

↳ Factors

↳ Prime nos.

↳ Calculate iterations

↳ Time and space complexity.

Q- Given a no.  $N$ , calculate the no. of factors of  $N$

$$N = 24 = 1, 2, 3, 4, 6, 8, 12, 24$$

→ (8) ✓

$$N = 36 = 1, 2, 3, 4, 6, 9, 12, 18, 36$$

⇒ (9)

count = 0 ✓ (3)

$$\min = 1, \max = N$$

int count = 0;

```
for (int i = 1; i <= N; i++) {
    if (N % i == 0)
        count++;
}
```

3

print(count)

(3)

$[1, N]$

$[1, \sqrt{N}]$

$$N = 9$$

$$i = 1 < 9 (T) \rightarrow 9 \% 1 = 0 (T)$$

$$i = 2 < 9 (T) \rightarrow 9 \% 2 \neq 0 (F)$$

$$i = 3 < 9 (T) \rightarrow 9 \% 3 = 0 (T)$$

$$i = 4 < 9 (T) \rightarrow 9 \% 4 \neq 0 (F)$$

$$i = 5 < 9 (T) \rightarrow 9 \% 5 \neq 0 (F)$$

$$i = 6 < 9 (T) \rightarrow 9 \% 6 \neq 0 (F)$$

$$i = 7 < 9 (T) \rightarrow 9 \% 7 \neq 0 (F)$$

$$i = 8 < 9 (T) \rightarrow 9 \% 8 \neq 0 (F)$$

$$i = 9 < 9 (F) \rightarrow 9 \% 9 = 0 (T)$$

$$i = 10 < 9 (F)$$

$$\Rightarrow \boxed{\text{in } 1 \text{ s} = 10^8 \text{ operations}} \Rightarrow 1 \text{ op} = \frac{1}{10^8} \text{ sec.}$$

$$\boxed{N = 10^{18}}$$

3

$$N = 10^9 \rightarrow ?$$

$$\Rightarrow 10^9 \times \frac{1}{10^8} \text{ sec} = \underline{\underline{10 \text{ sec.}}}$$

$$\Rightarrow \boxed{N = 10^{18}} \Rightarrow 10^{18} \times \frac{1}{10^8} = \frac{10^{10} \text{ sec}}{86400 \times 365} \approx \underline{\underline{317 \text{ yrs.}}}$$

↑

$$N = 24$$

1, 2, 3, 4, 6, 8, 12, 24

$i < \frac{N}{i}$   
 $1 * 24 = 24 \Rightarrow 2$   
 $2 * 12 = 24 \Rightarrow 12 < \frac{N}{i}$   
 $3 * 8 = 24 \Rightarrow 2$   
 $4 * 6 = 24 \Rightarrow 2$   
 $6 * 4 = 24$   
 $8 * 3 = 24$   
 $12 * 2 = 24$   
 $24 * 1 = 24$

$$a * b = N$$

$$b = \frac{N}{a}$$

$$a \Rightarrow N$$

$$b = \frac{24}{1} = 24$$

$$\frac{24}{2} = 12$$

$N = 36$   
 1, 2, 3, 4, 6, 9, 12, 18, 36

$i < \frac{N}{i}$   
 $1 * 36 = 36 \rightarrow 2$   
 $2 * 18 = 36 \rightarrow 4$   
 $3 * 12 = 36 \rightarrow 6$   
 $4 * 9 = 36 \rightarrow 8$   
 $6 * 6 = 36 \rightarrow 18$   
 $9 * 4 = 36$   
 $12 * 3 = 36$   
 $18 * 2 = 36$   
 $36 * 1 = 36$

⇒ To find all the factors of a no. 'N'

$$\Rightarrow \boxed{i \leq \frac{N}{i}}$$

$$\Rightarrow \boxed{i * i \leq N}$$

$$\Rightarrow i^2 = N$$

$$\Rightarrow \boxed{i = \sqrt{N}}$$

count = 4/2/4/8

```

count = 0;
for (int i = 1; i * i <= N; i++) {
    if (N % i == 0) {
        if (N / i == i)
            count += 1;
        else
            count += 2;
    }
}

