Note: Sum of degree of all the vertices of a graph G is Equal to twice the no. of edges in G.

Q. A graph G has 16 edges and all vertices of G are of degree 2. find the no. of vertices.

Sol: Let $v_1, v_2, v_3 \ldots v_n$ be the n vertices such that $\deg(v_i) = 2$; $1 \le i \le n$.

Since sum of deg. of all vertices is Equal to twice the no. of Edges i.e.

$$\Sigma \deg(v_i) = 2(\text{No. of Edges})$$

$\Rightarrow \deg(v_1) + \deg(v_2) + \ldots + \deg(v_n) = 2 \times 16$

$\Rightarrow \underbrace{(2+2+\ldots+2)}_{n \text{ times}} = 32$

$\Rightarrow 2n = 32$

$\Rightarrow \boxed{n = 16}$

$\therefore$ no. of vertices in graph G = 16.

Q. A graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. find the no. of vertices in G.

Sol. let n be no of vertices in G.

Since sum of deg. of all vertices is equal to twice the no. of edges

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i.e $\Sigma \deg(v_i) = 2$ (No. of Edges)

$\deg(v_1) + \deg(v_2) + \deg(v_3) + \dots + \deg(v_n) = 2$

$\Rightarrow \underbrace{4+4+4}_{3 \text{ times}} + \underbrace{(3+3+\dots+3)}_{(n-3) \text{ times}} = 42$

$\Rightarrow 12 + 3(n-3) = 42$

$\Rightarrow 12 + 3n - 9 = 42$

$\Rightarrow 3 + 3n = 42$

$\Rightarrow 3n = 39$

$\Rightarrow \boxed{n = 13}$

$\therefore$ No. of Vertices in Graph is 13.

Q) A graph has 5 Vertices, 2 of degree 3 and 3 of degree 2. find the no. of Edges.

Sol Since sum of deg. of all Vertices is equal to twice the no. of edges

$\therefore \Sigma \deg(v_i) = 2 \times$ no. of Edges

$\Rightarrow \deg v_1 + \deg v_2 + \deg v_3 + \deg v_4 + \deg v_5 = 2 \times$ no

$\Rightarrow 3+3+2+2+2 = 2 \times$ No. of Edges

$\Rightarrow 12 = 2 \times$ No. of Edges

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c) $\boxed{\text{No. of Edges} = 6}$

Q) How many nodes or vertices are required to construct a graph with exactly 6 Edges in which Each node is of degree 2.

Sol let n be no. of vertices.
Since sum of deg. of all vertices is Equal to twice the no of Edges
$\therefore \sum deg(v_i) = 2 \times \text{No. of Edges}$

$\Rightarrow deg\, v_1 + deg\, v_2 + deg\, v_3 + \cdots + deg\, v_n = 2 \times 6$

$\Rightarrow (2+2+2+ \cdots +2)\, n \text{ times} = 12$

$2n = 12$

$\boxed{n = 6}$

Q) Show that there does not Exist a graph with 5 vertices with degree 1, 3, 4, 2, 3 resp.

Sol Since sum of deg. of all vertices is Equal to twice the no. of edges.
of Edges
i.e. $\sum deg(v_i) = 2 \times \text{No. of Edges}$

$\Rightarrow deg\, v_1 + deg\, v_2 + deg\, v_3 + deg\, v_4 + deg\, v_5 = 2 \times \text{no. of Edges}$

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$1+3+4+2+3 = 2 \times$ No of Edges

$\frac{13}{2}$ = No of Edges

which is not possible.

Hence there does n't Exist a graph of this type.

Q Can there be a graph with 8 vertices and 29 edges.

Sol. Since $n = 8$ and $e = 29$

max. no. of Edges $= \frac{n(n-1)}{2} = \frac{8(8-1)}{2} = 28$

which is a Contradiction.

$\therefore$ Such a graph does n't Exist.

Q) How many vertices are there in a graph with 10 Edges if Each vertex has deg 2.

Sol. no. of edges i.e. $e = 10$

let n be no of vertices.

Sum of degree of all vertices is twice the no of Edges.

i.e. $\sum \deg(v_i) = 2 \times$ No of Edges

$\Rightarrow \deg v_1 + \deg v_2 + \dots + \deg(v_n) = 2 \times 10$

$\Rightarrow \underbrace{(2+2+ \dots +2)}$ n times $= 20$

$\Rightarrow 2n = 20$

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$\Rightarrow$ $\boxed{n=10}$

$\therefore$ There will be 10 vertices.

Q) Does there exist a graph with two vertices each of degree 4? If so, draw it.

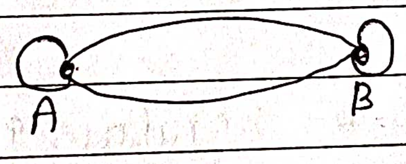
Sol^n: Here $n = 2$

Let e be no. of edges & each vertex is of degree 4.

Sum of deg of all vertices $= 2 \times e$

$$4+4 = 2e$$

$$\Rightarrow \boxed{e = 4}$$



Yes there exist a graph of this type.

Q) Determine whether it is possible to construct a graph with 12 edges such that 2 of the vertices have deg 3 and the remaining vertices have deg 4.

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Sol Let n be no. of vertices; then

$$\sum \deg(v_i) = 2 \times \text{no. of Edges}$$

$$\Rightarrow \deg(v_1) + \deg(v_2) + \deg v_3 + \deg v_4 + \cdots + \deg v_n = 2 \times 12$$

$$(3+3) + \underbrace{(4+4+\cdots+(4))}_{(n-2)\text{ times}} = 24$$

$$\Rightarrow 6 + 4(n-2) = 24$$

$$\Rightarrow 6 + 4n - 8 = 24$$

$$\Rightarrow 4n = 26$$

$n = \frac{26}{4} = \frac{13}{2}$ which is not possible.

Q Find k if a k-regular graph with 8 vertices has 12 Edges. Also draw k-regular graph.

Sol Given $V = 8$ & $E = 12$

$\therefore$ Sum of deg of all vertices $= 2 \times E$

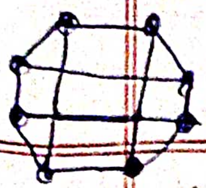
all vertices have same deg

let deg of each vertex is k

$$\Rightarrow \underbrace{\deg v_1 + \deg v_2 + \cdots + \deg v_8}_{k+k+\cdots\ 8\text{ times}} = 2 \times 12 \quad \Rightarrow 8k = 24$$

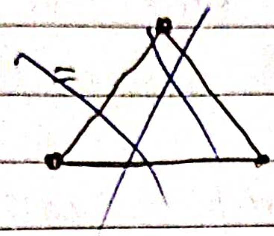
$$\Rightarrow \text{deg of Each vertex } (k) = \frac{24}{8}$$

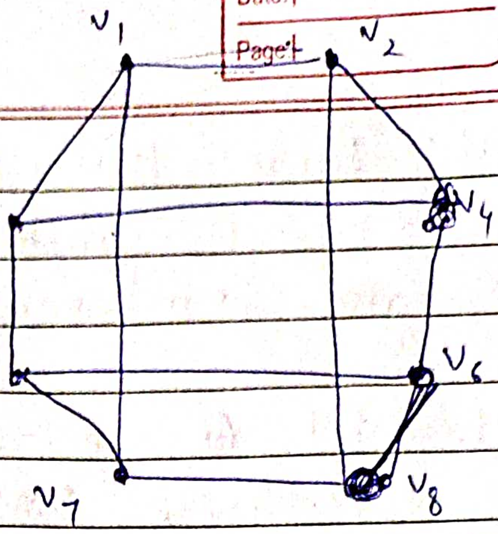
$$\Rightarrow \text{deg of Each vertex}(k) = 3$$



$\therefore \boxed{k = 3}$

$k_3 =$





Art: Prove that in a simple graph with n vertices, each vertex has maximum degree $n-1$.

