

PH3103 Mathematical Methods of Physics
Autumn Semester - 2025
Indian Institute of Science Education and Research, Kolkata
Instructor: Koushik Dutta

Homework: 1

Submission Date: 11/08/2025

The hand written solutions must be submitted at the start of the class

1. Consider the function $f(z) = x + 3iy$. Find out the limit of the function as $z \rightarrow 0$ along (a) x -axis (b) along y -axis (c) along the line $y = mx$ where m is real. What do you conclude?
2. In the list of following functions given below, identify the functions that are analytic functions of z in some region, and find their region of analyticity

$$(a) x \quad (b) r \quad (c) e^{i\theta} \quad (d) x^2 - y^2 - 2ixy \quad (e) (x - iy)/(x^2 + y^2) \quad (1)$$

3. Consider the following function:

$$f(z) = \begin{cases} \frac{x^3 - y^3 + i(3x^2y - y^3)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases} \quad (2)$$

First check Cauchy–Riemann conditions at the origin, and then find out whether it is differentiable at that point.

4. Find out the Cauchy-Riemann conditions in the polar coordinates and analyse the analyticity property of $f(z) = \sqrt{zz^*}$ both in polar coordinates and also in cartesian coordinates.
5. If a function $f(x, y)$ satisfies Laplace's equation, it is called a harmonic function. Show that $u = x^2 - y^2$ is harmonic. Find the function $v(x, y)$ such that $u + iv$ is an analytic function of z .