Sampling Distribution and Testing of Hypothesis

8.1. POPULATION OR UNIVERSE

An aggregate of objects (animate or inanimate) under study is called **population or universe.** It is thus a collection of individuals or of their attributes (qualities) or of results of operations which can be numerically specified.

A universe consisting of finite number of individuals or members is called a **finite inverse.** For example, the universe of the weights of students in a particular class or the universe of smokes in Rohtak district.

A universe with infinite number of members is known as an **infinite universe.** For example, the universe of pressures at various points in the atmosphere.

In some cases, we may be even ignorant whether or not a particular universe is infinite, *e.g.*, the universe of stars.

The universe of concrete objects is an **existent universe.** The collection of all possible ways in which a specified event can happen is called a **hypothetical universe.** The universe of heads and tails obtained by tossing a coin an infinite number of times (provided that it does not wear out) is a hypothetical one.

8.2. SAMPLING

The statistician is often confronted with the problem of discussing universe of which he cannot examine every member *i.e.*, of which complete enumeration is impracticable. For example, if we want to have an idea of the average per capita income of the people of India, enumeration of every earning individual in the country is a very difficult task. Naturally, the question arises: What can be said about a universe of which we can examine only a limited number of members? This question is the origin of the Theory of Sampling.

A finite sub-set of a universe is called a **sample.** A sample is thus a small portion of the universe. The number of individuals in a sample is called the **sample size.** The process of selecting a sample from a universe is called **sampling.**

The theory of sampling is a study of relationship existing between a population and samples drawn from the population. The fundamental object of sampling is to get as much information as possible of the whole universe by examining only a part of it. An attempt is thus made through sampling to give the maximum information about the parent universe with the minimum effort.

Sampling is quite often used in our day-to-day practical life. For example, in a shop we asses the quality of sugar, rice or any other commodity by taking only a handful of it from the bag and then decide whether to purchase it or not. A housewife normally tests the cooked products to find it they are properly cooked and contain the proper quantity of salt or sugar, by taking a spoonful of it.

8.3. TYPES OF SAMPLING

1. Purposive Sampling
2. Random Sampling

8.6. TEST OF HYPOTHESIS OR TEST OF SIGNIFICANCE

(P.T.U., Jan. 2010)

(a) Introduction (Test of hypothesis)

An important aspect of the sampling theory is to study the test of hypothesis which will enable us to decide, on the basis of the results of the sample, whether (i) the deviation between the observed sample statistic and the hypothetical parameter value and (ii) the deviation between the two sample statistics, is significant or might be attributed due to chance or due to the fluctuations of the sampling.

(b) Null Hypothesis (P.T.U., Jan. 2009)

For applying the tests of hypothesis, we first set up a hypothesis which is a definite statement about the population parameter called **Null hypothesis** denoted by $\mathbf{H_0}$. H_0 assists that there is no significant difference between sample statistic and population parameter and whatever the difference is there, is due to fluctuation of sampling or due to 'chance'.

(c) Alternative Hypothesis

Any hypothesis which is complementary to the null hypothesis $(\mathbf{H_0})$ is called **Alternative hypothesis** denoted by $\mathbf{H_1}$. It is set in such a way that rejection of H_0 implies acceptance of $\mathbf{H_1}$.

For example, if we want to test the null hypothesis that the population has a specified mean μ_0, then we have

$$\mathbf{H_0} : \mu = \mu_0$$

Alternative hypothesis will be

(i) $H_1 : \mu \neq \mu_0$ ($\mu > \mu_0$ *or* $\mu < \mu_0$) (two tailed alternative hypothesis).

(ii) $H_1 : \mu > \mu_0$ (right tailed alternative hypothesis (*or*) single tailed).

(iii) $H_1 : \mu < \mu_0$ (left tailed alternative hypothesis (*or*) single tailed).

Hence, alternative hypothesis helps to know whether the test is two tailed test or one tailed test.

8.7. ERRORS IN SAMPLING

(P.T.U., Jan. 2009, May 2011, Dec. 2013)

The main aim of the sampling theory is to draw a valid conclusion about the population parameters on the basis of the sample results. In doing this we may commit the following two types of errors:

Type I. Error. It involves rejection of null hypothesis H_0 when it should be accepted (true) probability of committing type I error = P(Reject H_0 when it is true) = P(Reject H_0/H_0) = α

α is called the size of the type I error also referred to as **producer's risk.**

Type II. Error. It involves acceptance of null hypothesis H_0 when it should have been rejected (wrong) probability of committing type II error = P(Accept H_0 when it is wrong) = P(Accept H_0/H_1) = β.

β is called the size of the type II error, also referred to as **consumer's risk.**

8.8. LEVEL OF SIGNIFICANCE

(P.T.U., Jan. 2009)

The level of significance is the maximum probability of making a type I error and is denoted by α *i.e.*, P(Rejecting H_0 when H_0 is true) = P(Rejecting H_0/H_0) = α or in other words level of significance is the probability of the value of the variate falling in the critical region. The best value for fixing the level of significance depends on the seriousness of the results of the types of error.

The commonly used level of significance in practice are 5% (0.05) and 1% (0.01). If we use 5% level of significance *i.e.*, $\alpha = 0.05$, we mean that there is probability of making 5 out of 100 type I error. Also it means that we are 95% confident that a correct decision has been made. Similarly 1% level of significances means that there is probability of making 1 out of 100 errors and 99% confident about correct decision.

Note 1. If no level of significance is given we always take $\alpha = 0.05$ (*i.e.*, 5%)

Note 2. Level of significance is always fixed in advance before collecting the sample information.

8.9(a). CRITICAL REGION OR REJECTION REGION

(P.T.U., Jan. 2009)

Suppose we take several samples of same size from a given population and compute some statistic t for each of these samples. Let $t_1, t_2, ..., t_k$ be the values of statistic for these samples. Each of these values may be used to test some null hypothesis H_0. Some values may lead to the rejection of H_0 while others may lead to acceptance of H_0. These sample statistic $t_1, t_2, ..., t_k$ may be divided into two (mutually) disjoint groups, one leading to the rejection of H_0 and other leading to acceptance of H_0.

