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1\. (i) $n! \in O(n^n)$ True

$O(n^n) = \{ \underbrace{f(n)}_{\text{set of f(n)'s}} :$ such that we can find $0 \le f(n) \le c \cdot n^n$ for some positive constant c & $n \ge n_0 \}$

For $n! \in$ set$\{f(n)\}$

$n!$ should satisfy

$$0 \le n! \le c \cdot n^n$$

lets take $c = 100$

$$0 \le n! \le 100 \cdot \underbrace{n \cdot n \cdot n \cdot n \cdot n \cdots n}_{n \text{ times}}$$

$$0 \le n! \le 100 \; n \cdot (n-1+1)(n-2+2)(n-3+3) \cdots (n-(n-1)+(n-1))$$

$$0 \le n! \le 100 \left[n(n-1)(n-2) \cdots + f(n) \right]$$

$$0 \le n! \le 100 \left[n! + f(n) \right]$$

for $n_0 > 0$
$0 \le n!$ Holds True

$n! \le 100 n! + 100 f(n)$

$-99 n! \le 100 \cdot f(n)$

L.H.S. is negative, R.H.S. is positive } so Holds true

Lets take $n_0 = 100$

for $c = 100$ & $n_0 = 100$ the condition holds true for every $n \ge n_0$

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(ii)

~~$2n^2 \cdot 2^n + n\log(n) \in \Theta(n^2 2^n)$~~

$2n^2 \cdot 2^n + n\log n \in \Theta(n^2 2^n)$ True

For above to hold true,

$$c_0 (n^2 2^n) \le 2n^2 \cdot 2^n + n\log(n) \le c_1 (n^2 2^n) \quad \text{—(1)}$$

for some positive c_0, c_1 & $n \ge n_0$

→ lets take right eq. from (1)

$$2n^2 \cdot 2^n + n\log(n) \le c_1 (n^2 2^n)$$

take $c_1 = 1$ million

$$n\log(n) \le c_1 (n^2 2^n) - 2 (n^2 2^n)$$

[as $c_1 >> 2$, so]

$$n\log(n) \le c_1 (n^2 \cdot 2^n)$$

$$\log(n) \le c \cdot n \cdot 2^n$$

$\boxed{n > 0}$

take $n_0 = 100$ & $c_1 = 1$ million the above holds true

- lets take left eq. from (1)

$$c_0 (n^2 \cdot 2^n) \le 2n^2 \cdot 2^n + n\log(n)$$

take $c_0 = \frac{1}{1 \text{ million}}$

$$c_0 (n^2 \cdot 2^n) - 2 (n^2 \cdot 2^n) \le n\log n$$

as c_0 is very small

$$-2 (n^2 \cdot 2^n) \le n\log n$$

take $n_0 = 100$ as before

$$-2n \cdot 2^n \le \log(n)$$

L.H.S is negative & R.H.S is positive so holds true.

~~Finally~~

We have found $c_0 = \frac{1}{1 \text{ million}}$, $c_1 = 1$ million & $n_0 = 100$ for which (1) is valid so Given Relationship is true.

(iii) $n^{1.001} + n\log(n) \in \Theta(n^{1.001})$ — True.

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for above to hold true

$$c_0 . n^{1.001} \le n^{1.001} + n\log(n) \le c_1 . n^{1.001} \quad -(1)$$

for some positive c_0 & c_1 & $n \ge n_0$

⇒ take right equation from (1)

$$n^{1.001} + n\log(n) \le c_1 . n^{1.001}$$

take c_1 = 1 million (Big number positive)

$$n \log n \le c_1 . n^{1.001}$$

for $n > 0$

$$\log n \le c_1 . n^{0.001}$$

$$\log n \le c_1 . (n)^{1/1000}$$

Lets take $n_0 = 10^{1000000}$

$$\log_{10} 10^{1000000} \le c_1 . 10^{1000}$$

$$1000000 \le c . 10^{1000}$$

True

$(n^{1/10})$

$\frac{1}{10} n^{(\frac{1}{1000}-1)}$

$\frac{1-1000}{1000}$

$\frac{999}{1000}$

$\boxed{\frac{1}{10} \frac{1}{n^{0.999}}}$

– Lets take left eq. from (1)

$$c_0 . n^{1.001} \le n^{1.001} + n\log(n)$$

$$c_0 = \frac{1}{1 \text{million}}$$

$$-n^{1.001} \le n\log(n)$$

$$-n^{0.001} \le \log(n) \quad n > 0$$

LHS is negative & RHS is positive so Holds true

for $n_0 = 10^{1000000}$ & $c_0 = \frac{1}{1 \text{million}}$ & c_1 = 1 million the eq. (1) Holds True. Hence True.

