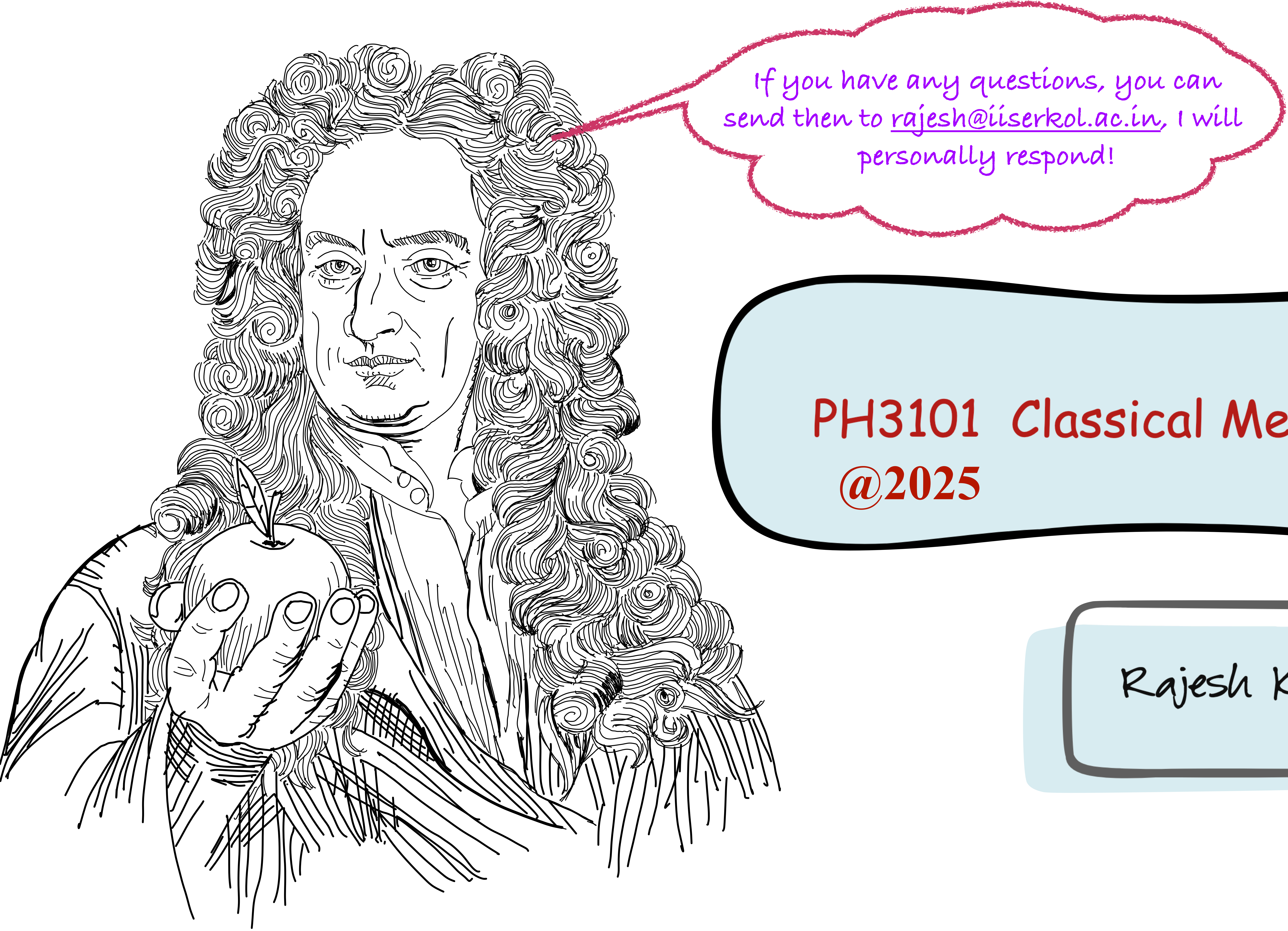

PH3101 Classical Mechanics @2025

Rajesh Kumble Nayak

Newtonian Mechanics

Objective is to describe the motion of objects

Time in Newtonian Mechanics

In Newtonian mechanics, time is treated as an absolute and universal parameter—a "background quantity that flows uniformly and independently of any physical processes or observers".

This view, rooted in Isaac Newton's *Philosophiæ Naturalis Principia Mathematica* (1687), underpins classical physics.

Newton postulated that time exists independently of space and matter. In his words, "Absolute, true, and **mathematical time**, of itself, and from its own nature, flows equably without relation to anything external."

Time in Newtonian Mechanics

This leads to:

- Time is the same for all observers, regardless of their state of motion.
- It provides a global parameter that allows the comparison of events at different locations.
- In equations of motion, time appears as an independent variable, *e.g.* $\vec{a} = \dfrac{d\vec{v}}{dt}$ and it enables deterministic evolution of systems

This absolute time enables determinism in Newtonian physics: given initial conditions at time $t = t_0$, one can predict the system's state at any later (or earlier) time

Time in Newtonian Mechanics

Although this concept of time aligns well with everyday experiences and suffices for many practical purposes, it was fundamentally challenged by Einstein's theory of relativity, where time becomes relative and dependent on the observer's motion and gravitational field.

Space in Newtonian Mechanics

In Newtonian mechanics, space is conceived as absolute, fixed, and independent—a passive stage on which physical events occur. This view is articulated most clearly in Newton's Principia, where he introduces the notion of absolute space.

Absolute Space:

Newton posited that space exists independently of the objects within it. That is, it has its own reality and structure, even in the absence of matter. He described it as "absolute, true, and mathematical space, which remains always similar and immovable."

Euclidean Geometry:

Newtonian space is three-dimensional and Euclidean. The geometry follows the familiar rules of flat space: straight lines, right angles, and the Pythagorean theorem all apply.

Space in Newtonian Mechanics

Independent of Matter:

Objects move through space, but do not affect it. Space is not influenced by the presence or motion of matter—a sharp contrast to general relativity, where mass and energy shape the geometry of spacetime.

Reference Frames:

Newtonian mechanics assumes the existence of an inertial reference frame—a privileged frame in which Newton's laws hold exactly. Motion can then be described relative to this absolute space.

A reference frame is a system—comprising spatial geometry and time—within which physical laws, such as Newton's laws of motion, can be formulated and applied.

In short

A reference frame provides the spatial and temporal structure necessary to formulate and apply physical laws.

Inertial Frames in Newtonian Mechanics

Definition

An inertial frame of reference is a frame in which Newton's first law (the law of inertia or Galilean Principle) holds true:

A body remains at rest or moves in a straight line at constant speed unless acted upon by a net external force.

This implies that in an inertial frame, if no force acts on a body, its velocity remains constant.

How to Identify an Inertial Frame?

To determine whether a given frame of reference is inertial, follow these steps:

Choose a reference frame (e.g., a lab, spacecraft, or moving vehicle).

Examine the motion of a test particle that is completely free from external forces.

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Choose a reference frame (e.g., a lab, spacecraft, or moving vehicle).

Examine the motion of a test particle that is completely free from external forces.

If the particle:

- ✓ Remains at rest if initially at rest, and
- ✓ Moves in a straight line at constant velocity if initially in motion, then the frame may be considered inertial.