Sol A simple graph is a graph without parallel edges and self loops.

If $G = [V, E]$ is a simple graph with only one vertex then no. of edges in G is zero.

$\therefore$ Maximum degree of a vertex in G
$= 1 - 1 = 0$

If $G = [V, E]$ is a simple graph with two vertices then no of edges in G is 1 i.e. $2-1$
and deg of each vertex is $2-1$.

. . . . — — — . . —

If $G = [V, E]$ be a simple graph with n vertices then maximum degree of each vertex is $n-1$.

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Art: Prove that maximum degree of edges in a graph G with n vertices and no multiple edges are $\frac{n(n-1)}{2}$.

Sol. Let G be a graph with n vertices, then degree of each vertex in G is $\leq n-1$.

$\therefore$ Sum of degree of n vertices in $G \leq n(n-1)$

$\Rightarrow \quad 2e \leq n(n-1)$ $\quad \because$ [Sum of deg of all vertices $= 2e$]

$\Rightarrow \quad e \leq \frac{n(n-1)}{2}$ which e is no. of edges in G.

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Path in a graph

A path of length n is a sequence of n+1 vertices of a graph in which each pair of vertices is an edge of the path

Simple path -> The path is called simple path if no edge is repeated in the path i.e. all the vertices are distinct except that first vertex equal to last vertex.

An Elementary path -> The path is called elementary if no vertex is repeated in the path i.e. all vertices are distinct.

circuit or closed path ->
The circuit or closed path is a path which starts & End at the same vertex i.e. $V_0 = V_n$.

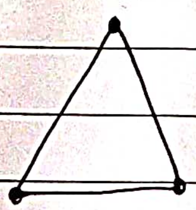
Simple circuit path: The simple circuit is a simple path which is a circuit.

Undirected complete Graph

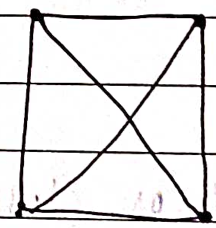
An undirected Complete graph $G=(V,E)$ of n vertices is a graph in which each vertex is connected to every other vertex i.e. an edge exists between every pair of distinct vertices. It is denoted by K_n.

A complete graph with n vertices has $\frac{n(n-1)}{2}$ edges.

E.g. of The complete graph is as follow.



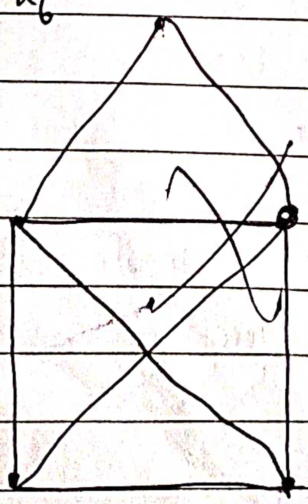
K_3



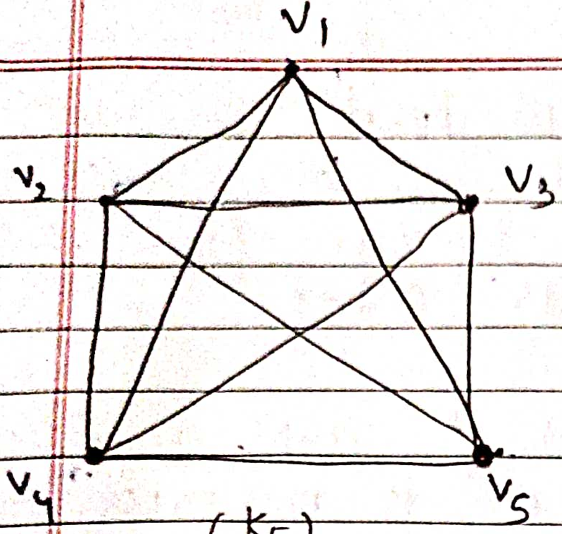
K_4

Q) Draw undirected Complete graphs ~~for~~ K_5 K_6

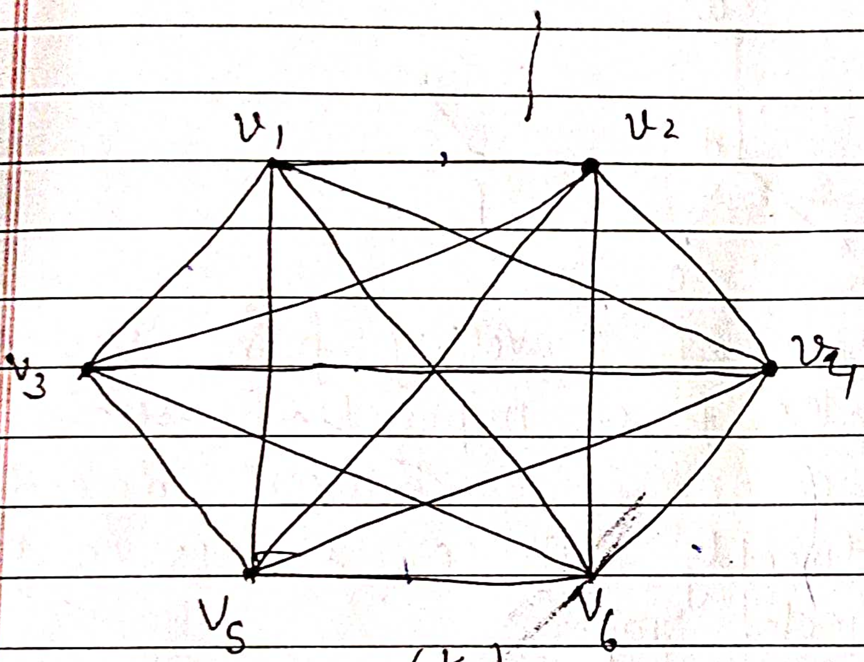
Sol



$\frac{5(4)}{2} = 10$



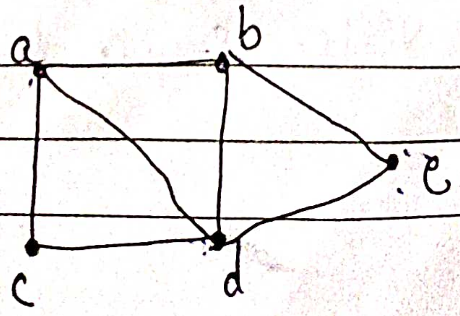
(K_5)



(K_6)

✓. <u>Connected Graph</u> → A graph is called connected if there is a path from any vertex u to v or vice-versa.

for eg.



