



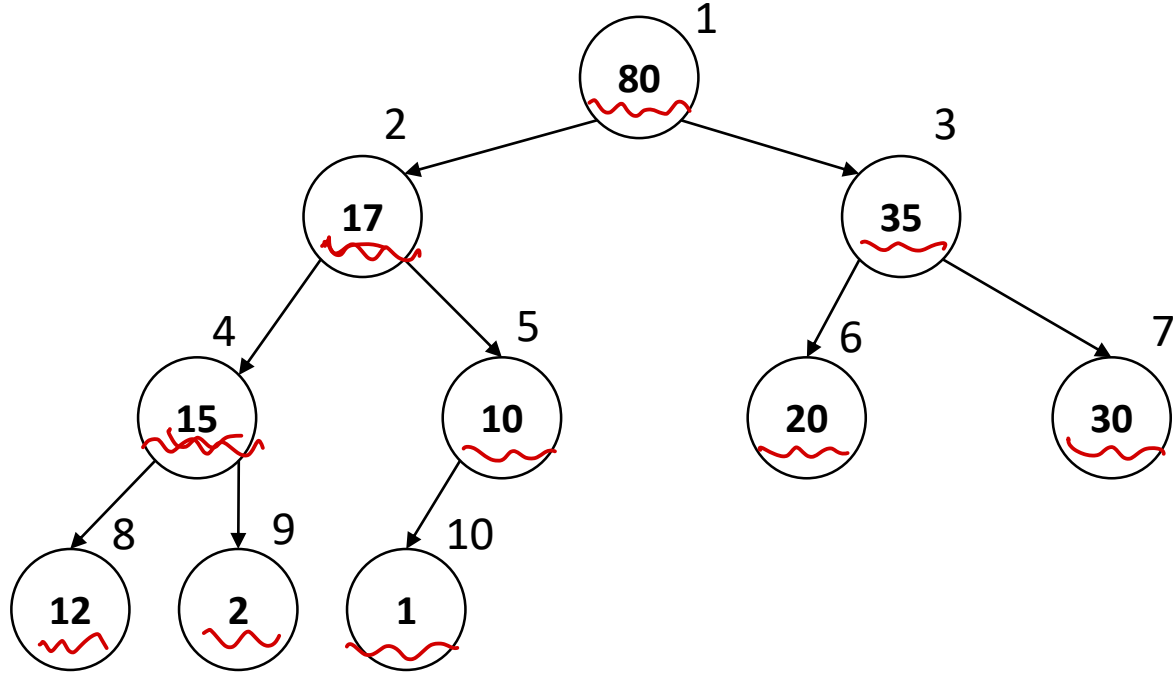
Data Structure & Algorithms

Sunbeam Infotech

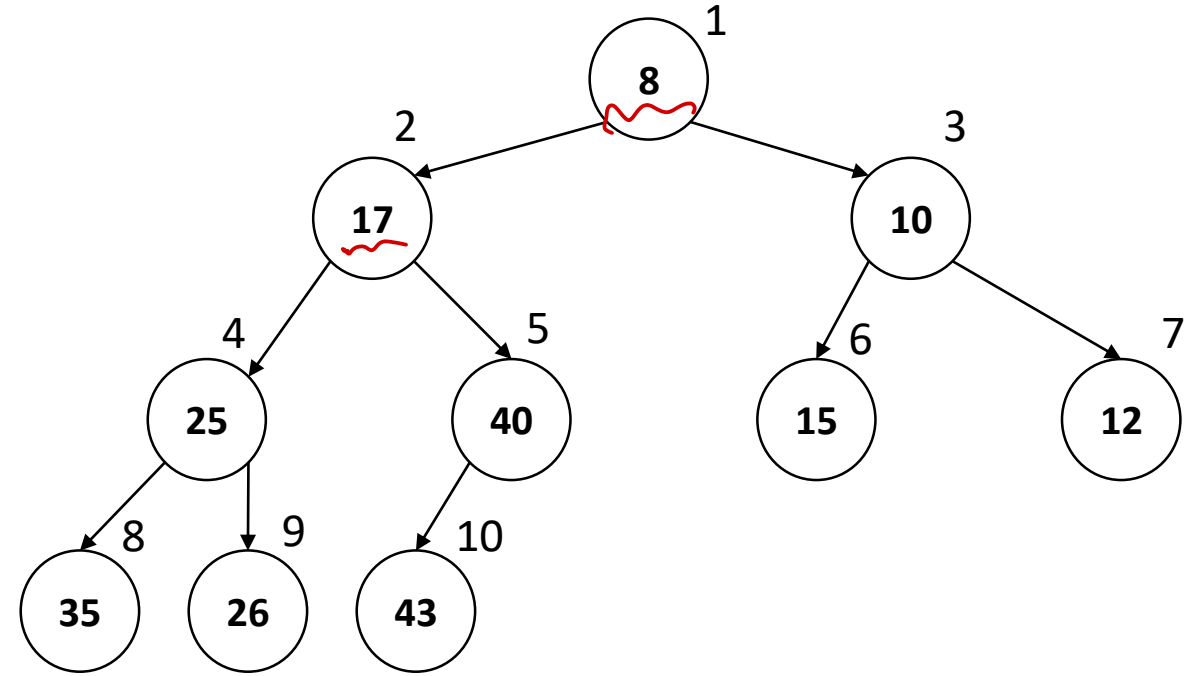
Nilesh Ghule



Max Heap & Min Heap



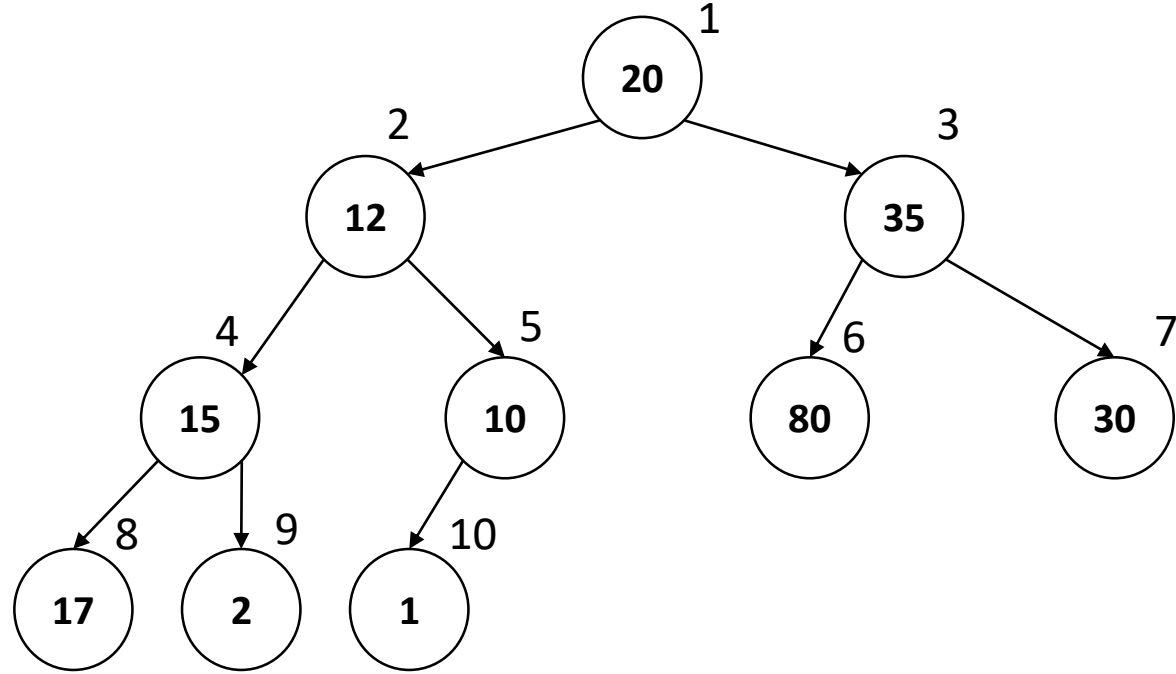
- Max heap is a heap data structure in which each node is greater than both of its child nodes.



