Tutorial 1

PH3102
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1 Continuity Equation in 1D

You have been introduced with the Schrodinger equation in class which in 1D reads:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x} + V(x)\psi(x) = \iota\hbar\frac{\partial\psi}{\partial t} \tag{1}$$

$P(x,t) = \psi^*(x,t)\psi(x,t)$ is the probability density i.e $\psi^*(x,t)\psi(x,t)dx$ is the probability for finding the particle in the range x to x+dx. Using the fact that V(x) is real (corresponds to potential energy), show that:

$$\frac{\partial P(x,t)}{\partial t} + \frac{\partial}{\partial x}\left[\frac{\hbar}{2\iota m}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)\right] = 0 \tag{2}$$

$J(x,t) = \frac{\hbar}{2\iota m}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right)$ is known as probability current density. Using the above equation show that:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{x_1}^{x_2} P(x,t)dx = J(x_1,t) - J(x_2,t) \tag{3}$$

This means that the rate of increase of probability of finding the particle in the range $x_1 < x < x_2$ is equal to net probability current entering through x_1 and leaving through x_2.

Aside Integrating from $-\infty$ to ∞ and using the fact wavefunction must vanishes at $\pm\infty$ gives:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\infty} P(x,t)dx = 0 \tag{4}$$

This means that the total probability of finding the particle in the whole space is always constant.

2 . Let x and p denote respectively , the coordinate and momentum operators satisfying the canonical commutation relation [x , p] = i , in natural units. Then prove that [x , p exp$\{-p\}$] is i(1-p)exp$\{-p\}$.