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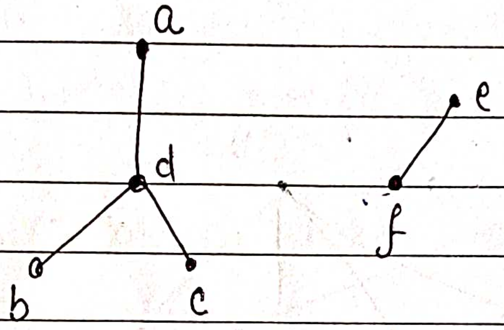
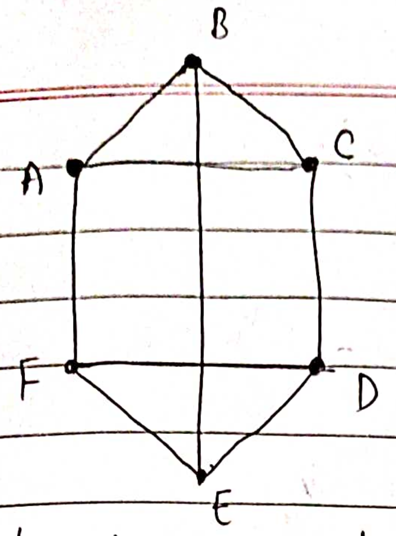
In this graph it is possible to travel from one vertex to any other vertex. for e.g. one can traverse from a to vertex e using path "a-b-e".

Disconnected graph → A graph is called disconnected if there is no path between any two of its vertices.

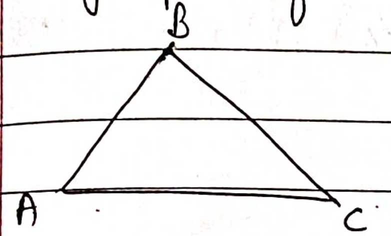


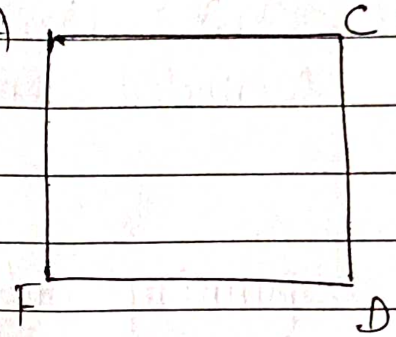
In above e.g. travelling from vertex a to vertex f is impossible because there is no path between directly or indirectly. Hence it is a disconnected graph.

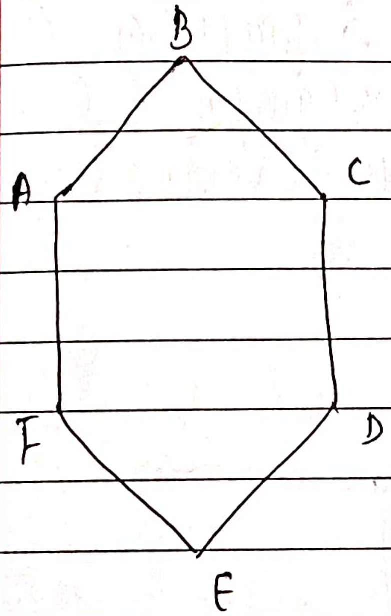
Subgraph → A subgraph of graph $G = (V, E)$ is a graph $G' = (V', E')$ where $V' \subseteq V$ & $E' \subseteq E$ & each edge of G' has same end vertices in G' as in graph G.
A single vertex is a subgraph.
e.g.

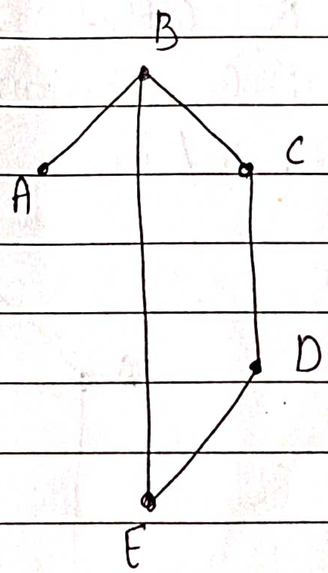


few Subgraph of above graph are:

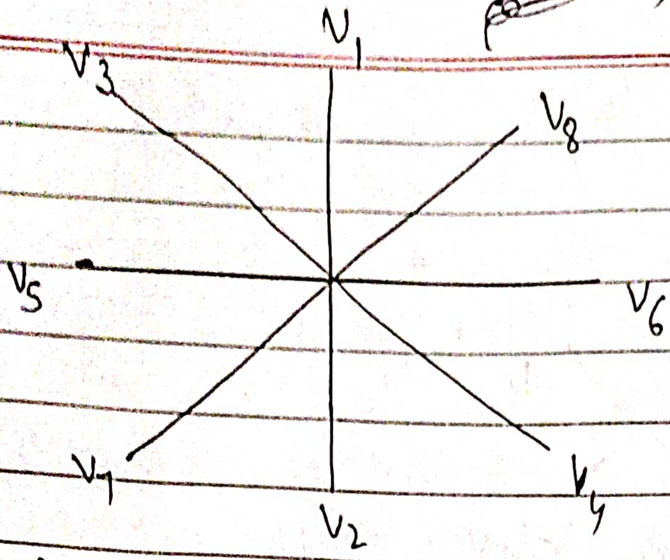








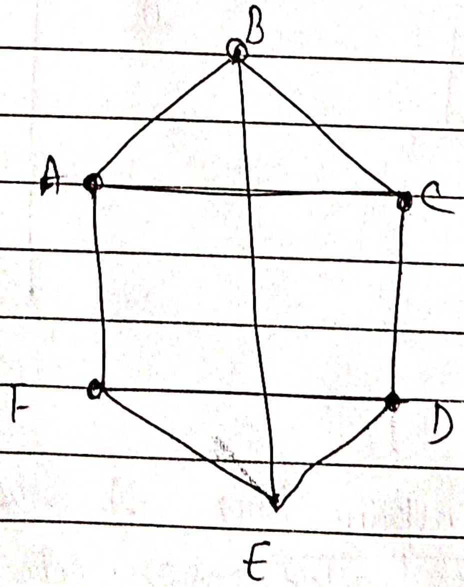
Connected Component → A subgraph of graph G is called connected Component of G, if it is not contained in bigger subgroup of G, which is Connected. It is defined by listing its vertices.



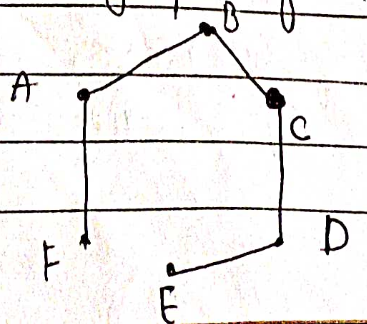
i.e. $\{v_1, v_2\}$, $\{v_3, v_4\}$, $\{v_5, v_6\}$, $\{v_7, v_8\}$ are connected components

Spanning Subgraph → A graph $G_1 = (V_1, E_1)$ is called spanning subgraph of $G = (V, E)$ if G_1 contains all the vertices of G & $E \neq E_1$.

