Sets

A set is a collection of well defined and differen[t] objects.

[we are given a rule with the help of ... we can say whet[her] object belong to ... not]

eg. The set of Even numbers

The set of days of a week.

Set Notations: Sets are generally denoted by capital letters A, B, C --- while Elements of set are mostly denoted by small letters a, b, c ---

Some standard sets:

N → Set of all Natural no's 1, 2, 3 ----

W → Set of all whole no's 0, 1, 2, 3 ----

Z or I → Set of all integers

Q → set of Rational no's

R → set of real no's.

Representation of Sets

A set can be specified in two ways:

- Roaster, Tabular or Enumeration method
- Set-builder, Rule method or selector method

(1) **Roaster Method** → When we represent a set by listing all its Elements within curly brackets { }, separated by commas, it is called roaster, Tabular or Enumeration method.

for. eg: i) Set of Vowels $A = \{a, e, i, o, u\}$

ii) Set of positive Even no's upto 10

$B = \{2, 4, 6, 8, 10\}$

iii) Set of odd natural no's

$C = \{1, 3, 5, 7, 9 ----\}$

Set Builder, Selector or rule method → In this method we don't list all the Elements but the set is represented by specifying the defining property.

for eg. $A = \{x : x$ is a vowel in English alphabet$\}$

$B = \{x : x$ is positive Even no upto $10\}$

$C = \{x : x$ is an odd positive integer$\}$

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e.g. Write the following set in Roaster form

(i) $\{x : x$ is a vowel before g in English alphabet$\}$

Sol: letters before g in English alphabet are a, b, c, d, e & f. but only a & e are vowels.

$\therefore$ Required set is $\{a, e\}$

Q) ii) $\{x \in N : x$ is a prime no. between 6 & 30$\}$

Sol: Prime no between 6 & 30 are

7, 11, 13, 17, 19, 23, 29

$\therefore$ required set is $\{7, 11, 13, 17, 19, 23, 29\}$

iii) $\{x \in N ; 3x + 5 < 31\}$

Sol. $3x + 5 < 31$

$3x < 31 - 5$

$3x < 26$

$x < \frac{26}{3} \Rightarrow x < 8.6$

$\cancel{x < 8\frac{2}{3}}$

$\therefore x = 1, 2, 3, 4, 5, 6, 7, 8$ as $x \in N$

$\therefore$ Required set is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Prime no. is which have only two factors one & number itself.

$3\overline{)26}(8.6$
$\quad 24$
$\quad \overline{\ 20}$
$\quad -18$
$\quad \overline{\ \ 2}$

(v) $\{x : x^2+5x+6=0\}$

$x^2+5x+6=0$ $\Rightarrow x^2+(3+2)x+6=0$

$x^2+3x+2x+6=0$

$x(x+3)+2(x+3)=0$

$\Rightarrow (x+2)(x+3)=0$

$\Rightarrow x=-2,-3$

$\therefore$ Required set is $\{-2,-3\}$

Practise Questions

Q) List Elements of following sets

(i) $A=\{x : x \text{ is an integer}, x^2 \leq 4\}$

ii) $B=\{x : x$ is a letter in the word "LOYAL"$\}$

iii) $C=\{x \in N : x^2=25\}$

iv) $D=\{x : x^2-3x+2=0\}$

v) $E=\{x : x$ is an integer & $-3 \leq x < 7\}$

Q) Express the following sets by using Set builder method.

(i) $A=\{1,3,5,7,9\}$

Sol^n: $A=\{x : x$ is an odd natural number less than $10\}$

ii) $B=\{2,4,6,8\}$

$=\{x : x$ is an Even natural number less than $10\}$

iii) $C=\{-1,1\}$

$=\{x : x$ is an odd integer & $|x|<2\}$

iv) $D=\{1,5,10,15...\}$

$=\{x : x$ is a natural no. multiple of 5 & $x=1\}$

v) $E=\{14,21,28,35,...\ 98\}$

$=\{x : x$ is a natural number multiple of 7 & $7<x<100\}$

Write following Sets in Set- Builder form
(i) $\{3, 6, 9, 12\}$
ii) $\{2, 4, 8, 16, 32\}$
iii). $\{2, 4, 6, 8, 10 ---- \}$

Types of Sets

1) Null set → A set which contains no Element is called null set. It is denoted by ϕ (phi)
In Roaster form ϕ. is denoted as $\{\ \}$.
Empty set or Null Set is also known as Void set.
E.g. $A = \{x : x$ is a positive integer satisfying $x^2 = \frac{1}{4}\}$.
Q $B = \{x : x$ is an Even prime no greater than $2\}$
Note: ϕ is subset of Every set.
eg. which of following are Examples of null set?
(i) set of odd natural no's divisible by 2.
Sol this set is null set as there is no odd natural number divisible by 2.
ii) Set of Even prime no.
Sol Set of Even prime no $= \{2\}$ this Set is not null set.
iii) $\{x : x$ is a natural no, $x < 5$ & simultaneously $x > 7\}$
Sol Null Set as there is no Element which is < 5 but > 7.

