Date.|
Page.|

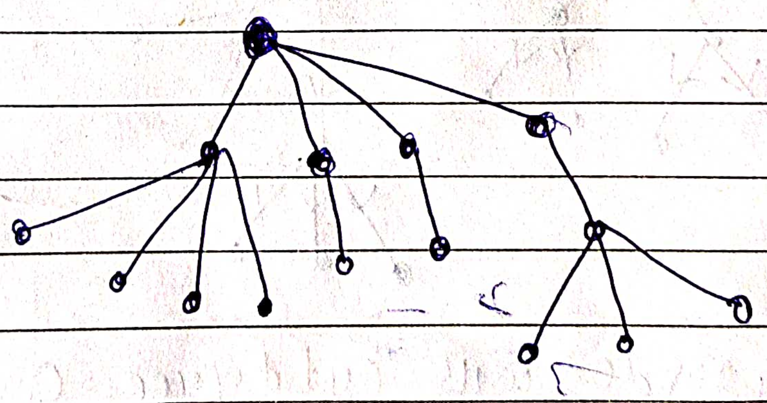
Trees

A graph which has no cycles is called an acyclic graph.

A tree is an acyclic graph or graph having no cycles.

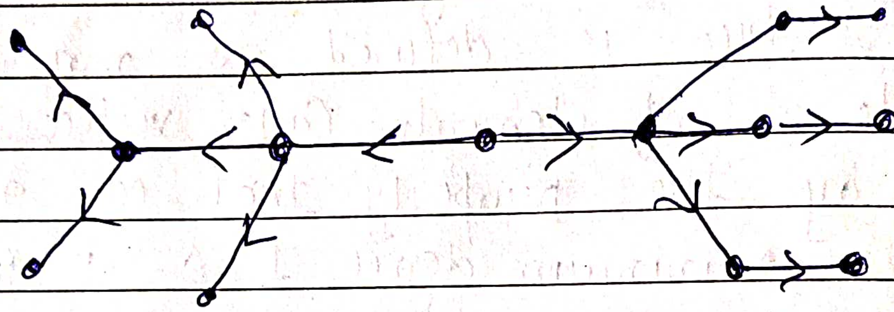
A tree is defined as non empty finite set of elements called vertices or nodes having the property that each node can have minimum degree 1 & maximum degree n.

It can be partitioned into n+1 disjoint sets such that 1st subset contains root of tree & remaining n subsets contains the elements of n subtree.

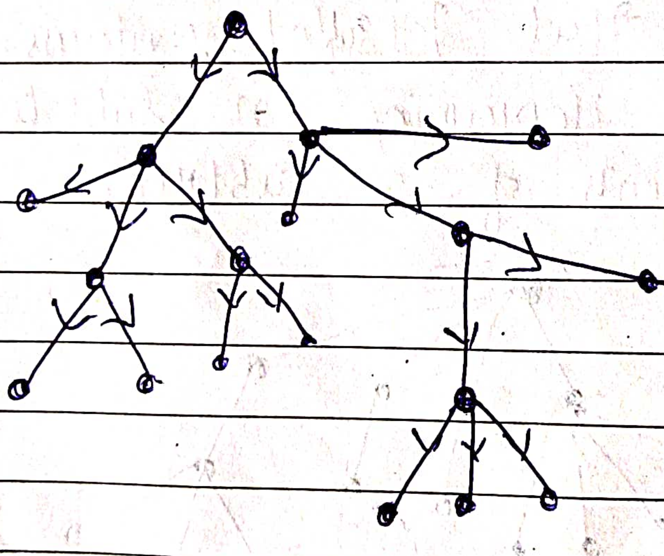


General - tree

Directed tree: It is an acyclic directed graph. It has one node with indegree 1 while other nodes having indegree & outdegree as shown in fig.



or

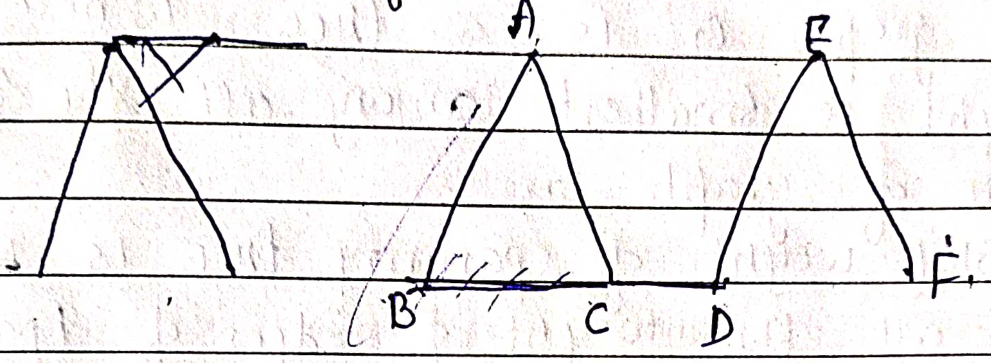


The node with outdegree 0 is called an external node or terminal node or a leaf.