For E.g.



Spanning Subgraph of above graph is

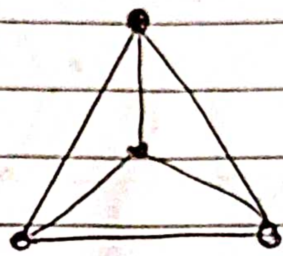


Complement of a subgraph :

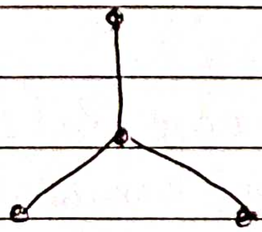
Let $G = (V, E)$ be a graph & S be a subgraph of G. If edge set of S be deleted from graph G, the graph so obtained is Complement of subgraph S. It is denoted by $\bar{S}$.

$\therefore \bar{S} = G - S$

e.g. Consider graph

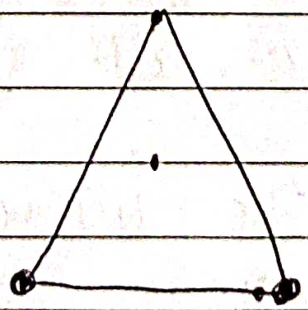


& its subgraph



Then Compliment of subgraph S is

$\bar{S} =$



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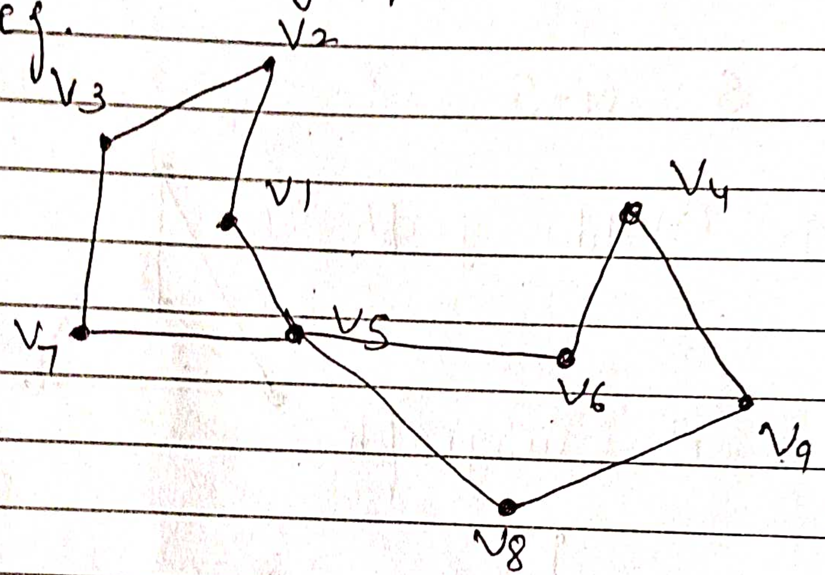
Cut-Set : consider a graph G = [V, E]. A cut set for G is a smallest set of edges such that removal of the set, disconnects the graph whereas the removal of any proper subset of this set, left a connected subgraph.
for eg.



for this graph the edge set {(V1, V5) (V7, V5)} is a cut set. After removal of this set we have left with a ~~subc~~ disconnected subgraph.
While after the removal of any of its proper subset we have left with a connected subgraph.

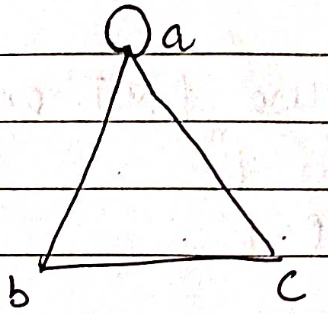
Cut points or cut vertices → consider a graph G = (V, E). A cut point for a graph G is a vertex v s.t G-v has more connected components than G or disconnected

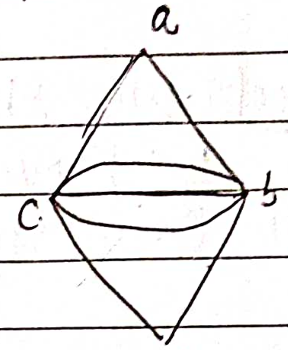
The subgraph G-v is obtained by deleting the vertex v from the graph & also deleting all the edges incident on v.

Order and Size of a graph →

Let G be a graph. The no. of vertices in graph G is called order of G. The no of edges in Graph G is called size of G.

for eg. Consider the graph



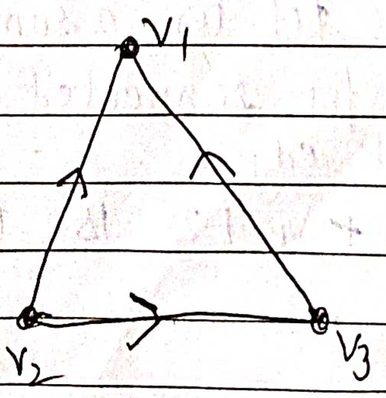


Order of G = 3
Size of G = 4

Order = 4
Size = 7

Directed complete graph → A directed complete graph G = (V, E) on n vertices is a graph in which each vertex is connected to every other vertex by an arrow. It is denoted by k_n.

for. Eg. Directed complete graph k_3.



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Q.1 State and prove Euler's theorem

Statement Let G be a connected planar simple graph with e edges and v vertices. Let r be the no. of regions then

$$r = e - v + 2$$

Proof We shall use P.M.I. on no. of Edges to prove this thm.

(i) Basic Step: $e = 1$

If $e=1$ then we have two vertices & one region i.e. $v = 2$ & $r = 1$

$\therefore \quad r_1 = e_1 - V_1 + 2$

$\Rightarrow \quad 1 = 1 - 2 + 2$

$\Rightarrow \quad 1 = 1$

which is true

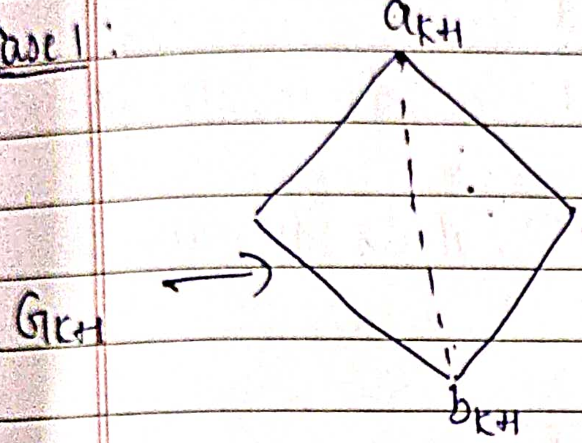
ii) Induction Step: Let us assume that formula holds for connected planar graph with k edges.

i.e. $r_K = e_K - V_K + 2$ is true for G_K.

iii) Verification for $e = k+1$ edges.