The statistic which leads to the rejection of H_0 gives us a **Critical Region (C)** or **Rejection Region (R)**, while those which lead to the acceptance of H_0 gives us a region called **Acceptance Region (A)**. Thus if $t \in C$ then H_0 is rejected and if $t \in A$, H_0 is accepted.

Note that $C \cap A = \phi$ and $C \cup A = S$ (Sample space).

8.9(b). CRITICAL VALUES OR SIGNIFICANT VALUES

Exact Sample Tests.

8.16. DEGREE OF FREEDOM

(P.T.U., Jan. 2009)

Degree of freedom (dof) is a positive integer denoted by ν (the greek letter nu) and equal to $n - k$, where n is the number of independent observations of the random sample and k is the number of population parameters which are calculated using the sample data. Thus dof (ν) $= n - k$ is the difference between n, the sample size and k the number of independent constraints imposed on the observations in the sample, *e.g.*, If we have to choose any four numbers whose sum is 50, we can exercise our independent choice for any three numbers only, the fourth being 50 minus the total of the three numbers selected. Thus though we were to choose any four numbers, our choice was reduced to three because of one condition imposed. There was only one restraint on our-freedom.

$\therefore$ Our degrees of freedom were $4 - 1 = 3$. Similarly if two restrictions were imposed, our degrees of freedom will be $4 - 2 = 2$.

When n variables be connected by one relation (say mean), the degrees of freedom will be $n - 1$.

8.17(a). STUDENTS *t*-DISTRIBUTION

(P.T.U., May 2003, 2004, 2007)

We have discussed in art. (8.13) that in the development of large sample test for mean, the statistic

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1) \qquad \ldots(1)$$

If the population variance is unknown then for large samples, its estimate provided by sample variance s^2 is used and normal test is applied. For small samples, an unbiased estimate of population variance σ^2 is given by

$$S^2 = n\frac{1}{n-1}\Sigma(x_i - \bar{x})^2$$

$$\therefore \qquad S^2 = \frac{1}{n-1}ns^2 \qquad \because \quad s^2 = \Sigma\frac{(x_i - \bar{x})^2}{n}$$

or $\qquad ns^2 = (n - 1)\, S^2$

It was customary to replace σ^2 by S^2 (for small samples) in (1) and then apply the normal test even for small samples. Thus W.S. Gosset obtained the sampling distribution of the statistic $\dfrac{\bar{x} - \mu}{S/\sqrt{n}}$ for small samples and showed that it was far from normality. This discovery started a new field, *viz*, 'Exact Sample Tests' in the history of statistical inference.

We write $\qquad t = \dfrac{\bar{x} - \mu}{S/\sqrt{n}} \sim t\,(n - 1)$ d.f.

Note. This *t*-distribution is used when sample size is ≤ 30 and the population variance is unknown.

The *t*-table

The *t*-table given at the end of the book is the probability integral of *t*-distribution. The *t*-distribution has different values for each degrees of freedom and **when the degrees of freedom are infinitely large, the *t*-distribution is equivalent to normal distribution and the probabilities shown in the normal distribution tables are applicable.**

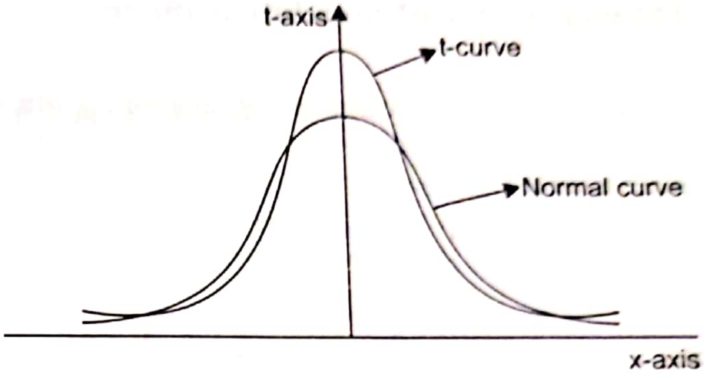
8.17(b). CRITICAL VALUE OF 't'

The critical value or significant value of t, at level of significance α, degree of freedom ν, for two tailed test is given by

$$P[|t| > t_\nu(\alpha)] = \alpha$$
$$P[|t| < t_\nu(\alpha)] = 1 - \alpha$$

The significant value of t, at level of significance α, for a single tailed test can be got from those of two tailed test by referring to the value 2α. For example $t_\nu(0.05)$ for single tailed test $= t_\nu(0.10)$ for two tailed test.

8.17(c). PROPERTIES OF t-DISTRIBUTION



1. t-distribution is used when sample size is very small *i.e.*, ≤ 30.
2. The curve is symmetrical about $t = 0$, like normal curve but it is more peaked than the normal curve with the same S.D.
3. t curve attains its maximum value at $t = 0$, so that its mode coincides with the mean and each $= 0$.
4. As t-distribution is symmetric about $t = 0$ $\therefore$ All the moments of odd order about mean are zero.
5. A very interesting property of the sampling distribution of t is that it does not depend upon the population parameters but depends only on $\nu = n - 1$ *i.e.*, depends on the sample size.

8.18. APPLICATION OF t-DISTRIBUTION

Some of the applications of t-distribution are given below:

1. *To test the significance of the mean of a random sample i.e., to test if the sample mean $\bar{x}$ differs significantly from the hypothetical value μ of the population mean.*
2. *To test the significance of difference between the means of two samples (samples may be independent or dependent).*
3. *To test the significance of observed partial and multiple correlation coefficients.*

8.19. TEST I: t-TEST OF SIGNIFICANCE OF SINGLE MEAN

This test is used to test whether the mean of a sample drawn from a normal population deviates significantly from specified value of the population mean (variance of population is unknown).

