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(1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Multiply row 1 by 2 & subtract from row 2

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Multiply row 1 by 3 & subtract from row 3

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Multiply row 1 by 6 & subtract from row 4

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Swap R_2 & R_3

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

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Multiply R_3 by $-4/3$ & $+R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Multiply R_2 by $-1/4$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$\times$ R_4 by -1 & add to R_2

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -9 & 17/4 \end{bmatrix}$$

$\times$ R_3 by $-1/3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -9 & 17/4 \end{bmatrix}$$

$\times$ R_4 by 3 & add to R_3

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

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$\therefore$ Non zero rows = 4

$\rho(A) = 4$

2. Let the standard base for symmetric 2×2 matrix be.

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

& the standard base of P_2 be

$B' = \{1, x, x^2\}$

$$[T]_B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

Apply row operations

$$[T]_B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ $\rho(T) = 2$

& Nullity = 1

3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$[A - \lambda I] = 0$

$$\therefore \left\{ \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} \right\}$$

$= (2-\lambda)^2 - 1$

$= \lambda^2 - 4\lambda + 3$

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$= (\lambda-1)(\lambda-3)=0$

$\therefore \boxed{\lambda = 1, 3}$

eigen values $= \lambda = 1, 3$

for $\lambda_1 = 1$

$(A - \lambda_1 I)v_1 = \left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

$v = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad v_1 = 0$

$\therefore v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$

$\lambda_2 = 3$

$(A - \lambda_2 I)v_2 = \left(\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\right) v_2$

$= \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \quad v_2 = 0$

$v_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

So, for A^{-1} then eigen values $= \frac{1}{1}$ & $\frac{1}{3} = \boxed{1 \text{ \& } \frac{1}{3}}$

& eigen vectors are same as A

for $A + 4I$ & $\lambda + 4 = 1+4$ & $3+4$
$= 5$ & $= 7$

& same eigen vectors as A

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(4)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = 19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

$$x(0) = y(0) = z(0) = 0$$

Iteration - 1

$$x^{(1)} = \frac{7.85 + 0.1y^{(0)} + 0.2z^{(0)}}{3} = \frac{7.85}{3} = 2.6167$$

$$y^{(1)} = \frac{-19.3 - (0.1)x^{(1)} + 0.3z^{(0)}}{7} = -2.7571$$

$$z^{(1)} = \frac{71.4 - 0.3x^{(1)} + 0.2y^{(1)}}{10} = 7.14$$

Iteration 2

$$x^{(2)} = \frac{7.85 + 0.1y^{(1)} + 0.2z^{(1)}}{3} = 2.8098$$

$$y^{(2)} = \frac{-19.3 - 0.1x^{(2)} + 0.3z^{(1)}}{7} = -2.9832$$

$$z^{(2)} = \frac{71.4 - 0.3x^{(2)} + 0.2y^{(2)}}{10} = 7.013$$

Iteration 3

$$x^{(3)} = \frac{7.85 + 0.1y^{(2)}}{3} = 0.22z^{(2)}$$

$$y^{(3)} = \frac{-19.3 - 0.1x^{(3)}}{7} + 0.3z^{(2)} = -2.9961$$

$$z^{(3)} = \frac{71.4 - 0.3x^{(3)}}{10} + 0.2y^{(3)} = 7.005$$

(5) A system is considered consistent if it has atleast one solution, a common solution satisfying all the equat

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If a system of equation does not have a common solution, then it is said to be inconsistent.

Now

$$x + 3y + 2z = 0$$
$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

$$\therefore [A \vdots B] = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \Rightarrow 2R_1 - R_2$; $R_3 \to R_1 - R_3$; $R_4 \to R_4 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_4$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 14 & 2 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & -7 & -1 & 0 \end{array}\right]$$

Multiply R_3 by -1 and add to R_2
Multiply R_3 by 1 & add to R_4

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$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 6 \\ 0 & 28 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A:B) = 2$

