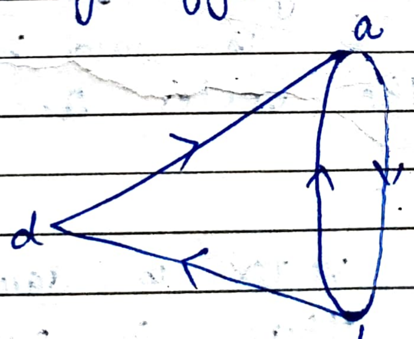
GRAPHS

The graphs consist of points or nodes called vertices which are connected to each other by way of lines called edges.
The lines may be directed or undirected.

Directed Graph: A directed graph is defined as an ordered pair (V, E) where V is a set and E is a binary relation on V.
A directed graph can be represented geometrically as a set of marked points V with a set of arrows E between pair of points.
Also Elements in V are called vertices.
The ordered pair in E are called edges.
While drawing a directed graph the edges are typically drawn as arrows indicating the direction, as shown in following figure.



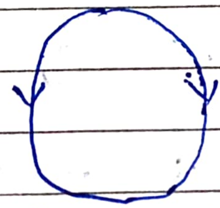
directed graph

Here a, b and d are vertices and edges are (a,b) (b,a) (b,d) (d,a)

A edge is said to be incident with the vertices it joins.
for e.g. the edge (a,b) is incident with vertices a and b. Also we can say that the edge (a,b) is incident from vertex a and incident into vertex b.

The vertex a is called initial vertex and b is called terminal vertex of (a,b)

Loop → An edge that incident from and into the same vertex is called loop or self-loop.
i.e.

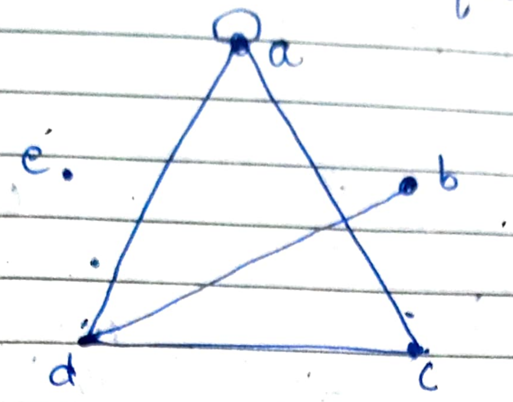


Degree of self loop is two as it is twice incident on the same vertex.

Corresponding to an edge (a,b), the vertex a is said to be adjacent to vertex b and the vertex b is said to be adjacent from the vertex a.

Isolated vertex : A vertex is said to be isolated vertex if there is no edge incident with it. or
A vertex of zero degree is called isolated vertex.

for e.g. consider the following graph

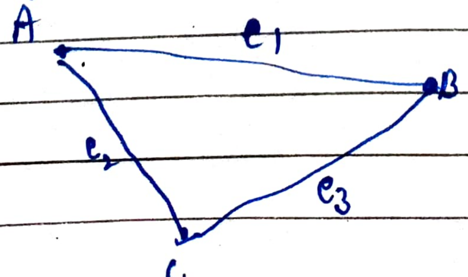


Vertex a has self loop so deg a = 2.

The vertex b is pendent vertex since only one edge is incident on it and the only edge which is incident with pendent vertex is called as pendent edge.
The vertex e is isolated vertex as it has no edge incident on it i.e. deg e = 0.

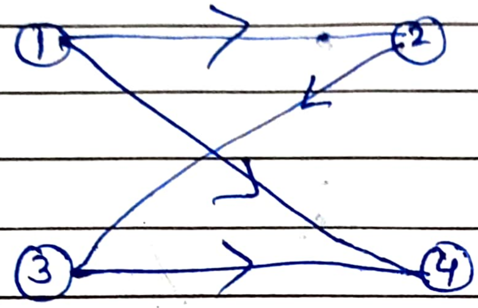
Undirected graph : An undirected graph G consists of set of vertices V and set of edges E. The edge set contains the unordered pair of vertices. If $(u, v) \in E$ then we say u and v are connected by an edge where u and v are vertices in set V.

for e.g. (A)———(B) the graph can be traversed from node A to B as well as from node B to A.



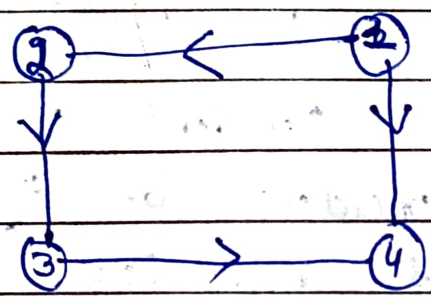
① e.g. Let $V = \{1, 2, 3, 4\}$ & $E = \{(1,2), (1,4), (3,4), (3,$
Draw the graph.