- ^{Min}~~Max~~ heap is a heap data structure in which each node is smaller than both of its child nodes.



Make Heap

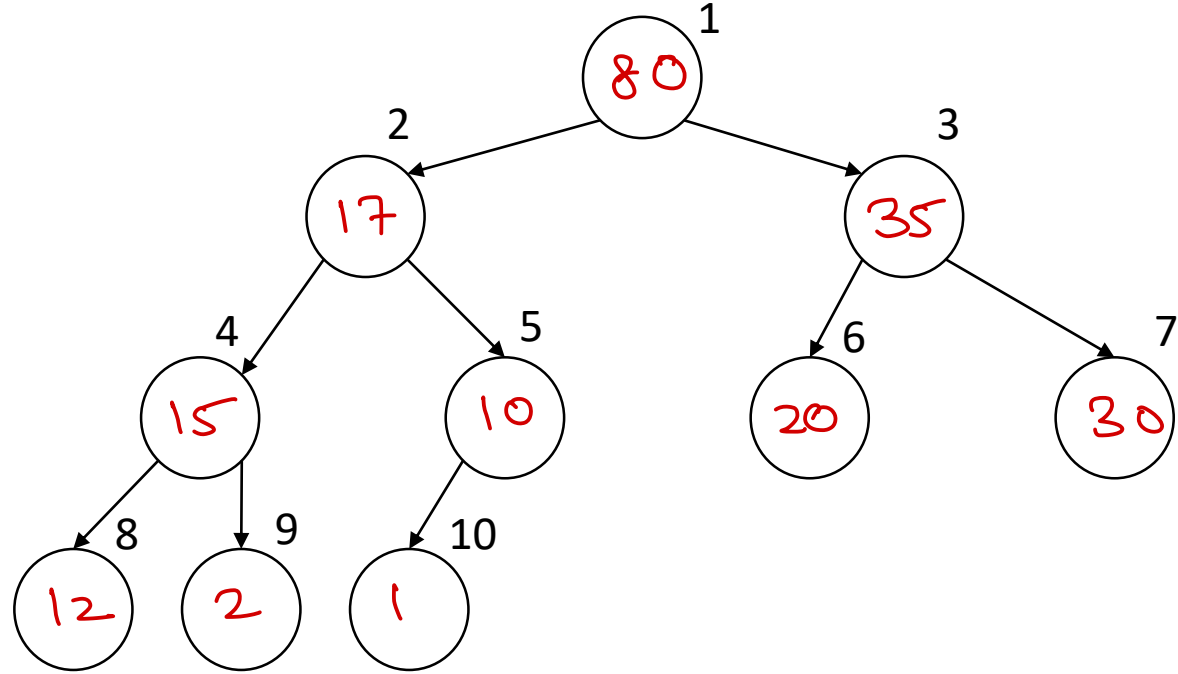


left child = parent * 2
right child = parent * 2 + 1
parent = child / 2

20	12	35	15	10	80	30	17	2	1
<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>



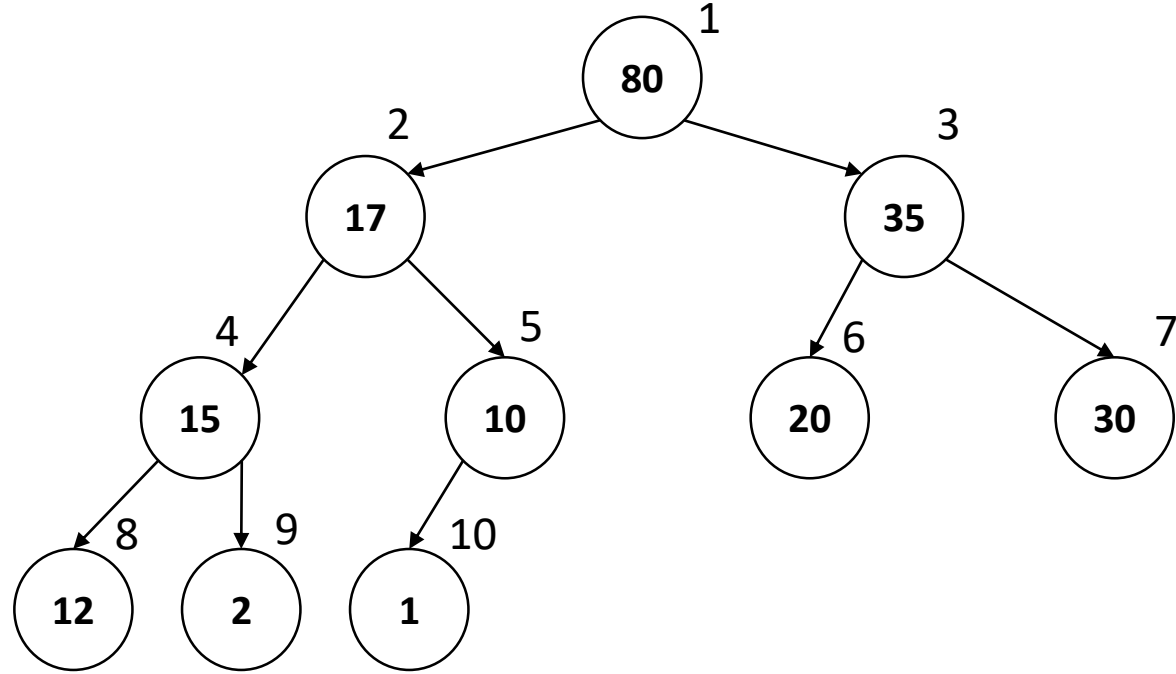
Max Heap – Initialize (heapify) 20 12 35 15 10 80 30 17 2 1