This behavior must hold:

How to Identify an Inertial Frame continued

This behavior must hold:

- ➡ At every location in space, not just at one point, and
- ➡ For all initial velocities of the test particle.

This ensures Newton’s first law holds globally and for all possible states of motion—defining the frame as truly inertial.

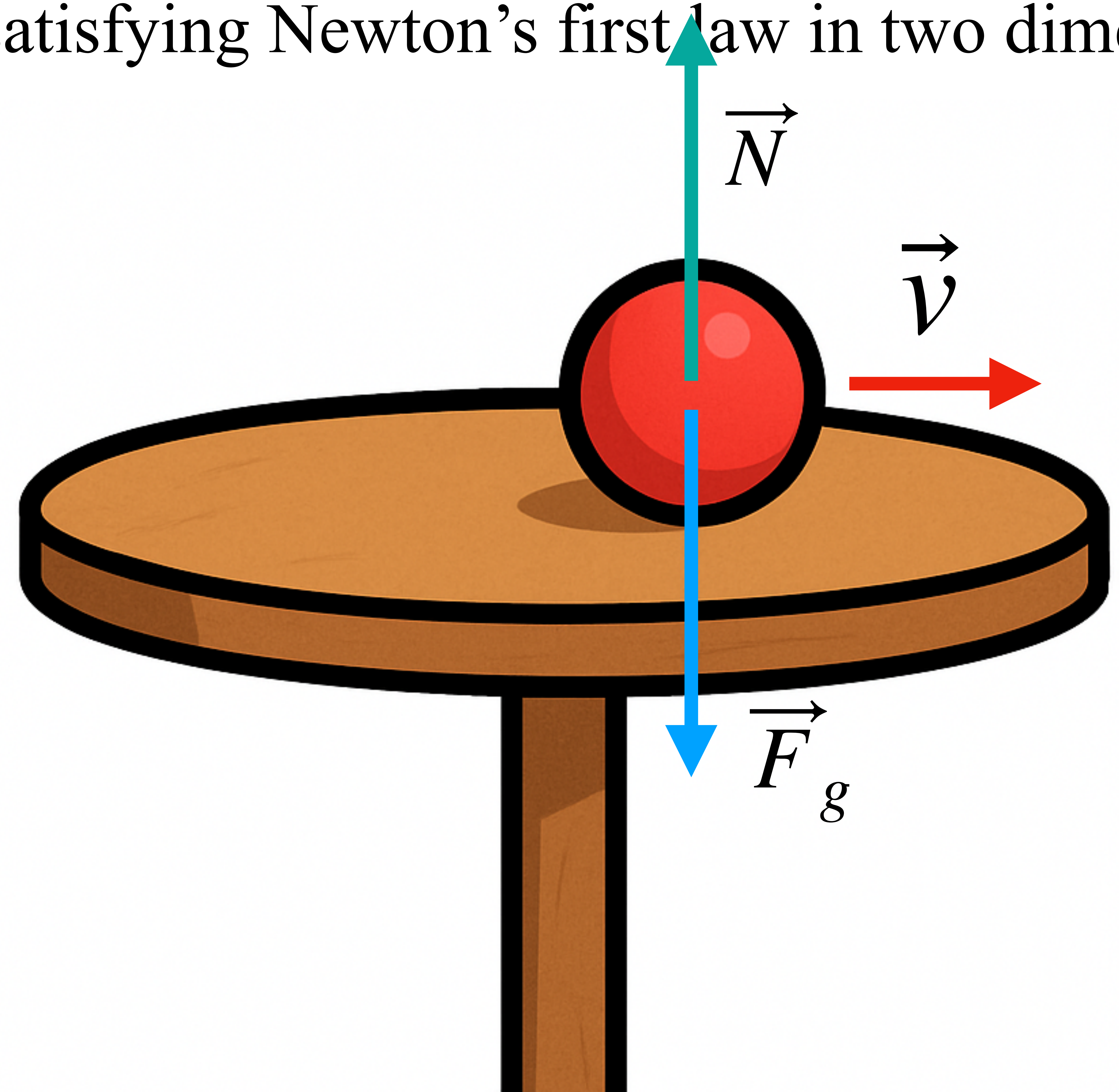
If, at any point or for any velocity, a free particle accelerates or changes direction without any applied force, the frame is non-inertial.

The Laboratory Frame on earth! (Not Strictly Inertial):

A lab fixed on Earth's surface is not a true inertial frame, because a free particle released from rest (e.g., at some height) will accelerate downward due to gravity. This violates Newton’s first law unless gravity is explicitly treated as an external force. Thus, the lab is only an approximately inertial frame, suitable for many practical purposes where gravitational and rotational effects are accounted for or negligible.

A Smooth Horizontal Table as a 2D Inertial Frame:

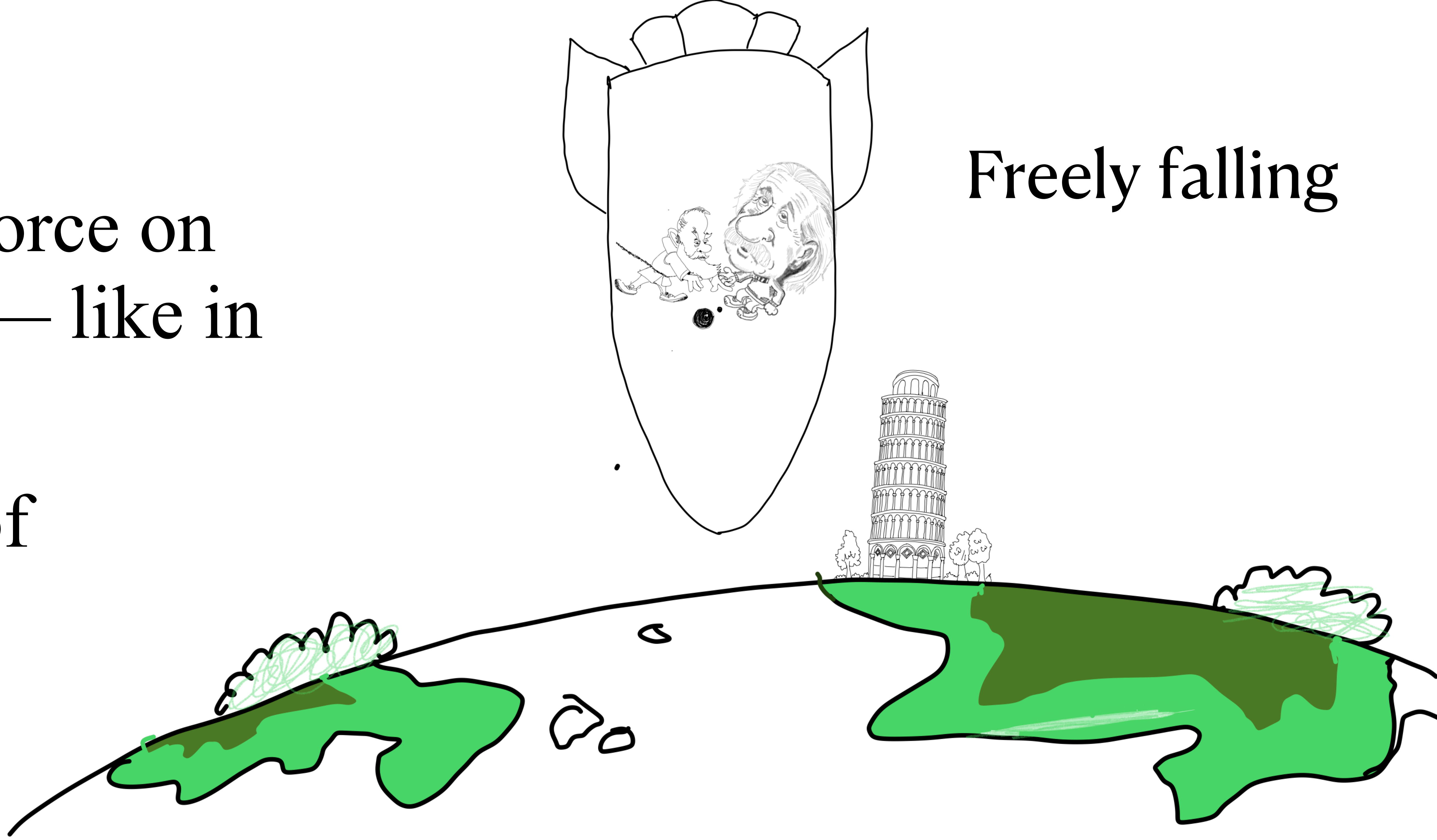
A smooth, level table in the lab can be treated as a 2-dimensional approximately inertial frame in the horizontal plane. Gravity is balanced by the table's normal force, so there is no vertical motion. In the absence of horizontal forces (e.g., friction), a particle moves with constant velocity, satisfying Newton's first law in two dimensions.

