

Problem Set 3

1. Would you consider a random variable which describes the height of each student in a statistics course to be discrete or continuous? (Assume that we are able to measure the heights with infinite precision).
2. Let the discrete random variable X represent the number of rooms in a randomly selected rental housing unit in Germany. Given the following table representing our random variable, calculate the mean & standard deviation. **Ans: 4.26, 1.293**

X	$P(X = x)$
1	0.01
2	0.03
3	0.25
4	0.35
5	0.20
6	0.10
7	0.04
8	0.02

3. A coin is tossed twice. X is the random variable of the number of heads obtained. What is the probability distribution of x ? **Ans: $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$**
4. Given a single roll of a six-sided die, find the expected value, variance and standard deviation of a given die roll. **Ans: 3.5, 2.92, 1.71**
5. Consider a gambling game which costs \$1 to play. The game consists of flipping 3 coins, if all 3 coins land on the same face the player wins \$3. Let the random variable X represent the number of heads in our 3 coin flip. What is the probability that the player wins the game? **Ans: 0.25**
6. Let $f(x) = kx^2$ for $x = 1, 2, 3$. Determine the constant k so that the $f(x)$ satisfies the conditions of being a probability mass function. **Ans: 0.0714**

7. Determine the constant **k** so that the following p.m.f. of the random variable **Y** is a valid probability mass function: $f(y) = k \cdot 0.25^y$ for $y = 1, 2, 3$ **Ans: 3**

8. Let **X** be a random variable, and $P(X=x)$ is the PMF given by:

X	0	1	2	3	4	5	6	7
$P(X = x)$	0	S	2S	2S	3S	S^2	$2S^2$	$7S^2+S$

- a. Determine the value of S. **Ans: 1/10**
- b. Find the probability i. $P(X \leq 6)$ and $P(3 < x \leq 6)$ **Ans: 0.83, 0.33**
9. The probability that a letter will be delivered anywhere in California in 2 days or less is 0.84. If a college graduate is sending job applications to 5 colleges, and they are all due in 2 days, find the probability that 4 arrive within 2 days. **Ans: 0.3983**
10. If a person receives an A in Intermediate Algebra, the probability of that person getting an A in Statistics is 0.4. If 4 people pass Intermediate Algebra with an A, find the probability of exactly 2 getting an A in Statistics. **Ans: 0.346**
11. The probability of a student graduating from college in four years is 0.4. If 7 students are selected at random, find the probability that at least 1 of them will not graduate in 4 years. (On Herald College 90210, all 7 managed to graduate in 4 years - only in Kollyhood!!!) **Ans: 0.998**
12. On a given day, each computer in a lab has at most one crash. There is a 5% chance that a computer has a crash during the day independent of the performance of any other computers in the lab. There are 25 computers in the lab. Find the probability that on a given day, there are;
- a. Exactly 3 crashes. **Ans: 0.093**
- b. At most 3 crashes. **Ans: 0.966**
13. Using the binomial model, and assuming that a success occurs with probability $\frac{1}{6}$ in each trial, find the probability that in 6 trials there are:
- a. 0 successes **Ans: 0.262**
- b. 3 successes **Ans: 0.0819**
- c. 2 failures **Ans: 0.01536**

14. It is expected that 10% of production from a continuous process will be defective. Find the probability that in a sample of 10 units chosen at random exactly 2 will be defective and at least 2 will be defective. **Ans: 0.1937, 0.2640**
15. Probability that a computer drawn from a batch of computer is defective is 0.1. If a sample of 6 computer is taken. Find the probability that it will contain; **Ans: 0.3487**
16. Let the probability of a crash on any single motorcycle ride is 0.005. What is the probability of having exactly 3 crashes in the first 1000 rides? **Ans: 0.1403**
17. If the probability of a crash is still 0.005, What is the probability of having 10 or more crashes in the first 1000 rides? **Ans: 0.0314**
18. If a patient is waiting for a suitable blood donor and the probability that the selected donor will be a match is 0.2, then find the expected number of donors who will be tested till a match is found including the matched donor.
19. Suppose you are playing a game of darts. The probability of success is 0.4. What is the probability that you will hit the bullseye on the third try? **Ans: 0.144**
20. A light bulb manufacturing factory finds 3 in every 60 light bulbs defective. What is the probability that the first defective light bulb will be found when the 6th one is tested? **Ans: 0.0368**
21. If $P(1) = P(5)$ in Poisson's distribution, find the value of mean **Ans: 3.31**
22. Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the books and x , the number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free of errors?
23. Jobs arrive at a facility at an average rate of 5 in an 8 hour shift. The arrival of the jobs follows Poisson distribution. The average service time of a job on the facility is 40 minutes. The service time follows exponential distribution. Idle time (in hours) at the facility per shift will be? **Ans: 14/3 hours**
24. Telephone calls arrive at an exchange according to the Poisson process at a rate $\lambda = 2/\text{min}$. Calculate the probability that exactly two calls will be received during each of the first 5 minutes of the hour. **Ans: 2.718**

“Do not mistake probability for truth, for it is a notorious liar”