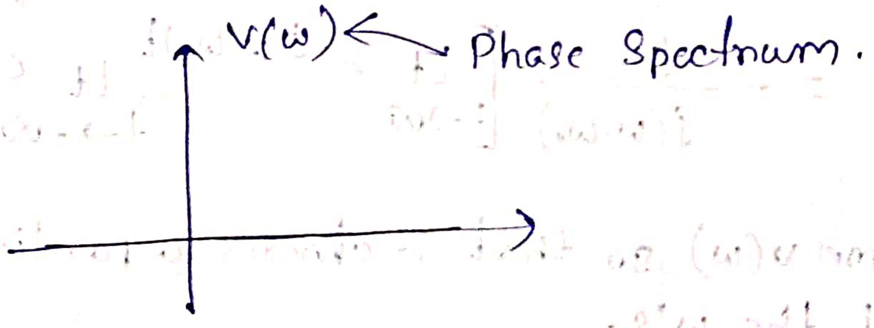
2) $v(t) = \cos(\omega_0 t) = \dfrac{\cancel{ej}\ e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$$V(\omega) = \frac{FT\{e^{j\omega_0 t}\} + F.T.\{e^{-j\omega_0 t}\}}{2}.$$

$$= \frac{2\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]}{2}$$ [From the Linearity of fourier transform]

$$= \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)].$$





3) $v(t) = \sin(\omega_0 t)$

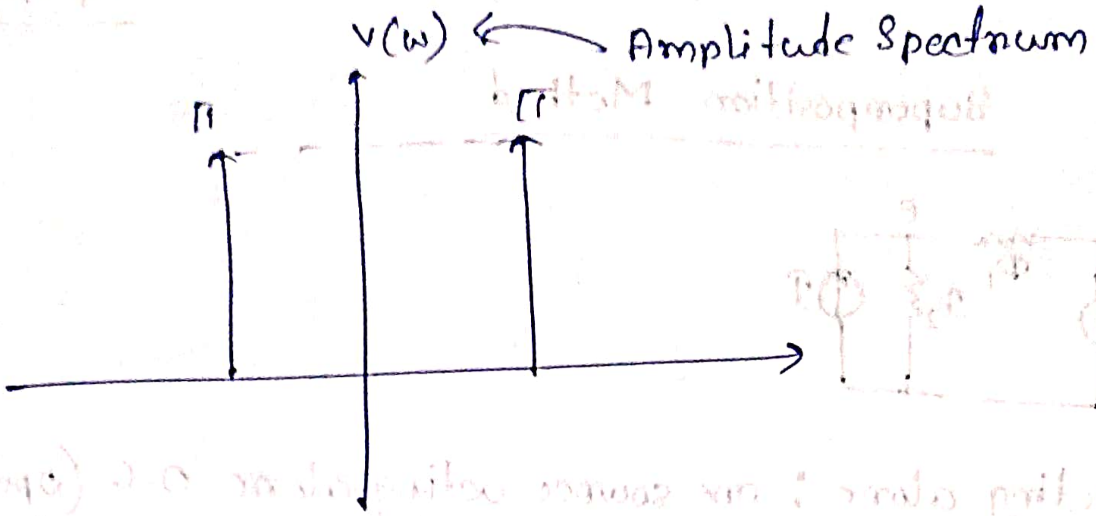
$$= \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

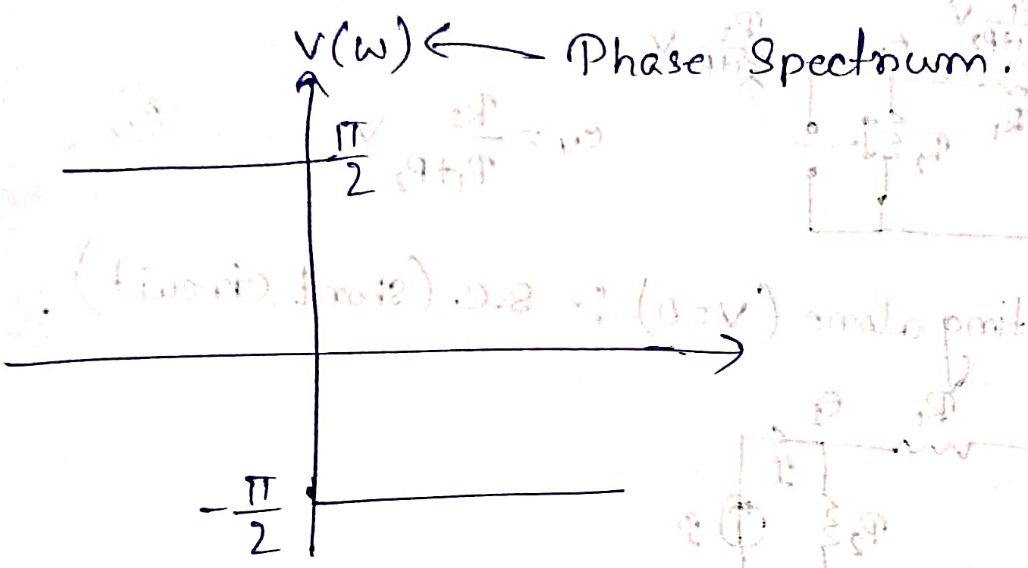
$$V(\omega) = \frac{[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)] \times 2\pi}{2j}$$

$$= -j\pi[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)].$$

$$= -j\pi\,\delta(\omega-\omega_0) + j\pi\,\delta(\omega+\omega_0)$$

$$= [e^{-\frac{j\pi}{2}}]\,\pi\,\delta(\omega-\omega_0) + \pi e^{\frac{j\pi}{2}}\,\delta(\omega+\omega_0)$$





☒ If $|v(\omega)| = |v'(\omega)|$ and $\angle v(\omega) \neq \angle v'(\omega)$ then $v(t)$ and $v'(t)$ are just phase shifted versions of each other.

4) <u>Constant function</u> :-

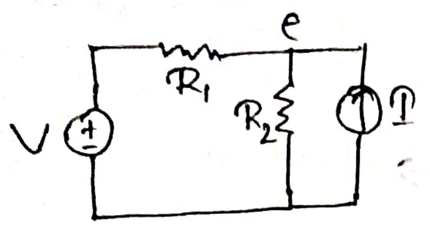
$v(t) = 1 = e^{j.0.t}$.

$\Rightarrow v(\omega) = 2\pi\delta(\omega)$.

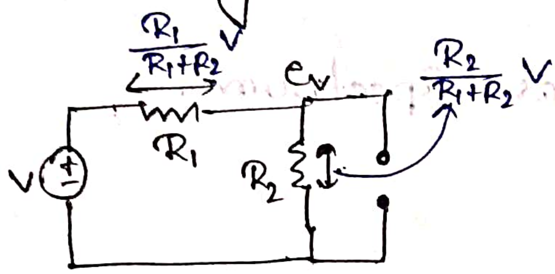
26/08/2025

Recap

Superposition Method

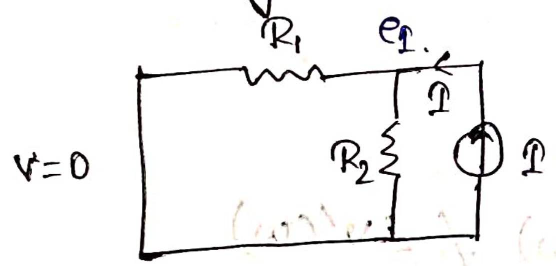


(a) 'V' acting alone : one source acting alone O.C. (open circuit)



$$e_V = \frac{R_2}{R_1+R_2} V \quad \text{---} \quad (a)$$

(b) 'I' acting alone (V=0) :- S.C (short circuit).



$$I_1 R_1 = I_2 R_2 \,, \quad I = I_1 + I_2$$

$$I_2 = \frac{I R_1}{R_1+R_2} \qquad e_I = \frac{R_1 I}{R_1+R_2} R_2$$

$$\Rightarrow e_I = \frac{R_1 R_2 I}{R_1+R_2} \quad \text{---} \quad (b)$$

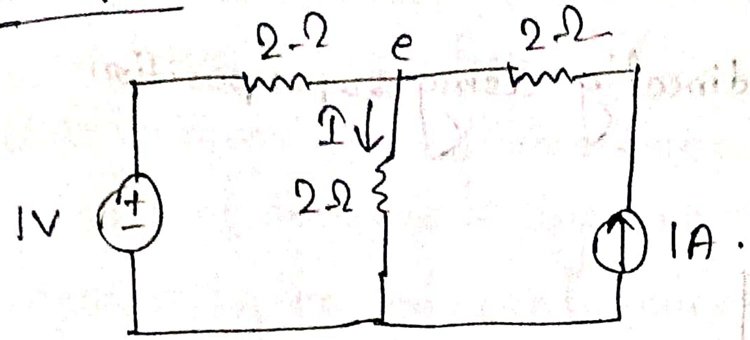
from (a) and (b)

$$e = e_I + e_V = \frac{R_2 V}{R_1+R_2} + \frac{R_1 R_2}{R_1+R_2} I \quad \text{----} \quad (c)$$

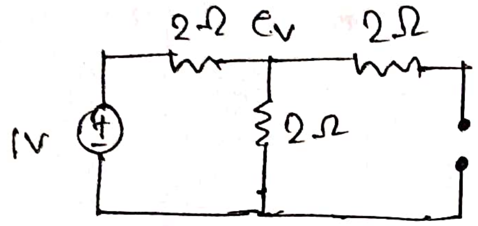
equation (c) is the same one as obtained using the node method (eq. (iii) of the lecture 18/8/2025)

Example :-

1)

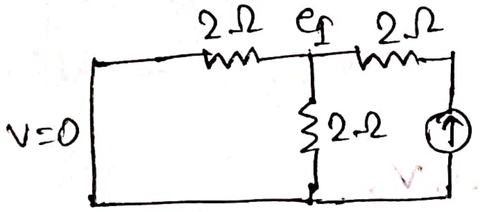


ⓐ $I = 0$.(o.c).