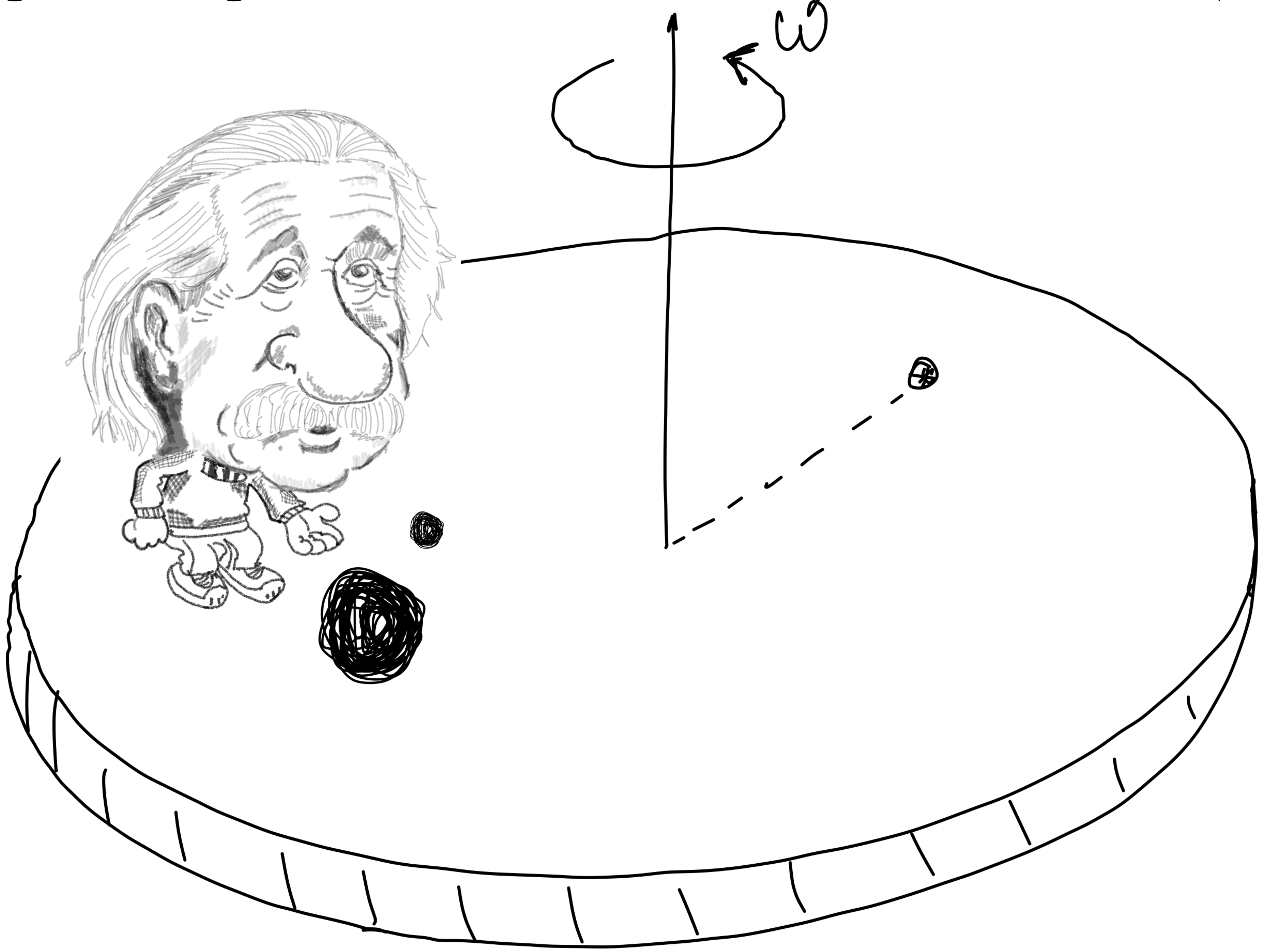


A Rocket in Free Fall (Inertial Frame)

An box in free fall experiences no normal force on objects inside. Everything appears to float — like in orbit.

This is an ideal inertial frame (in the limit of neglecting air resistance and tidal forces).





Rotating Merry-Go-Round (Non-Inertial)