(iv)

false

$10n^2+9 \in O(n)$ [In class Prof. said '$\in$' is correct relation instead of '$=$']

for above to hold true

$0 \le 10n^2+9 \le c_1 \cdot n$ for $n > n_0$ — (1)

& some c_1 (positive)

take left side of (1)

$0 \le 10n^2+9$

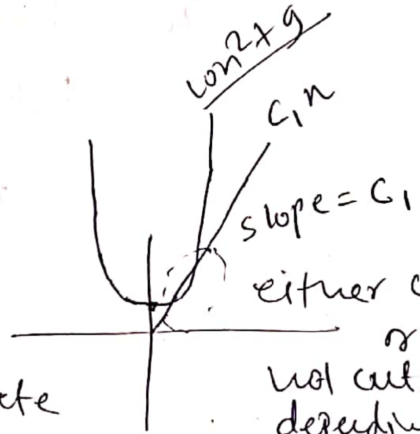
for $n_0 = 100$ Holds true

(✓)

take right side of (2)

$10n^2+9 \le c_1 \cdot n$

graphs of Both sides



either cuts or not cuts depending upon value of c_1.

~~But~~ Also If we differentiate Both side

$\boxed{20n}$ $\boxed{c_1}$

Left slope Right slope

So, for any c_1 there will be a point where the $(10n^2+9)$ will cross $c_1 \cdot n$ & we can't find any n_0 for which

$10n^2+9 \le c_1 \cdot n$ for all $n > n_0$

Not true.

So we can't find c_1 & n_0 for which (1) Holds true.

So not true

$10n^2+9 \notin O(n)$

(iv)

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$n^2 \log(n) \in \Theta(n^2)$ Result: False

For above to hold

$c_0 \cdot n^2 \le n^2 \log(n) \le c_1 \cdot n^2$ for some positive c_0 & c_1 & $n \ge n_0$ — (1)

So take left of (1)

$c_0 \cdot n^2 \le n^2 \log(n)$

$c_0 \le \log(n)$ $n > 0$

take $c_0 = \frac{1}{1 \text{ million}}$

Holds true for

$n_0 = 10000$

So, True

Now take Right of (1)

$n^2 \log(n) \le c_1 \cdot n^2$

$\log(n) \le c_1$ $n > 0$

We can take any c_1 But we will always able to take any $n \ge n_0$

$n \to \infty$

So False

So Given Relationship is false

$n^2 \log(n) \notin \Theta(n^2)$

(vi) False

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(6)

$n^3 2^n + 6n^2 3^n = O(n^3 2^n)$

for above to hold true,

$n^3 2^n + 6n^2 3^n \le c_1 n^3 . 2^n$ for some (+) c_1 & $n \ge n_0$ (1)

lets take c_1 Big positive number

$6n^2 . 3^n \le c_1 . n^3 . 2^n$

$6 . 3^n \le c_1 n . 2^n$ $n > 0$

$3^n \le c_1 . n . 2^n$

for any Bigger c_1 we can always increase $n \ge n_0$ so that conditions doesn't hold true.

as $3^n >>> n . 2^n$ for large n

so false

Hence eq. (1) doesn't hold True Hence Given Relationship is false

$n^3 . 2^n + 6n^2 . 3^n \notin O(n^3 . 2^n)$

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Q.2

(a) Computer can perform 10^{10} operations persecond

So in 1 hour

computer will perform

$= 60 \times 60 \times 10^{10}$ operations.

(i) n^2

So the time the Algo will take should not exceeds the limit of 1 hour operations

So $n^2 \leq 60 \times 60 \times 10^{10}$

$n \leq 60 \times 10^5$

$n = 60 \times 10^5$ Maximum Input size

largest input size.