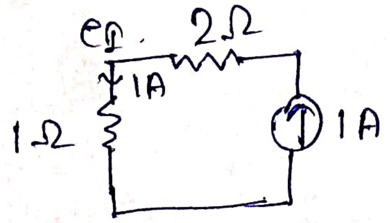


$$e_V = \frac{2}{2+2} \times 1 = \frac{1}{2} V = 0.5 V.$$

Step ⓑ: $V = 0$ (s.c.)



$$\frac{2\times 2}{2+2} = 1.$$

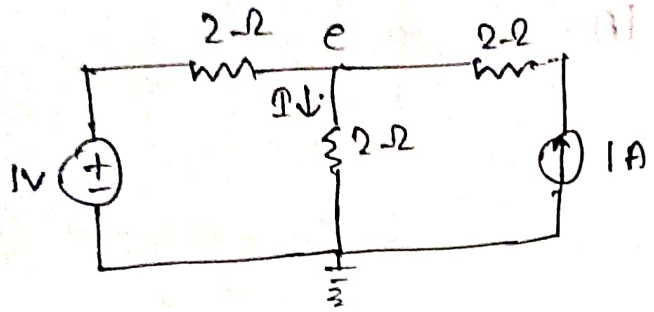


$$e_I = 1 \times 1 V = 1 V.$$

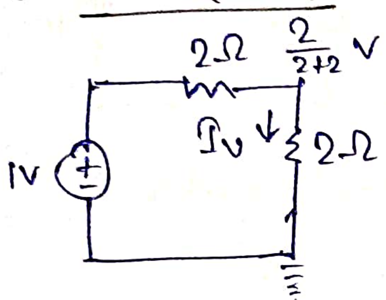
$$e = e_V + e_I = (0.5 + 1) V = 1.5 V. \text{ (Ans.)}$$

Method ⑪ :-

Evaluate current (I) directly using superposition method.



ⓐ $I = 0$ (o.c.) :-



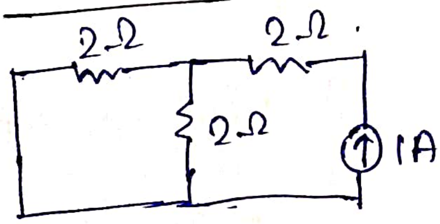
Voltage drops :- $\frac{2}{2+2} V = \frac{1}{2} V.$

$$I_v = \frac{\frac{1}{2}}{2} = 0.25 \text{ Amp.}$$

Voltage across 'a' and 'b' is 0.5V.

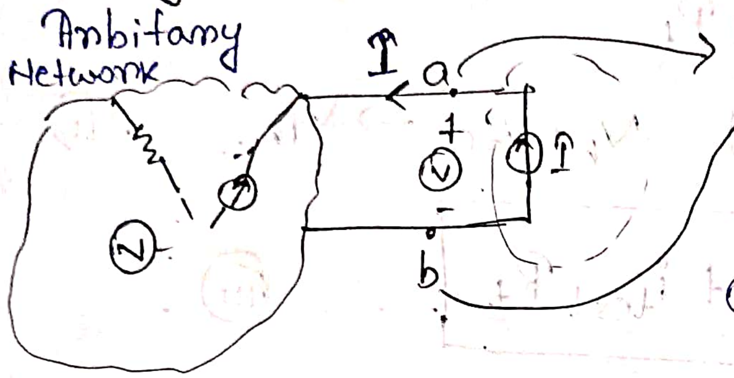
$$I_v = \frac{0.5V}{2A} = 0.25 A.$$

ⓑ $V = 0$ (s.c.) :-



Thevenin and Nortons equivalent

If we have a collection of (Linear) sources (voltage as well as current) and resistors, it can be represented by a pair of terminals by one voltage source and one resistor or by one current sources and one resistor.

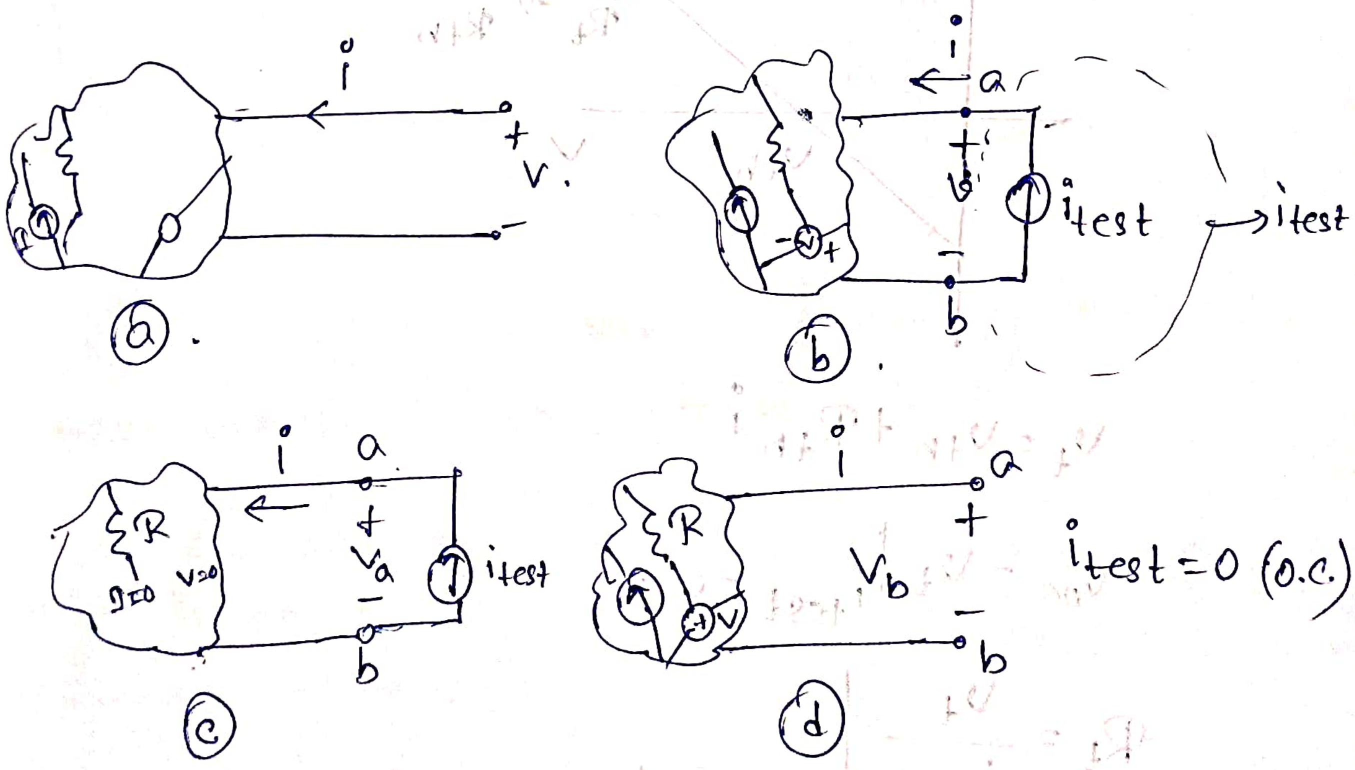


Two terminals

ⓐ Measure voltage across a and b.

ⓑ Measure equivalent resistance.

We want to establish a relationship between 'I' and 'V'. External excitation to this 2 point network is either by an external current source or an external voltage source.



$i_{test} = 0$ (o.c.)

(s.c.) all internal sources $\rightarrow 0$

voltage 'V_a'. Voltage across 'a' and 'b' is 'V_b'.

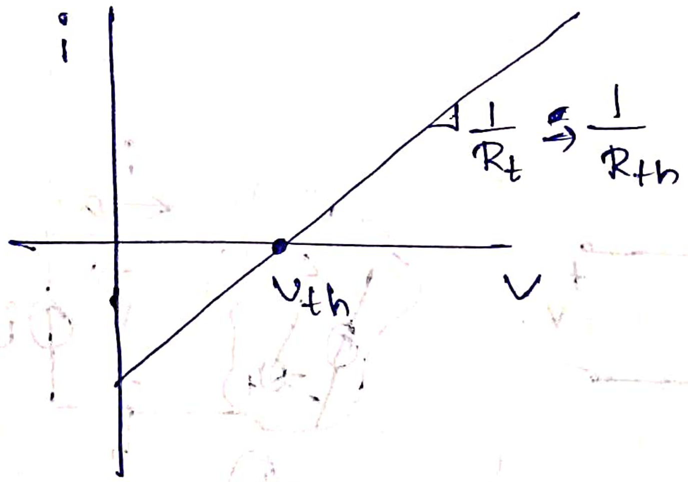
$$V_{total} = V_a + V_b = V_t .$$

(i) S.C. :- $V_a = i_{test} R_t$

(ii) O.C. :- $(i_{test} = 0)$ $V_b = V_{oc} \rightarrow V_{th}$:- - (ii)

$$\underbrace{\boxed{V_t = V_a + V_b = V_{oc} + i_{test} R_t .}}_{V_{th}} \quad \text{- - - (iii)} .$$

Graphical Representation :-



$$V_t = V_{th} + R_{th}\, i$$

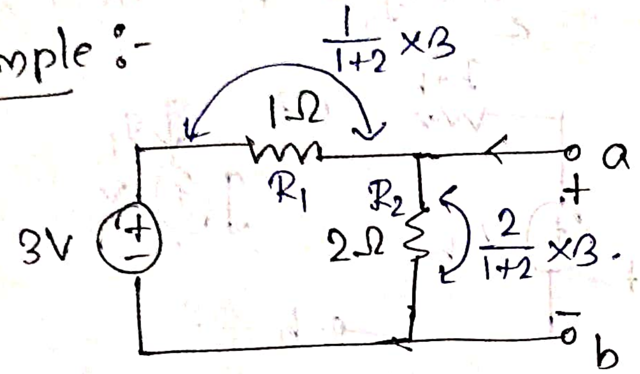
$$V_{oc} = V_t \Big|_{i_{test} = 0} ;$$

$$R_t = \frac{V_t}{i_{test}} \Bigg|_{\text{internal source} = 0} .$$

(i) V_{th} can be found by calculating the resistance of the open circuit voltage (V_{oc}) at the pair of terminals.