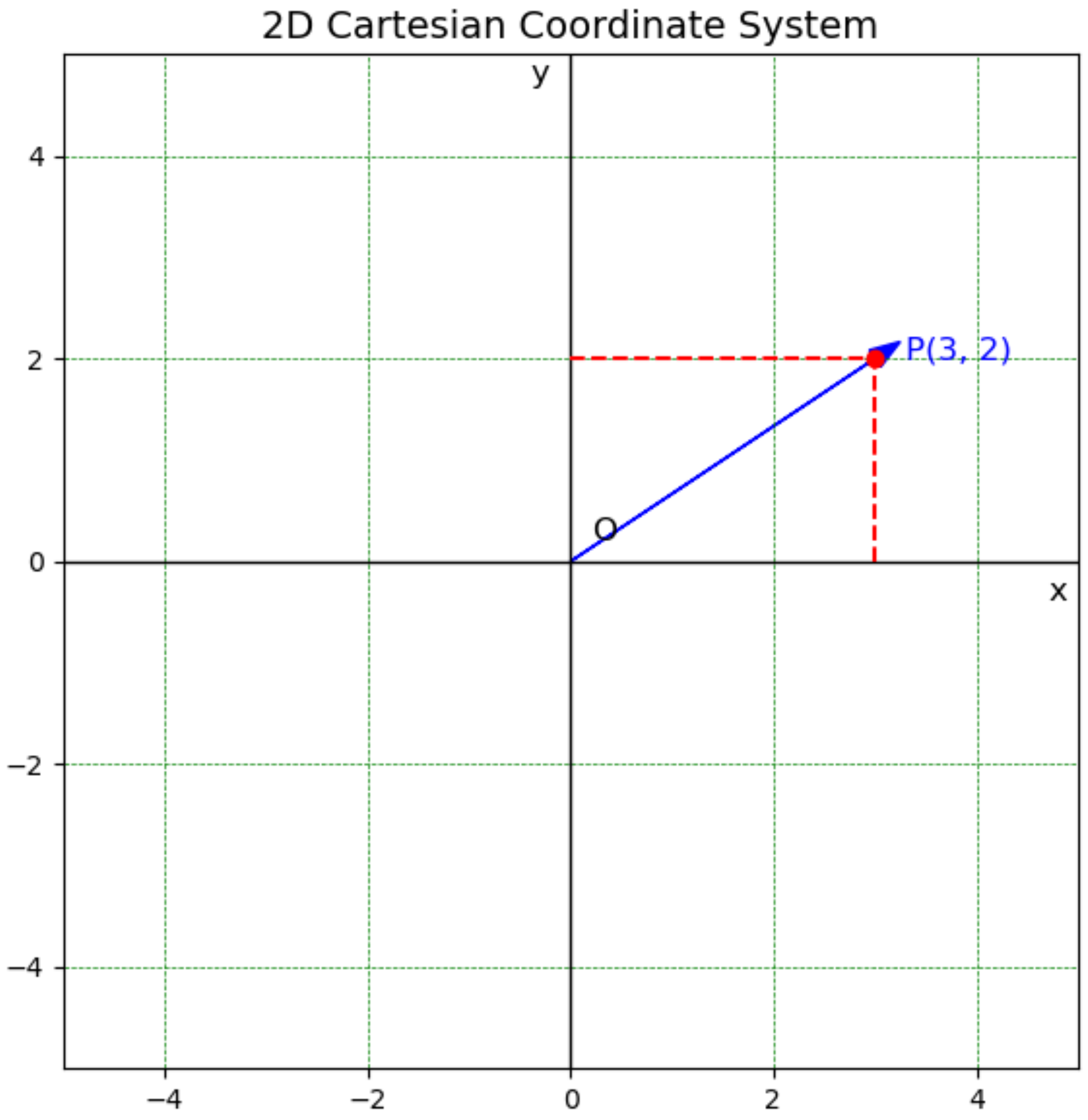
- On a rotating platform, objects seem to curve outward (centrifugal force) or deflect (Coriolis force).
- To apply Newton’s laws here, you must include **fictitious forces**, indicating a **non-inertial frame**.

Coordinate system

What Is a Coordinate System?

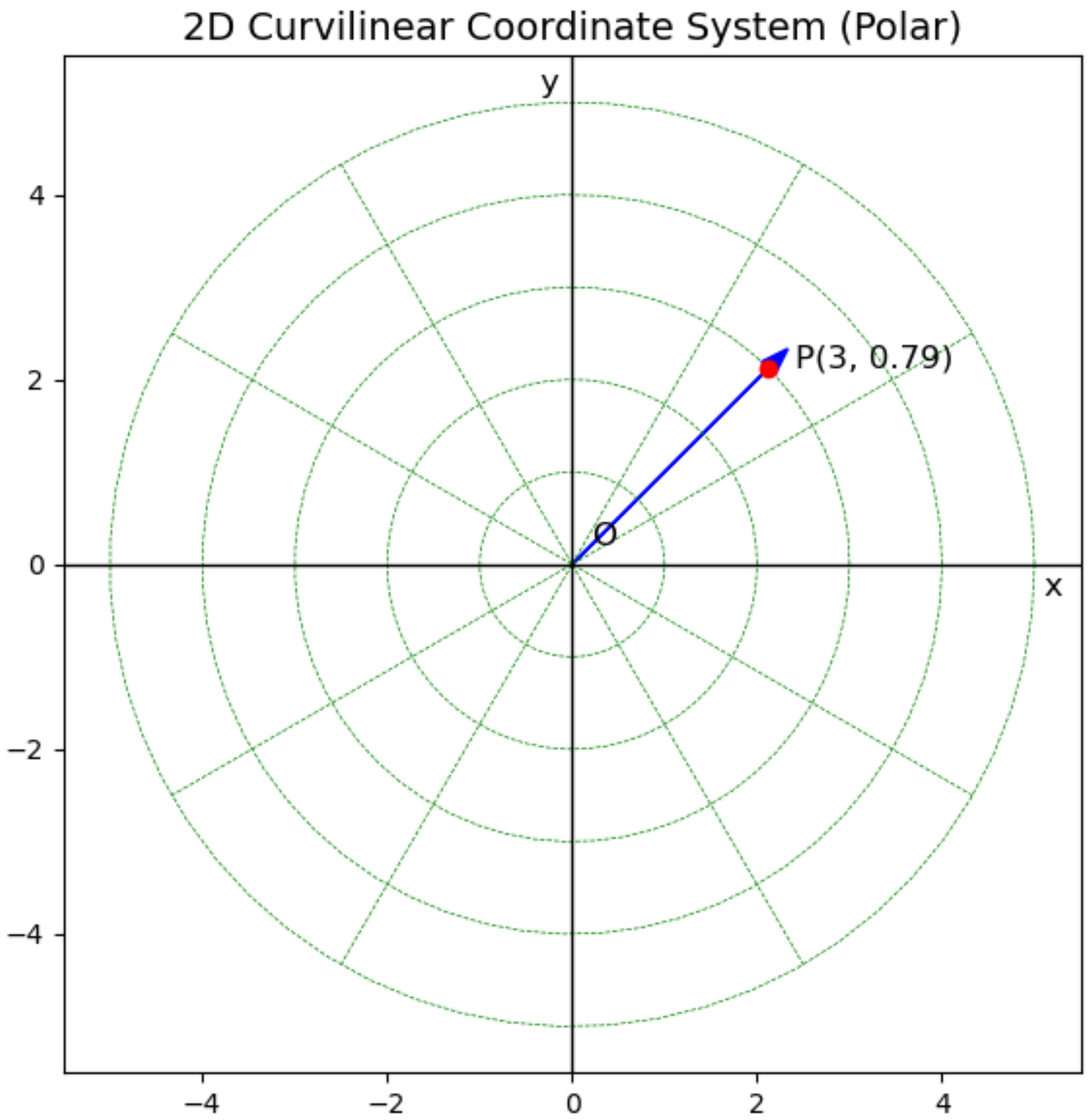
Once we have chosen an inertial frame for doing mechanics, we also need a way to measure or label positions and time. This is achieved using a coordinate system.

A coordinate system is a mathematical framework or construct that allows us to assign numerical labels (coordinates) to points in space at any time. It does not have any physical content—it's a tool to describe position, motion, and fields in space and time.



Think of it as a “grid” or “scaffold” we lay over space to do calculations—it helps us locate, compare, and differentiate physical quantities.