Sol.

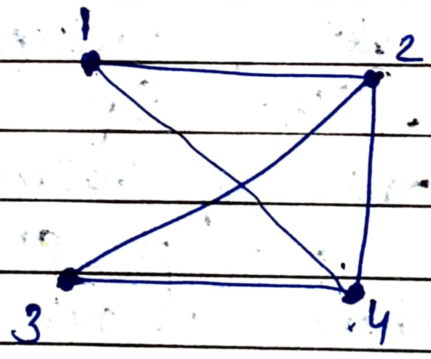


Or

i.e. graph can be drawn in several ways



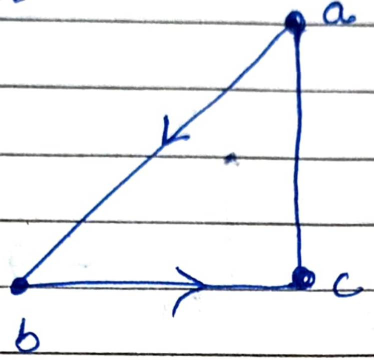
eg. ② Consider the foll. graph. find the Edge set & vertex set of this graph.



The vertex set $V = \{1, 2, 3, 4\}$

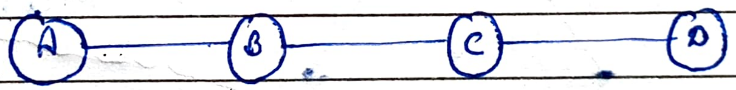
Edge set $E = \{(1,2)\ (1,4)\ (2,4)\ (2,3)\ (3,4)\}$

Mixed Graph : A graph $G=[V,E]$ in which some edges are directed and some are undirected is called a mixed graph. The graph shown in foll. fig. is a mixed graph.



Finite graph : A graph $G=[V,E]$ is said to be finite if V and E are finite sets.

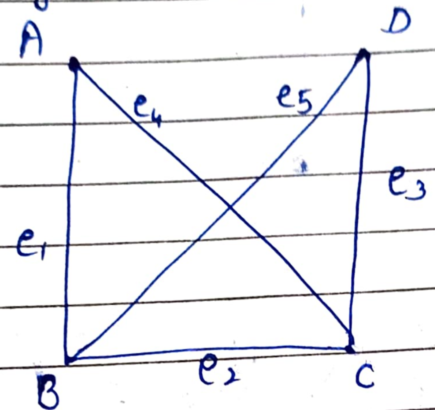
Linear Graph : A graph $G=[V,E]$ is said to be linear graph if its edges joining vertices lies along a line.
for Eg. :



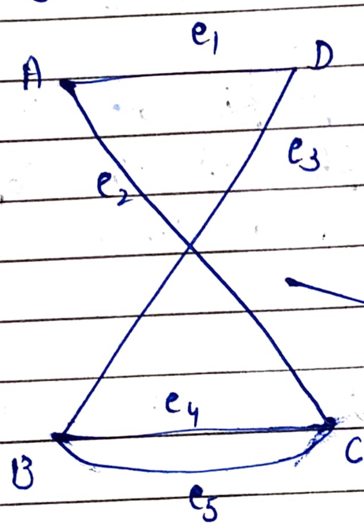
Discrete or Null graph: A graph containing only vertices and no edges is called a discrete or null graph.
The set E of edges in a graph $G=[V,E]$ is empty in a discrete graph. Also each vertex in a discrete graph is an isolated vertex.

Simple graph: A simple graph is one for which there is no more than one Edge directed from any one vertex to other vertex and no edge starts & ends at the same vertex. All other graphs are Multigraphs.

for E.g.



→ Simple graph

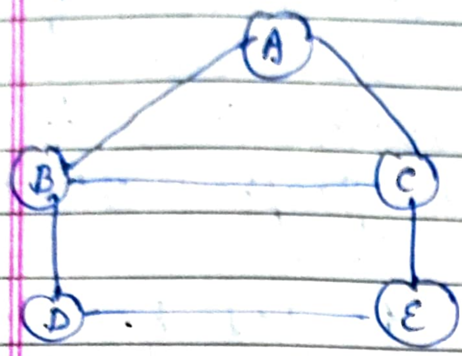


edges E_4 & e_5 are called Multi-Edges

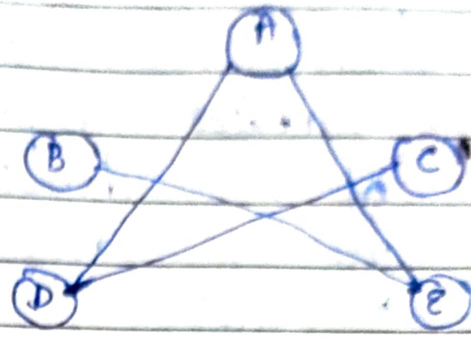
→ Multigraph.