80	17	35	15	10	20	30	12	2	1
1	2	3	4	5	6	7	8	9	10



Max Heap



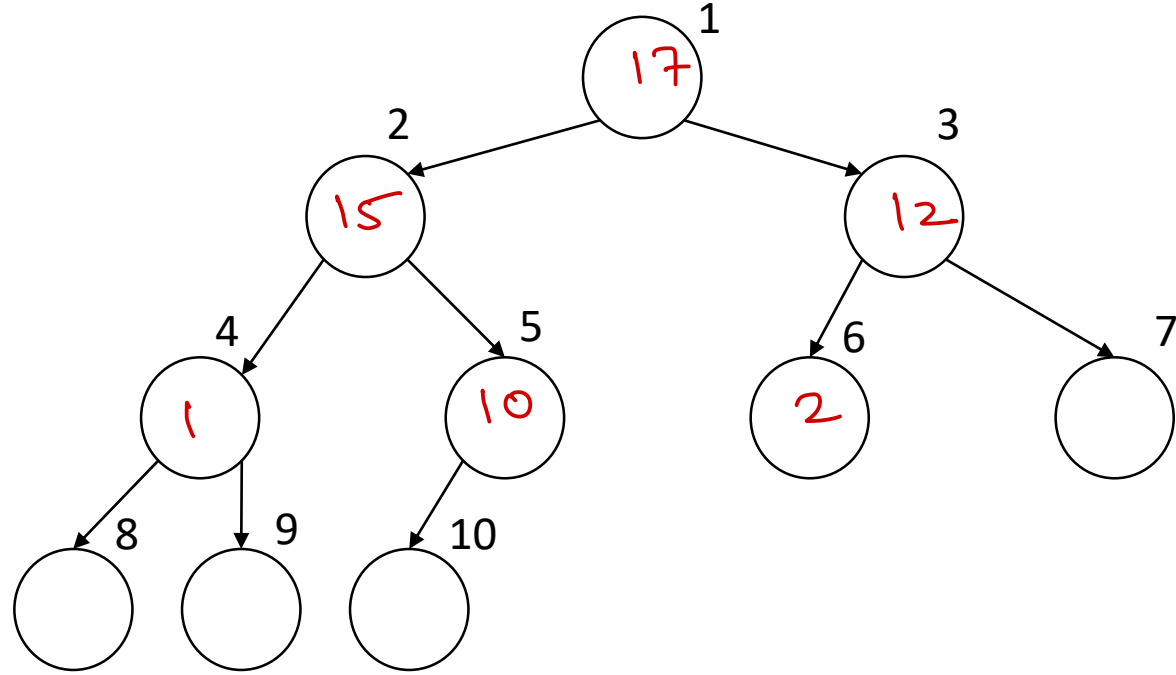
80	17	35	15	10	20	30	12	2	1
1	2	3	4	5	6	7	8	9	10

Heap is commonly used
as a priority queue.
Element with highest
priority comes out first.