Let (a_{k+1}, b_{k+1}) be the edges that is added to G_k.

Case 1:



both the vertices are in G_k.

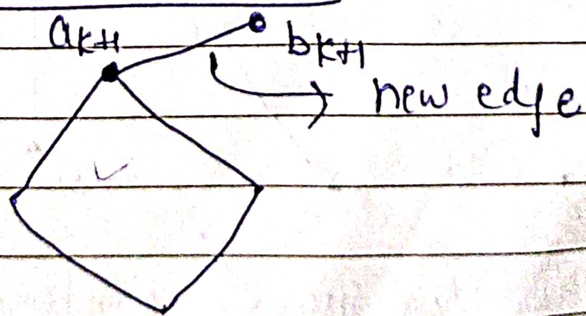
$$\therefore r_{k+1} = r_k + 1$$
$$e_{k+1} = e_k + 1$$
$$v_{k+1} = v_k \quad \because [\text{both the vertices are in } G_k]$$

$$\therefore r_{k+1} = e_{k+1} - v_{k+1} + 2$$

$$\Rightarrow r_k \cancel{+1} = e_k \cancel{+1} - v_k + 2$$

$$\Rightarrow \boxed{r_k = e_k - v_k + 2} \text{ which is true}$$

Case 2:



$$r_{k+1} = r_k$$
$$v_{k+1} = v_k + 1$$
$$E_{k+1} = E_k + 1$$

$$r_{k+1} = e_{k+1} - v_{k+1} + 2$$

2) $r_k = e_k + 1 - (v_k + 1) - 2$

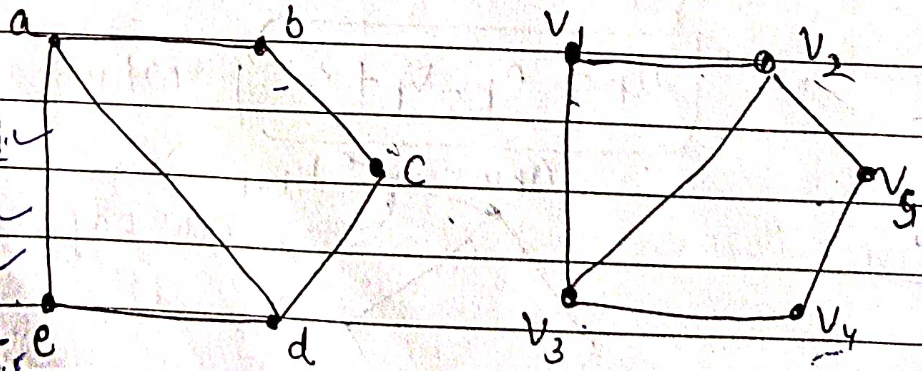
$\Rightarrow$ $\boxed{r_k = e_k - v_k + 2}$ $\rightarrow$ true

$\therefore$ By P.M.I. result is true for $e = k+1$

By induction method Euler's formula is proved.

Isomorphic graphs: Two graphs G_1 & G_2 are called isomorphic graphs if there is a one to one correspondence between their vertices and between their edges i.e. graphs have identical representation except that vertices may have different labels.

for e.g.



$d(a) = 3$ $d(v_1) = 2$

$d(b) = 2$ $d(v_2) = 3$

$d(c) = 2$ $d(v_3) = 3$

$d(d) = 3$ $d(v_4) = 2$

$d(e) = 2$ $d(v_5) = 2$

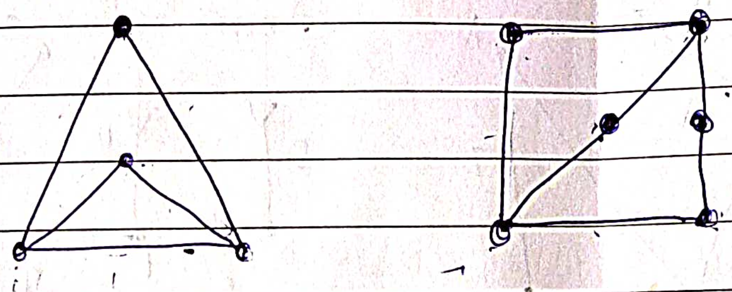
Since both the graph contains vertices having same deg i.e. $d(a) \leftrightarrow d(v_2)$, $d(d) \leftrightarrow d(v_3)$, $d(b) \leftrightarrow d(v_1)$, $d(e) \leftrightarrow d(v_4)$

$d(c) \leftrightarrow d(v_5)$.

Hence these graphs are isomorphic.

Homeomorphic graphs: Two graphs G_1 & G_2 are called homeomorphic graph if G_2 can be obtained from G_1 by a sequence of subdivisions of the edges of G_1. In other words we can introduce vertices of degree 2 in any edge of graph G_1.

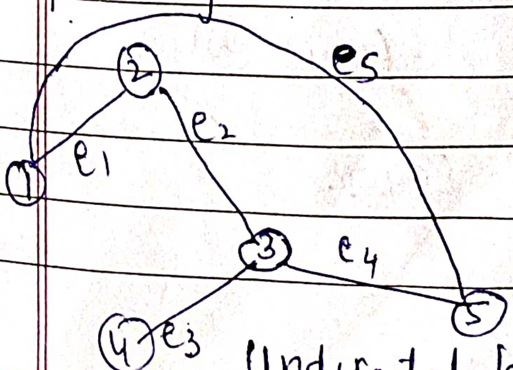
for E.g.



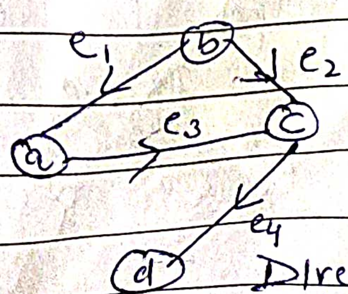
these are homeomorphic graphs.

Labelled graph: A graph $G=(V,E)$ is called labelled graph if its edges are labelled with some name or data so we can write these labels in place of an ordered pair in the edge set

for E.g.



Undirected labelled graph



Directed Lab. graph

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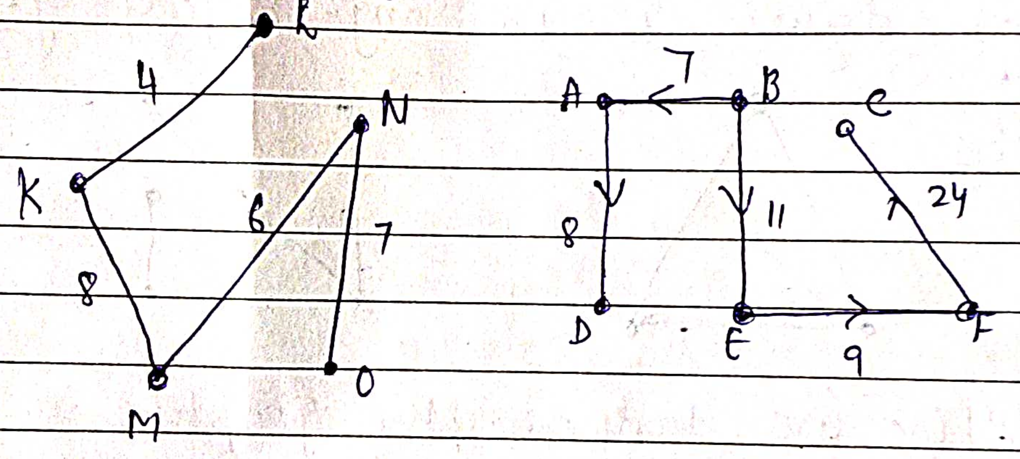
graph $G = \{\{1,2,3,4,5\}, \{e_1, e_2, e_3, e_4, e_5\}\}$ &
$G = [\{a,b,c,d\} \; \{e_1, e_2, e_3, e_4\}]$ are
labelled graphs.