(ii) R_{th} can be evaluated by finding the resistance of the O.C. network from the pair of terminals with all internal sources of the Network been set to zero.

Example :-

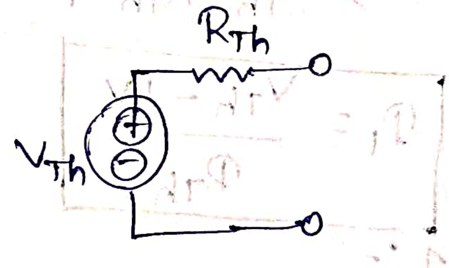


← Voltage divider.

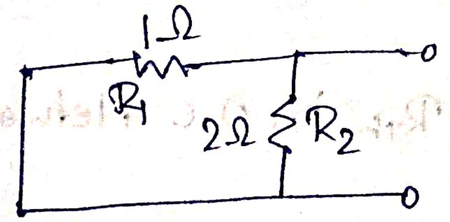
Step 1 :- V_{Th} is obtained by the O.C. voltage of the network at the 'a'-'b' point.

$$V_{Th} = \frac{2\Omega}{1\Omega + 2\Omega} \times 3\,V = 2V.$$

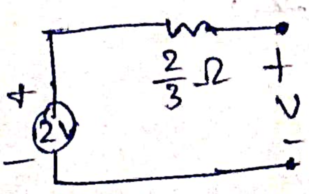
$$V_{Th} = \frac{R_2}{R_1 + R_2} \times 3\,V$$



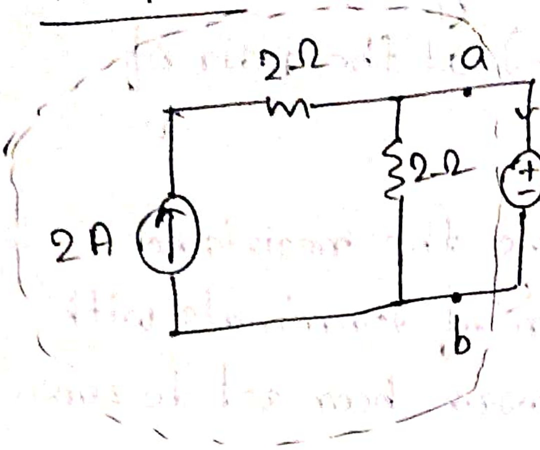
Step 2 :- The resistance R_{Th} is found by measuring the resistance of the network with $V = 0$.



$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{1 \cdot 2}{1+2} = \frac{2}{3}\Omega.$$

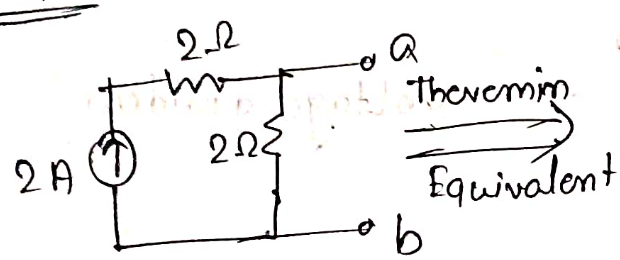


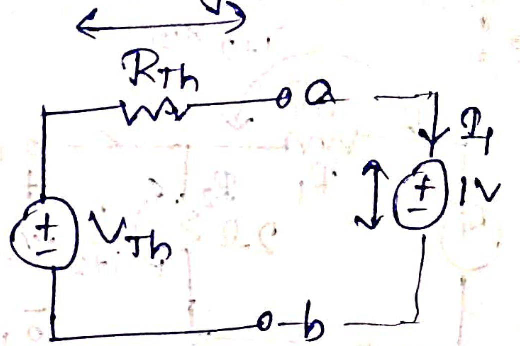
Example 2 :-



Determine the current I_1 flowing across the voltage source.

Step 1 :-





$$V_{Th} - R_{Th} I_1 - 1V = 0$$

$$\Rightarrow V_{Th} - R_{Th} I_1 = 1V.$$

$$\boxed{I_1 = \frac{V_{Th} - 1V}{R_{Th}}} \quad \text{---} \quad (1V)$$

Step 1 :-

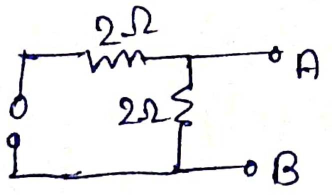
O.C. voltage measured / evaluated at the point 'a' 'b'.

∵ 2A is flowing in the loop, the voltage across 'a' and 'b' would be $2A \times 2\Omega = 4V$.

$$V_{oc} = 4V = V_{Th}.$$

Step 2 :- Measure the resistance R_{Th} in O-C Network.

$I = 0$



$R_{Th} = 2\Omega$.

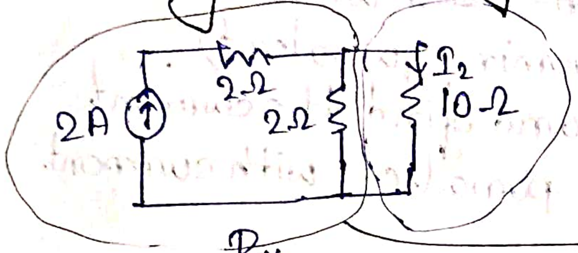
From eq. (v),

$$I_1 = \frac{V_{Th} - 1V}{R_{Th}} = \frac{4-1}{2} = \frac{3}{2} A.$$

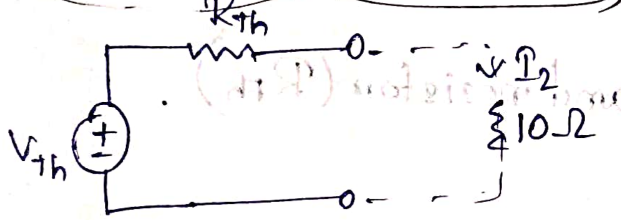
Calculate it by superposition method or node analysis method. (H.W.)

Another example :-

Replacing the voltage source by a resistor.



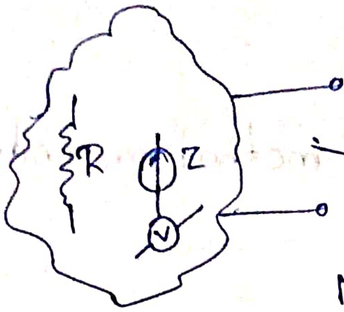
Same as last example.



$$\cancel{I_2 = \frac{V_{Th}}{R_{Th}}} \qquad I_2 = \frac{V_{Th}}{R_{Th} + 10\Omega} = \frac{4V}{(2+10)\Omega} = \frac{1}{3} A.$$

Lecture - 10

28/08/2025



Thevenin.

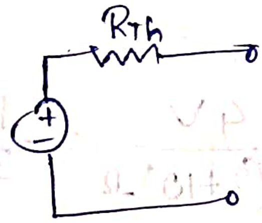
excitation source.

Norton

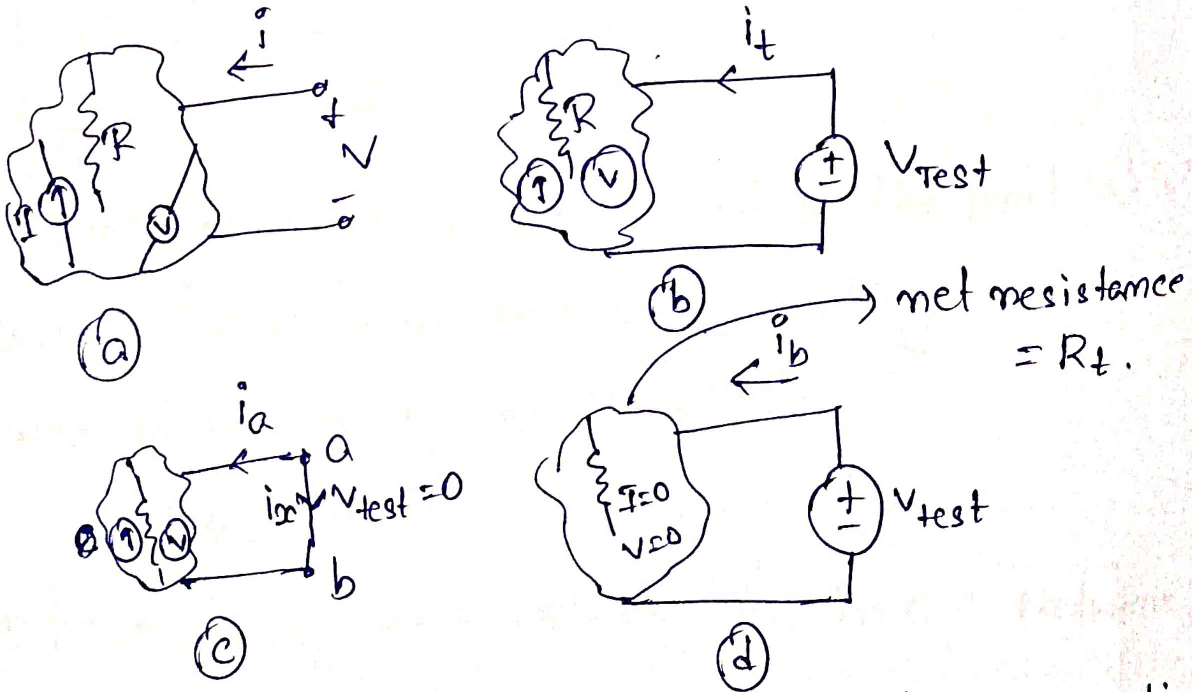
Excitation source → Voltage source.