Max Heap or min Heap



Max Heap – Delete Element → Del Max



80

35

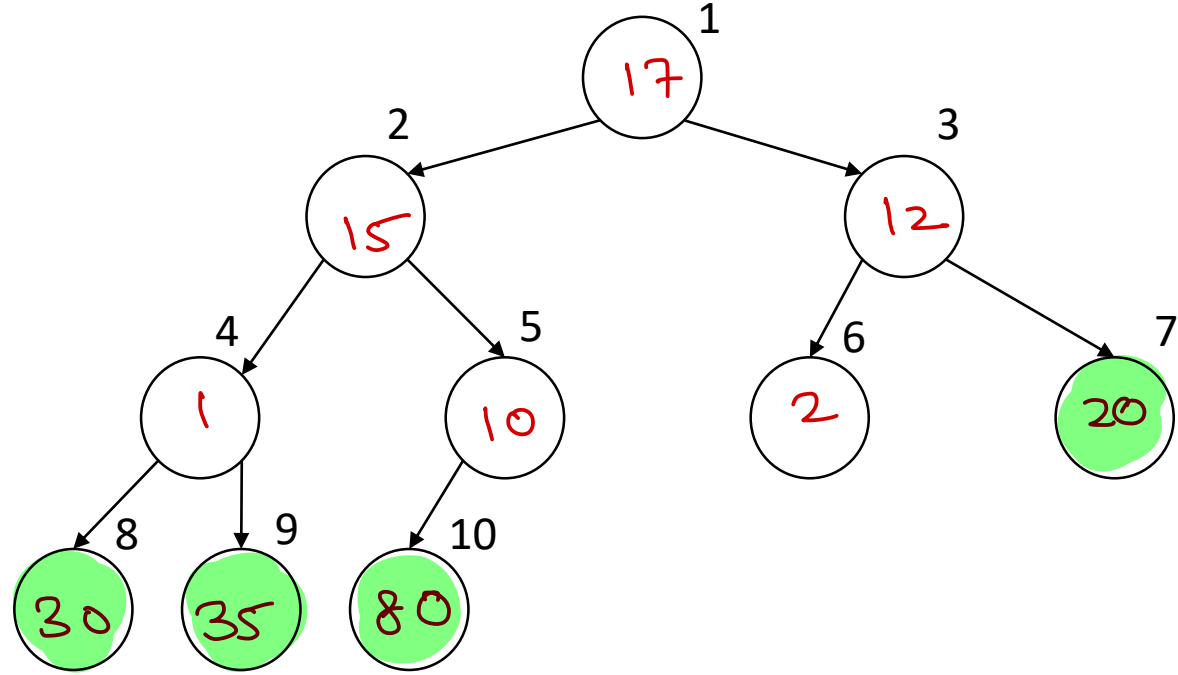
30

20

80	17	35	15	10	20	30	12	2	1
1	2	3	4	5	6	7	8	9	10



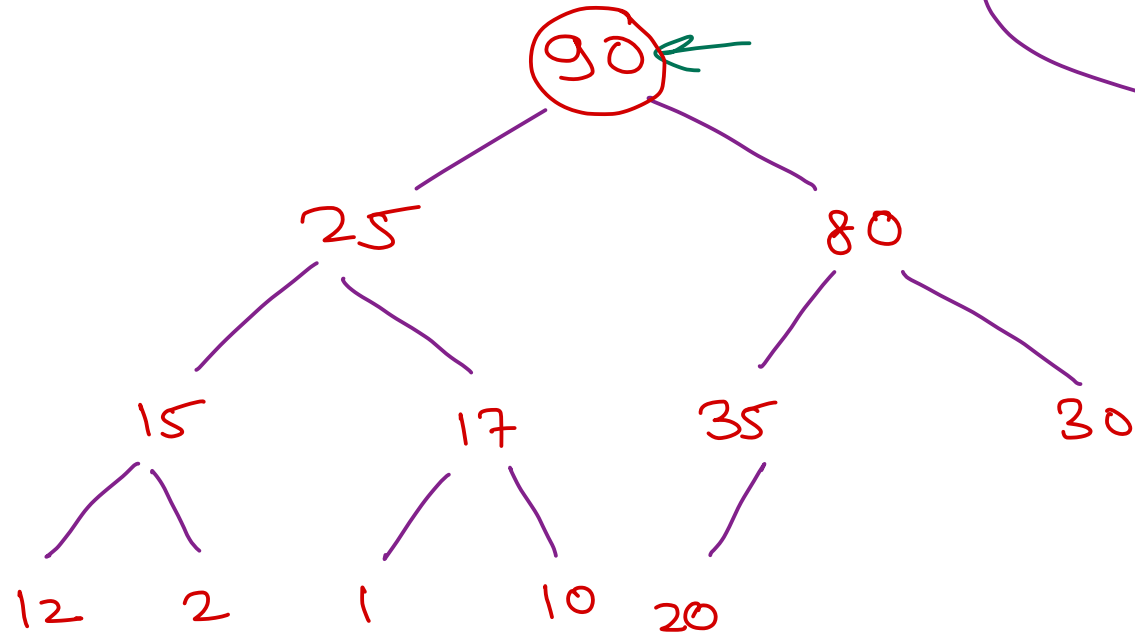
Heap Sort



17	15	12	1	10	2	20	30	35	80
1	2	3	4	5	6	7	8	9	10



Priority Queue \rightarrow insert/delete.



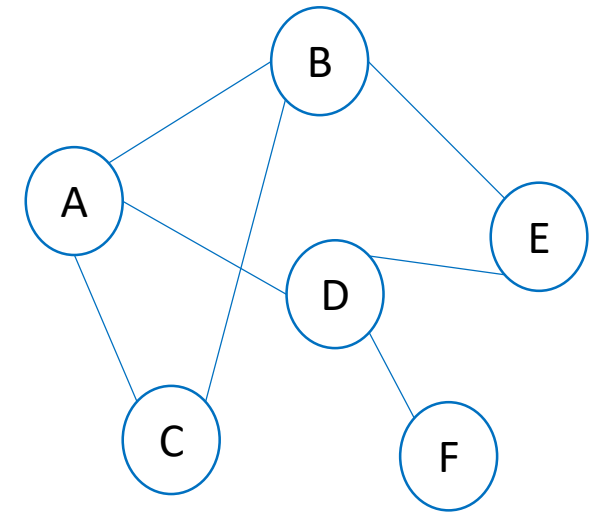
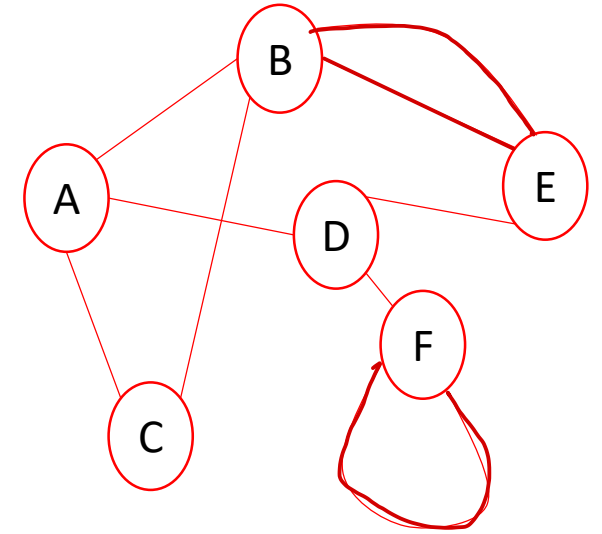
\rightarrow delete max

```
size = size + 1;  
i = size;  
while (i != 1) {  
    if (val <= arr[i/2])  
        break;  
    arr[i] = arr[i/2];  
    i = i / 2;  
}  
arr[i] = val;
```



Graph - impl $\begin{cases} \rightarrow \text{adj matrix} \\ \rightarrow \text{adj list} \end{cases}$

- Graph is a non-linear data structure.
- Graph is defined as set of vertices and edges. Vertices (also called as nodes) hold data, while edges connect vertices and represent relations between them.
 - $G = \{ V, E \}$
- ✓ Vertices hold the data and Edges represents relation between vertices.
- When there is an edge from vertex P to vertex Q, P is said to be adjacent to Q. (neighbour)
- Multi-graph
 - Contains multiple edges in adjacent vertices or loops (edge connecting a vertex to it-self).
- Simple graph
 - Doesn't contain multiple edges in adjacent vertices or loops.

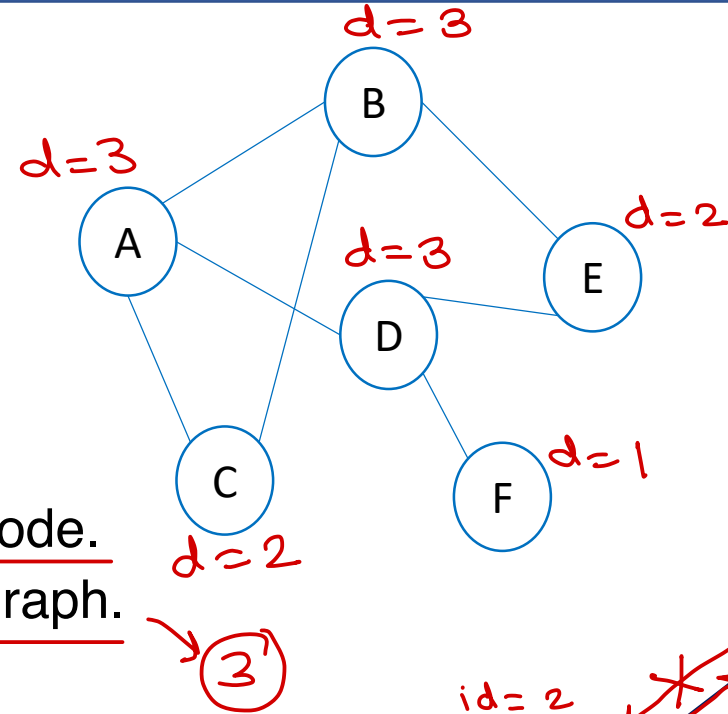


Graph

- Graph edges may or may not have directions.