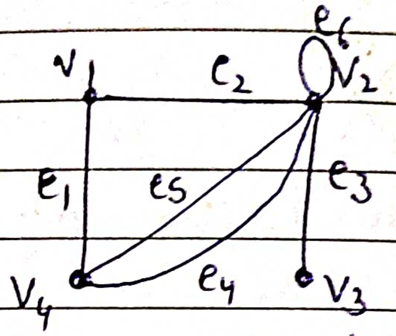
Weighted graphs: A graph $G = (V, E)$ is called weighted graph if each Edge of graph G is assigned a positive no. w called the weight of edge e. for e.g.



These are undirected & directed Weighted graphs.

Multiple edges: Two edges e_1 & e_1' which are distinct are said to be multiple edges if they connect the End points i.e. if $e_1 = (u,v)$ & $e_1' = (u,v)$ where e_1 & e_1' are Multiple Edges.

Multigraph: A multigraph $G=(V,E)$ consist of a set of vertices V & set of edges E such that edge set E may contains multiple edges & self loops.
for Eg.

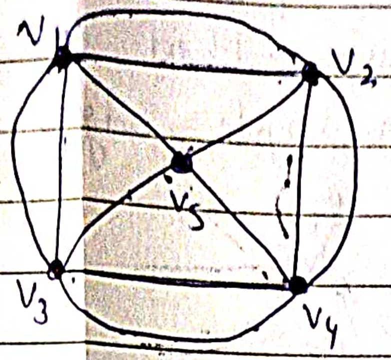


the above graph is an undirected Multigraph. where e_4 & e_5 are multiple Edges & e_6 is a self loop.

Traversable Multigraphs: consider a multigraph $G=(V,E)$. If the multigraph G consists a path which includes all the vertices & whose Edge set may Contain each Edge of graph exactly once. Then multigraph G is called traversable multigraph.

The sufficient & necessary condition for a multigraph to be traversable is that it should be connected & having zero or two vertices of odd degree.

Consider the multigraph



The multigraph has 3 Even deg. vertices i.e V_3, V_4 & V_5 & two odd deg. vertices V_1 & V_2.

Hence it is a traversable Multigraph.

Representation of graphs

Two ways to represent a graph G with matrices:

Adjacency matrix representation
Incidence matrix representation.

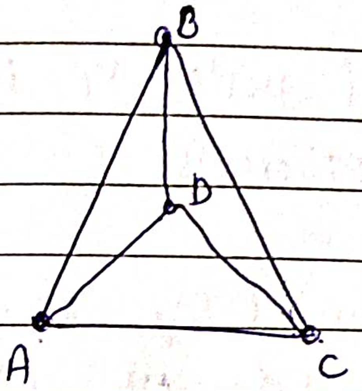
a) Representation of undirected graph

(i) Adjacency matrix Representation: If an undirected graph G consist of n vertices then adjacency matrix of graph is $n \times n$ matrix $A = [a_{ij}]$ & defined as

$$a_{ij} = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an Edge i.e. } v_i \text{ is adjacent to } v_j \\ 0 & \text{if there is no edges b/w } v_i \text{ \& } v_j \end{cases}$$

Since adjacency matrix contains 0 or 1, so it is also called Boolean matrix.

e.g.



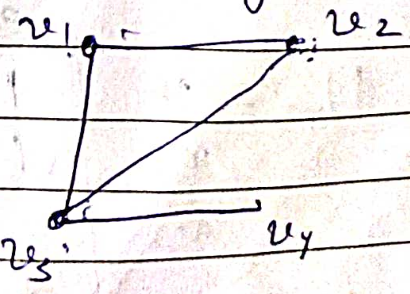
Since graph consist of 4 vertices so adjacency matrix will be a 4X4 matrix.

i.e.

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Adjacency List → In adjacency list we list each vertex followed by vertices adjacent to it.

for e.g.



v_1 ; v_2, v_3

v_2 ; v_1, v_3, v_4

v_3 ; v_1, v_2, v_4

v_4 ; v_3

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→ need not to be square matrix

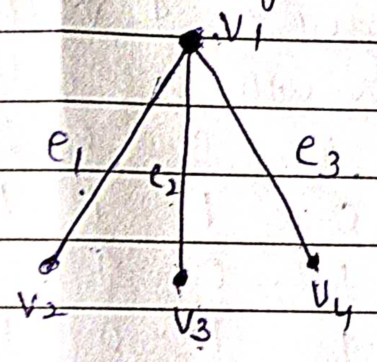
ii) Incidence Matrix representation →

If an undirected graph consist of n vertices & m Edges then incidence matrix ~~vertex~~ is an $n \times m$ matrix $C = [c_{ij}]$ defined by

$$c_{ij} = \begin{cases} 1; & \text{if vertex } v_i \text{ incident by Edge } e_j \\ 0, & \text{otherwise} \end{cases}$$

There is a row for every vertex & a column for every edge in incidence matrix

for e.g.



consider a graph $G = [V, E]$

$V = \{v_1, v_2, v_3, v_4\}$.

$E = \{e_1, e_2, e_3\}$

The Incidence matrix is

$$M_I = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

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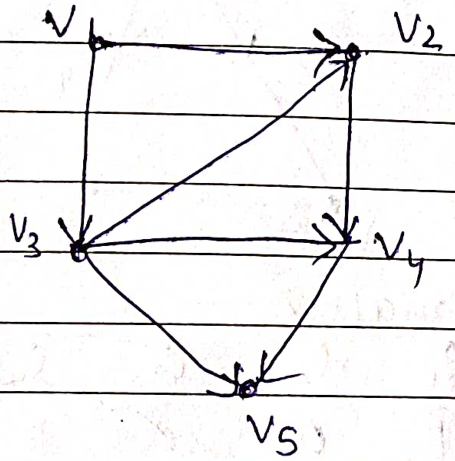
(b) Represenatation of Directed graph

