AMRITSAR COLLEGE OF ENGINEERING AND TECHNOLOGY, AMRITSAR
(AUTONOMOUS COLLEGE)

Roll No. ......................

Total No. of Questions: 09 | Total No. of Pages: 02

B.Tech. (C.Sc.) – 3rd Sem / 1st – Mid Sem

Mathematics III

SUBJECT CODE: BTAM 302

Batch(2012 *onwards*)

Time: 03 Hours | Maximum Marks: 60

**Instruction to Candidates:**

1) Section - A is Compulsory.
2) Attempt any Four questions from Section - B.
3) Attempt any Two questions from Section - C.

## Section - A

$(10 \times 2 = 20)$

Q1)

i) Solve : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$ .

ii) State Dirichlet's Conditions.

iii) State Fourier Series.

iv) Find Laplace transform of $2e^{3t} - e^{-3t}$.

v) What is formula for Euler's method .

vi) Write properties of Poisson distribution.

vii) Form partial differential equations from the following equation by eliminating the arbitrary constants: z = (x+a) (y+b).

viii) If the probability of male birth is 0.5, find the probability that in a family of 4 children there will be at least one boy.

ix) Write a short note on hypothesis testing and its uses.

x) Find the Inverse Laplace Transform of $\frac{4p+15}{16p^2-25}$.

## Section – B

Q2) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

Q3) Determine the analytic function $w = u + iv$, if $u = \log\sqrt{x^2 + y^2}$.

Q4) Find the Laplace transform of $(1 + te^{-t})^3$.

Q5) Solve the linear partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y \cos x.$$

Q6) Obtain Fourier series for function $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2 - x), & 1 \leq x \leq 2 \end{cases}$

## Section – C

(2 x

Q7) (a) Solve the following system of equation by Gauss elimination method

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 2 \\ -1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$$

(b) If on an average, one ship out of 10 wrecked, find the probability that out of 5 Ships expected to arrive the port, at least 4 will arrive safely.

Q8) Given $y' = x^2 - y$, $y(0) = 1$, find $y(0.1)$, $y(0.2)$ using Runge-Kutta methods of 4

Q9) Show that the function defined by $f(z) = \sqrt{|xy|}$ is not regular at the origin, although Cau Riemann equations are satisfied.

BTAM-302

Sec-A

1

The given eqn is

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$$

for S.F, put $\frac{\partial}{\partial x} = D$, $\frac{\partial}{\partial y} = D'$

∴ eqn in S.F is

$$(D^2 - DD' - 6D'^2)z = 0$$

for A.E, put $D = m$, $D' = 1$

∴ A.E is

$$m^2 - m - 6 = 0$$

$$(m-3)(m+2) = 0 \Rightarrow m = -2, 3$$

∴ C.F is $z = f_1(y-2x) + f_2(y+3x)$

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For expressing any function f(x) in Fourier series expansion, f(x) has to satisfies following conditions called Dirichlet's conditions

f(x) should be finite, single + periodic

f(x) should have finite points of Discontinuities

f(x) should have finite no. of maxima + minima

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State Fourier Series

Fourier Series of a function f(x) is the expansion of f(x) in a series in given time period. It can also be expressed in graphs in the form (i) wave form (ii) Triangular wave form (iii) Square wave form. For expansion any function in Fourier Series, we have

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where $a_0$, $a_n$, $b_n$ are called Euler Constants.

where $a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx$, $a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x).\cos nx\,dx$

$b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x).\sin nx\,dx$ are called Euler formula by the name fabinas a french mathematics.

(iv) Given $f(t) = 2e^{3t} - e^{-3t}$

$$L\{f(t)\} = 2L\{e^{3t}\} - L\{e^{-3t}\}$$

$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s+3}\right]$$ (by

(v) Euler method is given by

$$y_{n+1} = y_n + h f(x_n, y_n)$$

(vi) Write properties of poisson Distribution

ans The followings are the properties of poisson Distribution

(1) It is a discrete distribution. It takes values like $n=1, 2, 3, \ldots$

(2) In this distribution mean is equal mea

(3) In this distribution $n$ is very large.

(4) In this distribution ~~a~~ $p$ is very small, close to zero such that ~~np~~ $np=\lambda$ is Constant

③

The given eqn is

$$z = (x+a)(y+b) \quad \text{--- (1)}$$

Diff (1) partially w.r.t 'x', we get

$$\frac{\partial z}{\partial x} = 1\cdot(y+b)$$

$$\Rightarrow p = (y+b) \quad \text{--- (2)}$$

& w.r.t 'y', we get

$$\frac{\partial z}{\partial y} = (x+a)\cdot 1$$

$$\Rightarrow q = (x+a) \quad \text{--- (3)}$$

Multiplying (2) & (3), we get

$$pq = (x+a)(y+b) = z$$

is req. p.d.e

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$p = 0.5 = \frac{5}{10} = \frac{1}{2}$ = probability of boy.

