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Section-A

Q1. soln.- given that

$$(D^3-7D^2+6)y=0$$

$\therefore$ A.E is

$$D^3-7D^2+6=0$$

clearly $D=1$ is root of above eqn.

$\therefore$ synthetic division.

$$\begin{array}{c|cccc} 1 & 1 & -7 & 0 & 6 \\ & & 1 & -6 & -6 \\ \hline & 1 & -6 & -6 & 0 \end{array}$$

Remaining roots are given by

$$D^2-6D-6=0$$

$$D=\frac{6\pm\sqrt{36+24}}{2}$$

$$D=\frac{6\pm\sqrt{60}}{2}$$

$$D=\frac{\cancel{2}(3\pm\sqrt{15})}{\cancel{2}}=3\pm\sqrt{15}$$

$\therefore$ roots, $D=1,\ 3+\sqrt{15},\ 3-\sqrt{15}$

$\therefore$ C.F is

$$y_c=c_1e^{x}+c_2e^{(3+\sqrt{15})x}+c_3e^{(3-\sqrt{15})x}$$

Ans

Q2. solⁿ:- given that

$$(D^4+D^2+1)y=0$$

$\therefore$ A.E is

$$D^4+D^2+1=0$$

$$D^4+2D^2+1-D^2=0 \quad (\because \text{variable sepration})$$

$$(D^2+1)^2-D^2=0$$

$$(D^2+1+D)(D^2+1-D)=0$$

$D^2+1+D=0$	$D^2+1-D=0$
$D=\frac{1\pm\sqrt{1-4}}{2}$	$D=\frac{-1\pm\sqrt{1-4}}{2}$
$=\frac{1\pm i\sqrt{3}}{2}$	$=\frac{-1\pm i\sqrt{3}}{2}$

$\therefore$ same roots

$$\frac{1}{2}\pm i\frac{\sqrt{3}}{2}, \quad -\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$$

$\therefore$ C.F is

$$y_c=e^{\frac{1}{2}x}\left[c_1\cos\frac{\sqrt{3}}{2}x+c_2\sin\frac{\sqrt{3}}{2}x\right]+e^{-\frac{1}{2}x}\left[c_3\cos\frac{\sqrt{3}}{2}x+c_4\sin\frac{\sqrt{3}}{2}x\right]$$

Q3. solⁿ:- given that

$$(D^3-3D^2+4)y=e^{2x}$$

$\therefore$ A.E is

$$D^3-3D^2+4=0$$

Therefore clearly $D=-1$ is root of above eqⁿ.

$\therefore$ By synthetic division.

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 4 \\ & & -1 & 4 & -4 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

Remaining roots are given by

$$D^2-4D+4=0$$

$$D^2-2D-2D+4=0$$

$$D(D-2)-2(D-2)=0$$

$$(D-2)(D-2)=0$$

$$D=2,2$$

$\therefore$ roots, $D=-1,2,2$

$\therefore$ C.F is

$$y_c=c_1e^{-x}+e^{2x}[c_2+c_3x]$$

$$P.I=\frac{1}{D^3-3D^2+4}\cdot e^{2x} \quad \text{——} (1)$$

$$=e^{2x}\left[\frac{1}{(2)^3-3(2)^2+4}\right]=e^{2x}\frac{1}{8-12+4}=\frac{e^{2x}}{0}$$

$\therefore$ Rule fails in this case.

Again,

$$P.I=x\cdot\frac{1}{3D^2-6D}\cdot e^{2x}$$

$$=x\cdot\frac{1}{3(2)^2-6(2)}\cdot e^{2x}=x.e^{2x}\frac{1}{12-12}=xe^{2x}\frac{1}{0}$$

$\therefore$ Rule fails in this case.

Again

$$P.I=x^2\frac{1}{6D-6}\cdot e^{2x}$$

$$=x^2\frac{1}{6(2)-6}\cdot e^{2x}=x^2\cdot e^{2x}\left[\frac{1}{12-6}\right]$$

$$=x^2e^{2x}\left[\frac{1}{6}\right]$$

$\therefore$ CS = C.F + P.I

$$y=c_1e^{-x}+(c_2+xc_3)e^{2x}+x^2\frac{e^{2x}}{6}$$

Ans

Q4. Sol^n:- given that

$$P = \log(px - y).$$

~~$\log p = px - y$~~

$$\log_e(Px - y) = P$$

$$Px - y = e^P$$

differentiating both side with respect to x.