Instead of test current (~~shouldn't~~ ~~did~~ which we did in Thevenin equivalent) we will use test voltage V_{test} and find the current and a resistor which will be in parallel with current source.

Found the voltage (V_{Th}) and resistor (R_{Th}).

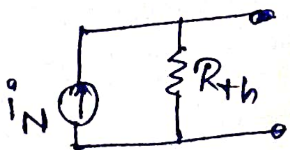


$V_t = V_a + V_b$.



$i_t = i_a + i_b$ → Principle of Superposition.

Norton
Sources of current



Superposition :-

(I) Circuit (C), $V_{test} = 0$, measure i_a.

(II) Circuit (A), All sources $= 0$, measure i_b.

$$i_t = i_a + i_b \quad \text{- - - - (1)}$$

In circuit (C) at node 'a' $i_a + i_{sc} = 0 \Rightarrow i_a = -i_{sc}$ - - (II).

For circuit (A),

$$i_b = \frac{V_{test}}{R_t}$$ (R_t is the net resistance measured across the port 'a' and 'b' ~~which where~~ with the internal sources are set to zero).

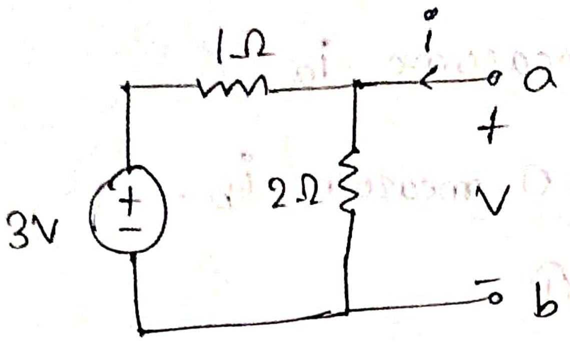
Steps :-

(I) i_N can be found by applying a short circuit at the terminals 'a' and 'b' and of the network and calculating the current through the S.C.

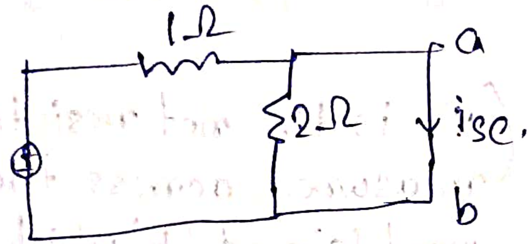
(II) R_N can be found in the same manner as done for R_{Th} by calculating/measuring the resistance of the O.C. network source from the designated terminal pair a and b with all internal sources set to zero.

Voltage Source → Short
Current Source → Open.

Example

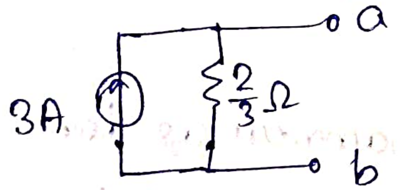


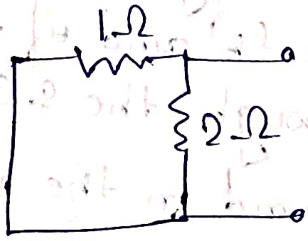
Step 1 :- Apply a S.C. to the terminal pair 'a' and 'b'.



$i_{sc} = \frac{3V}{1\Omega} = 3A.$ $\quad i_{sc} = i_N.$

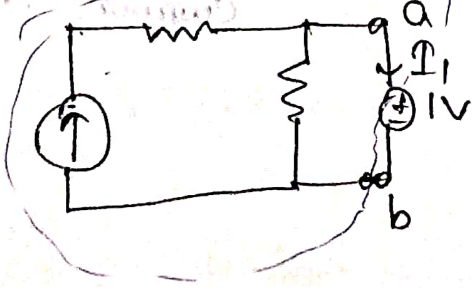
Step 2 :- Find R_N by measuring the resistance of the O.C. network with the internal voltage sources set to zero.

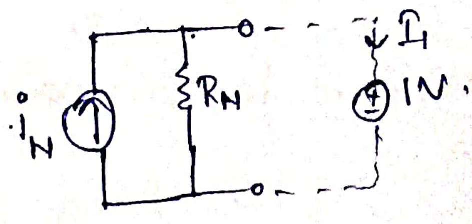




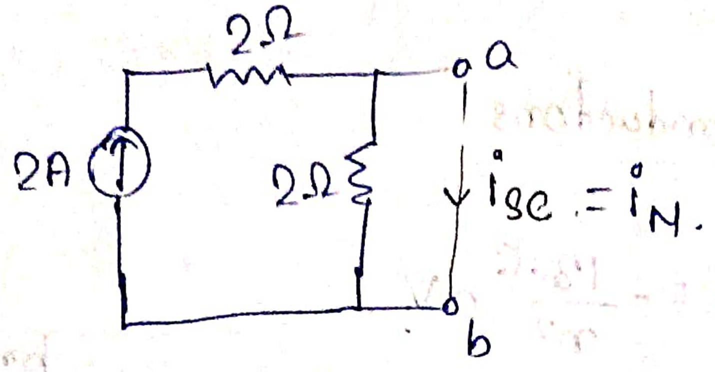
$R_{Th} = \frac{1 \cdot 2}{2+1}\Omega = \frac{2}{3}\Omega$

Example :-
Determine the current I through the voltage sources in the circuit.



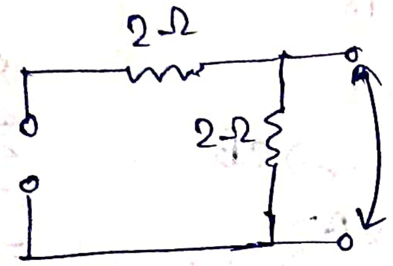


Step 1 :-

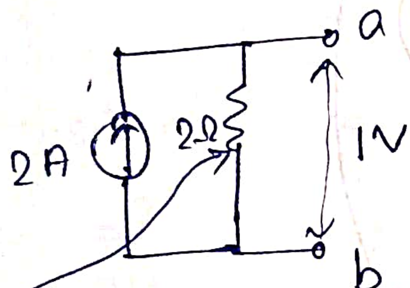


$i_{sc} = i_N = 2A$

Step 2 :- (S.C)



$R_N = 2\Omega$.



∴ The voltage drop across 'a'-'b' is 1V, the voltage apply across this 2Ω ~~resistance~~ resistor is 1V.

Current through the 2Ω resistor is 0.5 A.

$I_1 = 2A - 0.5A = 1.5A$.

