Practice Problem

Consider the following data for a one-factor economy. All portfolios are well diversified.

Portfolio	E(r)	Beta
A	10%	1.0
F	4%	0

Suppose another portfolio E is well diversified with a beta of 0.67 and an expected return of 9%. Would an arbitrage opportunity exist? If so, what would the arbitrage strategy be?

Since the beta for Portfolio F is zero, the expected return for Portfolio F equals the risk-free rate.

For Portfolio A, the ratio of risk premium to beta is: (10 – 4)/1 = 6
The ratio for Portfolio E is higher: (9 – 4)/(2/3) = 7.5

This implies that an arbitrage opportunity exists. For instance, by taking a long position in Portfolio E and a short position in Portfolio F (that is, borrowing at the risk-free rate and investing the proceeds in Portfolio E), we can create another portfolio which has the same beta (1.0) but higher expected return than Portfolio A. For the beta of the new portfolio to equal 1.0, the proportion (w) of funds invested in E must be: 3/2 = 1.5.

Portfolio Weight	**In Asset**	**Contribution to β**	**Contribution to Excess Return**
-1	Portfolio A	$-1 \times \beta_A = -1.0$	-1.0 x (10%- 4%) = -6%
1.5	Portfolio E	$1.5 \times \beta_E = 1.0$	1.5 x (9% - 4%) = 7.5%
-0.5	Portfolio F	$-0.5 \times 0 = 0$	0
Investment = 0		$\boldsymbol{\beta_{Arbitrage} = 0}$	$\boldsymbol{\alpha = 1.5\%}$

As summarized above, taking a short position in portfolio A and a long position in the new portfolio, we produce an arbitrage portfolio with zero investment (all proceeds from the short sale of Portfolio A are invested in the new portfolio), zero risk (because $\beta = 0$ and the portfolios are well diversified), and a positive return of 1.5%.