(i) Adjacency matrix representation :
If Directed graph G consist of n vertices then adjacency matrix of G is $n \times n$ matrix $A = [a_{ij}]$ defined by

$$a_{ij} = \begin{cases} 1 & \text{if there is an Edge b/w } v_i, v_j \text{ i.e. if } v_i \text{ is initial } \& \ v_j \text{ is final vertex} \\ 0 & \text{if there is no Edge b/w } v_i \ \& \ v_j \end{cases}$$

for. Eg. consider the directed graph



Since graph consist of 5 five vertices so matrix is

$$M_A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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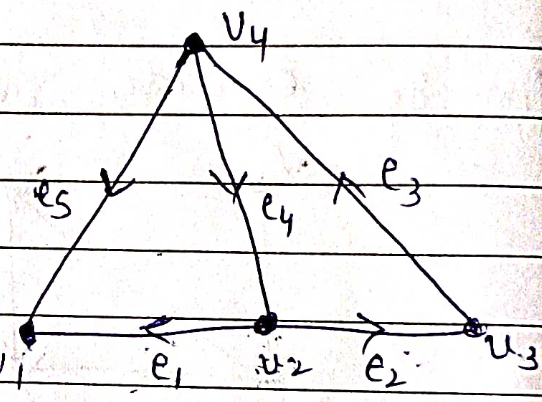
ii) Incidence Matrix Representation:

If directed graph consist of n vertices & m edges then incidence matrix is an $n \times m$ matrix $C = [c_{ij}]$ defined by

$$c_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is initial vertex of edge } e_j \\ -1 & \text{if } v_i \text{ is final vertex of edge } e_j \\ 0 & \text{if } v_i \text{ is not incident on edge } e_j \end{cases}$$

eg: Consider a graph



The incidence matrix

$$M_I = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & -1 \\ +1 & 1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \end{array}$$

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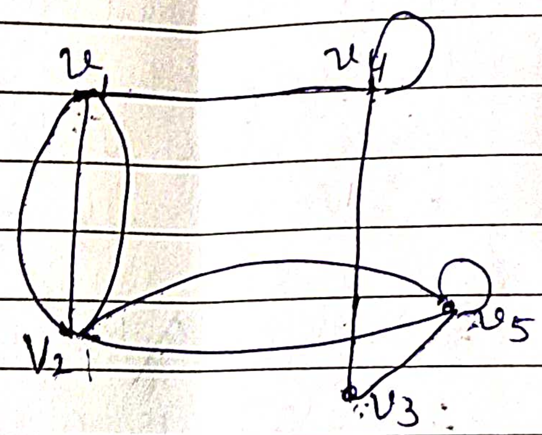
(c) Representation of Multigraph: It is only represented by adjacency matrix.

If Multigraph G Consist of n vertices then adjacency matrix is $n \times n$ matrix $A = [a_{ij}]$ defined as

$$a_{ij} = \begin{cases} N; & \text{if there are one or more than One edge b/w } v_i \text{ \& } v_j \text{ vertex where } N \text{ is no of Edges} \\ 0; & \text{otherwise.} \end{cases}$$

e.g. consider graph



$$M_A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

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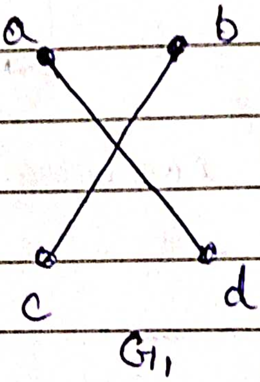
Bipartite graph: A graph $G=(V,E)$ is called bipartite graph if its vertices V can be partitioned into two subsets V_1 & V_2 s.t. Each Edge of G Connect a vertex of V_1 to vertex of V_2.
In other words no edge joining two vertices in V_1 or two vertices in V_2.
It is denoted by $K_{m,n}$ where m & n are no. of vertices in V_1 & V_2 resp.

for Eg. Consider a graph
the graph G_1 is bipartite graph have $V=\{a,b,c,d\}$
$V_1=\{a,b\}$ & $V_2=\{c,d\}$



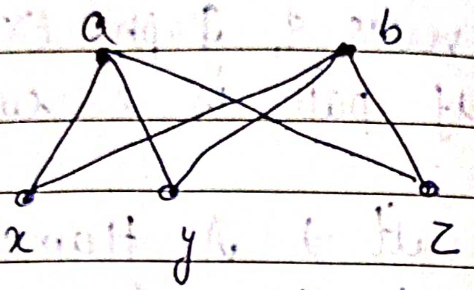
$\therefore V_1 \cup V_2 = \{a,b,c,d\}$ & $V_1 \cap V_2 = \phi$
$\therefore [V_1, V_2]$ is a partition of V.

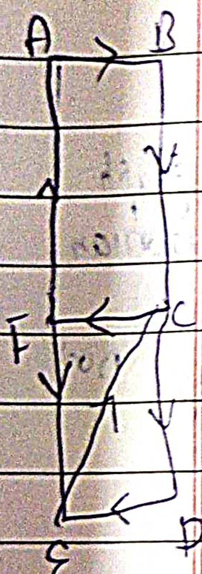
Complete Bipartite graph $\rightarrow$ A graph $G=(V,E)$ is called Complete bipartite graph if its vertices can be partitioned into two subsets V_1 & V_2 such that Each vertex of V_1 is Connected to Each vertex of V_2.
The no. of edges in Complete bipartite graph is $m.n$. It is denoted by K_{mn} & $m \leq n$.

for Eg $V = [a, b, x, y, z]$

$V_1 = [a, b]$ & $V_2 = [x, y, z]$

$V_1 \cap V_2 = \phi$ $\quad V_1 \cup V_2 = V$.





Euler Path: A Euler path through a graph is a path whose Edge list contains each edge of graph Exactly once.

Euler Circuit: An Euler circuit is a path through a graph, in which initial vertex appears second time as the terminal vertex.

Euler Graph: Euler graph is a Graph that possesses an Euler circuit. An Euler graph contains each Edge Exactly once but vertices may be repeated.

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Hamiltonian Path or Chain :

A hamiltonian path (or chain) through a graph is a path whose vertex list contain every vertex of graph exactly once, except if path is a circuit.

Hamiltonian Circuit → A Hamiltonian circuit is a path in which initial vertex appears a second time as the terminal vertex.

Hamiltonian Graph → A Hamiltonian graph is a graph that possesses a Hamiltonian path. A hamiltonian path uses each vertex exactly once but edges may not be included.

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Hamiltonian Path or chain

A hamiltonian path (or chain) through a graph is a path whose vertex list contain every vertex of graph exactly once, except if path is a circuit.

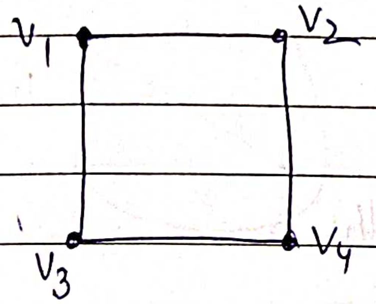
Hamiltonian Circuit → A Hamiltonian circuit is a path in which initial vertex appears a second time, as the terminal vertex.

Hamiltonian Graph → A Hamiltonian graph is a graph that possesses a Hamiltonian path. A hamiltonian path uses each vertex exactly once but edges may not be included.