$\rho(A) = \rho(A:B) \Rightarrow$ consistent system

& dimension $> \rho(A)$

infinitely many solution

let

$z = t$, so

$y = 0$

& $x = -3t$

are the parametric solution

6 To define linear transformation

i) Additivity

let $u = a_1 + b_1 x + c_1 x^2$

$v = a_2 + b_2 x + c_2 x^2$

$T(u+v) = T\left((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2\right)$

$= \left((a_1 + a_2) + 1\right) + \left((b_1 + b_2) + 1\right)x + \left((c_1 + c_2) + 1\right)x^2$

$T(u) + T(v) = (a_1 + 1) + (b_1 + 1)x + (c_1 + 1)x^2 + (a_2 + 1) + (b_2 + 1)x + (c_2 + 1)x^2$

$= \left((a_1 + 1) + (a_2 + 1)\right) + \left((b_1 + 1) + (b_2 + 1)\right)x + \left((c_1 + 1) + (c_2 + 1)\right)x^2$

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$= ((a_1+a_2)+1) + ((b_1+b_2)+1)x + ((c_1+c_2)+1)x^2$

$\therefore T(u+v) = T(u) + T(v)$

ii) Homogeneity

let $u = a+bx+cx^2$

& d be any scalar.

$T(du) = T(da + dbx + dcx^2)$
$= ((da+1) + (db+1)x + (dc+1)x^2)$

$dT(u) = d((a+1) + (b+1)x + (c+1)x^2)$
$= (da+d) + (db+d)x + (dc+d)x^2$

$\therefore T(du) \neq dT(u)$

$\therefore T: P_2 \to P_2$ is not a linear transformation as homogeneity is not satisfied.

7. Forming the equations

$a + 3b - 2c = 0$
$2a + b + c = 0$
$3a + 2b + 3c = 0$

let

$$A:B = \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 2 & 3 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$

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$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A : B only has trivial solution $(a = b = c = 0)$

$\therefore$ vectors in S are linearly independent.

Since we have 3 linearly independent vectors in $V_3(\mathbb{R})$, so S span the entire space

8.

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

$$x_0 = y_0 = z_0 = 1$$

Iteration - 1

$$x^{(1)} = \frac{1}{3}\left(23 - (-6 \times 1) - (2 \times 1)\right) = 6$$

$$y^{(1)} = \frac{1}{7}\left(-15 - (-4 \times 1) - (-1 \times 1)\right) = -10$$

$$z^{(1)} = \frac{1}{7}\left(16 - (1 \times 1) - (-3 \times 1)\right) = 2$$

Iteration - 2

$$x^{(2)} = \frac{1}{3}\left(23 - (-6(-10))\right) - (2 \times 2) = 7$$

$$y^{(2)} = \left(-15 - (-24) - (-1 \times 2)\right) = 1$$

$$z^{(2)} = \frac{1}{7}\left(16 - (1 \times 6) - (-3 \times (-10))\right) = 1$$

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Iteration 3

$$x^{3} = \frac{1}{3}(23-(-6\times1)-(2\times1))=6$$

$$y^{(3)} = {}^{1}(-15-(-4\times7)-(-1\times1)) = -12$$

$$z^{(3)} = \frac{1}{7}(16-(1\times7)-(-3\times1))=1$$

$$\boxed{\therefore\ x=6,\ y=-12,\ z=1}$$

Q. One application of matrix operations in image processing is convolution, where a small matrix is applied to each pixel in the image to perform operations like blurring, sharpening or edge detection.

For eg. A blur filter kernel could be used to blur an image by averaging pixel values in the neighborhood. This process is applied to every pixel.

Let say we have grayscale image represented by matrix of pixel values

We want to apply simple 3x3 blur filter to image.

So, Blur kernel $= \frac{1}{9}\times\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

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10 - Linear transformation play a crucial role in computer vision, particular in operations like rotating of images. These transformations involve applying a linear function to every pixel in the image result in a modified version of the original.
For rotating images affine transformation are commonly used because they maintain straight lines and parallelism. This is achieved by multiplying the coordinate of each pixel by the rotating matrix, which encodes the rotation angle & other parameters like translation & scaling.

By systematically applying this transformation to every pixel in the image, a new image is generated where each pixel is positioned according to the desired rotation angle. This process enables seamless rotation of images in various computer vision tasks including image processing, object detection & pattern recognition.