Let x_i $(i = 1, 2, \ldots, n)$ be a random sample drawn from a normal population with specified mean μ then $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

Under Null Hypothesis H_0, there is no significant difference between the sample mean and the population mean *i.e.*, the difference is due to fluctuation of sampling

Under H_0, the test statistic is $t = \dfrac{\bar{x} - \mu}{S.E(\bar{x})}$; we know that $S.E(\bar{x}) = \dfrac{S}{\sqrt{n}}$ (art. 8.5)

$$\therefore \qquad t = \frac{\bar{x} - \mu}{S/\sqrt{n}} \text{ with d.f. } \nu = n - 1 \qquad \ldots(I)$$

and S is given by $\quad S^2 = \dfrac{1}{n-1} \Sigma (x_i - \bar{x})^2$

Conclusion. Compute the value of t from (I) and compare it with the tabulated value of t for $(n-1)$ d.f. at derived level of significance. If calculated value of $|t|$ is > the tabulated value, we say H_0 is rejected ⇒ difference between sample mean and population mean is significant. If $|t| <$ the tabulated value of 't' then H_0 is accepted ⇒ difference is in significant.

Fiducial limits of population mean

If t_α is the table value of t at level of significance α at $(n-1)$ degree of freedom then $\left|\dfrac{\bar{x}-\mu}{S/\sqrt{n}}\right| < t_\alpha$ for acceptance

or $\qquad |\mu - \bar{x}| < t_\alpha \dfrac{S}{\sqrt{n}}$

or $\qquad \bar{x} - t_\alpha \dfrac{S}{\sqrt{n}} < \mu < \bar{x} + t_\alpha \dfrac{S}{\sqrt{n}}$

95% confidence limits (level of significance 5%) are $\bar{x} \pm t_{0.05} \dfrac{S}{\sqrt{n}}$

99% confidence limits (level of significance 1%) are $\bar{x} \pm t_{0.01} \dfrac{S}{\sqrt{n}}$.

Note 1. In numerical problems quite often we are given the sample S.Ds. Hence for such problems we can use $S^2 = \dfrac{n}{n-1} s^2$ and the test statistic changes to

$$t = \frac{\bar{x} - \mu}{\sqrt{\frac{n}{n-1} s^2} \cdot \frac{1}{\sqrt{n}}} = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \sim t(n-1)$$

Note 2. Computation of S^2 for numerical problems: if $\bar{x}$ is an integer then S^2 can be computed very easily by applying $S^2 = \dfrac{\Sigma(x-\bar{x})^2}{n-1}$. However if $\bar{x}$ **is a fraction and the values of x are small** then to compute S^2 we can use step deviation method given below:

We know that
$$\begin{aligned} \Sigma(x-\bar{x})^2 &= \Sigma(x^2 - 2x\bar{x} + \bar{x}^2) \\ &= \Sigma x^2 - 2\bar{x}\,\Sigma x + \Sigma \bar{x}^2 \\ &= \Sigma x^2 - 2\bar{x} . n\bar{x} + n\bar{x}^2 \\ &= \Sigma x^2 - n\bar{x}^2 \end{aligned} \qquad \left| \begin{aligned} \because \quad \bar{x} &= \frac{\Sigma x}{n} \\ \therefore \quad \Sigma x &= n\bar{x} \end{aligned} \right.$$

$$\therefore \qquad \Sigma(x-\bar{x})^2 = \Sigma x^2 - n\left(\frac{\Sigma x}{n}\right)^2$$

$$S^2 = \frac{1}{n-1}\left[\Sigma - \frac{(\Sigma x)^2}{n}\right]$$

$\therefore$

But if the values of x are not small and $\bar{x}$ is a fraction, then we use the following method. If $d = x - A$ where A is the assumed mean then $\frac{1}{n}\Sigma(x-\bar{x})^2 = \frac{1}{n}\Sigma d^2 - \left(\frac{\Sigma d}{n}\right)^2 \Rightarrow \Sigma(x-\bar{x})^2 = \Sigma d^2 - \frac{(\Sigma d)^2}{n}$

$$\therefore \quad S^2 = \frac{1}{n-1}\left[\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right] \text{ and } \bar{x} = A + \frac{\Sigma d}{n}.$$

ILLUSTRATIVE EXAMPLES

Example 1. *A random sample of size 16 has 53 as mean. The sum of squares of the derivation from mean is 135. Can this sample be regarded as taken from the population having 56 as mean? Obtained 95% and 99% confidence limits of the mean of the population.*

Solution. Given : $\bar{x} = 53, \mu = 56, n = 16, \Sigma(x-\bar{x})^2 = 135$

H_0: There is no significant difference between the sample mean and hypothetical population mean.

$$H_0 : \mu = 56; \quad H_1 : \mu \neq 56 \quad \text{(Two tailed test)}$$

Apply t-test

$$t : \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}} \sim t(n-1)\,\text{d.f.}$$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{135}{15}} = 3$$

$$t = \frac{53-56}{3/\sqrt{16}} = \frac{-3\times 4}{3} = -4$$

$$|t| = 4; \quad \text{d.f. } (v) = 16 - 1 = 15$$

Conclusion. For $v = 15$, $t_{0.05} = 2.13$ (See table at the end of the book.)

Since $|t| = 4 > t_{0.05} = 2.13$ *i.e.*, the calculated value of t is more than the table value. The hypothesis is rejected. Hence the sample mean has not come from a population having 56 as mean.

95% confidence limits of the population mean.

$$\bar{x} \pm \frac{S}{\sqrt{n}} t_{0.05}, = 53 \pm \frac{3}{\sqrt{16}}(2.13) = 51.41; 54.59$$

99% confidence limits of the population mean.

$$\bar{x} \pm \frac{S}{\sqrt{n}} t_{0.01}, = 53 \pm \frac{3}{\sqrt{16}}(2.95) = 50.79; 55.21.$$

Example 2. *The life time of electric bulbs for a random sample of 10 from a large consignment gave the following data:*

Item	1	2	3	4	5	6	7	8	9	10
Life in '000 hrs	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6

Can we accept the hypothesis the average lifetime of bulb is 4000 hr?

Solution. H_0 : There is no significant difference in the sample mean and population mean *i.e.*, $\mu = 4000$ hr.