$\therefore q = 1 - \frac{1}{2} = \frac{1}{2}$

$n = 4$

$\therefore$ Binomial distribution $= (p+q)^n = \left(\frac{1}{2}+\frac{1}{2}\right)^4$

$$P(\text{atleast one boy}) = 1 - P(\text{no boy})$$

$$= 1 - P(0)$$

$$= 1 - \left[{}^4C_0\, p^0 q^4\right]$$

$$= 1 - \left[1\cdot\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4\right]$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

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(1α) A hypothesis is an appropriate explanation that releates to the set of facts that can be tested for certain further in...

There are basical two types

(1) Null hypothesis and Alternative hy...

Research generally starts with the problem. Next these hypothesis provides researcher some specific restatement & clarif... of research problems. The criteria ... research problem informing null hypothesis & alternative hyp. should be express as a relation between two or more variables

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(α) Given $f(p) = \frac{4p+15}{16p^2-25} = \frac{4p+15}{16\left[p^2 - \frac{25}{16}\right]}$

$$= \frac{4p+15}{16\left[p^2 - \frac{5^2}{4^2}\right]} = \frac{4}{16}\left[\frac{p}{p^2-\left(\frac{5}{4}\right)^2}\right] + \frac{15}{16}$$

Taking L.I.T b/s, we get

$$L^{-1}\{f(p)\} = \frac{1}{4}\cosh\frac{5}{4}t + \frac{\cancel{15}^{3}}{\cancel{16}_{4}} \cdot \frac{\sinh\frac{5}{4}t}{\frac{5}{4}}$$

$$= \frac{1}{4}\cosh\frac{5}{4}t + \frac{3}{4}\sinh\frac{5}{4}t$$

Section B

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\,dx$$

$$= \frac{1}{\pi}\left(\frac{e^{-x}}{-1}\right)_0^{2\pi} = \frac{-1}{\pi}\left[e^{-2\pi} - e^{0}\right]$$

$$= \frac{-1}{\pi}\left(e^{-2\pi} - 1\right)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{-x}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}\left((-1)\cos nx + n\sin nx\right)\right]_0^{2\pi}$$

$$= \frac{\cancel{e^{-x}}\,1}{\pi(1+n^2)}\left\{\left[e^{-2\pi}(-1)\cos 2n\pi + n\sin 2n\pi\right] - e^{-0}\left[(-1)\cos 0 + n\sin 0\right]\right\}$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-2\pi} - 1\right]$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{-x}\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{(-1)^2+n^2}\left((-1)\sin nx - n\cos nx\right)\right]_0^{2\pi}$$

$$= \frac{1}{(1+n^2)\pi}\left[e^{-2\pi}\left(-\sin 2n\pi - n\cos 2n\pi\right) - e^{-0}\left(-\sin 0 - n\cos 0\right)\right]$$

$$= \frac{1}{\pi(1+n^2)}\left[-e^{-2\pi}n + n\right]$$

$$\therefore f(x) = -\frac{1}{2\pi}\left(e^{-2\pi}-1\right) + \frac{1}{\pi(1+n^2)}\left(-e^{-2\pi}-1\right)\cos nx + \frac{1}{\pi(1+n^2)}\left(-e^{-2\pi}n+n\right)$$

3. $u = \log\sqrt{x^2+y^2} = \frac{1}{2}\log(x^2+y^2)$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2}, \quad \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$$

$w = u + iv$

$$\frac{\partial w}{\partial z} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$= \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

$$\frac{dw}{dz} = \frac{z}{z^2} = \frac{1}{z}$$ (Replace $x$ by $z$ & $y$ by

Int, $w = \log z + c$

4. $L[1+te^{-t}]^3$

$= L[1 + t^3e^{-3t} + 3te^{-t} + 3t^2e^{-2t}]$

$= L(1) + L[t^3e^{-3t}] + 3L[te^{-t}] + 3L[t^2e^{-2t}]$

$$= \frac{1}{s} + \frac{6}{(s+3)^4} + \frac{3}{(s+1)^2} + \frac{3\times 2}{(s+1)^3}$$