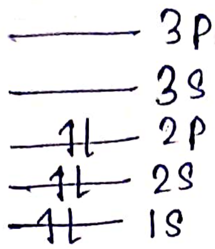
lecture-11

29/08/2025

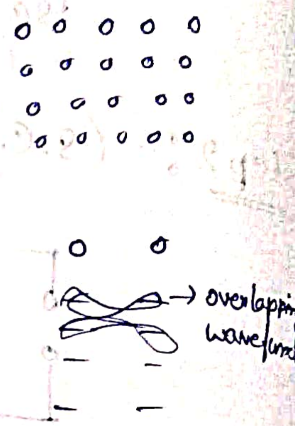
Semiconductors

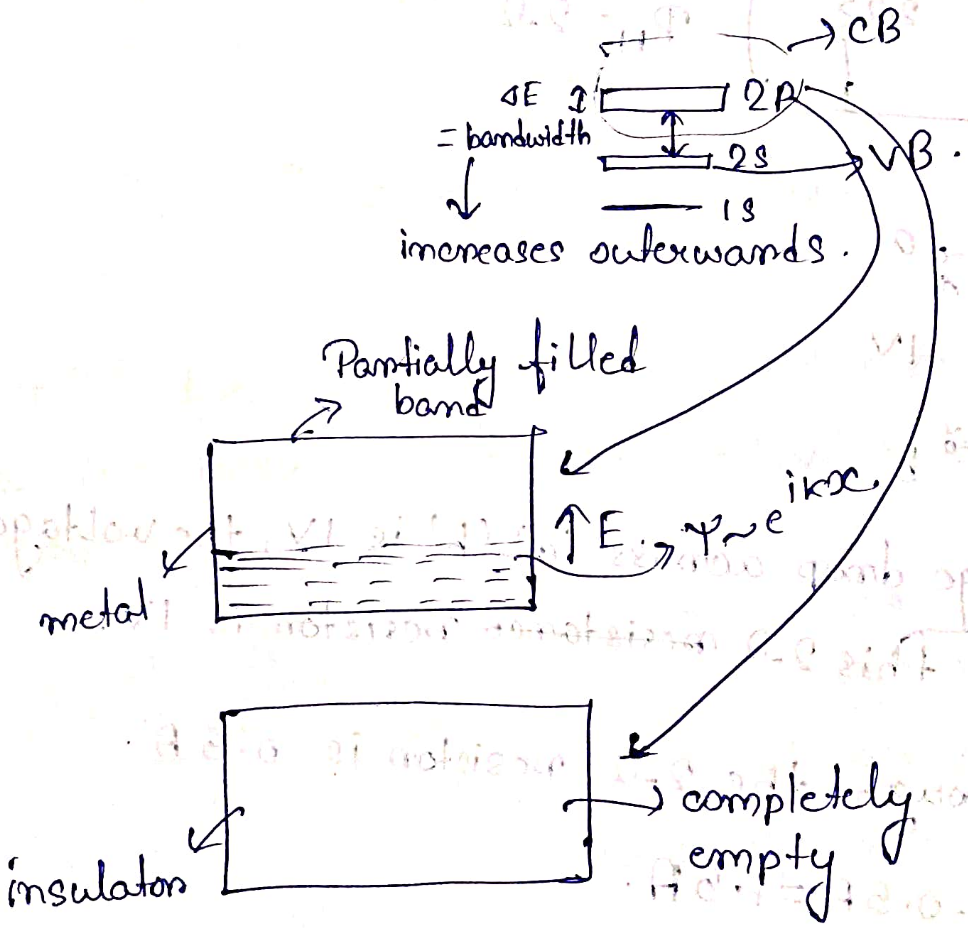
$$E_n = -\frac{13.6}{n^2}\ eV$$

orbitals



bring many atoms together





The energy gap between the C.B. and the V.B. in an insulator is large.

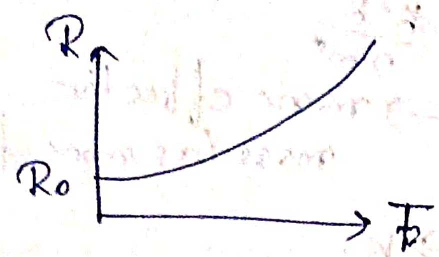
Diamond ~ E_G ~ 5.5 eV.

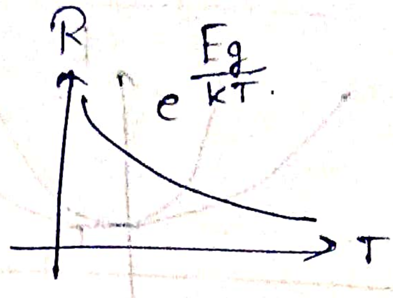
Semiconductor

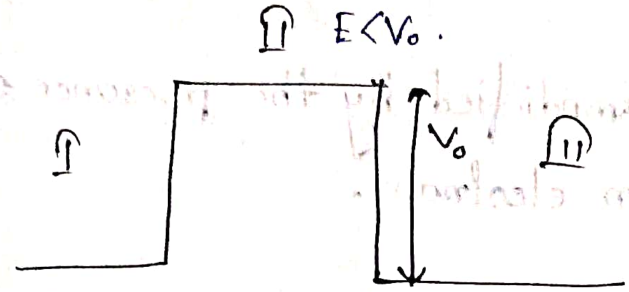
Si → 1.1 eV → indirect bandgap.

GaAs → 1.4 eV → direct bandgap.

$$R = R_0 (1 + \alpha t + \beta t^2)$$







$$\psi(t) = Ae^{-ikx} + Be^{ikx}.$$

$$E = \frac{\hbar^2 k^2}{2m}$$

Constant potential V_0

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V_0\psi(x) = E\psi(x).$$

$$\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + (E - V_0)\psi(x) = 0.$$

$E > V_0$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi(x) = 0$$

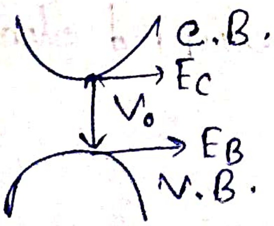
$$k^2 = 2m\left(\frac{E - V_0}{\hbar^2}\right)$$

$$E = V_0 + \frac{\hbar^2 k^2}{2m}.$$

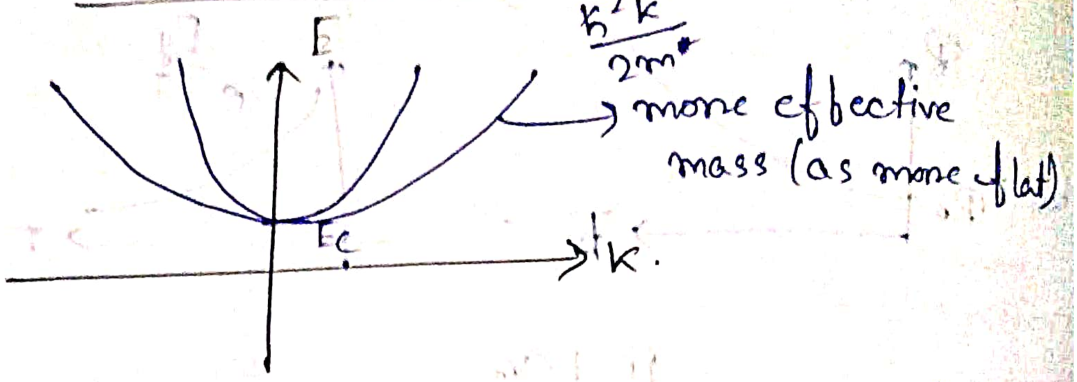
$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0.$$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}.$$

$$\psi(x,t) = Ae^{i(kx - \omega t)} + Be^{i(kx + \omega t)}.$$



Dispersion Relation



m^* :- effective mass modified by the presence of the nucleus and other electrons.

$$E = \hbar\omega.$$

$$U = E_C$$

$$m^* = \frac{\partial \omega}{\partial k} = \frac{1}{\hbar}.$$

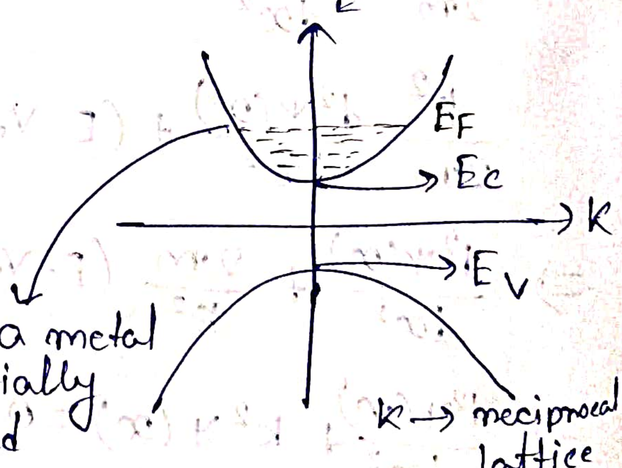
$$m^* = \frac{\hbar^2}{\left(\frac{\partial^2 E}{\partial k^2}\right)}.$$

$$\frac{\partial E}{\partial k} = \frac{\hbar^2 k}{m^*} \Rightarrow \frac{\partial^2 E}{\partial k^2} = \frac{\hbar^2}{m^*}$$

$$m = \frac{\hbar^2}{\left(\frac{\partial E}{\partial k}\right)}.$$

k → crystal momentum.

Band Structure



$$E_e = E_c + \frac{\hbar^2 k^2}{2m_e^*}.$$

$$E_h = E_v - \frac{\hbar^2 k^2}{2m_h^*}.$$

Read
Charles Kittle
Solid State Physics.

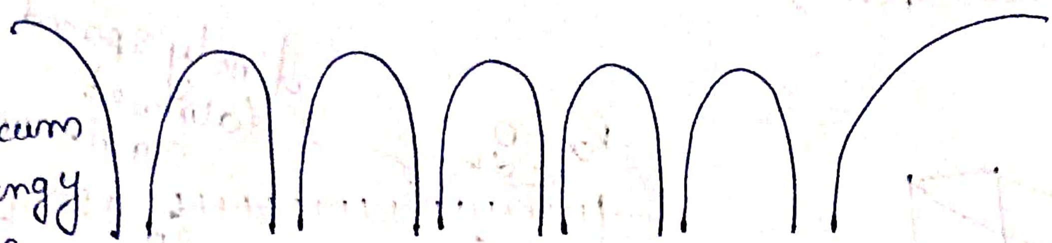
E_F → Fermi level is the highest occupied electronic state in the conduction Band.

$U_0 \longrightarrow U_c(x)$

$\hookrightarrow$ crystal potential

Faraway $\sim e^{i\vec{k}\cdot\vec{r}}$.

