PH3103 Mathematical Methods of Physics
Autumn Semester - 2025
Indian Institute of Science Education and Research, Kolkata
Instructor: Koushik Dutta

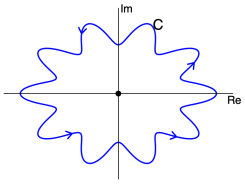
Homework: 2 Submission Date: 18/08/2025

The hand written solutions must be submitted at the start of the class

1. Evaluate the following integral

$$I_1 = \int_C \frac{e^z}{z^3} dz \tag{1}$$

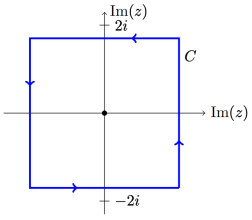
along the contour shown in the following figure.



2. Evaluate the following integral

$$I_2 = \int_C \frac{Cos\ z}{z(z^2+8)} dz \tag{2}$$

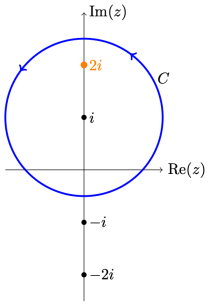
along the contour shown in the following figure.



3. Evaluate the following integral

$$I_3 = \int_C \frac{1}{(z^2+4)^2} dz \tag{3}$$

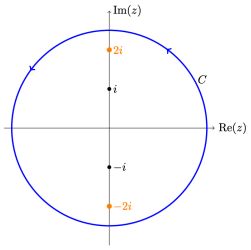
along the contour shown in the following figure.



4. Evaluate the following integral

$$I_3 = \int_C \frac{z}{(z^2+4)} dz \tag{4}$$

along the contour shown in the following figure. Repeat the calculations as done in the class, and also by doing partial fraction of the integrand.



5. By explicitly calculating over a contour, show the following identity

$$\frac{1}{2\pi i} \int_C \frac{dz}{(z-a)^{n+1}} = \delta_{n,0} \tag{5}$$

where n are positive integers, and C is a closed arbitrary simple (it does not intersect itself) contour encircling $z = a$ in the counterclockwise direction. This is one useful representation of the Kronecker delta.

6. Suppose $f(z)$ is analytic on the closed disk of radius r centered at z_0. Then, show the following identity

$$f(z_0) = \frac{1}{2\pi} \int_0^{2\pi} f(z_0 + re^{i\theta}) d\theta \ . \tag{6}$$