$H_1 : \mu \neq 4000$ hr

Applying the t-test : $t = \dfrac{\bar{x} - \mu}{\dfrac{S}{\sqrt{n}}} \sim (10-1)$ d.f.

$$\bar{x} = \frac{\Sigma x}{n} = \frac{44}{10} = 4.4$$

x	4.2	4.6	3.9	4.1	5.2	3.8	3.9	4.3	4.4	5.6
$x - \bar{x}$	−0.2	0.2	−0.5	−0.3	0.8	−0.6	−0.5	−0.1	0	1.2
$(x - \bar{x})^2$	0.04	0.04	0.25	0.09	0.64	0.36	0.25	0.01	0	1.44

$$\Sigma (x - \bar{x})^2 = 3.12$$

$$S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}} = \sqrt{\frac{3.12}{9}} = 0.589$$

$$t = \frac{4.4 - 4}{\dfrac{0.589}{\sqrt{10}}} = 2.123$$

For $\quad v = 9, t_{0.05} = 2.26$ (See table at the end of the book.)

Conclusion. Since the calculated value of t is less than table value $t_{0.05}$. ∴ The hypothesis $\mu = 4000$ hours is accepted.

i.e., The average lifetime of bulbs could be 4000 hours.

Example 3. *A machine is expected to produce nails of length 3″. A random sample of 25 nails gave an average length of 3.1″ with S. D 0.3. Can it be said that the machine is producing nails as per significations (value of $t_{0.05}$ for 24 d. f is 2.064)?* **(P.T.U., May 2006)**

Solution. Given, $n = 25, \bar{x} = 3.1'', s = 0.3, \mu = 3''$

H_0 : Machine is producing nails as per significations *i.e.*, $\mu = 3''$

$H_1 : \mu \neq 3''$

The test statistic $= \dfrac{\bar{x} - \mu}{S/\sqrt{n}} = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

∴ $\quad t = \dfrac{3.1 - 3}{0.3/\sqrt{24}} = \dfrac{0.1}{0.3}(4.9) = 1.63$

As calculated value of t *i.e.*, 1.63 is < the tabulated value of t *i.e.*, 2.064 (given)

∴ H_0 is accepted

Hence machines are producing nails as per significations.

Example 4. *The 9 items of a sample have the following value 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these values differ significantly from the assumed mean 47.5?* **(P.T.U., Dec. 2011, May 2014)**

Solution. Given, $n = 9$, $\mu = 47.5$

$H_0 : \mu = 47.5$ *i.e.*, There is no significant difference between the sample and population mean.

$H_1 : \mu \neq 47.5$ (two tailed test):

x	45	47	50	52	48	47	49	53	51
$x - \bar{x}$	−4.1	−2.1	0.9	2.9	−1.1	−2.1	−0.1	3.9	1.9
$(x - \bar{x})^2$	16.81	4.41	0.81	8.41	1.21	4.41	0.01	15.21	3.61

$$\bar{x} = \frac{\Sigma x}{n} = \frac{442}{9} = 49.11 ; S^2 = \frac{\Sigma(x - \bar{x})^2}{n-1} = \frac{54.89}{8} = 6.861 \quad \therefore \ S = 2.619$$

Apply t-test :
$$t = \frac{\bar{x} - \mu}{S/\sqrt{n}} = \frac{49.1 - 47.5}{2.619/\sqrt{9}} = \frac{(1.6)\sqrt{9}}{2.619} = 1.832$$

$$t_{0.05} = 2.31 \text{ for } \upsilon = 8$$

Conclusion. Since $|t| < t_{0.05}$, the hypothesis is accepted *i.e.*, there is no significant difference between their mean.

Note. We can compute the value of S by step deviation method using Note 2 of art. 8.19

As $\bar{x}$ is a fraction and the values of x are not small

$\therefore$ Let us consider assumed mean A = 50

x	$d = x - A$	d^2
45	− 5	25
47	− 3	9
50	0	0
52	2	4
48	− 2	4
47	− 3	9
49	− 1	1
53	3	9
51	1	1
	$\Sigma d = -8$	$\Sigma d^2 = 62$

$$\therefore \quad S^2 = \frac{1}{n-1}\left[\Sigma d^2 - \frac{(\Sigma d)^2}{n}\right] = \frac{1}{8}\left[62 - \frac{64}{9}\right]$$

$$= 6.86$$

Example 5. *Compute the value of students t for the values in a sample of size eight consisting of − 4, − 2, −2, 0, 2, 2, 3 and 3 taking the population mean as zero.* **(P.T.U., May 2005)**

Solution. First of all we will compute $\bar{x}$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{-4 - 2 - 2 + 0 + 2 + 2 + 3 + 3}{8} = \frac{2}{8} = \frac{1}{4}$$

x	-4	-2	-2	0	2	2	3	3
$x-\bar{x}$	$-\frac{17}{4}$	$-\frac{9}{4}$	$-\frac{9}{4}$	$-\frac{1}{4}$	$\frac{7}{4}$	$\frac{7}{4}$	$\frac{11}{4}$	$\frac{11}{4}$
$(x-\bar{x})^2$	$\frac{289}{16}$	$\frac{81}{16}$	$\frac{81}{16}$	$\frac{1}{16}$	$\frac{49}{16}$	$\frac{49}{16}$	$\frac{121}{16}$	$\frac{121}{16}$

$$\Sigma(x-\bar{x})^2 = \frac{792}{16}; \quad S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \left(\frac{792}{16}\right)\frac{1}{7} = 7.07$$

$$\therefore \quad S = \sqrt{7.07} = 2.66$$

Applying t-test

$$t = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} = \frac{\frac{1}{4}-0}{2.66}\sqrt{8} = \frac{\sqrt{8}}{4(2.66)}$$

$$t = 0.266.$$

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Example 6. *A certain stimulus administer to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, – 1, 3, 0, 6, – 2, 1, 5, 0, 4. Can it be concluded that the stimulus will be in general accompanied by an increase in blood pressure?*

Solution. To test whether the mean increase in blood pressure of all patients to whom the stimulus is administered will be positive. We have to assume that the population is normal with mean μ and S.D σ which are unknown

$$H_0 : \mu = 0, H_1 : \mu \neq 0$$

$$\bar{x} = \frac{5+2+8-1+3+0+6-2+1+5+0+4}{12} = \frac{31}{12} = 2.58 = 2.6 \text{ (approx.)}.$$

x	5	2	8	– 1	3	0	6	– 2	1	5	0	4
$x-\bar{x}$	2.4	– 0.6	5.4	– 3.6	0.4	– 2.6	3.4	– 4.6	– 1.6	2.4	– 2.6	1.4

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{104.92}{11} = 9.54, S = 3.08$$

Test statistic

$$t = \frac{\bar{x}-\mu}{\frac{S}{\sqrt{n}}} = \frac{2.6-0}{3.08}\sqrt{12} = \frac{2.6}{3.08}(3.464) = 2.924.$$

For $v = 11$ the tabulated value, $t_{0.05} = 1.796$

Another method to compute S^2 is $S^2 = \frac{1}{n-1}\left\{\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right\}$ $\because$ the values of x are small (see Note 1, art. 8.19)

$$\therefore \quad S^2 = \frac{1}{11}\left[185 - \frac{961}{12}\right] = 9.54$$

Conclusion. As calculated value of $|t| = 2.924$ is > the tabulated value 1.796 $\therefore$ H_0 is rejected *i.e.,* There is increase in the blood pressure.

