$gcd(a, b) = gcd(a \% b, b)$ ✗

$gcd(a, b) = gcd(b, a \% b)$ ✓

Case-1

$gcd(a,b) = gcd(a\%b, b)$

```
int gcd (a, b) {
    if (b == 0)
        return a;
    return gcd (a%b, b);
}
```

gcd (24, 16)

↓

gcd ( 8, 16)

gcd ( 0, 16)

gcd ( 8, 16)

gcd ( 8, 16)

$24\%16 = 8$

$8\%16 = 8$

Sujit wise Recursion

Case-2

gcd(a, b) = gcd(b, a%b)

```
int gcd (int a, int b) {
    if (b == 0)
        return a;

    return gcd (b, a%b);
}
```

gcd(24, 16)
↓
gcd(16, 8)
↓
gcd(8, 0)
↳ 8

gcd(16, 24)
↓
gcd(24, 16)

A & B > 0 A >= B

gcd(a, b) = gcd(a-b, b)
=> gcd(b, a%b)

T.C. = $\log_2 \max(a,b)$

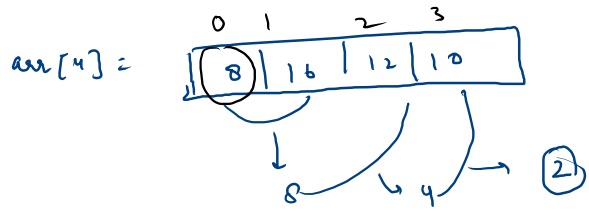
S.C. = $\log_2 \max(a,b)$

↳ Homework

Note:- If a < b, ⇒ gcd(a, b) = gcd(b, a%b)

one extra gcd step.

**Questions:-** Q.1) Given an array arr of size 'N', calculate the gcd of entire array.




ans = 8 8 4 2

gcd(8, 16) ⇒ 8

gcd(8, 12) ⇒ 4

gcd(4, 10) ⇒ 2

ans = arr[0]

```
for( int i = 1; i < arr.length; i++)
{
    ans = gcd(ans, arr[i]);
}
```

p.s.

```
int findGCD(int arr[]) {
    if (arr.length == 0)
        return 0;

    int ans = arr[0];

    for (int i = 1; i < arr.length; i++) {
        ans = gcd(ans, arr[i]);
    }

    return ans;
}
```

p.s.

```
int gcd(int a, int b) {
    if (b == 0)
        return a;

    return gcd(b, a%b);
}
```

T.C. $N \log_2 \max(arr[])$

# Prime Sieve | Sieve of Eratosthenes.

Print all prime nos from 1 - N

(1 - 50)

↳ 2, 3, 5, 7, 11, 13 — — — —

Idea-01

↳ Run a loop, from 1-N

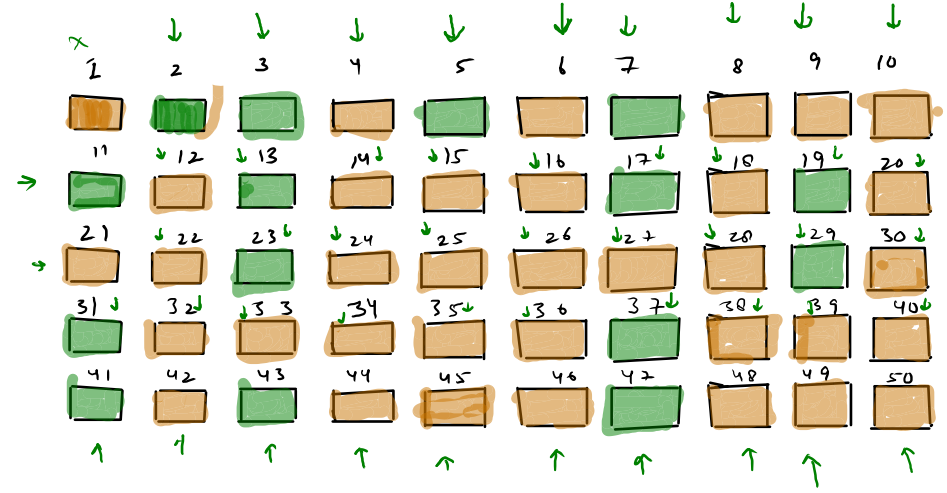
↳ Check a no. is prime or not.

↳ Find its factor — 2 ✓

— X

T.C. ⇒ $N$ $\sqrt{N}$

# Idea-02 Prime Sieve




x → is a prime

↳ all the multiple of x are not prime.

5 - prime

10 → 5, 1, 10

15

20

25

30

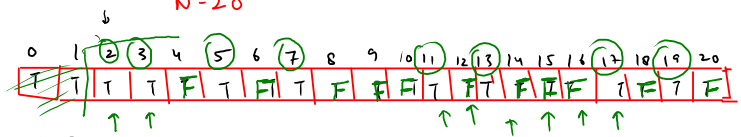
35

40

45,

1-20

N=20




3 → 2×3 = 6+3 = 9
5 → 2×5

```
fs void primeSieve (int N) {
    boolean isprime[] = new boolean[N+1];
    for (int i = 2; i<=N; i++) {
        if (isprime[i] == true) {
            for (int j = 2*i; j<=N; j=j+i)
            {
                isprime[j] == false;
            }
        }
    }
```