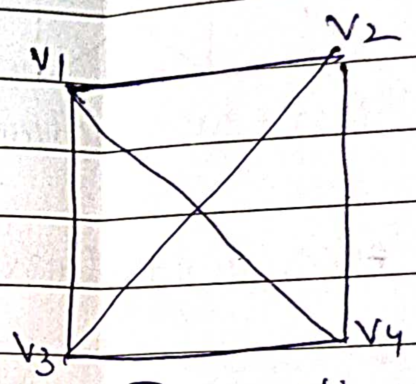
Regular graph: A graph is said to be regular or K-regular if all its vertices have same degree k. A graph whose all vertices have degree 2 is called 2-regular graph.

A complete graph K_n is a regular of degree $n-1$.

eg.

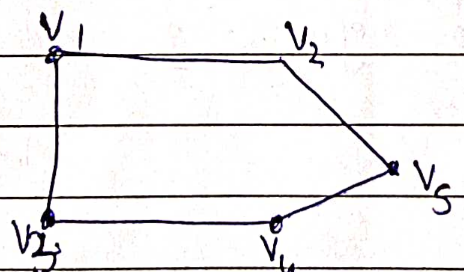


regular graph of deg-2 or 2-Regular graph.



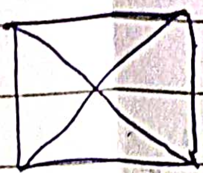
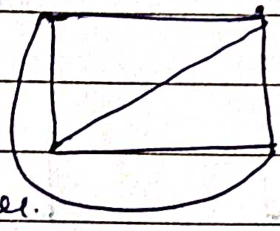
Reg. graph with degree 3

Q) Draw 2 Regular graph with 5 vertices.



Planar Graph → A graph is said to be planar if it can be drawn in a plane so that no edges cross.

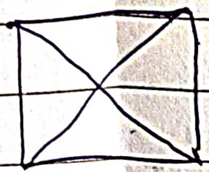
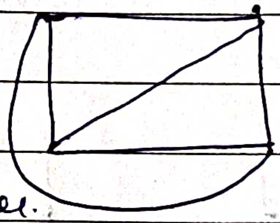
for E.g.  is a planar graph.

Also  is a planar graph as it can be redrawn as  in which edges don't cross each other.

Region of a graph : Consider a planar graph $G=(V,E)$. A region is defined to be an area of the plane that is bounded by edges and can't be further subdivided. A planar graph divides the plane into two or more regions.

Planar Graph → A graph is said to be planar if it can be drawn in a plane so that no edges cross.

for E.g.  is a planar graph.

Also  is a planar graph as it can be redrawn as  in which edges don't cross each other.

Region of a graph : Consider a planar graph G=(V,E). A region is defined to be an area of the plane that is bounded by edges and can't be further subdivided. A planar graph divides the plane into two or more regions.

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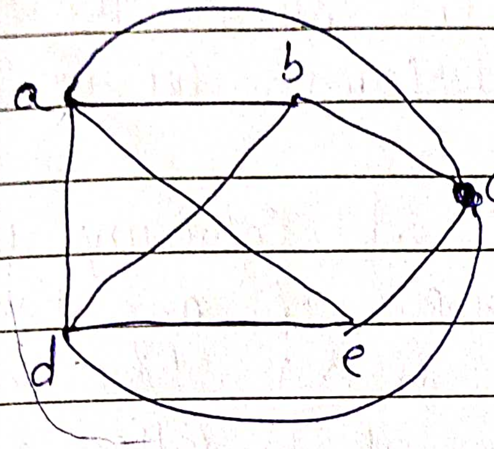
Non planar graphs: A graph is said to be non planar if it can't be drawn in a plane so that no edges cross.

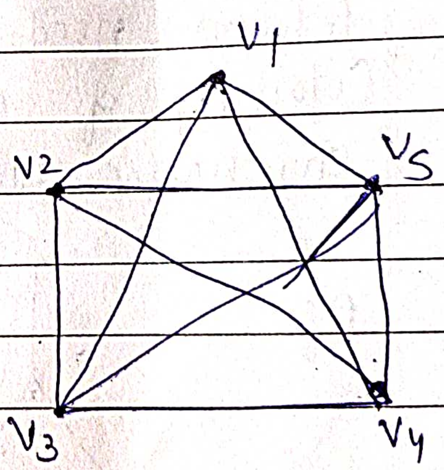
e.g.



Q. Show that K_5 is non-planar.

Sol Clearly K_5 is Connected. Also, we show K_5 is non-planar

for $v = 5$ & $e = 10$.



If K_5 is planar

$e \leq 3v - 6$

$10 \leq 3 \times 15 - 6$

$10 \leq 15 - 6$

$10 \leq 9$, a Contradiction.

$\therefore$ The Graph K_5 is non-planar.

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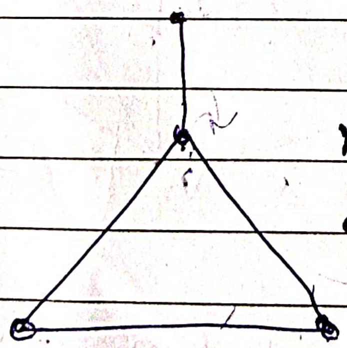
Graph Colouring → Suppose a graph $G=(V,E)$ with no multiple edges. A vertex Colouring of G is an assignment of Colours to the vertices of G such that adjacent vertices have different Colours.

Proper Colouring : A Colouring is said to be proper if any two adjacent vertices u & v have different Colours otherwise it is called improper Colouring.

CHROMATIC NUMBER → The minimum no. of Colours needs to produce a proper Colouring of a graph G is called the chromatic no. of G.

e.g.



This graph is minimum - 3 colourable so its chromatic no. is 3

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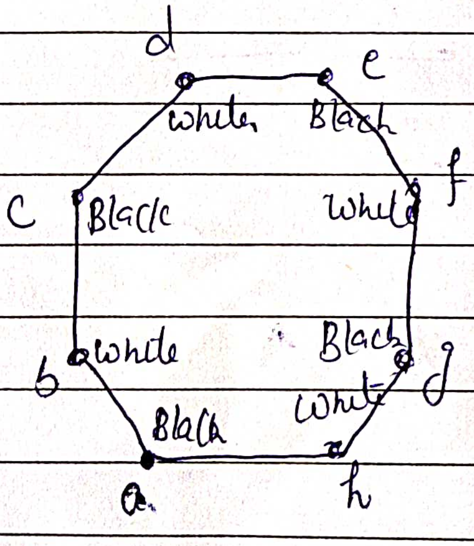
E.g. The chromatic number of a Graph C_n where C_n is cycle with n vertices is either 2 or 3.

Sol: Two colours are needed to colour C_n where C_n is Even.
To construct such a colouring simply pick a vertex & colour it Black. Then move around the graph in clock-wise direction colouring the second vertex white, the third vertex black & so on. The n^{th} vertex can be coloured white since two vertices are adjacent to it, namely $(n-1)^{th}$ & the 1st are both coloured black as shown in fig.



When n is odd & $n>1$ the chromatic no. of C_n is 3. To construct such a colouring pick initial vertex. first use

only 2 colours as graph is traversed in clockwise direction.
However nth vertex reached is adjacent to two vertices of different colours, the 1st & (n-1)th. Hence 3rd colour is needed as shown in fig.