Example 7. *A mechanist is making engines parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a S.D. of 0.04 inch. On the basis of sample, would you say that work is inferior.*

Solution. Here, we have $\mu = 0.7,\ \bar{x} = 0.742, s = 0.04, n = 10$

H_0 : Work is not inferior, *i.e.*, there is no significant difference between $\bar{x}$ and μ, *i.e.*, $\mu = 0.7$,

$H_1 : \mu \neq 0.7$

The test statistic
$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}} = \frac{0.742-0.700}{0.04}\sqrt{9} = \frac{(0.042)(3)}{0.04} = 3.15$$

Degree of freedom $v = 10 - 1 = 9$

For $v = 9$, the tabulated value, $t_{0.05} = 2.26$

Conclusion. As calculated value of t; $3.15 >$ the tabulated value 2.26

$\therefore$ H_0 is rejected which shows that the work is inferior.

Example 8. *The following results are obtained from a sample of 10 boxes of biscuits, mean weight content = 490 g, S.D of the weight = 9 g. Could the sample come from a population having a mean of 500 gm?*

Solution. Given, $n = 10,\ \bar{x} = 490, s = 9$ g, $\mu = 500$

H_0 : The sample comes from a population having a mean of 500 gm *i.e.*, $\mu = 500$,

$H_1 : \mu \neq 500$

The test statistic is
$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{\bar{x}-m}{\frac{s}{\sqrt{n-1}}} = \frac{490-500}{\frac{9}{\sqrt{9}}} = -\frac{10}{3} = -3.33$$

For $v = 9$, the tabulated value, $t_{0.05} = 2.26$

Conclusion. Since calculated value of $|t|$ *i.e.*, $3.33 >$ tabulated value 2.26

$\therefore$ H_0 is rejected *i.e.*, $\mu \neq 500$

$\therefore$ The sample could not have come from the population having mean 500 g.

Example 9. *A manufacturer intends that his electric bulbs have a life of 1000 hours. He tests a sample of 20 bulbs drawn at random from a batch and discovers that the mean life of the sample bulb is 990 hours with S.D of 22 hours. Does this signify that batch is not upto the mark?* **(P.T.U., Dec. 2005)**

Solution. Here, $\mu = 1000$ hours.

$n = 20,\ \bar{x} = 990$ hours, S.D $(s) = 22$

H_0 : the batch upto the mark *i.e.*, $\mu = 1000$

$H_1 : \mu \neq 1000$

Applying t-test

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{990-1000}{22/\sqrt{19}} = \frac{-10\sqrt{19}}{22} = -1.95$$

$|t| = 1.95$

Degree of freedom $v = 20 - 1 = 19$, for $v = 19$; tabulated value $t_{0.05} = 2.09$.

Conclusion. As calculated value of $|t| < 2.09$, the tabulated value

$\therefore$ H_0 is accepted

$\therefore$ Batch is upto the mark.

TEST YOUR KNOWLEDGE

1. Ten individuals are choosen at random from a normal population of students and their marks found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. In the light of these data discuss the suggestion that mean mark of the population of students is 66.
2. The following values gives the lengths of 12 samples of egyptian cotton taken from a consignment : 48, 46, 49, 46, 52, 45, 43, 47, 47, 46, 45, 50. Test if the mean length of the consignment can be taken as 46.
3. A sample of 18 items has a mean 24 units and standard deviation 3 units. Test the hypothesis that it is a random sample from a normal population with mean 27 units.
4. A random sample of 10 boys had the I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107 and 100. Do these data support the assumption of a population mean I.Q. of 160?
5. A filling machine is expected to fill 5 kg of powder into bags. A sample of 10 bags gave the following weights. 4.7, 4.9, 5.0, 5.1, 5.2, 4.6, 5.1, 4.6 and 4.7. Test whether the machine is working properly.
6. A drug is given to 10 patients and the increase in the blood pressure were recorded to be 3, 6, – 2, 4, – 3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood pressure.
7. Annual rainfall at a certain place is normally distributed with mean 45 cm. The rainfall for the last 5 years was 48 cm, 42 cm, 40 cm, 44 cm and 43 cm. Can it be concluded that the average rainfall during the last five years is less than the normal rainfall [**Hint:** $\bar{x} = 43.4$ s = 8.8]. **(P.T.U., Jan. 2008)**

ANSWERS

1. Accepted	**2.** Accepted	**3.** Rejected
4. Accepted	**5.** Accepted	**6.** Accepted
7. Accepted.		

8.20(a). TEST II: *t*-TEST FOR DIFFERENCE OF MEANS OF TWO SMALL SAMPLES (From a Normal Population)

This test is used to test if two independent samples have been drawn from two normal populations having the same means, the population variances being equal.

Let x_i $(i = 1, 2, 3, \ldots, n_1)$ and y_j $(j = 1, 2, 3, \ldots, n_2)$ be two independent random samples of size n_1 and n_2 respectively, drawn from two normal populations with equal means under the assumption that the population variances are unknown and equal *i.e.*, $\sigma_1^2 = \sigma_2^2 = \sigma^2$, we set up the null hypothesis.

H_0; the samples have been drawn from the normal populations with same means *i.e.*, $\mu_1 = \mu_2$ or in other words we can say that the sample means $\bar{x}$ and $\bar{y}$ donot differ significantly.