(ii) $n^3 \leq 60 \times 60 \times 10^{10}$

$n^3 \leq 6 \times 6 \times 10^{12}$

$n^3 \leq 6^2 \times 10^{12}$

$n \leq 6^{2/3} \times 10^4$

$n \leq 3.3019272 \times 10^4$

$n \leq 33019.272\ldots$

largest n is approximately

33019

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(8)

(iii) $100n^2$

Similarly

$$100n^2 \le 60 \times 60 \times 10^{10}$$

$$n^2 \le 60 \times 60 \times 10^{8}$$

$$n \le 60 \times 10^{4}$$

$$n = 60 \times 10^{4}$$

largest input size is
60×10^4

(iv) $2^n \cancel{= 60 \times 60 \times 10^{10}}$

$$2^n \le 60 \times 60 \times 10^{10}$$

$$n \le \cancel{\log} \frac{\ln(36 \times 10^{12})}{\ln(2)}$$

$$n \le \frac{31.21454005}{0.69314718}$$

$$n \le 45.033$$

$$n = 45$$

largest size of problem that
canbe solved is 45

3.

Hamiltonian cycle problem:

Instance: A Graph is Given

$$G = (V, E)$$

V = vertices
E = Edges

Questions: (a) Does G Contain a Hamiltonian cycle?

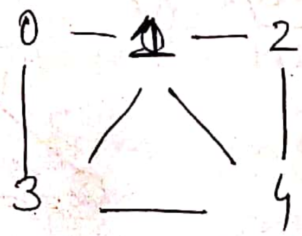
Explanation: (1) cycle: A cycle in a Graph $G = (V, E)$ is a sequence $\langle v_1, v_2, \ldots, v_k \rangle$ of distinct vertices of V such ~~tat~~ that $\{v_i, v_{i+1}\} \in E$ for $1 \leq i < k$ and such that $\{v_k, v_1\} \in E$.

(2) A Hamiltonian cycle in G is a simple cycle that includes all the vertices of G.

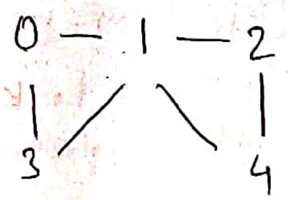
Hamiltonian path in an undirected graph is a path that visits each vertex exactly once. A Hamiltonian cycle ~~Travelling salesman problem:~~ (or Hamiltonian circuit) is a Hamiltonian path such that there is an Edge (in the graph) from the last vertex to the first vertex of the Hamiltonian path.

Ex.

(0, 1, 2, 4, 3, 0)



→ Hamiltonian cycle is there



→ Hamiltonian cycle is not there.

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Travelling salesman problem:

Given a set of cities and distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point.

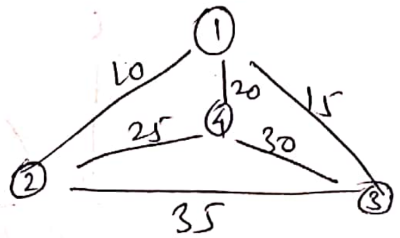
~~Note the diff b/w HC & TSP.~~

Instance: A finite set $C = \{c_1, c_2, \ldots c_m\}$ of cities, a distance $d(c_i, c_j) \in Z^+$ for each pair of cities $c_i, c_j \in C$ and a bound $B \in Z^+$ (where Z^+ denotes the positive integers).

questions: Is there a tour of all cities in C having total length no more than B, that is an ordering $\langle c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} \rangle$ of C such that,

$$\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)}) + d(c_{\pi(m)}, c_{\pi(1)}) \leq B$$

For example



TSP-tour in graph
1-2-4-3-1. is the shortest
Path is $10+25+30+15 = 80$

Transformation from HC problem to TSP & claim

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(11)

- Lets say: G is an unweighted ~~gra~~ graph G.

Hamiltonian Cycle ($G = (V, E)$)

lets say we construct a complete weighted graph $G' = (V', E')$ where $V' = V$.

$n = |V|$

```
For i = 1 to n do
    for j = 1 to n do
        if (i,j) ∈ E then w(i,j) = 1
        else w(i,j) = 2
Return the answer to
    Travelling-salesman
        ( G', n )
```

The above ~~al~~ algo ~~convert the~~ does a polynomial-time transformation from Hamiltonian cycle to problem to the Travelling salesman problem. (n^2)

The actual reduction is quite simple, with the traslation from unweighted graph easily performed in linear time. Further, this translation is designed to ~~argue~~ ensure that the answers of the two problems will be identical. If the graph G has a Hamiltonian cycle $\{v_1, \ldots v_n\}$, then this exact same tour will corresponds to n edges in E', each with weight 1.

