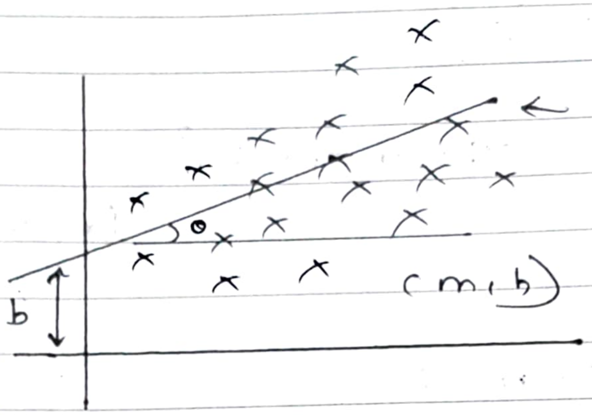
Simple Linear Regression



$y = mx + b$

where
m = slope
b = intercept

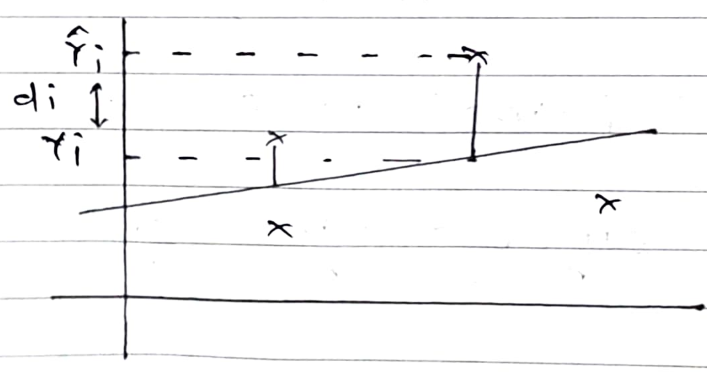
* To calculate b and m

$$b = \bar{Y} - m\bar{x}$$

$$m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\hat{Y}_1 = m x_i + b \quad \text{— (1)}$$

* Error function:



$$E = d_i + d_2 + d_3$$

$$E = \sum_{i=1}^{n} d_i^2$$

$$d_i = (Y_i - \hat{Y}_i)^2$$

$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$E(m,b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$ from 1

consider $b = 0$

$$E(m) = \sum_{i=1}^{n} (y_i - mx_i)^2$$

$$E(b) = \sum_{i=1}^{n} (y_i - x_i - b)^2$$

differentiate.

$$\frac{\partial E}{\partial b} = \frac{\partial}{\partial b} \sum_{i=1}^{n} (y_i - mx_i - b)^2 = 0$$

$$= \sum \frac{\partial}{\partial b} (y_i - mx_i - b)^2 = 0$$

$$= \sum -2(y_i - mx_i - b) = 0$$

$$= \sum (y_i - mx_i - b) = 0$$

$$\frac{\sum y_i}{n} - \frac{\sum mx_i}{n} - \frac{\sum b}{n} = \frac{0}{n}$$

$$\bar{y} - m\bar{x} - \frac{nb}{n} = 0$$

$$\boxed{b = \bar{y} - m\bar{x}} \quad \text{— } (2)$$

Page No.
Date

$$E = \Sigma (y_i - m x_i - \bar{y} + m\bar{x})^2 \quad \text{from } 2$$

$$\frac{\partial E}{\partial m} = \Sigma \frac{\partial}{\partial m} (y_i - m x_i - \bar{y} + m\bar{x})^2 = 0$$

$$\Sigma\, 2(y_i - m x_i - \bar{y} + m\bar{x})(-x_i + \bar{x}) = 0$$

$$= \Sigma - 2(y_i - m x_i - \bar{y} + m\bar{x})(x_i - \bar{x}) = 0$$

$$= \Sigma \left[(y_i - \bar{y}) - m(x_i - \bar{x})\right](x_i - \bar{x}) = 0$$

$$\Sigma \left[(y_i - \bar{y})(x_i - \bar{x}) - m(x_i - \bar{x})^2\right] = 0$$

$$\Sigma (y_i - \bar{y})(x_i - \bar{x}) = m\,\Sigma (x_i - \bar{x})^2$$

$$\boxed{m = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

We can use this formula to Create our own Simple Linear regression class.