Under H_0, the test statistic is

$$t = \frac{\bar{x} - \bar{y}}{S\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} \sim t(n_1 + n_2 - 2) \text{ d.f.}$$

where $$\bar{x} = \frac{\Sigma x_i}{n_1},\ \bar{y} = \frac{\Sigma y_j}{n_2} \text{ and } S^2 = \frac{1}{n_1 + n_2 - 2}\left[(x - \bar{x})^2 + (y - \bar{y})^2\right] \text{ is an} \quad \ldots(1)$$

Unbiased estimate of the common population variance σ^2 based on both the samples.

Conclusion. If the calculated value of $|t|$ is < the tabulated value of t for $n_1 + n_2 - 2$ degree of freedom at derived level of significance (usually 5%) then H_0 is accepted *i.e.*, the sample means donot differ significantly and if calculated value > tabulated value of t then H_0 is rejected.

Note. In numerical problems, quite often we are given standard deviations of the samples *i.e.*, s_1 and s_2 are given then S^2 can be expressed in s_1 and s_2: As we know $S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1}$, $S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2}$

$\therefore$ From (1) $$S^2 = \frac{1}{n_1+n_2-2}[n_1 s_1^2 + n_2 s_2^2]$$

or $$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}. \qquad \ldots(2)$$

8.20(b). PAIRED *t*-TEST FOR DIFFERENCE OF MEANS OF TWO SMALL SAMPLES

In the *t*-test for difference of means, the two samples were independent of each other. Let us now take a particular situation where:

(*i*) The sample sizes are equal *i.e.*, $n_1 = n_2 = n$ (say)

(*ii*) The sample observations $(x_1, x_2, \ldots, x_n)$ and $(y_1, y_2, \ldots, y_n)$ are not completely independent but they are dependent in pairs *i.e.*, the pairs of observations $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ correspond to the 1st, 2nd, ..., nth unit respectively. Such types of observations generally form 'before and after' type of data. For example we want to test whether advertisement is really effective in promoting sales of a particular product. Let $x_1, x_2, \ldots, x_n$ be the sales of product in n departmental stores for a certain period before advertisement campaign and $y_1, y_2, \ldots, y_n$ be the corresponding sales of the same product in the same departmental stores for the same period after advertisement. Now (x_i, y_j), $0 = 1, 2, \ldots, n$ is a pair of sales in the ith departmental store before and after advertisement. In such cases we can't apply the difference of means test (discussed in previous article). Here we apply paired t-test given below:

Let $d_i = x_i - y_i$ $(i = 1, 2, \ldots, n)$ denote the difference in the observations under null hypothesis that the increase in sale is not due to advertisement, it is just by chance we have H_0; $\mu_1 = \mu_2$

The test statistic is $t = \frac{\bar{d}}{S/\sqrt{n}} \sim t(n-1)$ d.f.

where $$d_i = x_i - y_j$$

and $$\bar{d} = \frac{\Sigma d_i}{n} \text{ and } S^2 = \frac{\sum_{i=1}^{n}(d_i - \bar{d})^2}{n-1} \text{ or } S^2 = \frac{1}{n-1}\left[\Sigma d_i^2 - \frac{(\Sigma d_i)^2}{n}\right].$$

ILLUSTRATIVE EXAMPLES

Example 1. *Two samples of sodium vapour bulbs were tested for length of life and the following results were got:*

	Size	*Sample mean*	*Sample S.D.*
Type I	*8*	*1234 hrs*	*36 hrs*
Type II	*7*	*1036 hrs*	*40 hrs*

Is the difference in the means significant to generalise that type I is superior to type II regarding length of life?

Solution. $H_0: \mu_1 = \mu_2$ *i.e.*, Two types of bulbs have same lifetime

$H_1: \mu_1 > \mu_2$ *i.e.*, Type I is superior to Type II (right tailed test)

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{8 \times (36)^2 + 7(40)^2}{8 + 7 - 2} = 1659.076 \qquad \therefore \quad S = 40.7317$$

The t-statistic
$$t = \frac{X - Y}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{1234 - 1036}{40.7317\sqrt{\frac{1}{8} + \frac{1}{7}}} = 18.1480 \sim t \; (n_1 + n_2 - 2 \text{ d.f.})$$

$t_{0.10}$ at d.f. 13 is 1.77 (One tailed test).

Conclusion. Since calculated $|t|$ is much $> t_{0.10}$ H_0 is rejected *i.e.*, H_1 is accepted.

$\therefore$ Type I is definitely superior to type II.

Example 2. *Samples of sizes 10 and 14 were taken from two normal populations with S.D 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% level.*

Solution. $H_0 : \mu_1 = \mu_2$ *i.e.*, The means of the two populations are the same.

$H_1 : \mu_1 \neq \mu_2$.

Given $\quad \bar{X} = 20.3, \; \bar{Y} = 18.6; \; n_1 = 10, \; n_2 = 14, \; s_1 = 3.5, s_2 = 5.2$

$$S^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{10(3.5)^2 + 14(5.2)^2}{10 + 14 - 2} = 22.775 \;\therefore\; S = 4.772$$

$$t = \frac{\bar{X} - \bar{Y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{20.3 - 18.6}{\left(\sqrt{\frac{1}{10} + \frac{1}{14}}\right)4.772} = 0.8604$$

the value of t at 5% level for $(n_1 + n_2 - 2 = 22)$ d.f. is $t_{0.05} = 2.0739$.

Conclusion. Since $|t| = 0.8604 < t_{0.05}$ the hypothesis is accepted *i.e.*, There is no significant difference between their means.

Example 3. *The height of 6 randomly chosen sailors are in inches are 63, 65, 68, 69, 71 and 72. Those of 9 randomly chosen soldiers are 61, 62, 65, 66, 69, 70, 71, 72 and 73. Test whether the sailors are on the average taller than soldiers.*

Solution. Let X and Y be the two samples denoting the heights of sailors and soldiers.

Given the sample size $n_1 = 6, n_2 = 9$, $H_0 : \mu_1 = \mu_2$.

i.e., The mean of both the population are the same.

$H_1 : \mu_1 > \mu_2$ (one tailed test).