Coordinate systems are used in both inertial and non-



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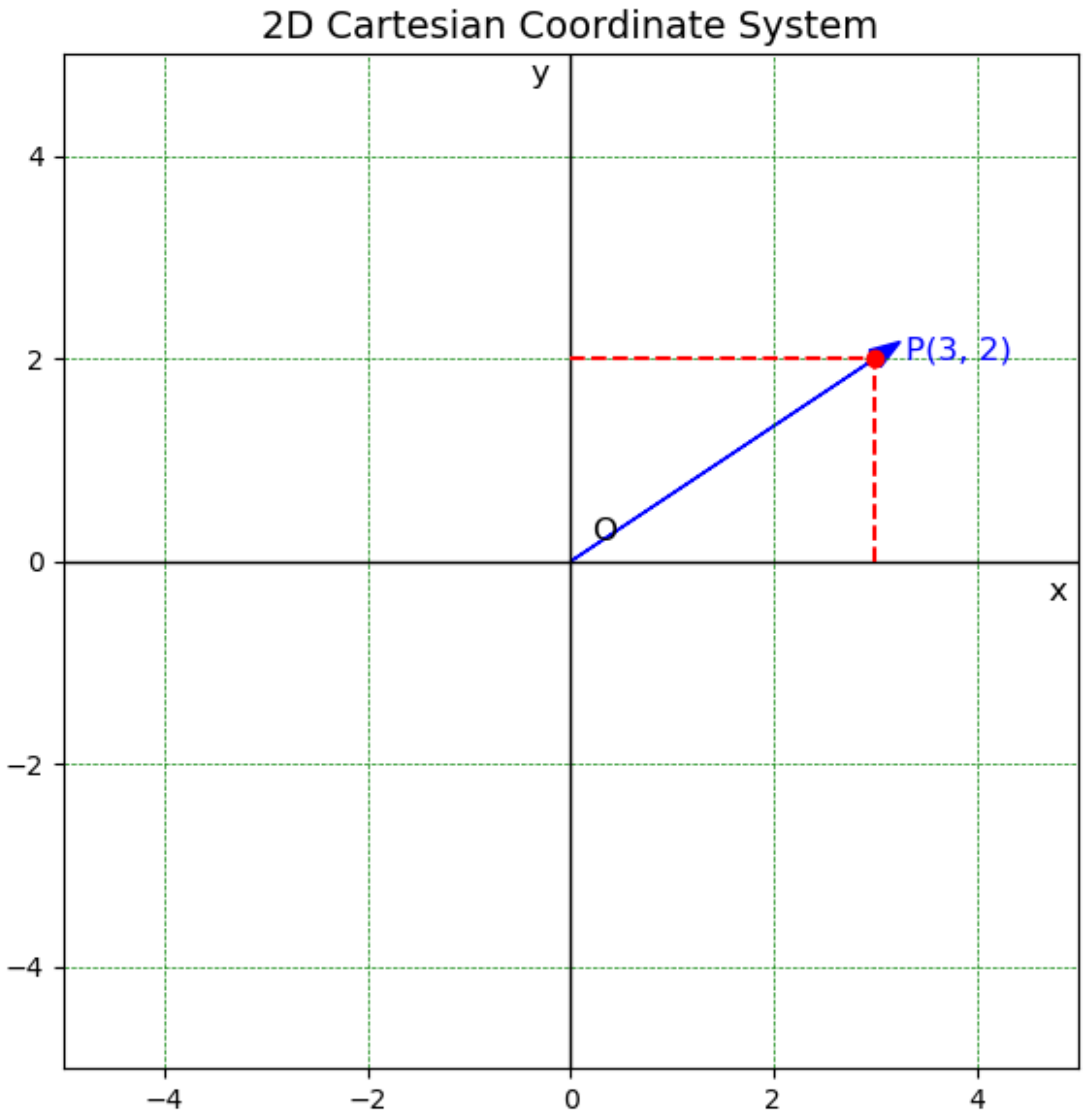
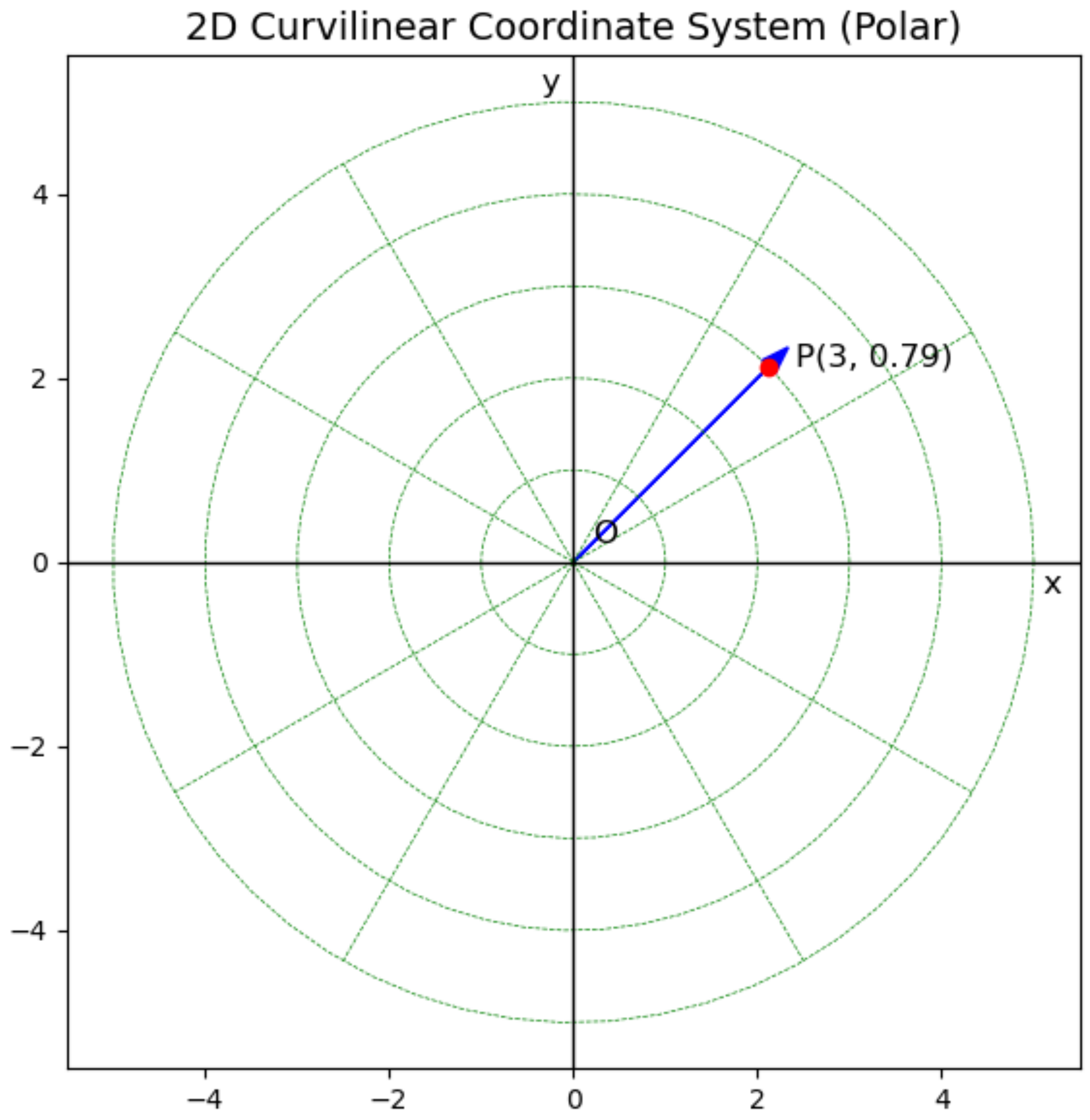


Coordinate systems are used in both **inertial and non-inertial frames.** Whether you use Cartesian or polar coordinates is unrelated to the inertial nature of the frame.