we get,

$$y = Px - e^P \quad \text{— (I)}$$

$$\frac{dy}{dx} = P + x\frac{dP}{dx} - e^P\frac{dP}{dx}$$

$$P = P + (x - e^P)\frac{dP}{dx} \qquad \left[\because \frac{dy}{dx} = P\right]$$

$$(x - e^P)\frac{dP}{dx} = 0 \quad \text{— (II)}$$

$(x - e^P) \neq 0$, when $\frac{dP}{dx} = 0$

$$\frac{dP}{dx} = 0$$

$$dP = 0\,dx$$

$\int dP = \int 0\,dx$ [$\because$ both side Integration]

$P = c$ [$\because c$ = Constant] — (III)

from eq^n (I) & (III)

$y = cx - e^c$ [$\because$ This is the general solution]

from (II), it $x - e^P = 0$

$$x = e^P$$

$$\log x = P$$

$$P = \log x \quad \text{— (IV)}$$

from (IV) & (I),

$$y = \log x \times x - e^{\log x}$$

$$= x\log x - x$$

$y = x(\log x - 1)$ [$\because$ This is the singular sol^n]

Q5 Soln: Given that

$$(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$$

which is of the from $Mdx+Ndy=0$

$M=x^2-4xy-2y^2$, $N=y^2-4xy-2x^2$

$\frac{\partial M}{\partial y}=-4x-4y$, $\frac{\partial N}{\partial x}=-4y-4x$

Now,

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$$

$\therefore$ given eqn. is exact diff. eqn.

$$\int\limits_{y\,\text{const}} M\,dx+\int(\text{terms of } N \text{ not containing } x)\,dy=c$$

$$\int(x^2-4xy-2y^2)dx+\int y^2dy=c$$

$$\frac{x^3}{3}-\frac{\cancel{4}^{2}x^2y}{\cancel{2}}-2xy^2+\frac{y^3}{3}=c$$

$$\frac{x^3}{3}-2x^2y-2xy^2+\frac{y^3}{3}=c$$

$$\frac{x^3-6x^2y-6xy^2+y^3}{3}=c$$

$$x^3-6x^2y-6xy^2+y^3=3c$$

$\therefore$ Hence, Required soln.

Q6. Solⁿ:- Given that

$$\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$$

which is of the from $Mdx + Ndy = 0$

$M = 1+e^{x/y}$, $N = e^{x/y}\left(1-\frac{x}{y}\right)$

$$\frac{\partial M}{\partial y} = e^{x/y}\left(-\frac{x}{y^2}\right),$$

$$\frac{\partial N}{\partial x} = e^{x/y}\left(\frac{1}{y}\right)\left(1-\frac{x}{y}\right) + e^{x/y}\left(-\frac{1}{y}\right)$$

$$\frac{\partial N}{\partial x} = e^{x/y}\left(\cancel{\frac{1}{y}} - \frac{x}{y^2} - \cancel{\frac{1}{y}}\right)$$

$$\frac{\partial N}{\partial x} = e^{x/y}\left(-\frac{x}{y}\right)$$

Now,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

∴ given Eqⁿ is Exact diff. Eqⁿ.

we solve is

$$\int_{y.con.} M dx + \int (\text{term of } N \text{ not Containg } x)\, dy = C$$

$$\int \left(1+e^{x/y}\right)dx + \int 0\, dy = C$$

$$\therefore x + e^{x/y} = C$$

Hence, Required eqⁿ <u>Ans</u>

Section-B

Q1. Solⁿ:- Given that

$$(D^2+D+1)y = (1+\sin x)^2$$

$\therefore$ A.E is

$$D^2+D+1=0$$

$$D = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$= -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$\therefore$ C.F is

$$y_c = e^{-\frac{1}{2}x}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$$

$$P.I = \frac{1}{(D^2+D+1)}[1+\sin x]^2$$

$$= \frac{1}{(D^2+D+1)}(1+\sin^2 x + 2\sin x)$$

$$= \frac{1}{(D^2+D+1)}\left(1+\frac{1-\cos 2x}{2} + 2\sin x\right)$$

$$= \frac{1}{(D^2+D+1)}\left[\frac{3}{2} - \frac{1}{2}\cos 2x + 2\sin x\right]$$

$$= \frac{3}{2}\frac{1}{(D^2+D+1)}\cdot e^{0x} - \frac{1}{2}\frac{1}{(D^2+D+1)}\cdot\cos 2x + 2\frac{1}{(D^2+D+1)}$$