Calculation of two sample mean :

X	63	65	68	69	71	72
$X - \bar{X}$	–5	–3	0	1	3	4
$(X - \bar{X})^2$	25	9	0	1	9	16

$$\bar{X} = \frac{\Sigma X}{n_1} = 68 ; \Sigma(X-\bar{X})^2 = 60$$

Y	61	62	65	66	69	70	71	72	73
$Y-\bar{Y}$	−6.66	−5.66	−2.66	1.66	1.34	2.34	3.34	4.34	5.34
$(Y-\bar{Y})^2$	44.36	32.035	7.0756	2.7556	1.7956	5.4756	11.1556	18.8356	28.5156

$$\bar{Y} = \frac{\Sigma Y}{n_2} = 67.66; \Sigma(Y-\bar{Y})^2 = 152.0002$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(X-\bar{X})^2 + \Sigma(Y-\bar{Y})^2\right]$$

$$= \frac{1}{6+9-2}[60+152.0002] = 16.3077 \quad \therefore \quad S = 4.038$$

Under H_0,
$$t = \frac{\bar{X}-\bar{Y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{68-67.666}{4.0382\sqrt{\frac{1}{6}+\frac{1}{9}}} = 0.3031 \sim t(n_1+n_2-2 \text{ d.f.})$$

The value of t at 10% level of significance ($\because$ the test is one tailed) for 13 d.f. is 1.77.

Conclusion. Since $|t| = 0.3031 < t_{0.05} = 1.77$ the hypothesis H_0 is accepted.

i.e., there is no significance difference between their average

i.e., the sailors are not on the average taller than the soldiers.

Example 4. *The following figures refer to observations in live independent samples.*

Sample I	*25*	*30*	*28*	*34*	*24*	*20*	*13*	*32*	*22*	*38*
Sample II	*40*	*34*	*22*	*20*	*31*	*40*	*30*	*23*	*36*	*17*

Analyse whether the samples have been drawn from the populations of equal means.

Solution. H_0 : The two samples have been drawn from the population of equal means *i.e.*, There is no significant difference between their means *i.e.*, $\mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (Two tailed test)

Given n_1 = Sample I size = 10 ; n_2 = sample II size = 10

To calculate the two sample mean and sum of squares of deviation from mean. Let X be the sample I and Y be the sample II.

X	25	30	28	34	24	20	13	32	22	38
$X-\bar{X}$	−1.6	3.4	1.4	7.4	−2.6	−6.6	−13.6	5.4	4.6	11.4
$(X-\bar{X})^2$	2.56	11.56	1.96	54.76	6.76	43.56	184.96	29.16	21.16	129.96
Y	40	34	22	20	31	40	30	23	36	17
$Y-\bar{Y}$	10.7	4.7	−7.3	−9.3	1.7	10.7	0.7	−6.3	6.7	−12.3
$(Y-\bar{Y})^2$	114.49	22.09	53.29	86.49	2.89	114.49	0.49	39.67	44.89	151.29

$$\bar{X} = \frac{\Sigma X}{n_1} = \frac{266}{10} = 26.6 \qquad Y = \frac{\Sigma Y}{n_2} = \frac{293}{10} = 29.3$$

$$\Sigma(X-\bar{X})^2 = 486.4 \qquad \Sigma(Y-\bar{Y})^2 = 630.08$$

$$S^2 = \frac{1}{n_1+n_2-2}\left[\Sigma(X-\bar{X})^2 + \Sigma(Y-\bar{Y})^2\right]$$

$$= \frac{1}{10+10-2}[486.4+630.08] = 62.026 \qquad \therefore\ S = 7.875.$$

Another method to find the values of $\Sigma(X-\bar{X})^2$, $\Sigma(Y-\bar{Y})^2$. As values of $\bar{X}$ and $\bar{Y}$ are fractions and also the values of X and Y are large.

∴ We can apply step deviation method (discussed in Note 2 art. 8.19)

Let the assumed mean for sample I be A = 28 and the assumed mean for sample II be B = 31.

X	$d_1 = X - A$	d_1^2	Y	$d_2 = Y - B$	d_2^2
25	– 3	9	40	9	81
30	2	4	34	3	9
28	0	0	22	– 9	81
34	6	36	20	– 11	121
24	– 4	16	31	0	0
20	– 8	64	40	9	81
13	– 15	225	30	– 1	1
32	4	16	23	– 8	64
22	– 6	36	36	5	25
38	10	100	17	– 14	196
	$\Sigma d_1 = -14$	$\Sigma d_1^2 = 506$		$\Sigma d_2 = -17$	$\Sigma d_2^2 = 659$

$$\therefore \quad \Sigma(X-\bar{X})^2 = \Sigma d_1^2 - \frac{(\Sigma d_1)^2}{n_1} = 506 - \frac{196}{10} = 486.4$$

$$\Sigma(Y-\bar{Y})^2 = \Sigma d_2^2 - \frac{(\Sigma d_2)^2}{n_2} = 659 - \frac{289}{10} = 630.1$$

Under H_0, the test statistic is given by

$$t = \frac{\bar{X}-\bar{Y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{26.6-29.3}{7.875\sqrt{\frac{1}{10}+\frac{1}{10}}} = -0.7666 \sim t(n_1+n_2-2 \text{ d.f.})$$

$$|t| = 0.7666.$$

Conclusion. The tabulated value of t at 5% level of significance for 18 d.f. is 2.1. Since the calculated value $|t| = 0.7666 < t_{0.05}$, H_0 is accepted.

i.e., there is no significant difference between their means

i.e., the two samples have been drawn from the populations of equal means.

Example 5. *Memory capacity of 9 students was tested before and after a course of meditation for a month. State whether the course was effective or not from the data below (in same units)*

Before	*10*	*15*	*9*	*3*	*7*	*12*	*16*	*17*	*4*
After	*12*	*17*	*8*	*5*	*6*	*11*	*18*	*20*	*3*

Solution. Since the data are correlated and concerned with the same set of students we use paired t-test.

H_0 : Training was not effective $\mu_1 = \mu_2$

H_1 : $\mu_1 \neq \mu_2$ (Two tailed test).