```

$$N = 24 \quad \sqrt{24} \approx 4$$

$i = 1 \angle = 4 (T)$        $24 \% 1 = 0 (T)$   
 $i = 2 \angle = 2 (T)$        $24 \% 2 = 0 (T)$   
 $i = 3 \angle = 4 (T)$        $24 \% 3 = 0 (T)$   
 $i = 4 \angle = 4 (T)$        $24 \% 4 = 0 (T)$   
 $i = 5 \angle = 4 (F)$

$$N = 36 \quad \sqrt{36} = 6$$

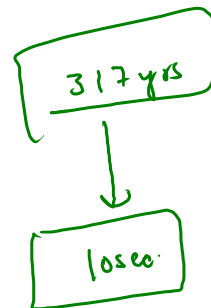
$i = 1 \angle = 6 (T) \rightarrow 36 \% 1 = 0 \rightarrow 2$   
 $i = 2 \angle = 6 (T) \rightarrow 36 \% 2 = 0 \rightarrow 4$   
 $i = 3 \angle = 6 (T) \rightarrow 36 \% 3 = 0 \rightarrow 6$   
 $i = 4 \angle = 6 (T) \rightarrow 36 \% 4 = 0 \rightarrow 8$   
 $i = 5 \angle = 6 (T) \rightarrow 36 \% 5 = 1 \rightarrow 8$   
 $i = 6 \angle = 6 (T) \rightarrow 36 \% 6 = 0 \rightarrow 10$   
 $i = 7 \angle = 6 (F)$

$$10^8 \xrightarrow{\text{ops.}} 1 \text{ sec.}$$

$$N = 10^9 \rightarrow \frac{\text{ops.}}{10^4 \sqrt{10}}$$

$$\Rightarrow \frac{10^4 \sqrt{10}}{10^8}$$

$$\sqrt{10} * 10^{-4} \Rightarrow \frac{10 \text{ sec.}}{\downarrow} \Rightarrow \underline{3.1 * 10^{-4} \text{ sec.}}$$



$$N = 10^{18} \rightarrow \frac{\text{ops.}}{\sqrt{10^{18}}}$$

$$= 10^9 \Rightarrow \frac{10^9}{10^8} = \boxed{10 \text{ sec.}}$$

$\Rightarrow$  Prime no.


$\Rightarrow N = 17 \rightarrow \begin{matrix} \text{true} \\ \text{false} \end{matrix}$

$N > 1$

$\hookrightarrow$  exactly 2 factors  $\rightarrow$  1, N

$\hookrightarrow$  5-6 mins -



$i = 2$   
  
 $1, N$

```

int count = 0;
for (int i = 1; i * i <= N; i++) {
    if (N % i == 0) {
        if (i == N / i)
            count += 1;
        else
            count += 2;
    }
    if (count > 2) return false;
}
if (count == 2) return true;
return false;

```

$1, N$

```

for (int i = 2; i * i <= N; i++) {
    if (N % i == 0)
        return false;
}
return true;

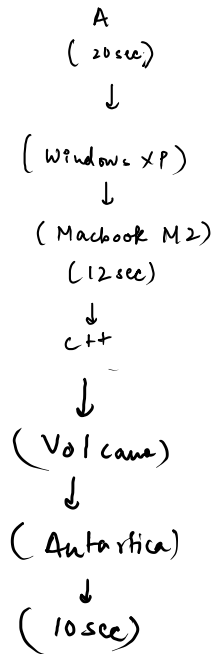
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# Comparing 2 Algo's

↳

(A)

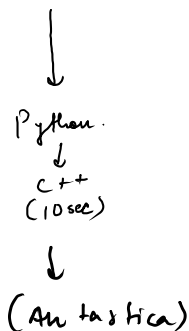
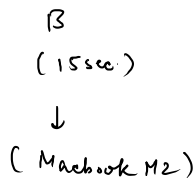
↳ Problem statement, test cases-



(B)

⇒ Is execution time, good parameter  
to judge

→ No



⇒ Calculating the no of  
iterations

Calculating iterations

```
for (int i = 1; i <= N; i++)  
{  
  
}
```

$$[a, b] \rightarrow b - a + 1$$

$$[1, N] \rightarrow N - 1 + 1 = N$$

↳ (N) iterations

Q- void fn (int N) {

int sum = 0;

for (i = 0; i < N; i++)  
sum = sum + i;



$[0, N] \Rightarrow N - 0 + 1 \Rightarrow \underline{(N+1) \text{ iterations}}$

}

}

Q:-

```
void fn (int N) {
```

```
    int sum = 0;
```

```
    for (i = 0; i * i <= N; i++)
```

```
        sum = sum + i;
```

```
}
```

$$[0, \sqrt{N}] \Rightarrow \sqrt{N} - 0 + 1$$

$$= (\sqrt{N} + 1)$$

Q.

void fn (int N) {

  i = N;

  while (i > 1) {

    i = i/2;

  }

}

→  $\log_2 N$

$$\frac{N}{2^k} = 1$$

$$N = 2^k$$

both sides  $\log_2$

$$\log_2 N = \log_2 2^k$$

$$K = \log_2 N$$

i	<sup>N/2</sup> after 'i' value
N	$\frac{N}{2} \rightarrow \frac{N}{2^1}$
$\frac{N}{2}$	$\frac{N}{4} \rightarrow$
$\frac{N}{4}$	$\frac{N}{8} \rightarrow$
...	
1	

$$\log_a a = 1$$