The node which has outdegree greater than or equal to one are called internal nodes or branch nodes.

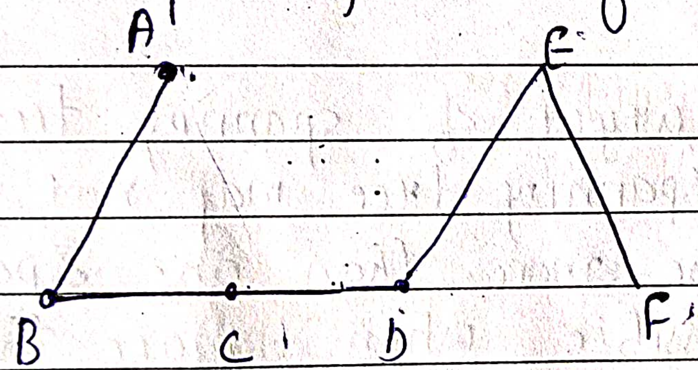
<u>Spanning tree</u>: Consider a Graph $G=(V,E)$
A Spanning tree T is defined as Subgroup of G if it is a Tree & <u>T includes all vertices</u> of G.

e.g

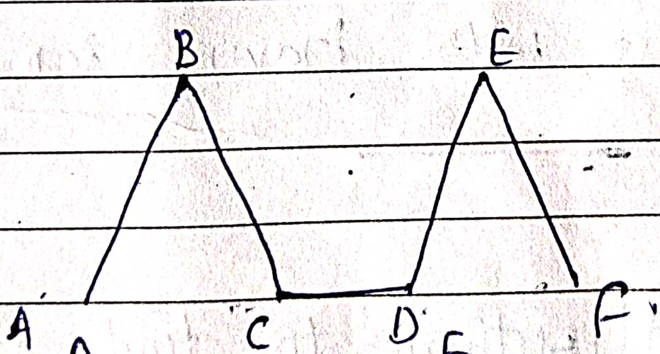


The Spanning trees of graph G are

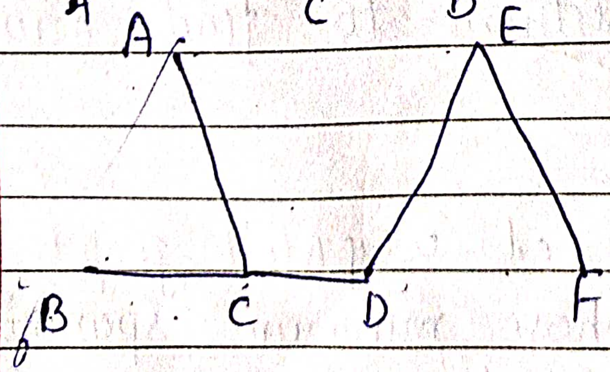
(i)



ii)



iii)



Date. |
Page. |

Minimum Spanning tree:

Consider a weighted connected graph $G=(V,E)$. A minimal Spanning tree T of graph G is a tree whose total weight is Smallest among all the Spanning tree of graph G.
Total weight of spanning tree is ~~sum~~ Sum of all the weight of Edges of Spanning trees.

The minimum weight of spanning tree is unique but spanning tree may not be unique because more than one Spanning tree are possible when more than one Edge Exist ~~with~~ having same weight

Kruskal's Algorithm to find minimum Spanning tree

1) Input the connected weighted graph G with n vertices whose minimum Spanning tree T, we want to find.

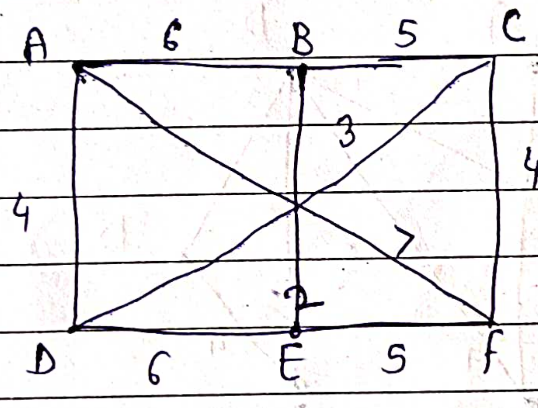
2) order all the Edges of the graph acc. to inc wts

Date.
Page.

3) Initialize T with all vertices but don't include any edge.