$$P.I = \frac{3}{2}(I_1) - \frac{1}{2}(I_2) + 2(I_3) \quad -(1)$$

Now, $I_1 = \frac{1}{D^2+D+1}\cdot e^{0x} = e^{0x}\frac{1}{0+0+1} = e^{0x} = 1$

Now $I_2 = \frac{1}{(D^2+D+1)} \cdot \cos 2x$

$= \frac{1}{(-4+D+1)} \cdot \cos 2x$

$= \frac{1}{D-3} \cdot \cos 2x$

$= \frac{D+3}{(D-3)(D+3)} \cdot \cos 2x$

$= \frac{(D+3)}{D^2-9} \cdot \cos 2x$

$I_2 = \frac{-\sin 2x(2) + 3\cos 2x}{-4-9}$

$= \frac{-2\sin 2x + 3\cos 2x}{-13}$

$I_3 = \frac{1}{D^2+D+1} \cdot \sin x$

$= \frac{1 \cdot \sin x}{\cancel{-1}+D+\cancel{1}} = \frac{1}{D} \cdot \sin x$

$= -\cos D$

put I_1, I_2, I_3 value in (1)

$= \frac{3}{2}(1) - \frac{1}{2}\left[\frac{-2\sin 2x + 3\cos 2x}{-13}\right] + 2[-\cos x]$

$= \frac{3}{2} + \frac{1}{26}[-2\sin 2x + 3\cos 2x] - 2\cos x$

$\therefore$ C.S = C.F + P.I

$= e^{-1/2 x}\left[c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right] + \frac{3}{2} + \frac{1}{26}[-2\sin 2x + 3\cos 2x] - 2\cos x.$

Hence, Required Solⁿ. Ans

Q2. Solⁿ:- ~~p~~ Given that,

$$(px-y)(py+x) = 2p \quad \text{--- (1)}$$

put $x^2 = X$, $y^2 = Y$

$\therefore 2x\,dx = dX$, $2y\,dy = 2dY$

$\therefore \frac{dY}{dX} = \frac{y\,dy}{x\,dx}$

$\therefore P = \frac{\sqrt{\cancel{x}}}{\sqrt{y}} p.$

$\therefore$ from (1) we get.

$$\left[\frac{\sqrt{x}}{\sqrt{y}} P.\sqrt{x} - \sqrt{y}\right]\left[\frac{\sqrt{x}}{\sqrt{y}} P.\sqrt{Y} + \sqrt{\cancel{x}}\right] = 2\frac{\sqrt{x}}{\sqrt{Y}} P.$$

$$\left[\frac{XP-Y}{\cancel{\sqrt{Y}}}\right]\cancel{\sqrt{X}}\left[P+1\right] = 2\frac{\cancel{\sqrt{X}}}{\cancel{\sqrt{Y}}} P$$

$$[XP-Y][P+1] = 2P$$

$$XP - Y = \frac{2P}{P+1}$$

$$\cancel{Y} = \cancel{XP - 2P}$$

$$Y = XP - \frac{2P}{P+1}$$

which is clairaut's diff. eqⁿ.

$$\therefore Y = Xc - \frac{2c}{c+1}$$

$\therefore$ sol. of given is

$$y^2 = x^2 c - \frac{2c}{c+1} \quad \text{Ans}$$

Q3. Sol^n:- Given that

$$(xy^3+y)\,dx + 2(x^2y^2+x+y^4)\,dy = 0 \quad \text{—— (1)}$$

which is of the from $Mdx+Ndy=0$

$\therefore M = xy^3+y \quad , \quad N = 2(x^2y^2+x+y^4)$

$\frac{\partial M}{\partial y} = 3xy^2+1 \quad , \quad \frac{\partial N}{\partial x} = 2(2xy^2+1) = 4xy^2+2$

Now

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

$\therefore$ given Eq^n is not Exact diff. Eq^n.

But

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{4xy^2+\cancel{2}-3xy^2-\cancel{1}}{xy^3+y}$$

$$= \frac{\cancel{xy^2+1}}{y(\cancel{xy^2+1})} = \frac{1}{y}$$

$\therefore I.F = e^{\int f(y)dy} = e^{\int \frac{1}{y}dy} = e^{\log y}$

$= y$

$\therefore$ Multiply (1) with $I.F = y$

$$y(xy^3+y)\,dx + 2y(x^2y^2+x+y^4)\,dy = 0$$

$$(xy^4+y^2)\,dx + (2x^2y^3+2xy+2y^5)\,dy = 0$$

which is Exact diff. Eq^n. & Its Sol^n is

$$\int_{y\text{ Const.}} Mdx + \int (\text{term of } N \text{ not Containg } x)\,dy = c$$

$$\Rightarrow \int (xy^4 + y^2)\,dx + \int 2y^5\,dy = C$$

$$\Rightarrow \frac{x^2y^4}{2} + xy^2 + \frac{\cancel{2}y^6}{\cancel{6}\,3} = C$$

$$\Rightarrow \frac{x^2y^4}{2} + xy^2 + \frac{y^6}{3} = C$$

Hence, Required Solution. Ans