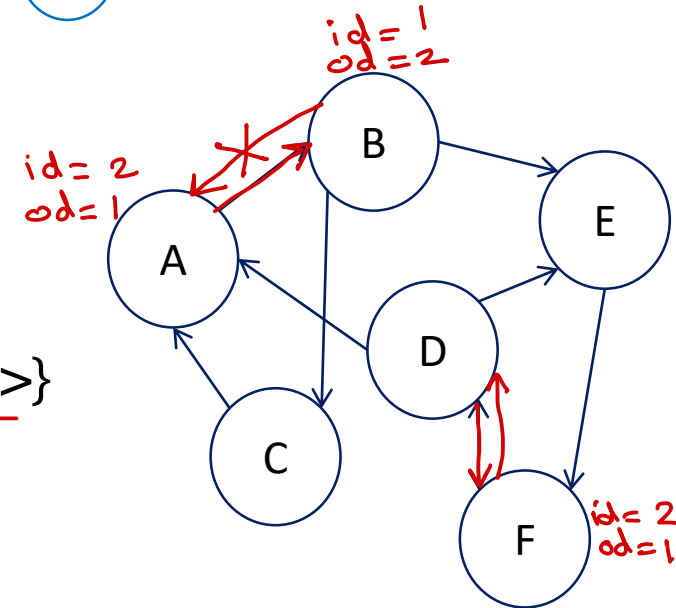
- Undirected Graph: $G = \{ V, E \}$

- ✓ $V = \{ A, B, C, D, E, F \}$
- ✓ $E = \{ \underline{(A,B)}, \underline{(A,C)}, \underline{(A,D)}, \underline{(B,C)}, \underline{(B,E)}, \underline{(D,E)}, \underline{(D,F)} \}$
- ✓ If P is adjacent to Q, then Q is also adjacent to P.
- ✓ Degree of node: Number of nodes adjacent to the node.
- ✓ Degree of graph: Maximum degree of any node in graph.



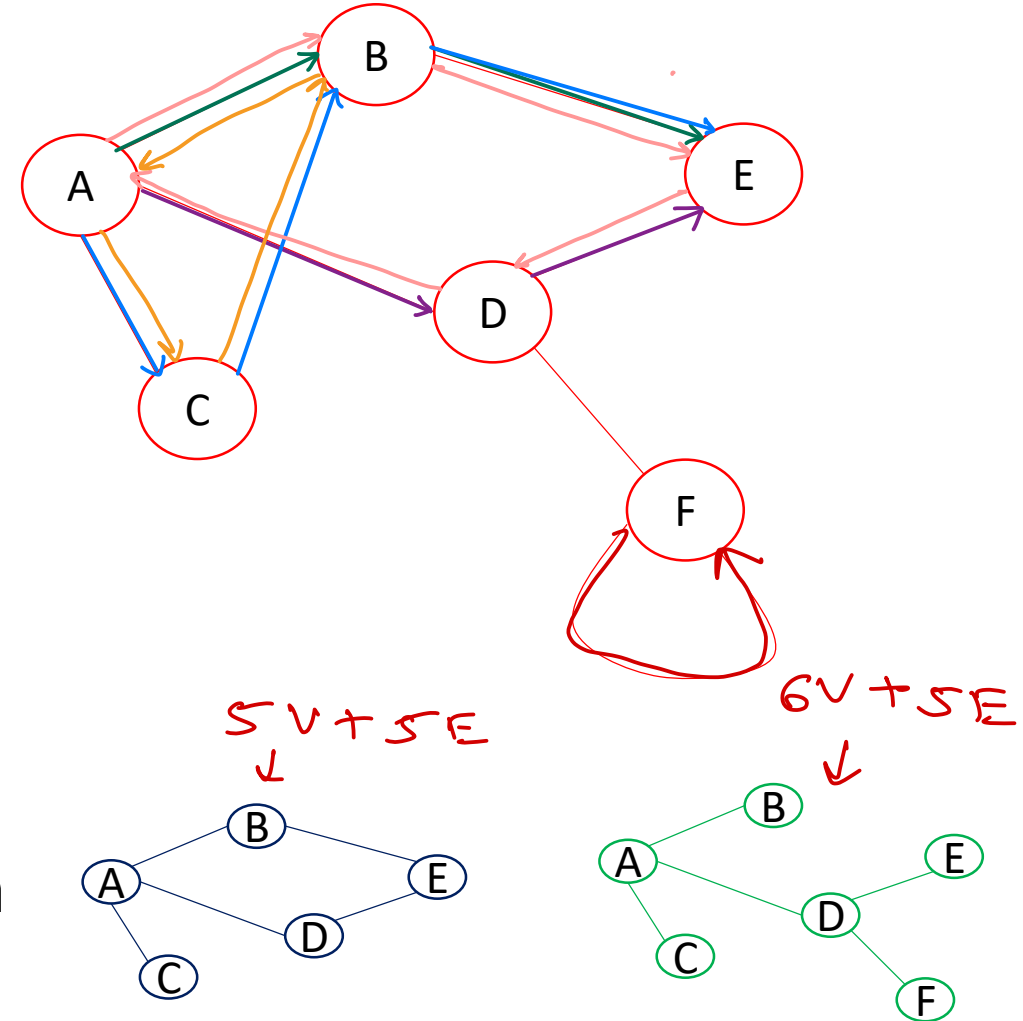
- Directed Graph: $G = \{ V, E \}$

- ✓ $V = \{ A, B, C, D, E, F \}$
- ✓ $E = \{ \underline{\langle A,B \rangle}, \underline{\langle B,C \rangle}, \underline{\langle B,E \rangle}, \underline{\langle C,A \rangle}, \underline{\langle D,A \rangle}, \underline{\langle D,E \rangle}, \underline{\langle D,F \rangle}, \underline{\langle E,F \rangle}, \underline{\langle F,D \rangle} \}$
- ✓ If P is adjacent to Q, then Q may or may not be adjacent to P.
- ✓ Out-degree: Number of edges originated from the node
- ✓ In-degree: Number of edges terminated on the node



Graph

- Path: Set of edges between two vertices. There can be multiple paths between two vertices.
 - ✓ A – D – E
 - ✓ A – B – E
 - ✓ A – C – B – E
- Cycle: Path whose start and end vertex is same.
 - A – B – C – A
 - A – B – E – D – A
- Loop: Edge connecting vertex to itself. It is smallest cycle.
 - F – F
- Sub-Graph: A graph having few vertices and few edges in the given graph, is said to be sub-graph of given graph.