Inertial Frame VS Coordinate System

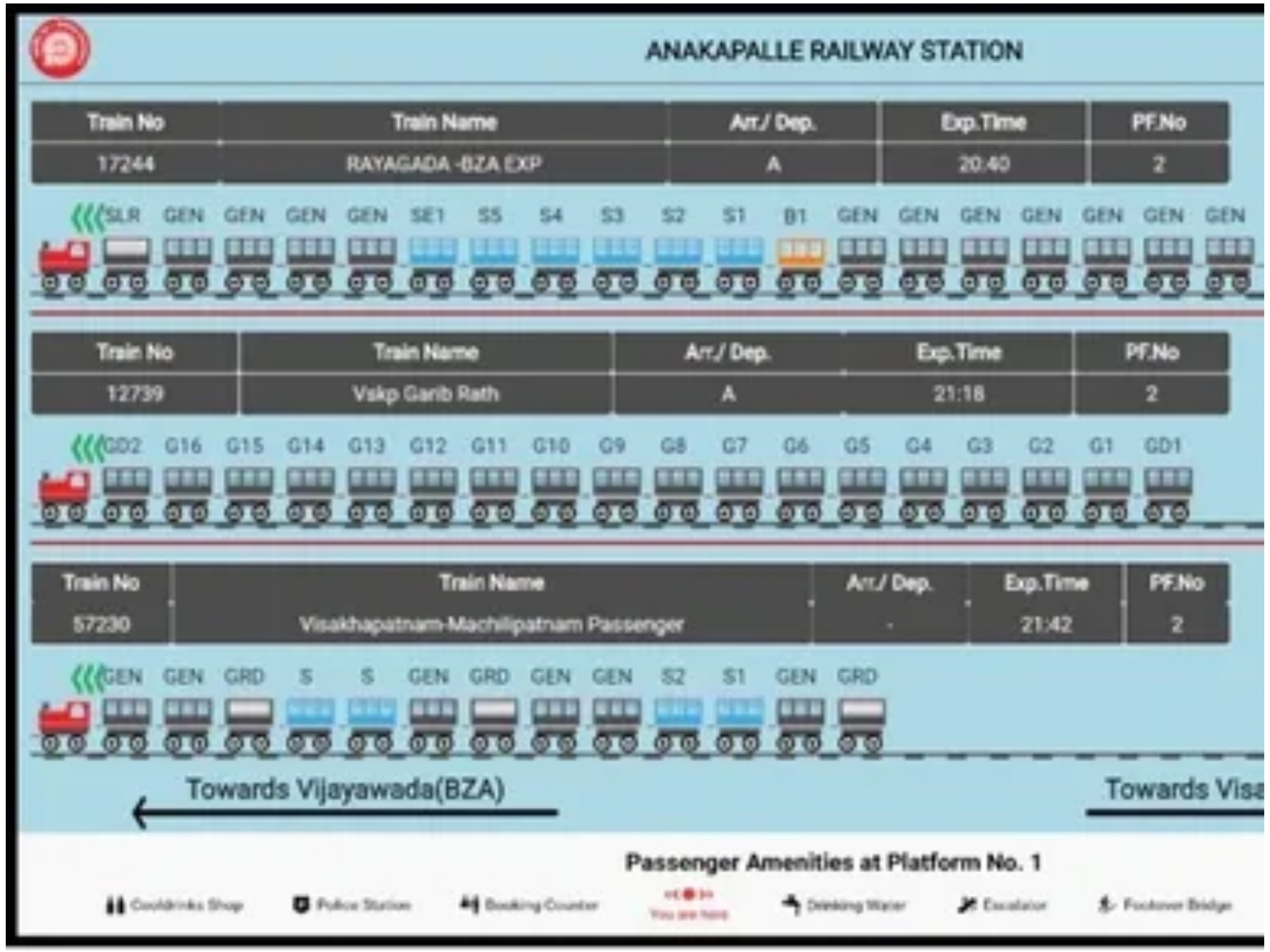
Concept	Inertial Frame	Coordinate System
What it is	A *physical* reference frame (state of motion) in which Newton's laws hold	A *mathematical tool* for labeling positions and directions
Defined by	The *motion* of the frame (e.g., at rest or moving at constant velocity)	The *geometric grid* you overlay (Cartesian, polar, etc.)
Affects physics?	Yes—laws like NLOM hold only in inertial frames	No—physics is coordinate-independent
Examples	Lab at rest (approx. inertial), rotating merry-go-round (non-inertial)	Cartesian axes, spherical coordinates