② Singleton set → A set containing only one Element is called singleton set.
eg. $A = \{x : x$ is a perfect square & $30 \le x \le 40\}$
$= \{36\}$.

$B = \{x : x \text{ is a positive integer satisfying } x^2 = 4\}$
$= \{2\}$.

(3) finite Set → A set is said to be finite if it has finite no. of elements.

e.g's (i) $A = \{2, 4, 6, 8\}$

ii) $B = \{x : x$ is a student of D.A.V. College amritsar$\}$

iii) $\{x : x \in N \ \& \ 5 < x < 11\}$

(4) Infinite set → A Set is said to be infinite if it has infinite no. of elements.

e.g. (i) $A = \{x : x$ is an odd integer$\}$

ii) $B = \{x : x$ is a multiple of 6$\}$

iii) $C = \{1, 2, 3, 4, 5 \ ----\}$

e.g. State which of following sets are finite and which are infinite.

(i) $A = \{x : x \in Z \ \& \ x^2 + 7x + 12 = 0\}$

Sol. Since $x^2 + 7x + 12 = 0$

$\Rightarrow x^2 + 4x + 3x + 12 = 0$

$\Rightarrow x(x+4) + 3(x+4) = 0$

$\Rightarrow (x+4)(x+3) = 0$

$\Rightarrow x = -3, -4$

$\therefore A = \{-3, -4\}$ So A is a finite set.

(ii) $B = \{x : x \in Z$ and x^2 is Even$\}$

$= \{----, -6, -4, -2, 0, 2, 4, 6, \ ----\}$

B is an infinite set.

iii) $C = \{x : x \in Z$ and $x^2 = 64\}$

$= \{-8, 8\}$

$\therefore$ C is finite set.

iv) $D = \{x : x \in Z$ & $x > -6\}$

$= \{-5, -4, -3, -2 \ldots\}$

$\therefore$ D is an infinite set.

Equal Sets → Two Sets A and B are said to be equal if they have Same ~~number~~ of Elements.

In other words two sets A & B are said to be equal when Every element of A is an Element of B & Every Element of B is element of A.

e.g. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{x : x$ is a natural no. & $1 \leq x \leq 10\}$.

Here $A = B$.

Equivalent Sets: Two Sets A & B are said to be equivalent if total no of Elements in A is Equal to total no. of Elements in B.

or Two finite sets are Equivalent iff their cardinal numbers are ~~safe~~. same.

e.g. $A = \{1, 2, 3, 4, 6\}$; $B = \{1, 2, 7, 9, 12\}$

$\therefore o(A) = 5 = o(B)$.

2) A and B are Equivalent Sets.

Show that the set of letters needed to spell "CATARACT" and set of letters needed to spell "TRACT" are Equal.

Sol: Let X be Set of letters in CATARACT

$\therefore X = \{C, A, T, A, R, A, C, T\}$

i.e. $X = \{C, A, T, R\}$

Let Y be set of letters in TRACT.

$Y = \{T, R, A, C, T\}$

$Y = \{T, R, A, C\}$

Since every letter of X is in Y & every letter of Y is in X.

$\therefore \boxed{X = Y}$

Q) From the following sets; Identify Equal & Equivalent sets.

Sol. (i) $A = \{0, a\}$ $B = \{1, 2, 3, 4\}$ $C = \{4, 8, 12\}$

$D = \{3, 1, 2, 4\}$ $E = \{1, 0\}$ $F = \{8, 4, 12\}$

$G = \{1, 5, 7, 11\}$ $H = \{a, b\}$

Sol: A, E & H has same no. of elements.

$\therefore$ A, E & H are equivalent sets.

B, D & G have same no. of elements

$\therefore$ B, D & G are equivalent sets.

Also $B = D$ so B & D are Equal sets.

$C = F$ so C & F are equal sets.

Disjoint Sets : If A and B be two given sets such that $A \cap B = \phi$, then the sets A & B are said to be disjoint.

e.g. Let $A = \{a, b, c, d\}$, $B = \{l, m, n, p\}$

$\therefore A \cap B = \phi$

Hence A & B are disjoint sets.

e.g's which of the following pair of sets are disjoint?