Graph

✓ Weighted graph

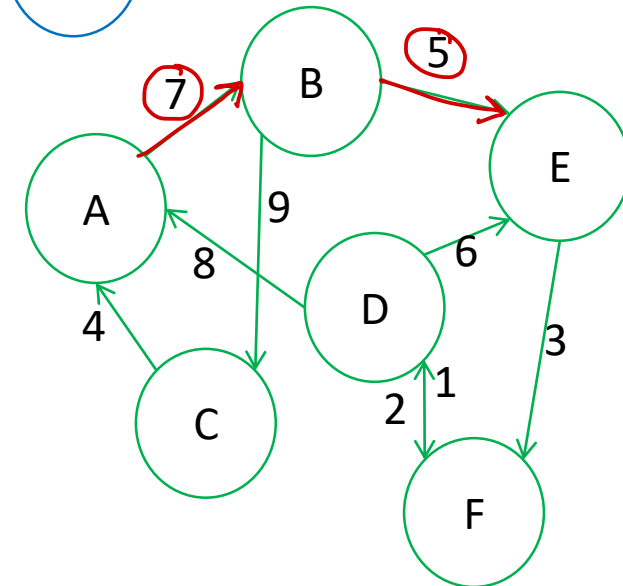
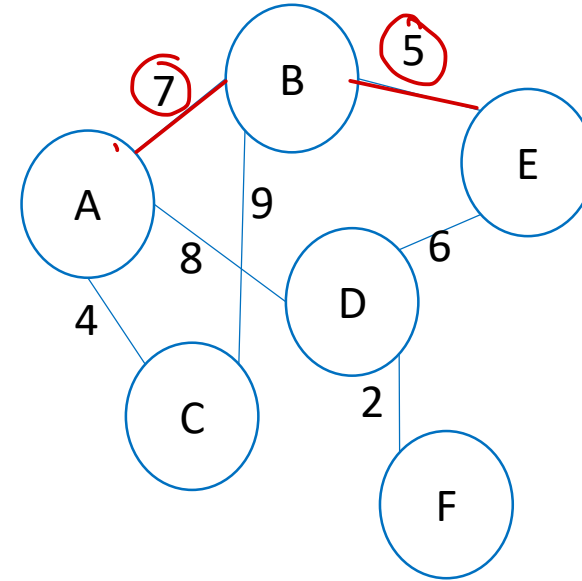
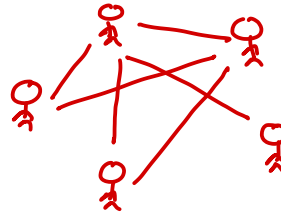
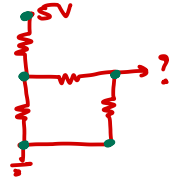
- Graph edges have weight associated with them.
- Weight represent some value e.g. distance, resistance.

✓ Directed Weighted graph (Network)

- Graph edges have directions as well as weights.

• Applications of graph

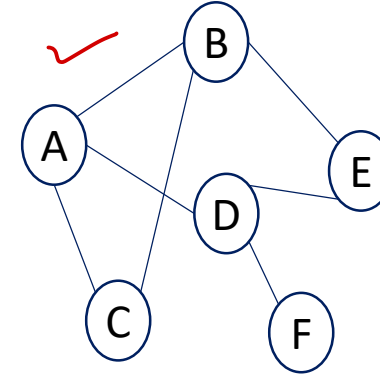
- ✓ Electronic circuits
- ✓ Social media
- ✓ Communication network
- ✓ Road network
- ✓ Flight/Train/Bus services
- ✓ Bio-logical & Chemical experiments
- ✓ Deep learning (Neural network, Tensor flow)
- ✓ Graph databases (Neo4j)



Graph

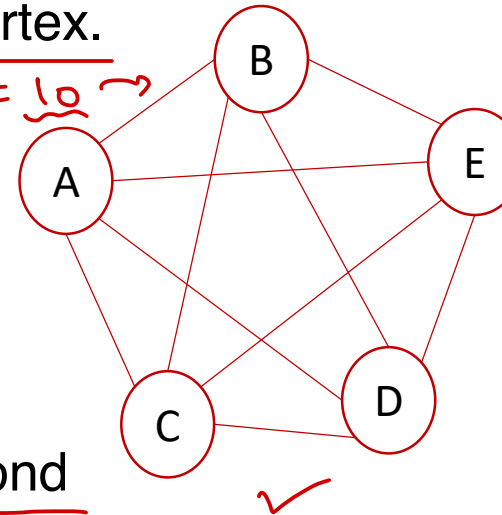
- Connected graph

- From each vertex some path exists for every other vertex.
- Can traverse the entire graph starting from any vertex.



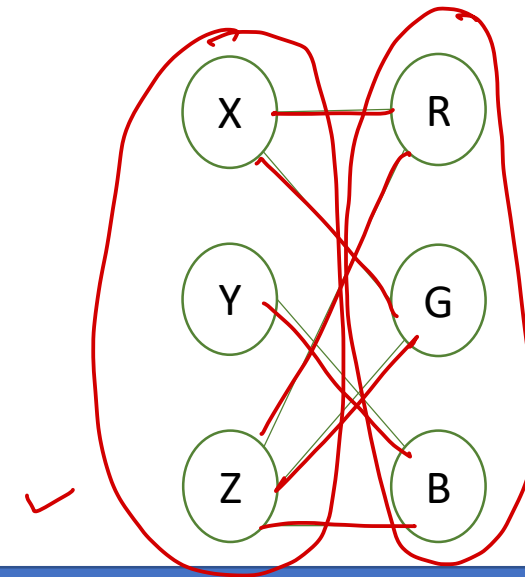
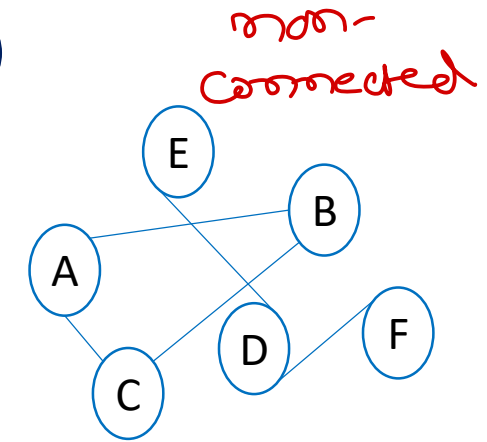
- Complete graph

- Each vertex of a graph is adjacent to every other vertex.
- Un-directed graph: Number of edges = $n(n-1)/2 = 10$
- Directed graph: Number of edges = $n(n-1) = 20$



- Bi-partite graph

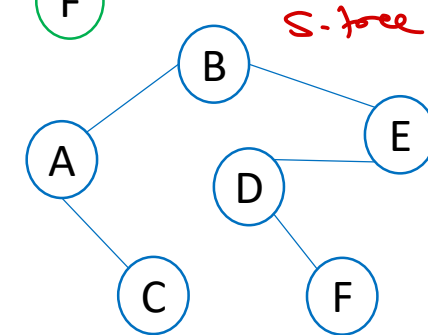
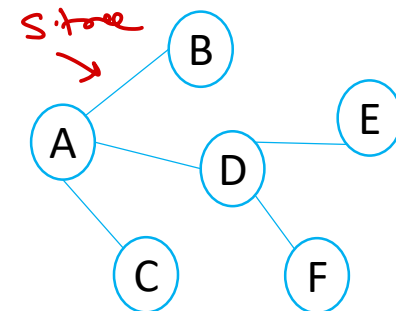
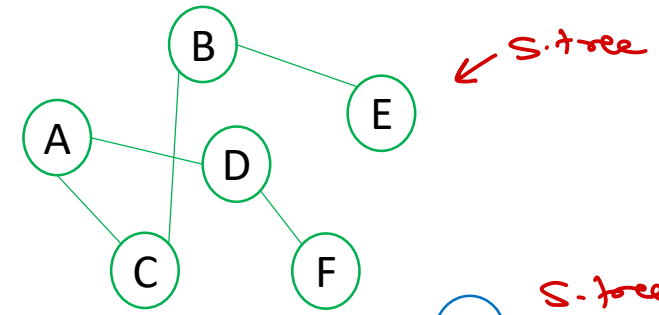
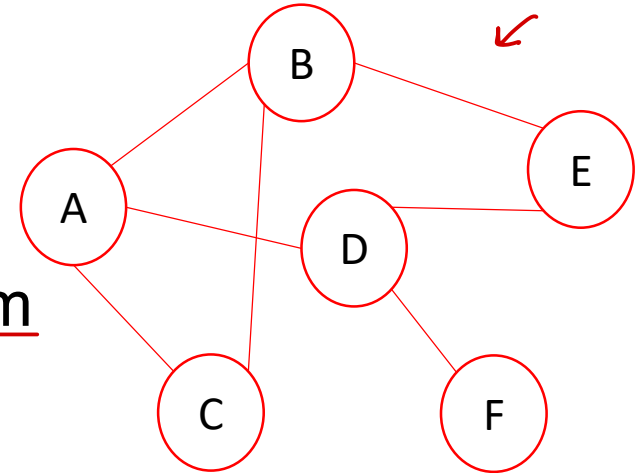
- Vertices can be divided in two disjoint sets.
- Vertices in first set are connected to vertices in second set.
- Vertices in a set are not directly connected to each other.