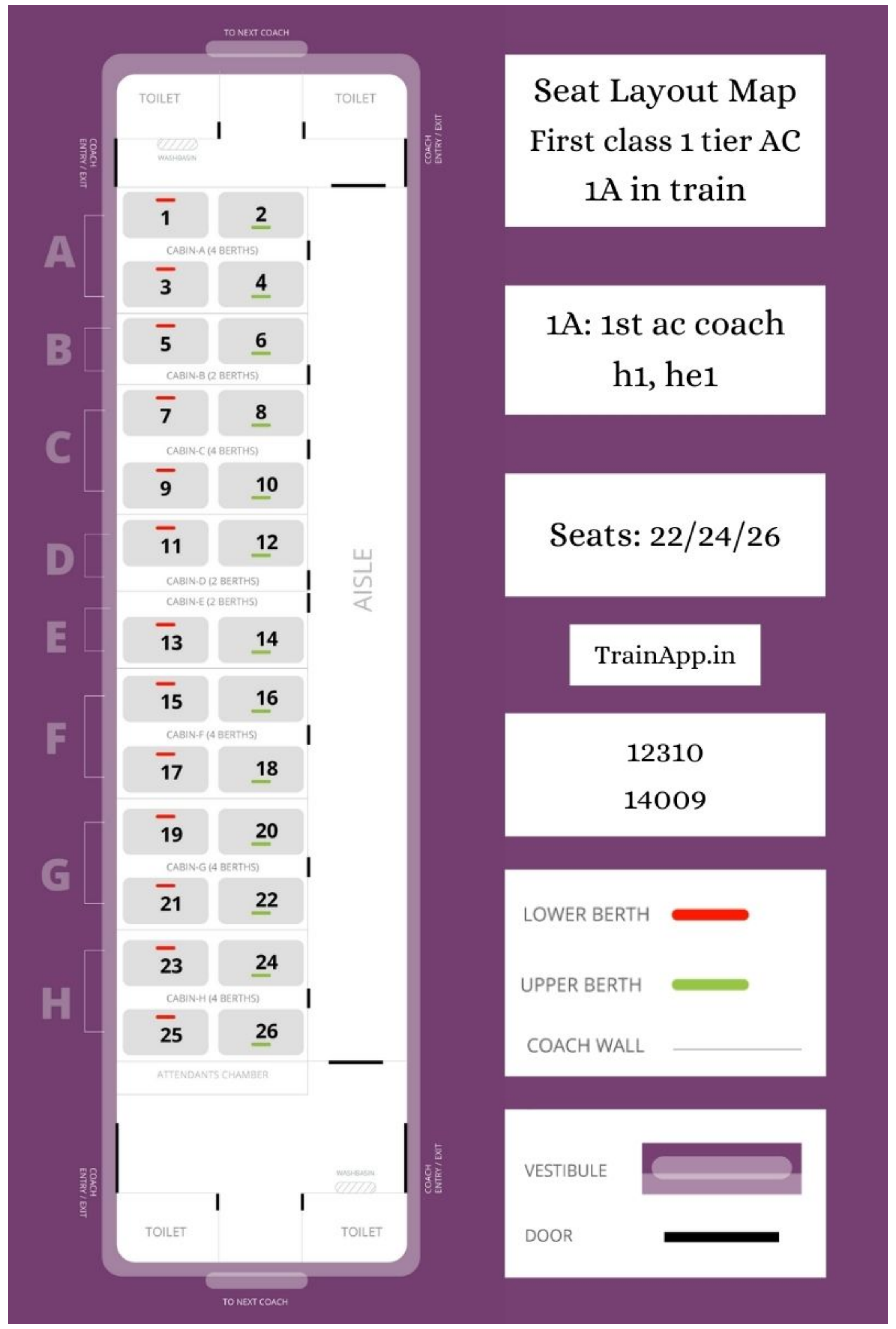
Inertial Frame VS Coordinate System

Analogy:

Think of an inertial frame as the platform you're observing from—like standing on solid ground or on a moving train.

A coordinate system is the grid you draw on that platform to describe positions—it's your ruler, compass and stop-watch, not the ground itself.

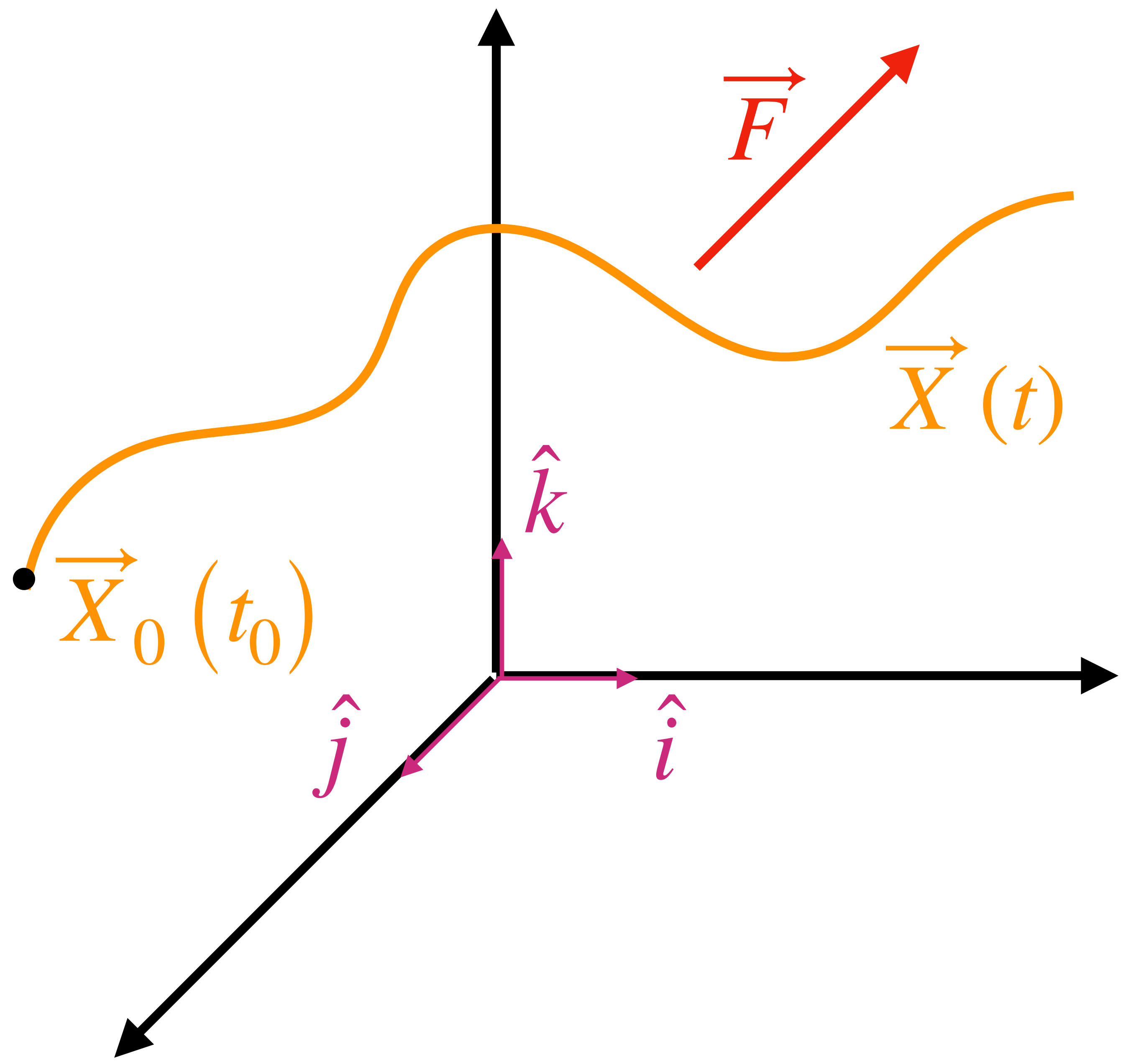




Classical Mechanics

Aim of the "Mechanics" is to predict the motion of an oobject or physical systems by applying the laws of force or law of motion.

We start with the simplest of mechanical system is a point particle. A point particle is represented by its mass, m, position, $\overrightarrow{X}$ at any given time.



We use $\mathbb{R}^3$ as basic geometry and Cartesian coordinate system for measurement. In this case position is represented by a three vector $\overrightarrow{X}$ at any time t. Starting from position $\overrightarrow{X}_0(t_0)$ at time t_0. As particle moves, it covers a three-dimensional curve or trajectory given by $\overrightarrow{X}(t)$. If if particle has a mass m and a force $\overrightarrow{F}$ is acting in the frame, we would like to understand the trajectory $\overrightarrow{X}(t)$.