Therefore, This gives a TSP tour of G' of weight exactly n. If G does not have a Hamiltonian cycle, then there can be no such TSP tour in G', because the only way to get a tour of cost 'n' in G would be to use only edges of weight 1, which implies a Hamiltonian cycle in G.

~~This redu~~

Given reduction is both efficient and truth preserving.

A fast Algorithm for TSP would imply a fast Algorithm for Hamiltonian cycle, while a hardness proof for Hamiltonian cycle would imply that TSP is Hard. Since the latter is the case, the reduction shows that TSP is Hard, at least as hard as Hamiltonian cycle.

<u>So</u> If Hamiltonian cycle is "Hard" (i.e. NP-Complete) then Travelling Salesman problem must also be "Hard" or "Harder"!

4.

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13

Ascending order of growth rate.

$$(2n)^{0.7} \le n^3+100 \le n^{3.4} \le 20^n \le (250)^n$$

$f_1(n) = n^{3.4}$

$f_2(n) = (2n)^{0.7}$

$f_3(n) = n^3+100$

$f_4(n) = (20)^n$

$f_5(n) = (250)^n$

$$\underbrace{f_2(n) \le f_3(n)}_{1} \le f_1(n) \le f_4(n) \le f_5(n)$$

2: $f_3(n) \le f_1(n)$; 3: $f_1(n) \le f_4(n)$; 4: $f_4(n) \le f_5(n)$

lets consider 1

$$(2n)^{0.7} \le n^3+100$$

$$(2n)^{0.7} \in O(n^3+100)$$

$$0 \le (2n)^{0.7} \le c_1 \cdot (n^3+100)$$

$$c_1 = 100,\ n_0 = 10^3$$

we can find positive c_1 & n_0 for above case so

equation holds true for all $n \ge n_0$

② $f_3(n) \le f_1(n) \Rightarrow$

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$$n^3 + 100 \le n^{3.4}$$

$$n^3+100 \in O(n^{3.4})$$

$0 \le n^3+100 \le c_1 \cdot n^{3.4}$ for ~~atleast~~ some positive c_1 & $n \ge n_0$

$c_1 = 10^3$

$n = 10^9$

$$10^{27} + 100 \le 10^3 \cdot 10^{30.6}$$

atleast for all $n \ge 10^9$

So we can find c_1 & n_0 so True.

also we can differeiate

1 time $3n^2 \le 3.4\, c_1 \cdot n^{2.4}$

2nd time $6n \le 3.4 \times 2.4 \cdot c_1\, n^{1.4}$

3rd time $6 \le 3.4 \times 2.4 \times 1.4 \times c_1\, n^{0.4}$

Rate will be more

So for Bigger n ~~we~~ here the $f_1(n)$ will increase faster.

③. $f_1(n) \le f_4(n)$

$n^{3.4} \le 20^n$

$n^{3.4} \in O(20^n) \Rightarrow 0 \le n^{3.4} \le c_1 \cdot 20^n$ for some positive c_1 & $n \ge n_0$

$0 \le n^{3.4} \le c_1 \cdot 20^n$

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1.5

20^n will increase at very high rate ~~far~~ compared to $n^{3.4}$.

to proof differentiate

$\frac{d_1}{dn}$ $\quad 3.4 \cdot n^{2.4} \le c_1\, 20^n \log(20)$

$\frac{d^2}{dn^2}$ $\quad 3.4 * 2.4 * n^{1.4} \le c_1 \cdot 20^n . 20^n . \log(20)$

RHS will increase at very high rate.

SO true. for $n \ge n_0$

also

We can find a set of values $\rightarrow$ $c_1 = 10^3$ & $n_0 = 10^3$ such that above condition Holds true ~~true above~~ for all $n \ge n_0$

So ~~it~~ True.

(4) $f_4(n) \le f_5(n)$

$20^n \le 250^n$

$20^n \in O(250^n)$

$0 \le 20^n \le c_1 \cdot (250)^n$ for some positive c_1 & $n \ge n_0$ — (1)

for $c_1 = 1000$ & $n_0 = 10^3$ the condition ~~of~~ (1) Holds true.

So True.

Hence finally [$(2n)^{0.7} \le n^3 + 10 \le n^{3.4} \le 20^n \le 250^n$

$f_2(n) \le f_3(n) \le f_1(n) \le f_4(n) \le f_5(n)$]

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