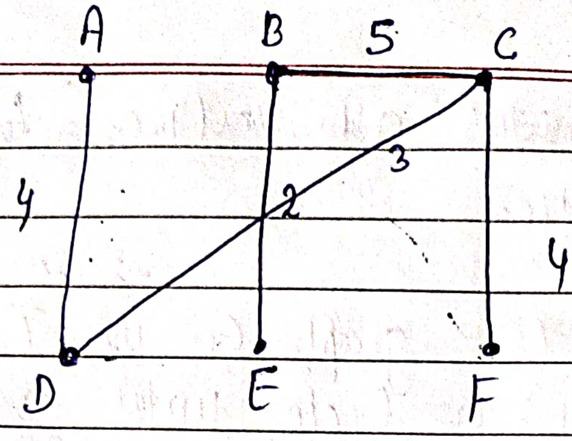
4) Add Each edge of the graph G in T which does n't form a Cycle until (n-1) Edges are added.

Q 1) Determine Minimum Spanning tree of wt. graph shown below:



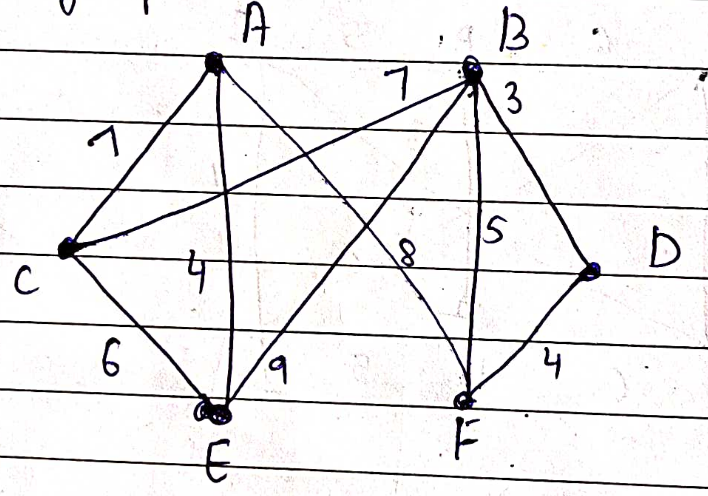
Sol.

Edges	weights	Added or not
(B, E)	2	Added
(C, D)	3	Added
(A, D) (C, F)	4	Added added
(B, C)	5	added
(E, F)	5	Not Added (N/A)
(A, B)	6	N.A
(D, E)	6	N.A
(A, F)	7	N.A



Minimum Spanning tree.

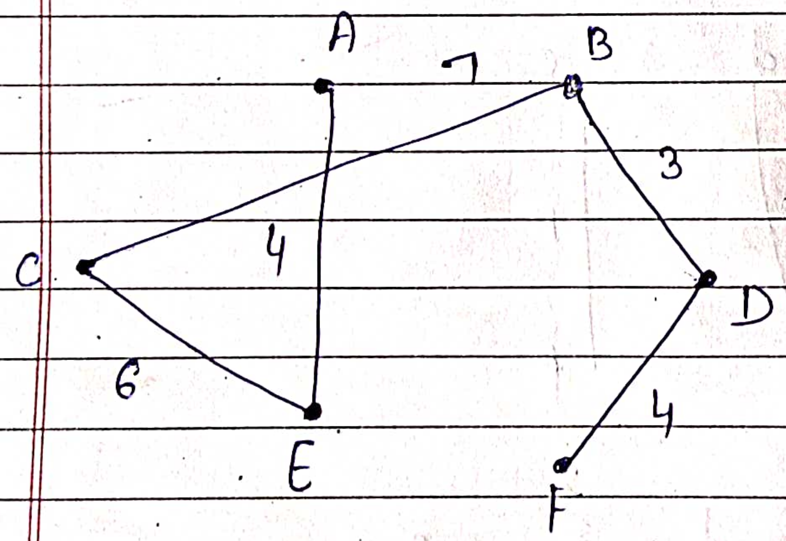
Q. find Minimum Spanning tree of foll. graph



Sol 1st Arrange all Edges in inc. order and initialize Spanning tree with all vertices of graph G.