(i) $\{1, 2, 3, 4\}$ & $\{x : x$ is a natural no and $4 \le x \le 6\}$

Sol let $A = \{1, 2, 3, 4\}$

$B = \{4, 5, 6\}$

$A \cap B = \{4\} \neq \phi$

Hence A & B are not disjoint sets.

ii) $\{a, e, i, o, u\}$ & $\{c, d, e, f\}$

Sol Let $A = \{a, e, i, o, u\}$

$B = \{c, d, e, f\}$

Since $A \cap B = \{e\} \neq \phi$

Hence A & B are not disjoint sets.

iii) $\{x : x$ is an Even integer$\}$ & $\{x : x$ is an odd int$\}$

Sol let $A = \{x : x$ is an Even integer$\}$

$= \{ \ldots -6, -4, -2, 2, 4, 6 \ldots \}$

& $B = \{x : x$ is an odd integer$\}$

$= \{ \ldots -5, -3, -1, 1, 3, 5 \ldots \}$

$\therefore A \cap B = \phi$

A & B are disjoint sets.

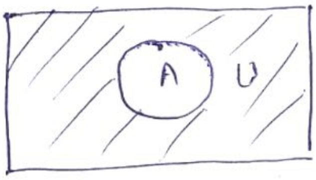
Subs
Set

Universal Set → If all the sets under consideration are subsets of a fixed set U then U is called a universal set. e.g. When we are using sets containing natural no's then N is the Universal set. & when we are using intervals on Real line then set R of real no's is taken as universal set.

Operations with sets :

Venn diagrams :→ The relationship between sets can be illustrated by certain diagrams called venn diagrams.

In Venn diagram universal set U is represented by rectangle and any subset of U is represented by a circle within a rectangle U.



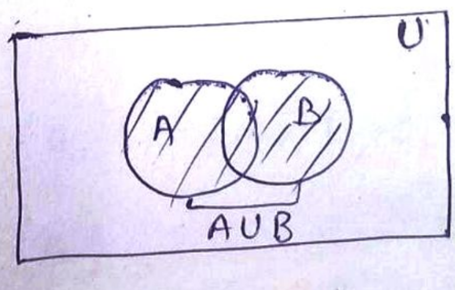
Union of Two Sets

If A and B are two sets then their union is the set consisting of all elements of A together with all elements in B.

We should not repeat the elements

The union of set A and B is written as A ∪ B.

In symbols: $A \cup B = \{x : x \in A \text{ or } x \in B\}$



E.g. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ $C = \{5, 6, 7$ Inte
$D = \{7, 8, 9, 10\}$ find (i) $A \cup B$ (ii) $A \cup C$ iii) (
iv) $B \cup D$ (v) $A \cup B \cup C$ (vi) $A \cup B \cup D$ vii) $B \cup C \cup$

Sol Let $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$
$C = \{5, 6, 7, 8\}$ $D = \{7, 8, 9, 10\}$

(i) $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

iii) $B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$
$= \{3, 4, 5, 6, 7, 8\}$

iv) $B \cup D = \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

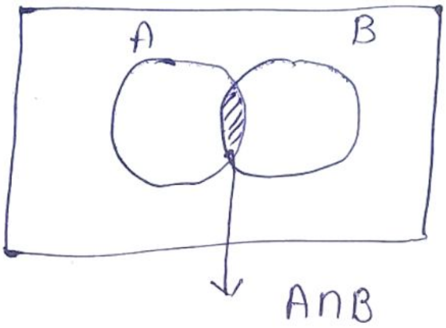
v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

Intersection of sets: If A and B are two sets then their intersection is the set consisting of elements common to A and B. The intersection of two sets A and B is denoted as $A \cap B$.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$



e.g. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ & $B = \{2, 3, 5, 7\}$ find $A \cap B$ & prove that $A \cap B = B$.

Sol: Since $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

& $B = \{2, 3, 5, 7\}$

$\therefore A \cap B = \{2, 3, 5, 7\}$

Hence $\boxed{A \cap B = B}$

Q find the union and intersection of each of following pair of sets

(i) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$

Sol $A \cup B = \{a, b, c, e, i, o, u\}$

$A \cap B = \{a\}$

iii) $A = \{x : x \text{ is a natural no. \& multiple of } 3\}$

$B = \{x : x \text{ is a natural no. less than } 6\}$

Sol : $A = \{3, 6, 9, 12, 15, 18 \ldots\}$

$B = \{1, 2, 3, 4, 5\}$

$\therefore A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, 15, 18, \ldots\}$