Vaccum Energy $E=0$



$U_c(\vec{r}+\vec{a}) = U_c(\vec{r})$

$\hookrightarrow$ Lattice Spacing.

$$U_c(x+a) = U_c(x).$$

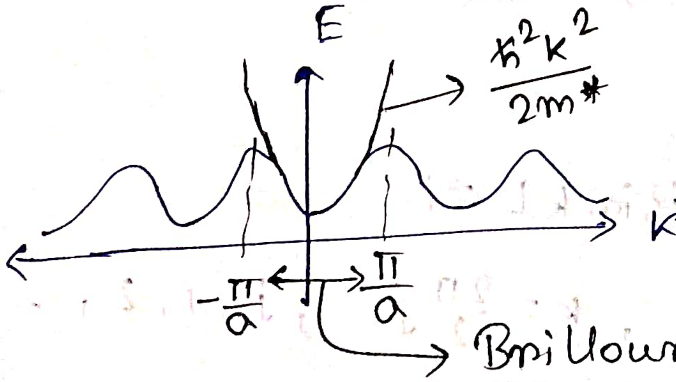
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U_c(x)\,\psi(x) = E\psi(x). \quad \text{Block function.}$$

$\psi(x) \sim e^{ikx} \rightarrow$ for free electron.

$$\psi(x) = U_k(x)\, e^{ikx}$$

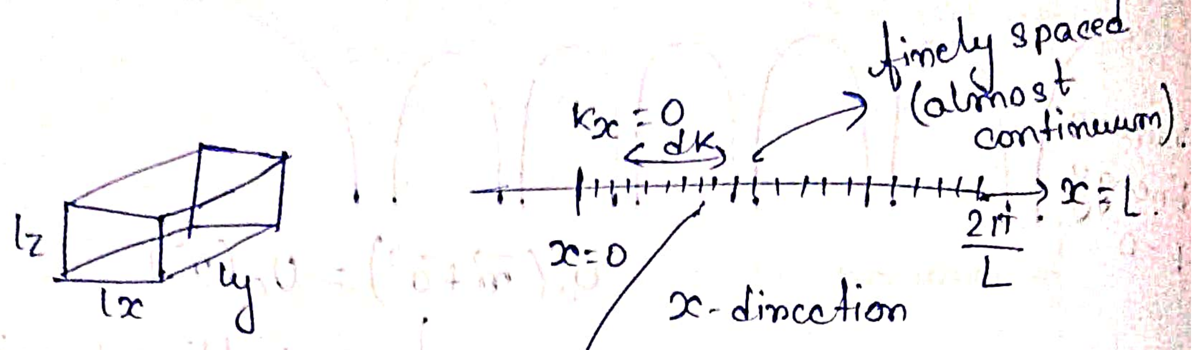
$U_k(x+a) = U_k(x)$.

$\hookrightarrow$ periodicity in real space.



Density of States

k-space
(reciprocal space).



$V = l_x l_y l_z$.

Spacing between each state in k-space is $\frac{2\pi}{L}$.

Solution to the schrödinger equation in a periodic potent

$\psi(x) = U_k(x) e^{ikx}$.

B.C. $\psi(x=0) = \psi(x=L)$

$\psi(x=L) = U_k(0) = U_k(x+L) e^{ikL}$.

$e^{ikL} = 1$

$e^{ikL} = \cos kL + i\sin kL = 1$

$\cos kL = 1$, $k = \frac{2\pi}{L} j$; $j = 1, 2, \ldots$

$\Delta k = \frac{2\pi}{L}$.

fermion with spin $\frac{1}{2}$.

no. of states in dk = $\frac{dk}{\frac{2\pi}{L}} \times 2 = N_k\, dk$

No. density of states in k-space.

$N_k = \frac{L}{\pi}$

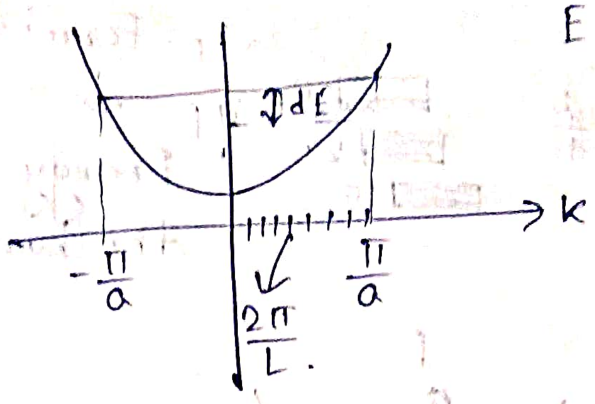
$L = Na$ ... (ii) $k = \frac{2\pi}{a} \frac{j}{N}$ (from (i) and (ii)).

Total no. of lattice points.

$j_{max} = N.$

Brillouin zone $(0 < k < \frac{2\pi}{a})$, $k_{max} = \frac{2\pi}{a}$.

$E = \frac{\hbar^2 k^2}{2m}.$



To find ~~the~~ out density of states in energy space.

Density of states in Energy Space

$$D(E)\,dE = N(k)\,dk \quad \text{--- (iii)}$$

Total no. of states in $N(k)dk$ should correspond to total no. of states $D(E)dE$.

$$E = E_c + \frac{\hbar^2 k^2}{2m^*}$$

$$dE = \frac{\hbar^2 k\,dk}{m^*} \quad \text{----- (iv)}$$

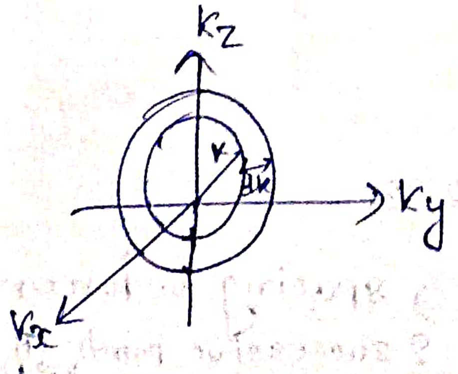
$$D(E)\,dE = N(k)\,dk.$$

$$D(E) \cdot \frac{\hbar^2 k\,dk}{m^*} = \frac{\frac{L}{\pi}dk}{L} = \frac{1}{\pi}dk.$$

what is relevent is density of states per ~~for~~ unit volume

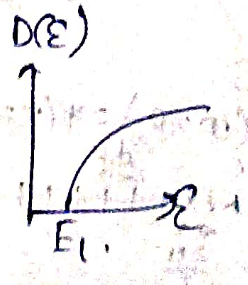
$$D(E) = \frac{1}{\pi\hbar}\sqrt{\frac{m^*}{2(E-E_c)}}$$

$$E_k = \frac{\hbar k_x^2}{2m^*} + \frac{\hbar k_y^2}{2m^*} + \frac{\hbar k_z^2}{2m} = \frac{\hbar k^2}{2m}$$



$$4\pi k^3 dk = D(E)\,dE.$$

$$D(\varepsilon) \approx \varepsilon^{\frac{1}{2}} = \sqrt{\varepsilon}$$



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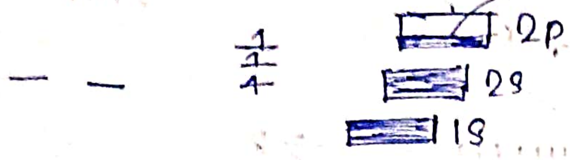
~~30/08/0~~ 02/09/2025

Recap

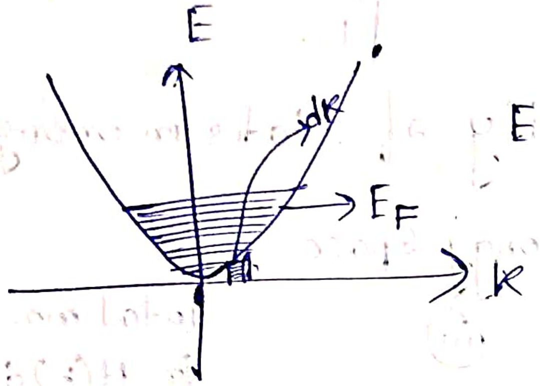


$\to E_F$ = Fermi level.

$\downarrow$

Energy level upto which the bands is filled.

DOS



$$E = \frac{\hbar^2 k^2}{2m}.$$

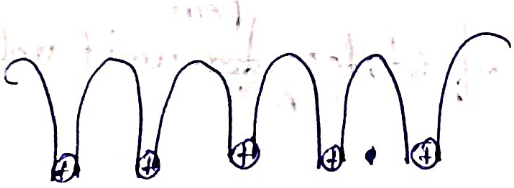
Block function :-

$$\psi(x) = U_k(x)\, e^{ikx}$$

$\downarrow$

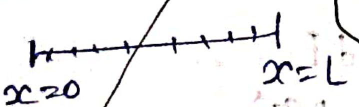
$$U_k(x+a) = U_k(x).$$

Block's function



$$\psi(0) = U_k(0)\, e^0 = U_k(L)\, e^{ikL}.$$

$$U_k(0) = U_k(L).$$



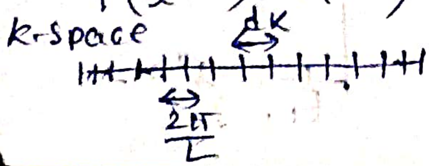
PBC

$\psi(x=0) = \psi(x=L)$

$$e^{ikL} = 1 = e^{i \times 2\pi m}$$

$$k = \frac{2\pi}{L} j, \quad j = 1, 2, 3, \ldots$$

k-space



$\Delta k = \frac{2\pi}{L} \longrightarrow$ spacing between 2 successive points in k-space.

no. density of states in the region $dk = 2 \frac{dk}{\frac{2\pi}{L}} = g_k \frac{dk}{\frac{2\pi}{L}}$

$\uparrow$ electron $\downarrow$ degeneracy.