5. AE $m^2 + m - 6 = 0$

$m = 2, -3$

CF $f_1(y+2x) + f_2(y-3x)$

$$PI = \frac{1}{D^2 + DD' - 6D'^2}\, y\cos x$$

$$= \frac{1}{(D+3D')(D-2D')}\, y\cos x$$

$$= \frac{1}{(D+3D')}\left[\frac{1}{D-2D'}(y\cos x)\right] \qquad m = 2,\ c = y+2x$$

$$= \frac{1}{D+3D'}\int (c-2x)\cos x\, dx$$

$$= \frac{1}{D+3D'}\left[(y+2x)\sin x - 2x\sin x - 2\cos\right.$$

$$= \frac{1}{D+3D'}[y\sin x - 2\cos x] \qquad m = -3 \quad c = y-$$

$$= \int \left[ (c+3x) \sin x - 2\cos x \right] dx$$

$$= -y \cos x + \sin x$$

C.S $z = f_1(y+2x) + f_2(y-3x) - y\cos x + \sin x$

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$a_0 = \frac{1}{1} \int_0^2 f(x)\, dx = \pi$$

$$a_n = \int_0^2 f(x) \cos n\pi x\, dx$$

$$= \int_0^1 \pi x \cos n\pi x\, dx + \int_1^2 \pi(2-x) \cos n\pi x\, dx$$

$$= \left[ \frac{\cos n\pi}{n^2\pi} - \frac{1}{n^2\pi} \right] + \left[ -\frac{\cos 2n\pi}{n^2\pi} + \frac{\cos n\pi}{n^2\pi} \right]$$

$$= \frac{2}{n^2\pi} \left[ (-1)^n - 1 \right]$$

$= 0$ or $-\frac{4}{n^2\pi}$ acc. as $n$ is even or odd

$$b_n = \int_0^2 f(x) \sin n\pi x\, dx$$

$$= \int_0^1 \pi x \sin n\pi x\, dx + \int_1^2 \pi(2-x) \sin n\pi x\, dx$$

$$= 0$$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \cdots \right]$$

## Sec C

$$(A:B) = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ 1 & -1 & 2 & -2 \\ -1 & 2 & -1 & 2 \end{array}\right]$$

$R_{12}$
$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 2 & 1 & -1 & 4 \\ -1 & 2 & -1 & 2 \end{array}\right]$$

$R_2 - 2R_1,\ R_3 + R_1$
$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 3 & -5 & 8 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_3 - \frac{1}{3} R_2$
$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 3 & -5 & 8 \\ 0 & 0 & 8/3 & -8/3 \end{array}\right]$$

$\Rightarrow$
$$x - y + 2z = -2$$
$$3y - 5z = 8$$
$$\frac{8}{3} z = -\frac{8}{3}$$
$$\Rightarrow \quad z = -1, \quad y = 1, \quad x = 1$$

2 $p = 1 - \frac{1}{10} = \frac{9}{10}, \quad q = \frac{1}{10}$

Probability atleast four ships out of five arrive safely $= P(4) + P(5)$

$$= {}^5C_4 \left(\tfrac{1}{10}\right)^1 \left(\tfrac{9}{10}\right)^4 + {}^5C_5 \left(\tfrac{1}{10}\right)^0 \left(\tfrac{9}{10}\right)^5$$
$$= 0.91854$$

$k_1 = h\, f(x_0, y_0) = -0.1$

$k_2 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = -0.09475$

$k_3 = h\, f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.9050$

$k_4 = h\, f(x_0 + h, y_0 + k_3) = -0.0894987$

$= -0.0948372$

$Y = 4048 - 0.1059821$

$K_1 = -0.0895162$
$K_2 = -0.0837904$
$K_3 = -0.0840767$
$K_4 = -0.0781085$
$K = -0.0838931$
$y_2 = 0.8212695$

9. $f(z) = u(x,y) + iv(x,y) = \sqrt{|xy|}$
$u = \sqrt{|xy|}$ , $v = 0$

At $(0,0)$ $\frac{\partial u}{\partial x} = \lim_{x \to 0} \frac{u(x,0) - u(0,0)}{x - 0} = 0$

$\frac{\partial u}{\partial y} = \lim_{y \to 0} \frac{u(0,y) - u(0,0)}{y - 0} = 0$

$\frac{\partial v}{\partial x} = \lim_{x \to 0} \frac{v(x,0) - v(0,0)}{x - 0} = 0$

$\frac{\partial v}{\partial y} = \lim_{y \to 0} \frac{v(0,y) - v(0,0)}{y - 0} = 0$

i.e. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Hence CR eqns are satisfied at origin

$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z} = \lim_{z \to 0} \frac{\sqrt{|xy|} - 0}{x + iy}$

If $z \to 0$ along line $y = mx$

$f'(0) = \lim_{z \to 0} \frac{\sqrt{|mx^2|}}{x(1+im)} = \lim_{z \to 0} \frac{\sqrt{|m|}}{1+im}$

This limit is not unique since it depends upon $m$
i.e. $f'(0)$ does not exist
Hence $f(z)$ is not regular at origin.