Now add Edges of G in T which doesn't form a Cycle & having minimum wt. until $n-1$ edges are not added where n is no. of vertices

Edges	Weights	Added or not
(B, D)	3	added
(A, E)	4	added
(D, F)	4	added
(B, F)	5	Not added
(C, E)	6	added
(A, C)	7	Not added
(B, C)	7	Added
(A, F)	8	Not added
(B, E)	9	Not added



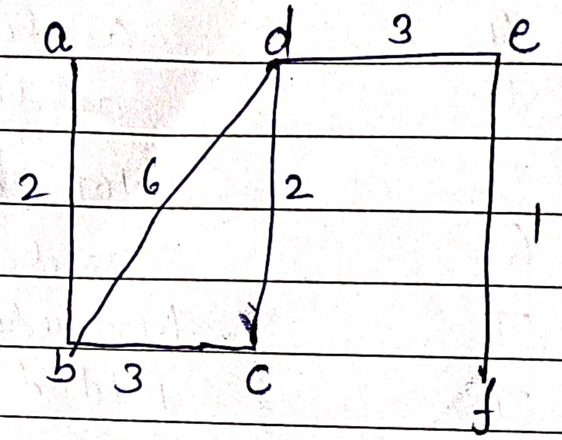
The min. wt of Spanning tree is

$3+4+4+6+7 = 24.$

Date.

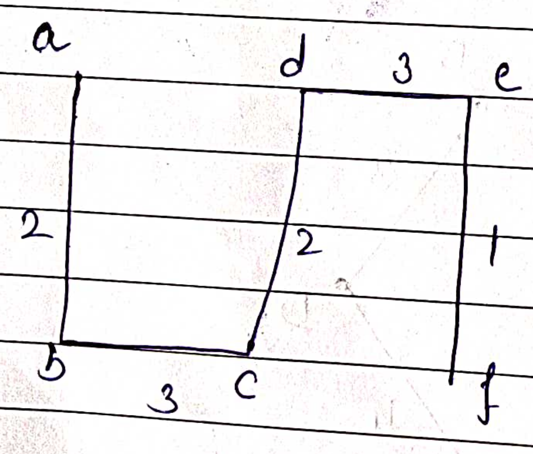
Page.

Q) find all spanning tree of graph G & find which is minimal spanning tree of G.

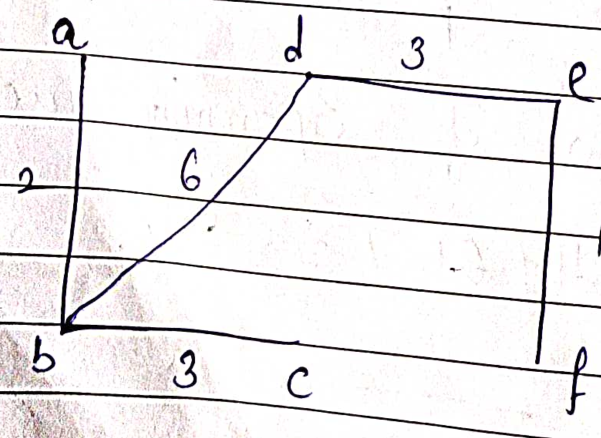


Sol Spanning tree of above graph are

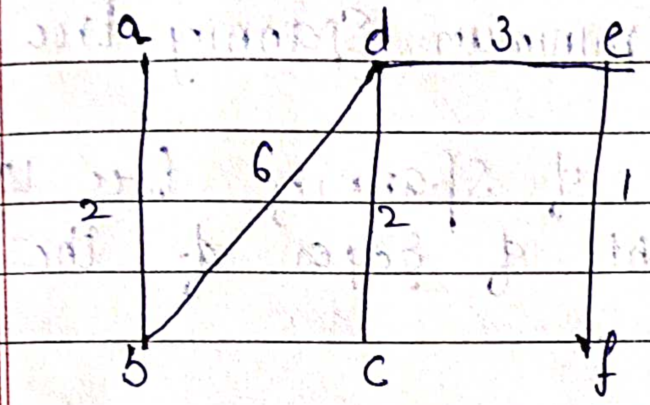
(i)



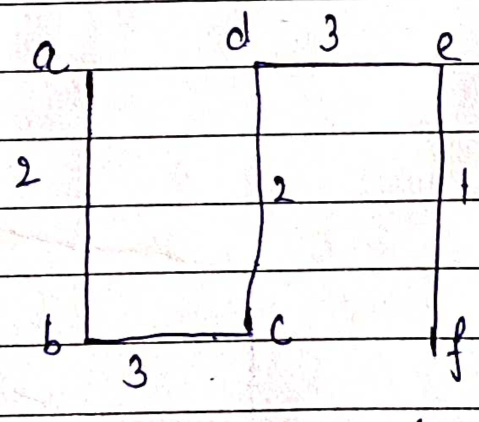
ii)



III)



Edges	wts.	added or not
(E,F)	1	added
(A,B)	2	added
(C,D)	2	added
(B,C)	3	added
(D,E)	3	added
(B,D)	6	N.A.



This is minimum spanning tree having weight $= 2+3+2+3+1 = 11$

Date.
Page.

Properties of minimum Spanning tree

(i) The total wt of Spanning tree is sum of weight of Edges of the spanning tree.

ii) The minimum weight of Spanning tree is unique.