

Problem Set-2

What, Why and How of Probability.

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Instructions:

- Solve all the Problem.

1 EXPERIMENT, SAMPLE SPACE, EVENTS AND COUNTING

1. Write a sample space for the Given experiment:

- a) A Die is rolled.
 - b) A die is rolled and coin is tossed
 - c) A penny and nickel are tossed.
 - d) A jar contains four marbles numbered 1,2,3,and 4. Two marbles are drawn.
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2. A Ultima has five identical ear-buds for shipment, unknown to person-in charge for shipping two of the five ear-buds are defective. A particular order calls for two of the ear-buds and is filled randomly selecting two of the five available:

- a) List the sample space for this experiment.
 - b) Let **A** denote the event that the order is filled with two non-defective ear-buds.List the sample points in A.
 - c) Assign probabilities to all possible simple events, and verify all the axioms are met.
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3. Every person's blood type is A,B,AB,or o.In addition, each individual either has the Rhesus(RH) factor +ve or -ve. A medical technician records a person's blood and Rh factor. List the sample space for this experiment.

4. Assume to win a bet at MotoGP, you need to specify the riders that finish in the top three spots in the exact order in which they finish. If eight riders enter the race, how many different ways can they finish in the top three spots?

5. There are 18 faculty members in the department of Mathematics and Statistics. Four people are to be in the executive committee. Determine how many different ways this committee can be created.

6. The director of a research laboratory needs to fill a number of research positions; two in biology and three in physics. There are seven applicants for the biology positions and 9 for the physicist positions. How many ways are there for the director to select these people?

2 PROBABILITY OF AN EVENT.

Hint: Problem Solving Strategies.

- Problem formalization:

1. Describe a sample space Ω .
2. Describe an event $A \subset \Omega$
3. Count the number of elements in the event A (use combinatorics)
4. Compute:

$$P(A) = \frac{A}{\Omega}$$

1. n cards are drawn from a standard deck of 52. What is the probability that:

- a) $n = 8$, all card are queens or kings?
- b) $n = 8$, all cards contain exactly two rankes?
- c) $n = 6$, all cards contain only two spades and 1 heart?
- d) $n = 5$, all cards contain all 4 suits(spade, heart, diamond, clubs)?
- e) $n = 4$, all cards contain only 2 spades and 1 king?

Please don't forget to describe the universe, space and the events!

2. A fair die is rolled 100 times and the results are written as $\{x_1, x_2, \dots, x_{100}\}$. Find the probability that:
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- a) Probability that $\max\{x_i\} < 4$:
- b) Probability that $\min\{x_i\} < 3$:
- c) Probability that $\max\{x_i\} = 4$:
- d) Probability that $\max\{x_i\} < 5$ and $\min\{x_i\} < 3$:

Hint:

For Probability that $\max\{x_i\} < 4$:

- Each roll of the die has a probability of $\frac{1}{2}$ of resulting in a number less than 4 i.e. 1, 2, 3 are the possible outcomes.
- Since the die is rolled 100 times independently , the probability of this condition is

$$= \prod_{i=1}^{100} P(\max\{x_i\} < 4) = \frac{1}{2}^{100}$$

3. A fair die is rolled 2 times and the results are written as (x_1, x_2) . What is the probability that :
 $x_1 \times x_2 = 20$ if $x_1 + x_2 = 9$? Explain your answer.

4. A box contains 10 white and 10 black marbles. Following experiment were performed: randomly drawing out, with replacement, two marbles in succession and noting the color each time. (To draw “with replacement” means that the first marble is put back before the second marble is drawn.)
- Construct a sample space for the experiment.
 - List the outcomes that comprise each of the following events.
 - At least one marble of each color is drawn.
 - No white marble is drawn.
 - Assuming that each outcome is equally likely, find the probability of each event at (b).
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5. For the experiment of rolling a single six-sided die once, define events

- T: the number rolled is three.**
- G: the number rolled is four or greater**

Solve following:

- List the outcomes that comprise T and G.
 - List the outcomes that comprise $T \cap G$, $T \cup G$, T^c , and $(T \cup G)^c$.
 - Assuming all outcomes are equally likely, find $P(T \cap G)$, $P(T \cup G)$, and $P(T^c)$.
 - Determine whether or not T and G are mutually exclusive. Explain why or why not.
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6. A prize is placed at random behind one of three doors. You pick a door (suppose you always pick door 1) Now Ramesh chooses one of the other two doors, opens it and shows you that it is empty. He then gives you the opportunity to keep your door or switch to the other unopened door. Should you stay or switch? Explain your answer.