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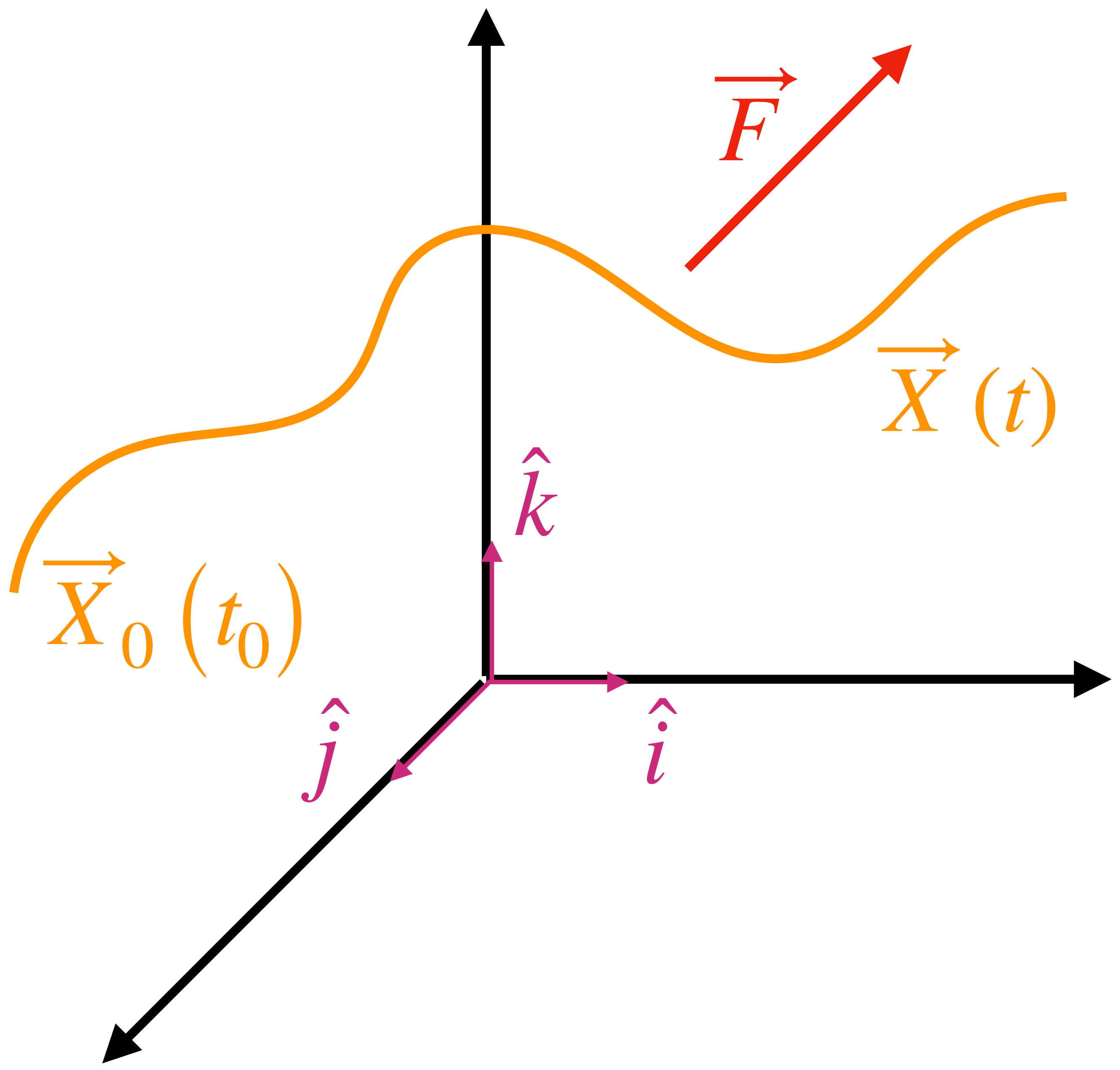

Any ~~position~~ vector can be written in terms of basis vectors

$\gamma = \overrightarrow{X}(t)$ is called trajectory of the particle and our goal is to find, it under the influence of a force field

$\overrightarrow{X} = x\,\hat{i} + y\,\hat{j} + z\,\hat{z}$ in a Cartesian coordinate system

$\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$ and $\hat{k} = (0, 0, 1)$.

Characterising the Curves

A curve is characterised by velocity and acceleration, jerk, etc

The velcity of the particle is the rate of change of poistion with respect to time t

$$\vec{v} = \frac{d\overrightarrow{X}}{dt} = \frac{d}{dt}\left(x\,\hat{i} + y\,\hat{j} + z\,\hat{k}\right) = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$$

Here $\dot{\hat{i}} = \dot{\hat{j}} = \dot{\hat{k}} = 0$

We also use the notation $\dot{x} = \dfrac{dx}{dt}$

The acceleration of the particle is the rate of change of velocity with respect to time t

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}\right) = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} + \ddot{z}\,\hat{k}$$

Let's see Newton's first law or Galilean Principle

In an inertial frame, if $\overrightarrow{F} = 0$, then the trajectory γ will have constant velocity!

Since velocity is a vector, $\dfrac{d\vec{v}}{dt} = 0 \implies \dot{x} = \dot{y} = \dot{z} = 0$

Newton's Second Law of Motion

In an inertial frame, the motion of a particle under the influence of a force is such that the rate of change of momentum equals the applied force.

mathematically, $\dfrac{d\vec{P}}{dt} = \vec{F}$ For a force field $\vec{F}$ and momentum $\vec{P}$

Momentum describes the staste of physical system. If the physical system is a particle then

m is the mass of the particle

$\vec{P} = m\vec{v}$

$\vec{v}$ is the velocity of the particle, w.r.t to a inertial frame.

In a more complex system, $\vec{P}$, may have to be defined suitably to incorporate all the physics!

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$\vec{v}$ is the velocity of the particle, w.r.t to a inertial frame.

This is the popular form of the Newtons law of motion!

If mass is not changing with time *i.e.* $\frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$

Newton's Second Law of Motion continued ...

The role of a coordinate system

Let $\mathcal{O}$ be an inertial frame, with a cartesian coordinates system, the basis vector for such a coordinate system is given by,

$$\hat{i} = (1, 0, 0), \hat{j} = (0, 1, 0) \text{ and } \hat{k} = (0, 0, 1),$$

Which are constants *i.e.* $\dfrac{d\hat{e}}{dt} = 0$ for $\hat{e} = \left\{ \hat{i}, \hat{j}, \hat{k} \right\}$ this simplifies lots of complexcity

In addition they are orthonormal $\hat{e}_i \cdot \hat{e}_j = \delta_{ij}$ here i, j are $\{1, 2, 3\}$

and is identity matrix or $\delta_{ij} = \begin{cases} = & 1 & \text{if } i = j \\ = & 0 & \text{ig } i \neq j \end{cases}$

For a given a force field $\overrightarrow{F} = f(x, y)\,\hat{i} + g(x, y)\,\hat{j}$, here $f(x, y)$ is compoment of force in the $x-$direction or along $\hat{i}$ and here $g(x, y)$ is component of force in the $y-$direction or along $\hat{j}$, the motion can be determined by the equation

Newton's Second Law of Motion continued ...

The role of a coordinate system

For a given a force field $\vec{F} = f(x, y)\,\hat{i} + g(x, y)\,\hat{j}$, here $f(x, y)$ is compoment of force in the $x-$direction or along $\hat{i}$ and here $g(x, y)$ is compoment of force in the $y-$direction or along $\hat{j}$, the motion can be determined by the equation

$$\boxed{\ddot{x} = \frac{1}{m} f(x, y) \ \text{ and } \ddot{y} = \frac{1}{m} g(x, y)}$$

Two coupled differential equations.

Coupled differential equations are not easy to solve analytically and numerical solutions don't give much insight in to the problem!