$A \cap B = \{3\}$

iv) $A = \{x : x \text{ is a natural no. \& } 1 < x \leq 6\}$

$B = \{x : x \text{ is a natural no. \& } 6 < x < 10\}$

Sol $A = \{2, 3, 4, 5, 6\}$

$B = \{7, 8, 9\}$

$\therefore A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

& $A \cap B = \phi$

(a) Let $A = \{x : x \text{ is a natural no}\}$

$B = \{x : x \text{ is an Even natural no.}\}$

$C = \{x : x \text{ is an odd natural no.}\}$

$D = \{x : x \text{ is a prime no}\}$ find (i) $A \cap B$

ii) $A \cap C$ iii) $A \cap D$ iv) $B \cap C$ v) $B \cap D$ vi) $C \cap D$

Sol Let $A = \{1, 2, 3, 4, 5, 6, 7 \ldots\}$

$B = \{2, 4, 6, 8, 10 \ldots\}$

$C = \{1, 3, 5, 7, 9 \ldots\}$

$D = \{2, 3, 5, 7, 11, 13 \ldots\}$

(i) $A \cap C = \{1, 3, 5, 7, 9 \ldots\}$

$A \cap D = \{2, 3, 5, 7, 11, 13 \ldots\}$

iii) $A \cap B = \{2, 4, 6, 8, 10 \ldots\}$

iv) $B \cap C = \phi$

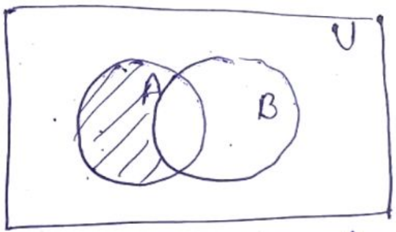
v) $B \cap D = \{2\}$

vi) $C \cap D = \{3, 5, 7, 11, 13 \ldots\}$

Difference of Sets

The difference of two sets A and B is the set of those Elements of A which donot belong to B. We denote this by $A - B$.

$A - B = \{x : x \in A \ \& \ x \notin B\}$ It is also written as $A \setminus B$.



- Shaded portion denote $A - B$.

Q) Let $A = \{1, 2, 3, 4, 5, 6\}$ & $B = \{2, 4, 6, 8\}$ then find $A - B$ & $B - A$.

Sol. Since $A = \{1, 2, 3, 4, 5, 6\}$

$B = \{2, 4, 6, 8\}$

So $A - B = \{1, 3, 5\}$

& $B - A = \{8\}$

(Q) let $V = \{a, e, i, o, u\}$ & $B = \{a, i, k, u\}$

find $V - B$ & $B - V$

Sol. $V - B = \{e, o\}$

$B - V = \{k\}$

(Q) let $A = \{3, 6, 12, 15, 18, 21\}$

$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ & $D = \{5, 10, 15, 20\}$

find (i) $A - B$ ii) $A - C$ iii) $A - D$ iv) $B - A$ v) $C - A$

$D - A$, $B - C$, $B - D$, $C - B$, $D - B$, $C - D$ & $D - C$

Q. which of following pair of sets are disjoint?

(i) $\{1, 2, 3, 4\}$ & $\{x : x$ is a natural no. & $4 \le x \le 6\}$

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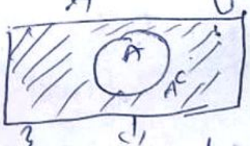
* Compliment of a set

Let A be a subset of universal set U. then the compliment of A is set of all those Elements of U which donot belong to A and We write compliment of A by A^c or A'.



We can write $A^c = \{x : x \in U ; x \notin A\}$

Shaded portion denote A^c

e.g If $U = \{2, 4, 6, 8, 10\}$ $A = \{4, 8\}$

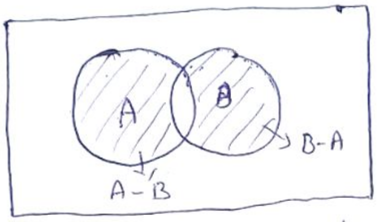
then $A^c = \{2, 6, 10\}$

Symmetric Difference of Sets → If A and B are two sets then the set $(A-B) \cup (B-A)$ is called symmetric difference of A & B and is denoted by $A \Delta B$.

In symbols:

$$A \Delta B = \{\{x : x \in A \text{ and } x \notin B\} \text{ or } \{x \in B \text{ \& } x \notin A\}\}$$

or $A \Delta B = \{x : x \notin A \cap B\}$



Shaded portion denote $A \Delta B$.

e.g. If $A = \{x : x \in N : 0 < x < 3\}$, $B = \{x \in N : 1 \leq x \leq 5\}$ ~~and~~ then find $A \Delta B$.