i=2 j=4 j=6 j=8 j=10 j=12 j=14 j=16 j=18 j=20

i=3 j=6 j=9 j=12 j=15 j=18 j=21

i=4 j=8 j=12 j=16 j=20

i=5 j=10 j=15 j=20 j=25

i=6 j=12 j=18

i=7 j=14 j=21

i=8 j=16

i=9 j=18

i=10 j=20

i=11 j=22

i=12 j=24

i=13 j=26

i=14 j=28

i=15 j=30

i=16 j=32

i=17 j=34

i=18 j=36

i=19 j=38

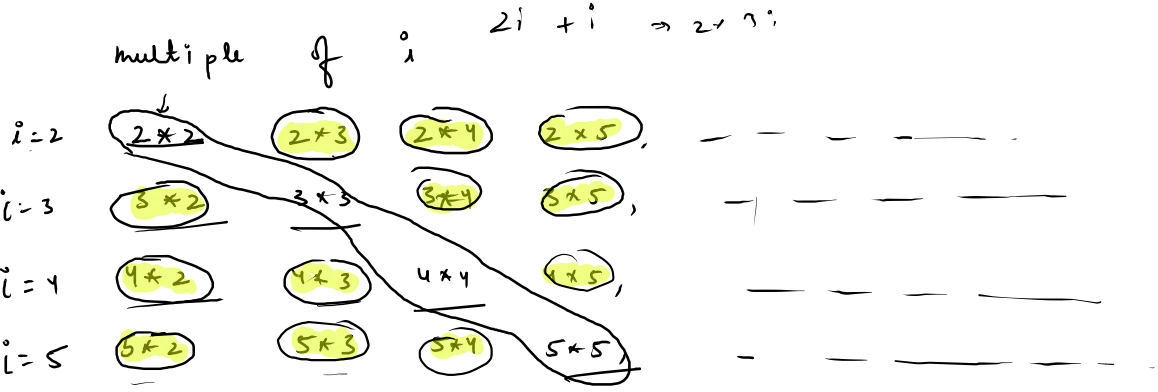
i=20 j=40

i=21 j=42

→ for (int i=2; i<=N; i++) {
    if (isprime[i] == true)
        sysout(———)
}

T.C. ⇒ $O(N \cdot \log\log(N)) < N\sqrt{N}$

S.C. $O(N)$




for(int j=2i ; j<=N; j=j+1) ⇒ for(int j= i*i ; j<=N; j+1)

T.C.

| i | j | no. of iterations |
|---|---|---|
| 2 | multiple of 2 | $\frac{N}{2}$ |
| 3 | mult of 3 | $\frac{N}{3}$ |
| 4 | → | |
| 5 | mul of 5 | $\frac{N}{5}$ |
| 6 | → | |
| 7 | mult of 7 | $\frac{N}{7}$ |
| 8 | → | |
| 9 | → | |
| N | | |

⇒ Total no. of iterations

$\frac{N}{2} + \frac{N}{3} + \frac{N}{5} + \frac{N}{7} + \frac{N}{11} + \cdots$

⇒ $N\left[\underbrace{\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \cdots}\right]$

↓

Sum of reciprocals of prime nos. till N.

⇒ $\log(\log N)$

⇒ $N \log\log(N)$

```
p.s. void primeSieve (int N) {

  boolean isprime[] = new boolean[N+1];

  for (i=2; i*i<=N; i++) {
      if (isprime[i] == true) {
          for( j= i*i; j<=N; j=i+j) {
              isprime[j] = false;
          }
      }
  }

  for (int i=2; i<=N; i++) {
      if (isprime[i] == true)
      {  print( i )
      }
  }
}
```

→ Prime Sieve

T.C. ⇒ $O(N \log\log(N))$

S.C. ⇒ $O(N)$

% ⇒

A%B ⇒ Remainder when A is divided by 'B'

14%5 = 4

40%7 = 5

```
   2
5 ) 14 (
    10
   ---
    4
```

40%40

5 → 0, 1, 2, 3, 4, (5)

a%b → (0, b-1)

a%10

↳ 0 → 0
↳ 1
↳ 2
3
4
5
6
7
8
9 → 9 (10-1)

↳ A%B ⇒ { min: 0 , max: B-1 }

# Modular Arithmetic

% → (+, −, /, *)

$$(a+b) \% m = (a \% m + b \% m) \% m$$

(a+b) → (m-1)

(a%m) → (m-1)

(b%m) → (m-1)

$$\Rightarrow \boxed{(a+b) \% m = (a \% m + b \% m) \% m}$$

$$(a * b) \% m = (a \% m * b \% m) \% m$$

$(a*b) \% m$ : $(m-1)$

$(a \% m)$ : $(m-1)$

$(b \% m)$ : $(m-1)$

$> m-1$

↳

$$\boxed{(a*b) \% m = (a \% m * b \% m) \% m}$$

$(a - b) \,\%\, m = (a \,\%\, m - b \,\%\, m) \,\%\, m$.

Ex. $a = 13$
$b = 9$
$m = 5$

LHS

$(13 - 9) \,\%\, 5$

$4 \,\%\, 5$

$= 4$

RHS

$(13 \,\%\, 5 - 9 \,\%\, 5) \,\%\, 5$

$(3 - 4) \,\%\, 5$

$(-1) \,\%\, 5$

$\Rightarrow -1$

$(-1 + 5)$

$\hookrightarrow (0, 4)$

$(a - b) \,\%\, m = (a \,\%\, m - b \,\%\, m + m) \,\%\, m \Rightarrow$

$(13 \,\%\, 5 - 9 \,\%\, 5 + 5) \,\%\, 5$

$(3 - 4 + 5) \,\%\, 5$

$(4 \,\%\, 5) = 4$

$(a/b)\ \%\ m = \ ?$