Section-A Sumit Kumar Giri

Q1.

(i) Define basic counting principle.

Ans:- There are mainly two Counting principle namely

(i) Sum rule.

(ii) product rule.

There two principles form the basies of permutations and Combinations and heree known as basic counting principles.

(ii) Define Sum and product rule.

Ans:- Sum rule:- If there are two jobs such that they Can be performed independently in maade n ways. Then number of ways in which either of the two jobs Can be performed is m+n.

Product rule:- If there are two jobs such that one of them Can be done in m ways and n ways. and when it has been done, second job Can be done in n ways then the two jobs Can be done in mxn ways.

(iii) A snack bar serves 5 different Sandwhiches and 3 different bererage. How many different lunches Can a person order?

Ans:- The person Can place his order by

(i) asking a sandwhich among 5 sandwiches.

(ii) asking a beverage among 3 berenages.

∴ By product Rule' The total number of ways of placing a order is $5 \times 3 = 15$

iv) In a monthly test the teacher decides that there will be three questions, one from each chapter I, II and III of the book. ~~If~~ If there are 12 questions in chapter I, 10 in chapter II and 6 in chapter III. In how many ways can three questions be selected.

Ans:- Given that teacher decides in monthly test there will be three question.

If there are 12 question in chapter I, 10 in chapter II and 6 in chapter III then question can be selected as

$$12 \times 10 \times 6 = 720$$

So, there are 720 ways in which three question can be selected.

v) How many three digit can be formed without using the digit 0, 2, 3, 4, 5, 6?

Ans:- The digits without the digits are 0, 2, 3, 4, 5, 6 are 1, 7, 8, 9

So, threee digit can be formed. and we get

$$n = 4$$

The value of n will be 3, as we need a from 3 digit n only.

$$^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{4!}{(4-3)!} = 4! = 4 \times 3 \times 2 \times 1$$

$$= 24$$

vi) Define Boolean Ring.

Ans:— A Boolean Ring R is a ring for which $x^2 = x$ for all x in R, i.e. a ring that consists only a indempotant elements. A simple example of a boolean Rings is Z2.

Q2. If R is Communication, then R/I is also Commutative.

Ans:- By definition if I is an Ideal of R then

$R/I := \{a+I : a \in R\}$

Let, $a+I, b+I \in R/I$ and Consider

$$(a+I)(b+I) = ab+I$$

$$= ba+I \quad \because (a, b \in R \text{ and } R \text{ is Commutative})$$

$$= (b+I)(a+I)$$

$\therefore R/I$ is Commutative.

Q3. Solve $s(k) - 4s(k-1) + 4s(k-2) = 3k + 2^k$.

Soln:- Given equation is $s(k) - 4s(k-1) + 4s(k-2) = 3k + 2^k$ —(1)

The characteristic equation is given by

$$x^k + 4x^{k-1} + 4x^{k-2} = 0$$

Divide given equation by x^{k-2}

$$x^2 + 4x + 4 = 0$$

$$x^2 - 2x - 2x + 4 = 0$$

$$x(x-2) - 2(x-2) = 0$$

$$(x-2)(x-2) = 0$$

$$x = 2, 2$$

The homogenous solution is given by

$$(C_1+C_2k)\cdot 2^k$$

To find the particular solution of eqn ①. we observe that the RHS of ① contains the terms $3k$ and 2^k, But the terms 2^k and $k\cdot 2^k$ also occurs in $s(k)$.

hence we divide the particular solution in two parts.

∴ particular solution corresponding to the form $3k$ is

$$s'_p(k) = d_0 + d_1 k$$

$$s'_p(k-1) = d_0 + d_1(k-1)$$

$$s'_p(k-2) = d_0 + d_1(k-2)$$

using all these values in ①, we get

$$d_0 + d_1 k - 4(d_0 + d_1(k-1) + 4(d_0 + d_1(k-2))) = 3k$$

$$d_0(1-\cancel{4}+\cancel{4}) + d_1(k-4(k-1)+4(k-2)) = 3k$$

$$d_0 + d_1(k-\cancel{4k}+4+\cancel{4k}-3) = 3k$$

$$d_0 + d_1(k-4) = 3k$$

Equating the cofficient of constant terms we get, $d_0 - 4d_1 = 0$ — ②

Equating coefficient of k, we get $d_1 = 3$ — ③

from eqn ②

$$d_0 = 12$$

$$\boxed{S_{(p)} = 12 + 3k} \text{ — ④}$$

particular solution Corresponding to term 2^k

let,

$$S_p'(k) = d_2 k^2 . 2^k$$

$$S_p'(k-1) = d_2 (k-1)^2 . 2^{k-1}$$

$$S_p'(k-2) = d_2 (k-2)^2 . 2^{k-2}$$

using all these values in eqⁿ ①, we get

$$d_2 k^2 . 2^k - 4d_2 (k-1)^2 . 2^{k-1} + 4d_2 (k-2)^2 . 2^{k-2} = 2^k$$

$$d_2 . 2^{k-2} (4k^2 - (8k^2 - 1)^2 + 4(k-2)^2)) = 2^k$$

$$d_2 . 2^{k-2} (4k^2 - 8k^2 - 8 + 16k + 4k^2 + 16 - 16k) = 2^k$$

$$d_2 . 2^{k-2} (8) = 2^k$$

$$\frac{8d_2}{4} = 1$$

$$d_2 = 1/2$$

$$S_p(k) = S_p'(k) + S_p'(k)$$

$$= 12 + 3k + k^2 . 2^{k-1}$$

$\therefore$ Complete solution $= S(h) + S(p)$

$$\boxed{CS = (c_1 + c_2 k) . 2^k + 12 + 3k + k^2 . 2^{k-1}}$$

1. Find the generating function from the recurrence relation given by $s(k)-6s(k-1)+5s(k-2)=0$ where $s(0)=1$, $s(1)=2$

Soln- The given relation is,

$$s(k)-6s(k-1)+5s(k-2)=0 \quad \text{—} \; ①$$

order $= k-(k-2)=2$

multiply between of eqn ① by z^R and take Summation for $R=2$ to ∞, we get

$$\sum_{R=2}^{\infty} s(k)z^R - 6\sum_{R=2}^{\infty} s(k-1)z^R + 5\sum_{R=2}^{\infty} s(k-2)\, z^R = 0 \quad \text{—} \; ②$$

Consider $G(s,z) = \sum_{R=0}^{\infty} s(k)z^R$

$$= s(0)+s(1).z+s_2(z)^2+s_3(z)^3+ \text{-------}$$

$$= s(0)+z.s(1)+\sum_{R=2}^{\infty} s_R(z)^R$$

$\therefore \sum_{R=2}^{\infty} s(R)z^R = G(s,z)-1-2z \quad \left[\because s(0)=1, s(1)=2 \text{ given}\right]$

$\therefore$ eqn (2) becomes

$$G(s,z) - 1-2z-6\left[z\sum_{R=1}^{\infty} s(k-1)z^{R-1} - s_0\right] + \left[sz^2\sum_{R=2}^{\infty} s(k-2).z^{R-2}\right]=0$$

$$G(s,z)-1-2z-6z\left[G(s,z)-1\right]+sz^2G(s,z)=0$$

$$G(s,z)\left[1-6z+5z^2\right]-1-2z+6z=0$$

$$\boxed{G(s,z) = \frac{1-4z}{6-6z+5z^2}}$$

Hence, This is required Solution.