Spanning Tree

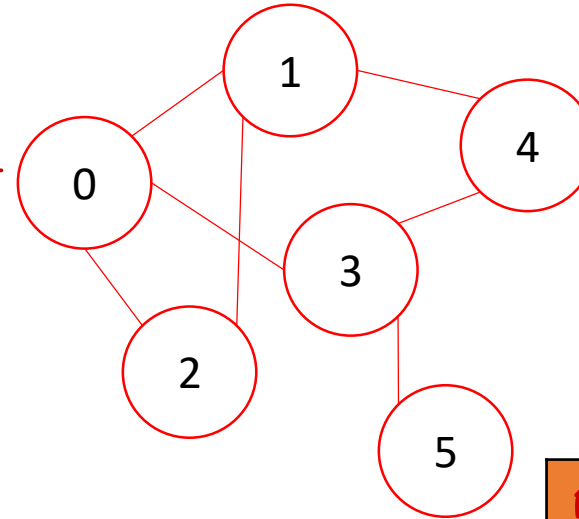
- Tree is a graph without cycles.
- Spanning tree is connected sub-graph of the given graph that contains all the vertices and sub-set of edges ($V-1$).
- Spanning tree can be created by removing few edges from the graph which are causing cycles to form.
- One graph can have multiple different spanning trees.
- In weighted graph, spanning tree can be made who has minimum weight (sum of weights of edges). Such spanning tree is called as Minimum Spanning Tree. (MST)
- Spanning tree can be made by various algorithms.
 - ✓ BFS Spanning tree
 - ✓ DFS Spanning tree
 - Prim's MST ✓
 - Kruskal's MST ✓

← { optimised resource planning



Graph Implementation – Adjacency Matrix

- If graph have V vertices, a $V \times V$ matrix can be formed to store edges of the graph.
- Each matrix element represent presence or absence of the edge between vertices.
- For non-weighted graph, 1 indicate edge and 0 indicate no edge.
- For un-directed graph, adjacency matrix is always symmetric across the diagonal.
- Space complexity of this implementation is $O(V^2)$.

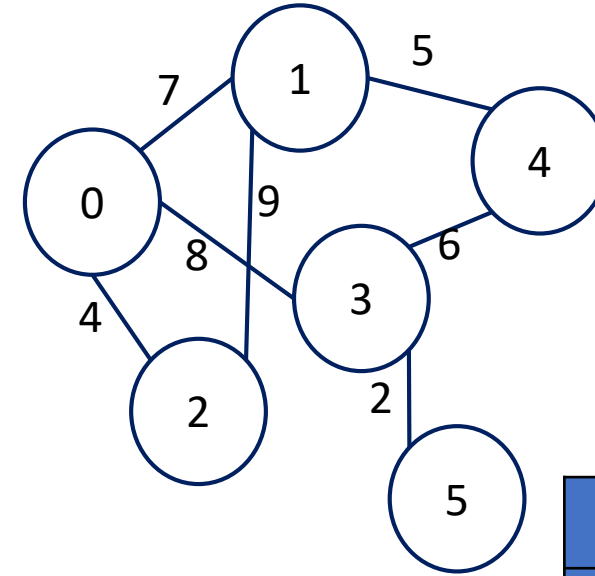


	0	1	2	3	4	5
0	0	1	1	1	0	0
1	1	0	1	0	1	0
2	1	1	0	0	0	0
3	1	0	0	0	1	1
4	0	1	0	1	0	0
5	0	0	0	1	0	0



Graph Implementation – Adjacency Matrix

- If graph have V vertices, a $V \times V$ matrix can be formed to store edges of the graph.
- Each matrix element represent presence or absence of the edge between vertices.
- For *weighted graph*, weight value indicate the edge and infinity sign ∞ represent no edge.
- For un-directed graph, adjacency matrix is always symmetric across the diagonal.
- Space complexity of this implementation is $O(V^2)$.

