ASSIGNMENT-Ist

Discrete Structures

Q1.

(i) If $A = \{1, 2, \{1, 3\}, \phi\}$ determine $A - \{1, 2\}$.

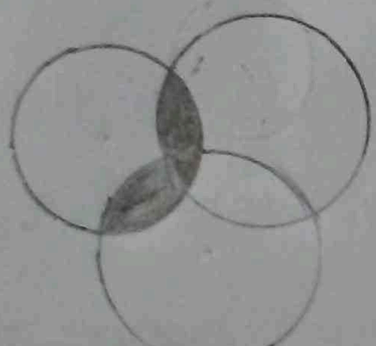
Solution:- $A = \{1, 2 \{1, 3\}, \phi\}$

Now,

$A - \{1, 2\}$

$= \{\{1, 3\}, \phi\}$

(ii) Draw the venn diagram of $A \cap (B \cup C)$.



(iii) Let R be the set of a relation on $A = \{2, 3, 4, 5, 6\}$ defined by 'x is relatively prime to y'. write R as a set of ordered pair.

Solution:- Every ordered pair in the set

$\{(2,3), (2,2), (2,4), (2,5), (2,6), (3,3), (3,2), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), (5,2), (5,3), (5,4), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

But given the condition is,

'x is relatively prime to y'.

so,

$R = \{(2,3), (3,2), (2,5), (5,2), (3,4), (4,3), (3,5), (5,3), (4,5), (5,4), (5,6), (6,5)\}$

(iv) Let R be relation on $A=\{1,2,3,4\}$ defined by 'x is less than y'. write R as a set of ordered pairs Find the inverse of the relation R. Can inverse of R be defined in words.

Sol^n:- ~~we know that~~

Given set,

$$A = \{1,2,3,4\}$$

$\therefore A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3)$
$(2,4), (3,1), (3,2), (3,3), (3,4), (4,1)$
$(4,2), (4,3), (4,4)\}$

$\because x < y$

$\therefore R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

So,

$R = \{(2,1), (3,1), (4,1), (3,2), (3,2), (4,2), (4,3)\}$

(v) Define into function with example.

Solution :- Let $A = \{0,1,2\}$ and $B = \{0,1,2,3,4\}$ and $f: A \to B$ is defined by $f(x) = x^2$

Then ~~ff~~ $f = \{(0,0), (1,1), (2,4)\}$

Range of $f = f(A) = \{0,1,4\} \neq$ co-domain B.

$2 \in B$ and $3 \in B$ but 2 and 3 are not the image of any element of the domain A.

Such functions are called into function.

If the function $f: A \to B$ is such that there is at least one element in B which is not f-image of any element in A. i.e. $f(A) \neq B$ then we say 'f' is a function from A into B or 'f' maps A into.

(vi) Let R be the relation defined on set $X = \{0,1,2,3,----\}$ of a non negative integers defined by the equations $x^2+y^2=25$, write R as a set of ordered pairs.

Solution :- ~~The relation R from~~ to ~~is given as~~

Given the equation ~~is~~

$$x^2+y^2=25$$

Now, put the value of x in equations.

$x=0$	$x=1$
$0+y^2=25$	$(1)^2+y^2=25$
$y=\sqrt{25}$	$y^2=25-1$
$y=\pm 5$	$y=\sqrt{24}$
$y=5$	$y=2\sqrt{6}$

$x = 2$

$(2)^2 + y^2 = 25$

$y^2 = 25 - 4$

$y^2 = 21$

$y = \sqrt{21}$

$x = 3$

$(3)^2 + y^2 = 25$

$y^2 = 25 - 9$

$y = \sqrt{16}$

$y = \pm 4$

$y = 4$

$x = 4$

$(4)^2 + y^2 = 25$

$y^2 = 25 - 16$

$y = \sqrt{9}$

$y = \pm 3$

$y = 3$

$x = 5$

$(5)^2 + y^2 = 25$

$y^2 = 25 - 25$

$y = \sqrt{0}$

$y = 0$

$\therefore R = \{(0,5), (3,4), (4,3), (5,0)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

Q2. Let $A = B = \{1,2,3,4,5\}$ Define functions $F: A \to B$ such that (i) F is one-one and onto

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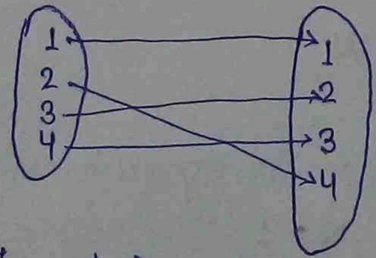
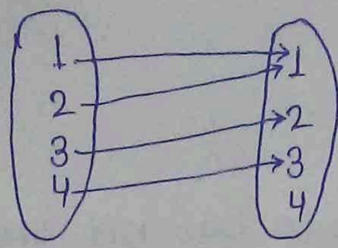


The ~~d~~ function $F = \{(1,1), (2,4), (3,2), (4,3)\}$ is one - one and onto

(ii) f is neither one-one nor onto.



The function $f = \{(1,1), (2,1), (3,2), (4,3)\}$

is neither one-one nor onto.

(iii) f is one-one but not onto.

→ The function is one-one but not possible to onto function.

Let function $f : N \to N$

and $f(x) = 2x$

Calculate, $f(x_1) = 2x_1$ and $f(x_2) = 2x_2$

Now,

$f(x_1) = f(x_2)$

$2x_1 = 2x_2$

$x_1 = x_2$

∴ The function f is one-one.

Now,

$f(x) = 2x$

Let,

$f(x) = y$ such that $y \in N$

$2x = y$

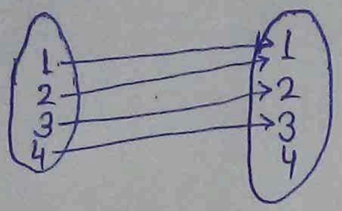
$x = \frac{y}{2}$

if, $y = 1 \Rightarrow x = \frac{1}{2} = 0.5 \notin N$

Hence f is not onto function.

(iv) F is onto but not one-one.

Sol^n:-



The function f which is onto but not one-one is not possible on the set $A = B = \{1, 2, 3, 4\}$

3. Prove that $A \cup (B-A) = A \cup B$

Sol^n:- To prove: $A \cup (B-A) = A \cup B$

L.H.S

$A \cup (B-A)$

$\Rightarrow A \cup (B \cap A^c)$ $\quad (\because A-B = A \cap B^c)$

$\Rightarrow (A \cup B) \cap (A \cup A^c)$

$\Rightarrow (A \cup B) \cap (U)$

$\Rightarrow A \cup B = R.H.S$

Hence, proved.