5CS037 Concepts and Technologies of AI,
Cohort-10; Herald College
University of Wolverhampton

Problem Set-5
Continuous Random Variable and There Distributions.

Siman Giri

December 18, 2023

Instructions:

- Solve all the Problem.

1 Review of Anti-derivative - Integration.

Evaluate each of the following integrals.

1. $\int_1^6 12x^3 - 9x^2 + 2\,dx$ **[3250]**
2. $\int_{-2}^1 5z^2 - 7z + 3\,dz$ $[\mathbf{\frac{69}{2}}]$
3. $\int_{-4}^{-1} x^2(3-4x)\,dx$ **[318]**
4. $\int_2^1 \frac{2y^3-6y^2}{y^2}\,dy$ **[3]**

---

2 PDF and CDF.

1. The length of time X, needed by students in a particular complete a 1 hour exam is a random variable with PDF given by:

$$f_X(x) = \begin{cases} k(x^2+x) & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

For the Random Variable **X,** Find:

a) Find the value k that makes $f_X(x)$ a **PDF.**

b) Find the **CDF of X**

c) Use the **CDF** to find:

- $\mathbf{P}(\mathbf{X} \le \mathbf{0})$
- $\mathbf{P}(\mathbf{X} \le \mathbf{1})$
- $\mathbf{P}(\mathbf{X} \le \mathbf{2})$

d) Find the probability that a randomly selected student will finish the exam in less than half an hour.

e) Find the mean time needed to complete a 1 hour exam.

f) Find the variance and standard deviation of **X**

---

2. Let **X** be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} ce^{-x} & \text{for } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

where **c** is a positive constant.

a) Find **c**.

b) Find the **CDF** of **X**

c) Find P(1 < X < 3).

---

3. The probability density function of **X** is given by:

$$f_X(x) = \begin{cases} kxe^{-2x} & \textbf{for x > 0} \\ 0 & \textbf{for x} \leq \textbf{0} \end{cases}$$

Find the value of k.

4. The probability density function of **X** is:

$$f_X(x) = \begin{cases} x & \mathbf{0 < x < 1} \\ 2 - x & \mathbf{1 \leq x < 2} \\ 0 & \textbf{Otherwise} \end{cases}$$

Find: (i) $\mathbf{P(0.2 \leq X < 0.6)}$ (ii) $\mathbf{P(1.2 \leq X < 1.8)}$ (iii) $\mathbf{P(0.5X < 1.5)}$

3 Uniform Probability Distribution

1. The average weight that is gained by a person in Harvard University over the winter months is uniformly distributed and it ranges from 0 to 30 lbs. What will be the probability of a person that he will gain between 10 and 15 lbs in the winter months? [$\mathbf{\frac{1}{6} \approx 0.1667}$].

2. **X** is a uniformly distributed random variable that takes a value between 0 and 1. What will be the value of $\mathbf{E(X^3)}$?

3. Bus is uniformly late between **2and10**minutes. How long can you expect to wait and with what standard deviation? If it is greater than **7**minutes you will be late for class. What is the probability of you being late?
Hint:
For a uniform distribution on the interval **Expectation and Variance** are given as:

$$\mathbf{E[X]} = \frac{\mathbf{a+b}}{\mathbf{2}}$$

$$\mathbf{var(X)} = \frac{\mathbf{(b-a)^2}}{\mathbf{12}}$$

$$\mathbf{SD(X)} = \sqrt{\mathbf{var(X)}} = \frac{\mathbf{(b-a)}}{\sqrt{\mathbf{12}}}$$

4. You arrive into a building and are about to take an elevator to your floor. Once you call the elevator, it will take between 0 and 40 seconds to arrive to you.We will assume elevator arrives uniformly between 0 and 40 seconds after your press the button. Find the expectation and variance of the distribution. Also find the probability elevator will arrive in less than 15 seconds.

4 Normal and Standard Normal Distribution

1. If the value of a random variable is 2, the mean is 5 and the standard deviation is 4, then find the probability density function of the Gaussian distribution.
   Hint:
   The probability density function (PDF) of a Gaussian distribution is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

2. Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination, otherwise they drop their module. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT?
   Hint:
   Probability for z-score can be calculated using a standard normal table or calculator, you would look up the value for z score and then subtract that value from 1. You can also use following python script.

Listing 1: Python script to calculate probability from a given z-score

```
from scipy.stats import norm
def probability_from_z_score(z_score):
    """
    Calculate the probability from a given z-score using the
    standard normal distribution.
    Parameters:
        - z_score: The z-score for which to find the probability.
    Returns:
        - probability: The probability corresponding to the given z-score.
    """
    probability = norm.cdf(z_score)
    return probability
    # Example: Find the probability for a z-score of -27/112
    z_score_example = -27/112
    resulting_probability = probability_from_z_score(z_score_example)
```

3. The length of life of an instrument produced by a machine at X factory has a normal distribution with a mean of 12 months and a standard deviation of 2 months. Find the probability that an instrument produced by this machine will last
   (i)**less than 7 months** (ii) **between 7 and 12 months**