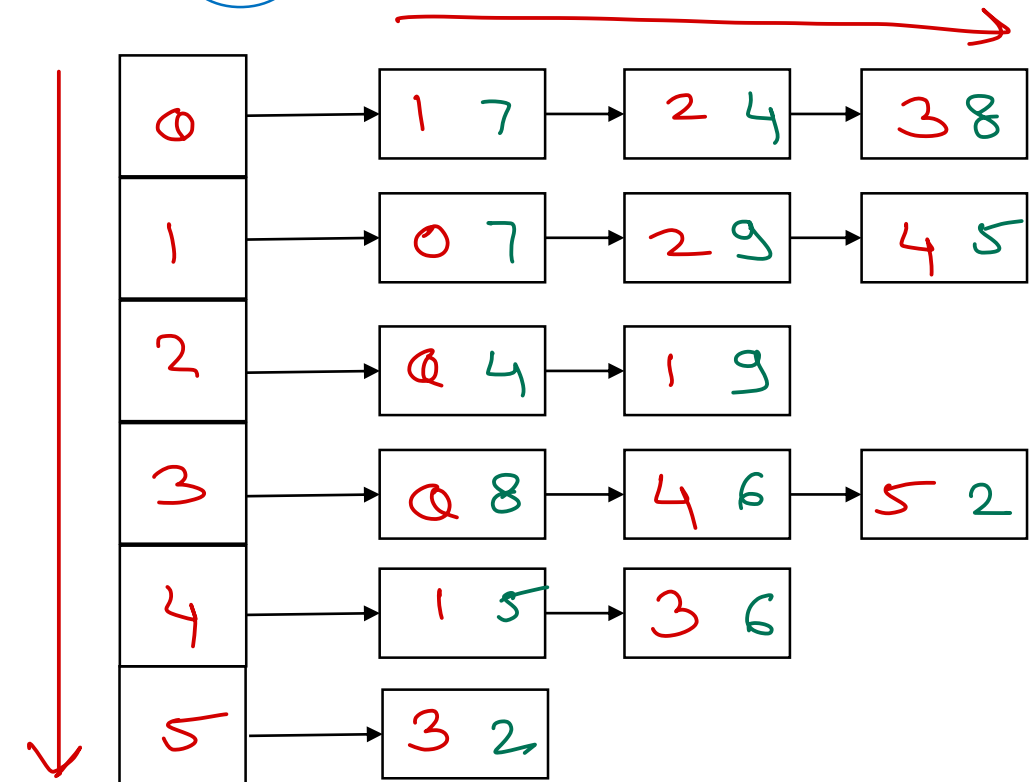
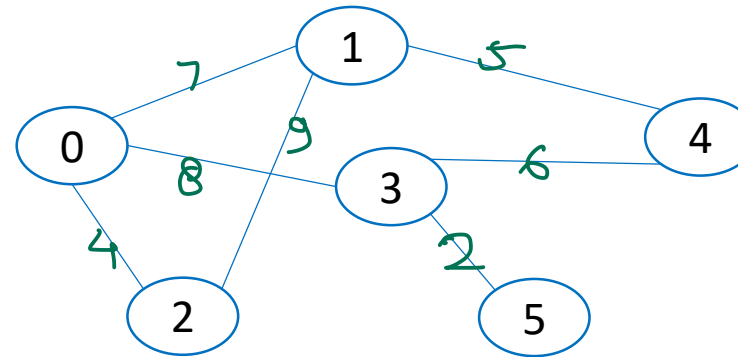


	0	1	2	3	4	5
0	∞	7	4	8	∞	∞
1	7	∞	9	∞	5	∞
2	4	9	∞	∞	∞	∞
3	8	∞	∞	∞	6	2
4	∞	5	∞	6	∞	∞
5	∞	∞	∞	2	∞	∞



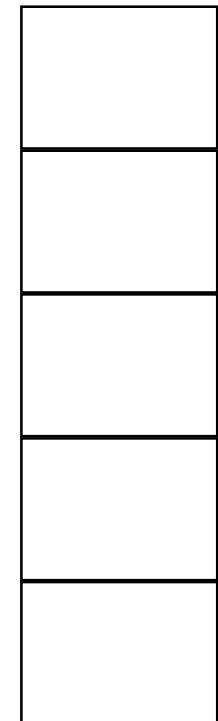
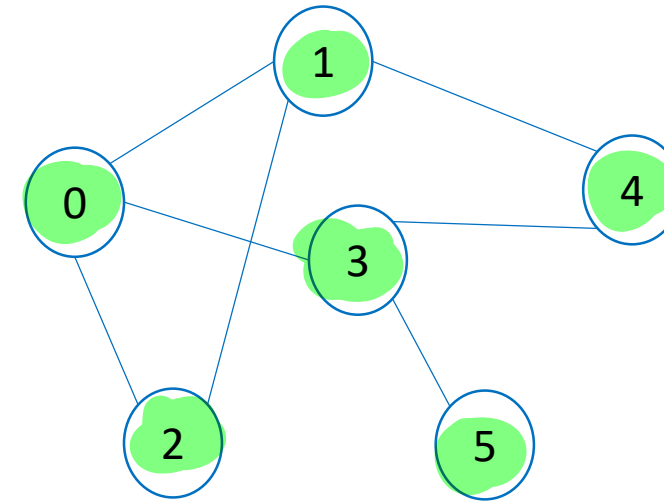
Graph Implementation – Adjacency List

- Each vertex holds list of its adjacent vertices.
- For non-weighted graphs only, neighbour vertices are stored.
- For weighted graph, neighbour vertices and weights of connecting edges are stored.
- Space complexity of this implementation is $O(V + E)$.
- If graph is sparse graph (with fewer number of edges), this implementation is more efficient (as compared to adjacency matrix method).



Graph Traversal – DFS Algorithm

1. Choose a vertex as start vertex.
2. Push start vertex on stack & mark it.
3. Pop vertex from stack.
4. Visit (Print) the vertex.
5. Put all non-visited neighbours of the vertex on the stack and mark them.
6. Repeat 3-5 until stack is empty.



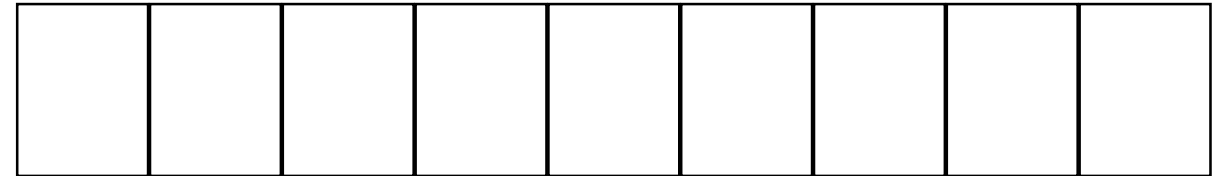
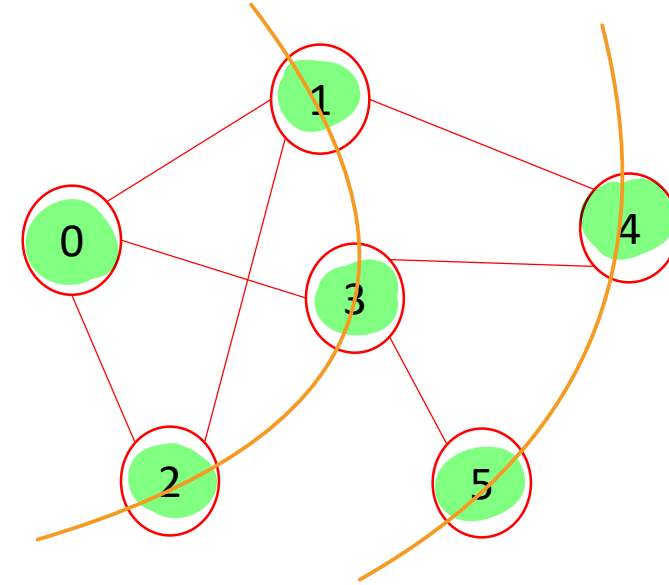
① 3 5 4 2 1



Graph Traversal – BFS Algorithm

1. Choose a vertex as start vertex.
2. Push start vertex on queue & mark it.
3. Pop vertex from queue.
4. Visit (Print) the vertex.
5. Put all non-~~visited~~^{marked} neighbours of the vertex on the queue and mark them.
6. Repeat 3-5 until queue is empty.

- BFS is also referred as level-wise search algorithm.



Q 1 2 3 4 5





Thank you!

Nilesh Ghule <nilesh@sunbeaminfo.com>