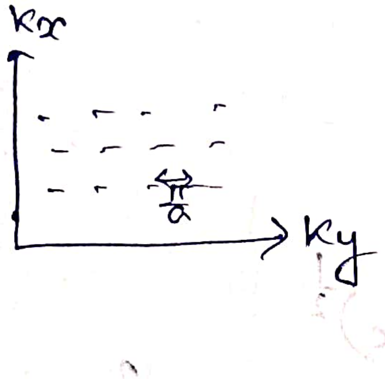
density of state in k-space $N_k = \frac{L}{\pi}$. per unit length or per unit volume.

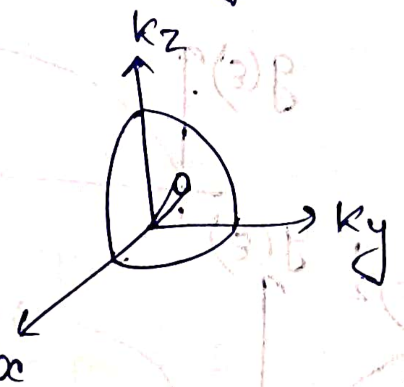
In particle in a box problem -

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right) = \frac{\hbar^2}{2m}\left(n_x^2 + n_y^2 + n_z^2\right) \times \frac{\pi^2}{a^2}.$$

$k_x = \frac{n\pi x}{a}$ $\qquad$ $n_x = 1, 2, 3.$

$n_y = 1, 2, 3.$





no. of points in the space contained will be $= \frac{1}{8} 4\pi k^2 dk.$

$$g_k dk = 2 \times \frac{1}{8} \frac{4\pi k^2 dk}{\left(\frac{\pi}{a}\right)^3} \quad \ldots (1)$$

$\uparrow$ density of states in 3-D. $\qquad$ $\uparrow$ electron

$\downarrow$ ~~distance~~ dimension of single cell in k-space in 3D.

$$k^2 = \frac{2mE}{\hbar^2} \quad \cdots (ii)$$

$$k = \frac{\sqrt{2mE}}{\hbar}.$$

$$dk = \frac{\sqrt{2m}}{\hbar} \frac{1}{\sqrt{E}} \quad \cdots (iii).$$

~~Substracting~~ Substituting k^2 from (i) and dk from (iii) into equation (i) —

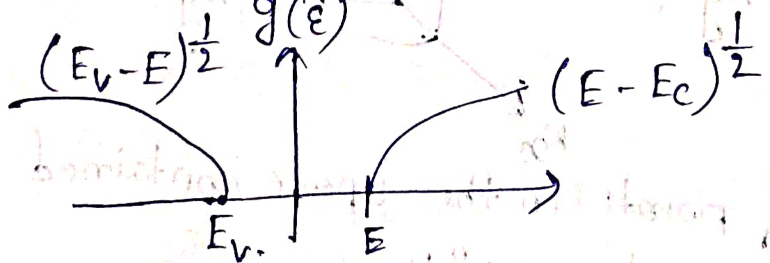
$$g_k dk = \frac{a^3 \pi \times 2mE}{\pi^3 \hbar^2} \cdot \frac{1}{\hbar} \sqrt{\frac{m}{2E}}\, dE.$$

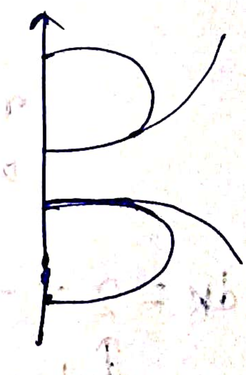
$$= g(E)\, dE.$$

DOS per unit volume $g(E) = \frac{\sqrt{2}}{\pi^2 \hbar^3} m^{\frac{3}{2}} \sqrt{E}$.

$g(E) \sim E^{\frac{1}{2}}$ for C.D.

$(E - E_c)^{\frac{1}{2}}$.





Probability of occupancy of a state for fermions

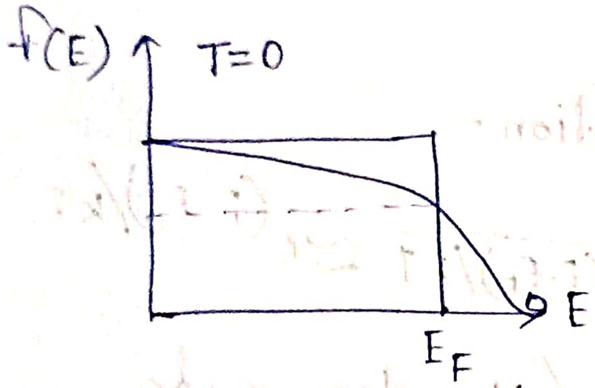
$$f(E) = \frac{1}{1+e^{(E-E_F)/k_BT}}$$

$k_B \rightarrow$ Boltzmann Constant
$T \rightarrow$ Temperature.

$\rightarrow$ Fermi dirac distribution.

ⓐ $T=0$:-

(i) $E < E_F$, $f(E) = 1$.
(ii) $E > E_F$, $f(E) = 0$.



ⓑ $T \neq 0$:-

$E = E_F$

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

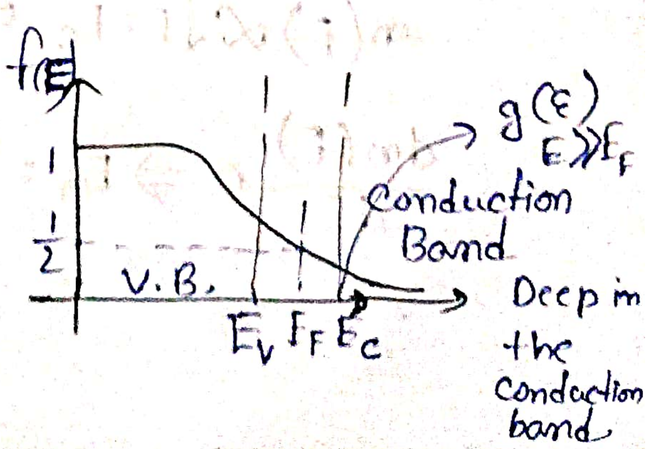
$$= \frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}$$

$E = E_F$ is the level where the probability of occupancy is $\frac{1}{2}$.

For Semiconductor :-

Electrons in Conduction Band.

$$f_c(E) = \frac{1}{1+e^{(E-E_F)/kT}} \simeq e^{(E_F-E)kT}$$



Silicon bandgap of 1.1 eV :

$T = 300\,K$, $k_B T = 0.026\,eV$.

For Conduction Band -

$$f(E_c) = e^{(E_F - E_c)/kT}$$

$$f(E_v) = e^{(E_v - E_F)/kT}.$$

E_c

$E_G = 1.12\,eV$

E_v

For hole, distribution function -

$$f_h(E) = 1 - f_c(E) = 1 - \frac{1}{1 + e^{(E - E_F)/kT}} \simeq e^{(E - E_F)/kT}$$

<u>Intrinsic Semiconductor</u> (Non degenerate Semiconductor)

E_c ———— $n_o = n_i$

- - - - - - - $E_F = E_i$

E_v ———— $P_o = P_i$

$$f_c(E) \approx e^{(E_F - E)/kT}$$

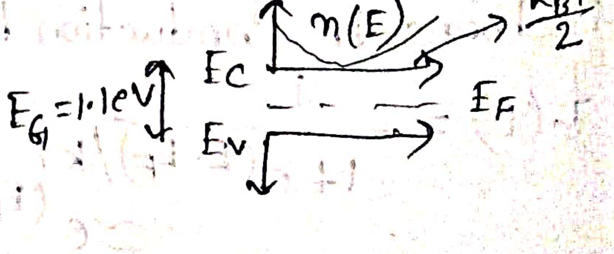
$$n_o(E)\,dE = f(\varepsilon)\,g(\varepsilon)\,d\varepsilon.$$

$$f(\varepsilon) \to e^{-(E - E_F)/kT} \qquad g(\varepsilon) \to \sqrt{E - E_c}$$

$$n(E) \propto \sqrt{E - E_c}\; e^{-(E - E_F)/kT}.$$

$$\frac{dn(E)}{dE} = 0 \Rightarrow E_n = \frac{k_B T}{2}$$

Distribution of Electron in Conduction Band



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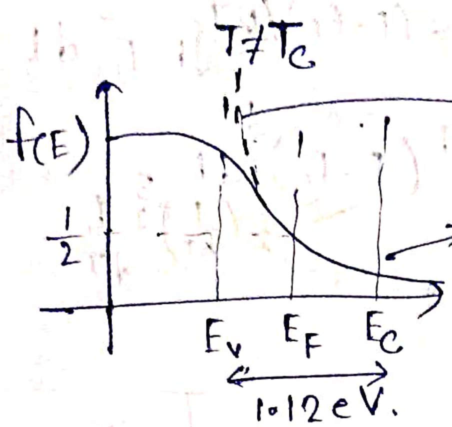
$$f(E) = \frac{1}{1+e^{(E-E_F)/KT}}$$



$f(E) = e^{-(E-E_F)/KT}$ definite limit

$k_B T \approx 0.026\, eV$.