Sol: Since $A \Delta B = (A-B) \cup (B-A)$

$A = \{1, 2\}$ & $B = \{1, 2, 3, 4, 5\}$

$A - B = \phi$ & $B - A = \{3, 4, 5\}$

$\therefore A \Delta B = \{3, 4, 5\}$

Q Let $A = \{1, 2, 4\}$ & $B = \{1, 2, 3, 5, 6\}$ find $A \Delta B$

Sol Since $A \Delta B = (A-B) \cup (B-A)$

$= \{4\} \cup \{3, 5, 6\}$

$$\boxed{A \Delta B = \{3, 4, 5, 6\}}$$

eg. let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ & $C = \{3, 4, 5, 6\}$ find (i) A' (ii) B' (iii) $(A \cap C)'$ (iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B-C)'$

Sol.

Since $U = \{1, 2, 3, ---- 9\}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6\}$

(i) $A' = \{5, 6, 7, 8, 9\}$

ii) $B' = \{1, 3, 5, 7, 9\}$

iii) $A \cap C = \{3, 4\}$

$(A \cap C)' = \{1, 2, 5, 6, 7, 8, 9\}$

iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = \{5, 7, 9\}$

v) $(A')' = ?$

$A' = \{5, 6, 7, 8, 9\}$

$(A')' = \{1, 2, 3, 4\}$

vi) $B - C = \{2, 8\}$

$(B-C)' = \{1, 3, 4, 5, 6, 7, 9\}$

are said

Subset → If Every member of set A is member of set B then A is called subset of B & B is Called super set of A.

or if $x \in A \Rightarrow x \in B$ then A is called subset of B

i.e. $A \subseteq B$ or $A \subset B$ or $A \subset B$
$\Rightarrow \{x; x \in A \Rightarrow x \in B\}$

?

Eg (i) Let $A = \{1, 2, 3, 4, 5, 6, 8, 10\}$
& $B = \{2, 4, 6, 10\}$.

Now Every Element of B is an Element of A.

$\therefore B \subseteq A$

ii) $A = \{1, 2, 3, 4, 5, 6, 8, 10\}$ & $C = \{1, 2, 7, 8\}$

Since $7 \in C$ but $7 \notin A$

$\therefore$ C is not subset of A.

i.e. $C \not\subset A$.

Note → If a set has n Elements no. of subsets 2^n.

e.g. List all the subsets of set $\{-1, 0, 1\}$

Sol let $A = \{-1, 0, 1\}$

Subsets of A are ϕ, $\{-1\}$, $\{0\}$, $\{1\}$, $\{-1, 0\}$, $\{-1, 1\}$,
$\{0, 1\}$, $\{-1, 0, 1\}$

Proper subset → A non Empty subset A is said to be proper subset of B if $A \subset B$ & $A \neq B$.

Note: ϕ and A are called improper subsets of A.

e.g Write down all proper subsets of $\{2, 4, 6\}$.

Sol let $A = \{2, 4, 6\}$

Proper subsets of A are $\{2\}$, $\{4\}$, $\{6\}$, $\{2, 4\}$,
$\{2, 6\}$, $\{4, 6\}$

or

If A is subset of B and $A \neq B$ then A is said to be proper subset of B; then B is not subset of A i.e. there is atleast one element which is not in A. e.g. $A = \{2, 3, 4\}$, $B = \{2, 3, 4, 5\}$ then A is proper subset of B e.g. let $A = \{1,2,3,4\}$ $B = \{1,2,3\}$ $C = \{2,4\}$

find all sets X such that

(i) $X \subset B$ and $X \subset C$

ii) $X \subset A$ but $X \not\subset B$

Sol $A = \{1,2,3,4\}$ $B = \{1,2,3\}$ $C = \{2,4\}$

$X \subset B$ means X is subset of B

Subsets of B are $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

$X \subset C$ means X is subset of C

$\phi, \{2\}, \{4\}, \{2,4\}$

When $X \subset B$ and $X \subset C$, we have

$X = \phi, \{2\}$

ii) $X \subset A$ means X is subset of A

$\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}$.

Subsets of B are $\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

When $X \subset A$ & $X \not\subset B$ we have

$X = \{4\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}$.

Improper subset: If A is subset of B and $A = B$ then A is called improper subset of B. e.g. $A = \{2,3,4\}$, $B = \{2,3,4\}$ then A is improper subset of B.

Power Set → The power set of finite set is set of all subsets of the given set. Power set of set A is denoted by P(A). If a set has n elements then P(A) has 2^n elements.

e.g. let $A = \{1,2,3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Note: Every set is improper subset of itself.