Complement graph: The compliment of graph is defined to be a graph which has the same no. of vertices as in Graph G and has two vertices Connected iff they are not connected in the graph G.

for Eg.



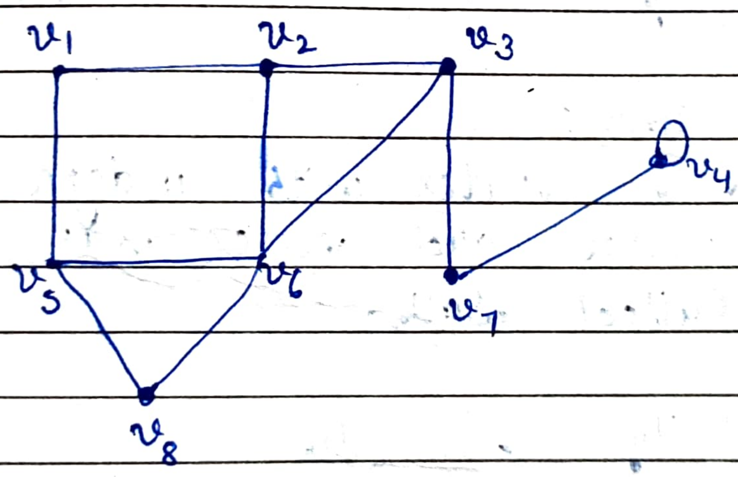
(Graph G)



(Compliment graph $\bar{G}$)

<u>Degree</u> : Let v be a vertex of an undirected graph. The degree of v, denoted by $d(v)$, is the no. of Edges that connect v to other vertices in the graph. The degree of a graph cannot be negative.

for eg.



$\deg(v_1) = 2$

$\deg(v_2) = 3$

$\deg(v_3) = 3$

$\deg(v_4) = 3$

$\deg(v_5) = 3$

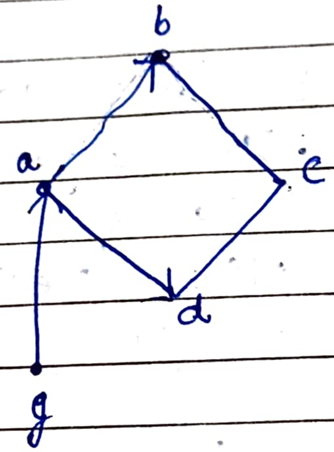
$\deg(v_6) = 4$

$\deg(v_7) = 2$

$\deg(v_8) = 2$

Outdegree & Indegree: If v is a vertex of directed graph, then the outdegree of v, denoted by outdeg (v), is the no. of edges of the graph that initiate v.

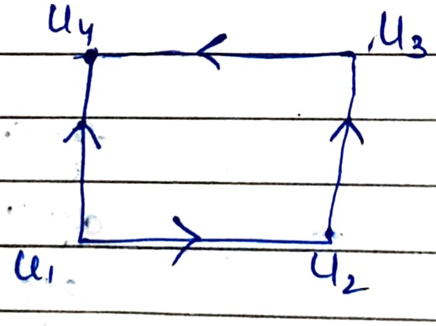
The indegree of v, denoted by indeg(v) is the no. of edges that terminate at v.



Since Vertex a has two edges ab & ad which are going outward so outdegree is 2. Edge ga coming towards a. Hence indegree of a is 1.

Source & Sink: A vertex with indegree 0 is called source and vertex with outdegree 0 is called sink.

for Eg.

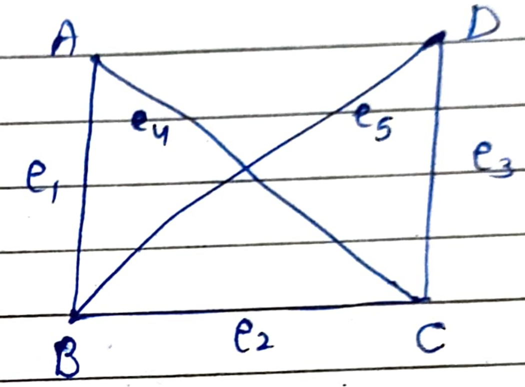


here u_4 is sink.

Even and odd vertex

A vertex is said to be Even vertex if its degree is an even no.

A vertex is said to be odd vertex if its degree is an odd no.



for e.g.

Since deg (A) = 2
& deg (D) = 2
So A and D are Even vertices.

Also deg (B) = 3 & deg (C) = 3
Since deg. of B & C is odd so A and C are odd vertices.

Adjacent vertices: Two vertices are called adjacent if they are connected by an edge. If there is an Edge (e_1, e_2) then we can say that e_1 is adjacent to vertex e_2 and e_2 is adjacent to vertex e_1.