For electron –

$$f_n(E) = \frac{1}{1+e^{(E-E_F)/KT}}$$

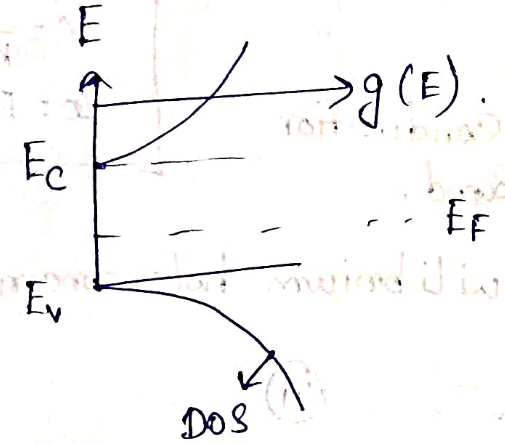
For hole –

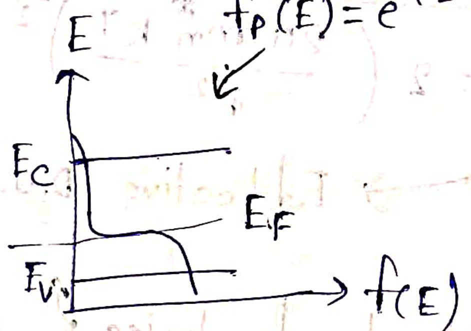
$$f_p(E) = 1 - f_n(E) = \frac{1}{1+e^{-(E-E_F)/KT}}$$

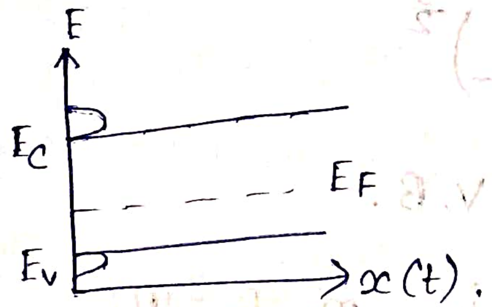
$1 - \frac{1}{1+e^{x}} = \frac{1}{1+e^{-x}}$

$E << E_V \rightarrow$

$$f_p(E) = e^{(E-E_F)/KT}$$







Intrinsic Semiconductor –

$n_0 = P_0 = n_i$

n_0 → electron density at equilibrium

P_0 → hole

Now let's calculate concentration (no. density) of electrons in Conduction Band –

$$n_0 = \int_{E_c}^{\infty} f(E)\,g(E)\,dE = \int_{E_c}^{\infty} \frac{1}{1+e^{(E-E_F)/kT}} \sqrt{E-E_c}\,dE.$$

equilibrium

$$= \int_{E_c}^{\infty} e^{-(E-E_F)/kT} \sqrt{E-E_c}\,dE.$$

$$\Rightarrow n_0 = \frac{1}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} \int_{E_c}^{\infty} e^{-(E-E_c)/kT}(E-E_c)^{\frac{1}{2}}\,dE \quad \frac{1}{2}\left(\frac{1}{kT}\right)^{\frac{3}{2}}\frac{E_c}{\sqrt{\pi}}.$$

Prefactor of $g(E)$.

$$n_0 = 2\left(\frac{2\pi m^* k_B T}{\hbar^2}\right)^{\frac{3}{2}} e^{(E_F-E_c)/kT}$$

$$\int_0^{\infty} e^{-\alpha x} x^{\frac{1}{2}} dx = \frac{1}{2\alpha}\sqrt{\frac{\pi}{\alpha}}\left(\frac{1}{\alpha}\right)^{\frac{3}{2}}$$

$$\alpha = \frac{1}{kT}$$

$$x = E - E_c$$

$$\boxed{n_0 = N_c\, e^{-(E_c-E_F)/k_BT}.} \quad \text{--- (I)}$$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{\hbar^2}\right)^{\frac{3}{2}}$$

→ Effective D.O.S. in Conduction Band.

Similarly for holes p_0 (equilibrium hole concentration) –

$$P_0 = N_v\, e^{-(E_F-E_c)/k_BT} \quad \text{--- (II)}$$

$$\Rightarrow N_v = 2\left(\frac{2\pi m_p^* k_BT}{\hbar^2}\right)^{\frac{3}{2}}$$

↓

Effective D.O.S. in V.B.

For intrinsic Semiconductor, $n_0 = P_0 = n_i$

$$n_i^2 = n_0 P_0 = N_c N_v\, e^{-(E_c-E_F)/kT} \times e^{-(E_F-E_c)/kT}.$$

$$n_i^2 = N_c V_c\, e^{-(E_c-E_v)/kT} = N_c N_v e^{-E_G/kT}. \quad \text{--- (III)}$$

Position of E_F

Let E_i be the position of fermi level for Intrinsic S.C.

$E_F \rightarrow E_i$ (let's say E_F has lies at some intrinsic value).

$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$p_i = N_v e^{(E_v - E_i)/kT}$$

For Intrinsic Semiconductor,

$$n_i = p_i = N_c e^{-(E_c - E_i)/kT} = N_v e^{(E_v - E_i)/kT}$$

$$\ln(N_c) - \frac{E_c - E_i}{kT} = \ln N_v + \frac{E_v - E_i}{kT}$$

$$\frac{2E_i}{k_B T} = \frac{E_c + E_v}{k_B T} + \ln\left(\frac{N_v}{N_c}\right)$$

$$E_i = \frac{E_c + E_v}{2} + \frac{k_B T}{2} \ln\left(\frac{N_v}{N_c}\right)$$

if $m_p^* \approx m_n^*$ $N_c \approx N_v$.

$E_i = \frac{E_G}{2}$

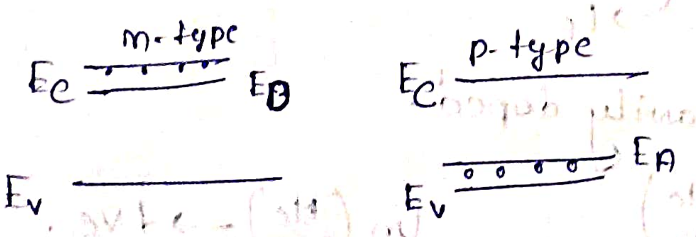
For Si, 1.12 eV, T = 300 K.

$n_i \approx 1 \times 10^{10} /cm^3$

Doping

$Si \rightarrow P \rightarrow$ n-type

$\rightarrow B \rightarrow$ p-type



p N_A^-
n N_D^+

$n_0, p_0, N_D, N_A \rightarrow$ Acceptor Density.

N_D ↓ Donor Density

Net charge density : $\rho = p_0 + n_0 + N_A^- + N_D^+ = 0$ -- (V)

space charge ~~density~~ neutrality

In equilibrium

$n_0 p_0 = n_i^2$ ... (vi)

(v) and (vi), $p_0 = \frac{n_i^2}{n_0}$

$\Rightarrow \frac{n_i^2}{n_0} + n_0 + N_A^- + N_D^+ = 0$ ... (vii)

$\Rightarrow p_0 - \frac{n_i^2}{p_0} + N_D - N_A = 0$ ... (viii)

2 quadratic equation -

N-type, $N_D >> N_A$.

$$n_0 = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right].$$

$$p_0 = \frac{n_i^2}{n_0}$$

For heavily dope

$N_D >> N_A$

$N_D >> n_i$

$n_0 \approx N_D$.

$n_0 p_0 = n_i^2$

minority carrier!

$$p_0 = \frac{n_i^2}{n_0} \approx \frac{n_i^2}{N_D}$$

Now, $x_0 = N_c e^{-(E_c - E_i)/kT}$ $\rightarrow E_F$

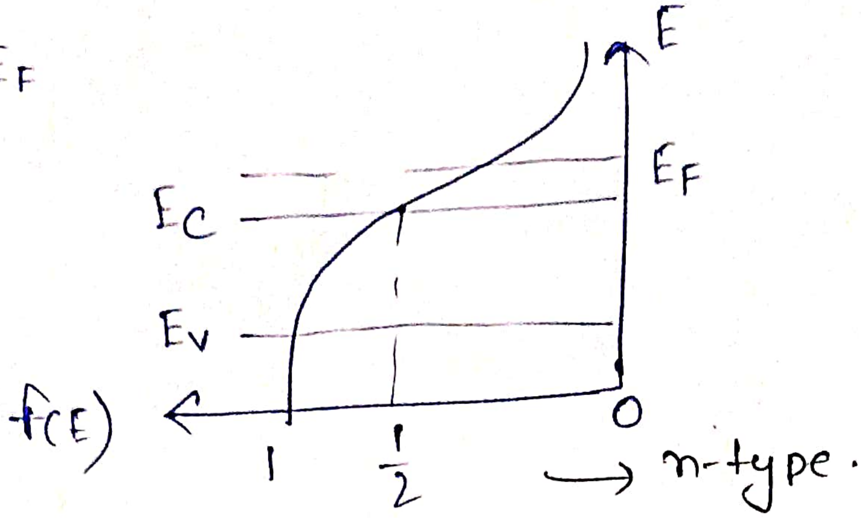
$n_0 \approx N_D \rightarrow$ for heavily doped.

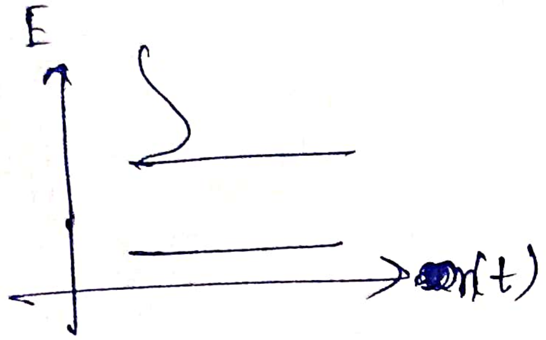
$$E_i = E_F = E_c - K_B T \ln\left(\frac{N_c}{N_D}\right)$$

$\frac{N_c}{N_D}$.

$\ln\left(\frac{N_c}{N_D}\right) \rightarrow +ve$

$\hookrightarrow$ Become smaller and smaller as N_D increases.





For hole