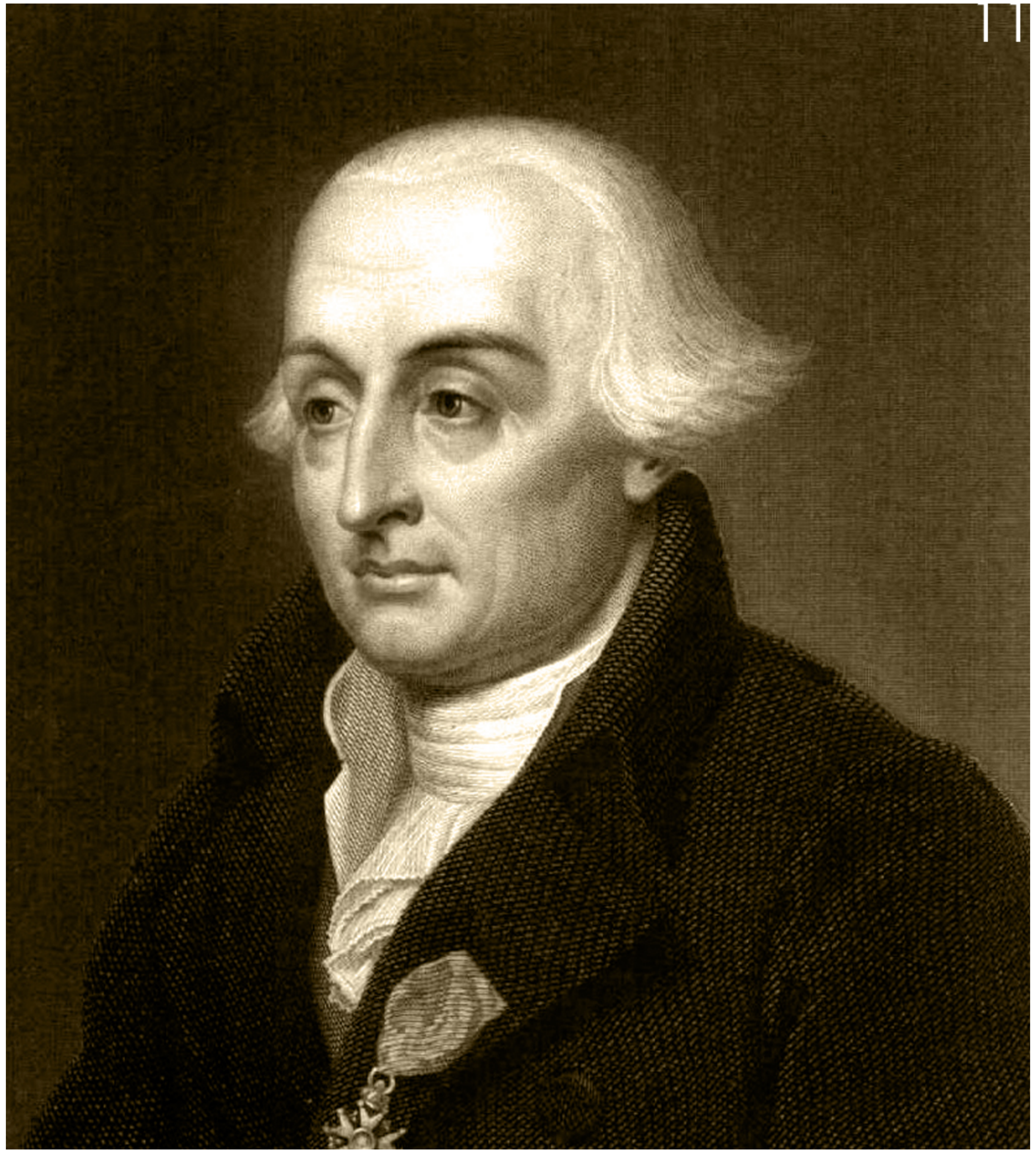
PH3101 Classical Mechanics @2025

Rajesh Kumble Nayak

Lecture 07

Lagrange Equation of Motion



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if thee basis vectors $\left\{\hat{q}_j\right\}$ are not constant, *i.e.* $\dfrac{d\hat{q}_j}{dt} \neq 0$, we have seen this results in the many terms and often difficult to deal with.

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Reaction $\frac{d\vec{p}}{dt}$

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As per Newton, these should balance—that is, the third law of motion. The total external force and the reaction/restoring force offered by the system are in equilibrium.

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D'Alembert's Principle: D'Alembert realised that, it is not need to have real displacement , it sufficient to use virtual displacement and virtual work results in the priciple of virtual work

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Here, Coefficient of δq represent the EOM

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Why Virtual Work ?

$$W = \sum_{i=1}^{N} \vec{F}_i \cdot \vec{dr}_i = \sum_{i=1}^{3N} F_i dx_i$$

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Virtual work give’s more freedom to work with only one of the virtual displacement is real

Lagrange Equation of Motion

Lagrange Equation of Motion

We start with

$$\sum_{i=1}^{N}\left(\vec{F}_i - \frac{d\vec{p}_i}{dt}\right)\cdot\delta\vec{r}_i = 0$$

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It is just transformation of components of vector under the coordinate transformation

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The remaining term is $\displaystyle\sum_{i=1} \frac{d\vec{p}_i}{dt} \cdot \delta\vec{r}_i = \sum_{i=1} m_i \ddot{\vec{r}}_i \cdot \delta\vec{r}_i$

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Expanding second term, 2 , we get

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Note this is
$\dot{r}_i = v_i$

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