Before training (X)	*After training* (Y)	$d = X - Y$	$d - \bar{d}$	$(d - \bar{d})^2$
10	12	-2	-1.22	1.49
15	17	-2	-1.22	1.49
9	8	1	1.78	3.17
3	5	-2	-1.22	1.49
7	6	1	1.78	3.17
12	11	1	1.78	3.17
16	18	-2	-1.22	1.49
17	20	-3	-2.22	4.92
4	3	1	1.78	3.17
		$\Sigma d = -7$		$\Sigma(d - \bar{d})^2 = 23.56$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-7}{9} = -0.7778 = -0.78 \text{ approx.}$$

$$S^2 = \frac{\Sigma(d - \bar{d})^2}{n-1} = \frac{23.56}{8} = 2.945$$

$$t = \frac{\bar{d}}{\frac{S}{\sqrt{n}}} = \frac{-0.78}{\sqrt{2.945}}\sqrt{9} = \frac{-2.34}{1.71} = -1.37..$$

The tabulated value of $t_{0.05}$ at 8 d.f. is 2.31.

Conclusion. Since $|t| = 1.359 < t_{0.05}$, H_0 is accepted *i.e.*, Training was not effective in improving performance.

Example 6. *Eleven school boys were given a test in drawing. They were given a month's further tution and a second test of equal difficulty was held at the end of it. Do the marks given evidence that the students have benefited by extra coaching?*

Boys	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
Marks in 1st Test	*23*	*20*	*19*	*21*	*18*	*20*	*18*	*17*	*23*	*16*	*19*
Marks in 2nd Test	*24*	*19*	*22*	*18*	*20*	*22*	*20*	*20*	*23*	*20*	*17*

(P.T.U., May 2004)

Solution. Since the data are correlated and concerned with the same set of students we use paired t-test.

H_0 : Test provides no evidence that student are benefited by extra coaching *i.e.*, $\mu = 0$

$H_1 : H_1 \neq H_2$ (two tailed test) *i.e.*, $\mu \neq 0$.

Marks in 1st test	*Marks in 2nd test*	$d = X - Y$	$d - \bar{d}$	$(d - \bar{d})^2$
23	24	– 1	0	0
20	19	1	2	4
19	22	– 3	– 2	4
21	18	3	4	16
18	20	– 2	– 1	1
20	22	– 2	– 1	1
18	20	– 2	– 1	1
17	20	– 3	– 2	4
23	23	0	1	1
16	20	– 4	– 3	9
19	17	2	3	9
				$\Sigma (d - \bar{d})^2 = 50$

$$\bar{d} = \frac{\Sigma d}{n} = \frac{-11}{11} = -1$$

$$S^2 = \frac{\Sigma(d - \bar{d})^2}{n - 1} = \frac{50}{10} = 5$$

$\therefore$
$$S = 2.24$$

$$t = \frac{\bar{d}}{\frac{S}{\sqrt{n}}} = \frac{-1}{2.24}\sqrt{11} = -1.48$$

$$|t| = 1.48.$$

Tabulated values of $t_{0.05}$ at $(11 - 1 = 10)$ d.f. $= 2.228$.

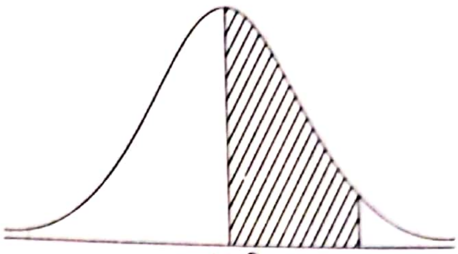
Conclusion. Since $|t| = 1.48 < t_{0.05}$ at 10 *i.e.*, (2.228)

H_0 is accepted *i.e.*, the test provides no evidence that the students are benefited by extra coaching.

Table 1: NORMAL TABLE

AREAS UNDER THE STANDARD NORMAL

$$\text{CURVE} = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}}\, dz$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2485	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.8729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4930	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4999	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993

Table 2 : SIGNIFICANT VALUES $t_v(\alpha)$ OF t-DISTRIBUTION (TWO TAIL AREAS) $[|t| > t_v(\alpha)] = \alpha$

d.f. (v)	*Probability (Level of Significance)*					
	0.50	*0.10*	*0.05*	*0.02*	*0.01*	*0.001*
1	1.00	6.31	12.71	31.82	63.66	636.62
2	0.82	0.92	4.30	6.97	6.93	31.60
3	0.77	2.32	3.18	4.54	5.84	12.94
4	0.74	2.13	2.78	3.75	4.60	8.61
5	0.73	2.02	2.57	3.37	4.03	6.86
6	0.72	1.94	2.45	3.14	3.71	5.96
7	0.71	1.90	2.37	3.00	3.50	5.41
8	0.71	1.80	2.31	2.90	3.36	5.04
9	0.70	1.83	2.26	2.82	3.25	4.78
10	0.70	1.81	2.23	2.76	3.17	4.59
11	0.70	1.80	2.20	2.72	3.11	4.44
12	0.70	1.78	2.18	2.68	3.06	4.32
13	0.69	1.77	2.16	2.05	3.01	4.22
14	0.69	1.76	2.15	2.62	2.98	4.14
15	0.69	1.75	2.13	2.60	2.95	4.07
16	0.69	1.75	2.12	2.58	2.92	4.02
17	0.69	1.74	2.11	2.57	2.90	3.97
18	0.69	1.73	2.10	2.55	2.88	3.92
19	0.69	1.73	2.09	2.54	2.86	3.88
20	0.69	1.73	2.09	2.53	2.85	3.85
21	0.69	1.72	2.08	2.52	2.83	3.83
22	0.69	1.72	2.07	2.51	2.42	3.79
23	0.69	1.71	2.07	2.50	2.81	3.77
24	0.69	1.71	2.06	2.49	2.80	3.75
25	0.68	1.71	2.06	2.49	2.79	3.73
26	0.68	1.71	2.06	2.48	2.78	3.71
27	0.68	1.70	2.05	2.47	2.77	3.69
28	0.68	1.70	2.05	2.47	2.76	3.67
29	0.68	1.70	2.05	2.46	2.76	3.66
30	0.68	1.70	2.04	2.46	2.75	3.65
∞	0.67	1.65	1.96	2.33	2.58	3.29

Table 3 : E. VALUES OF e^{-m} (For Computing Poisson Probabilities) (0)

(0 < m < 1)

m	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0.0	1.0000	.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
0.1	0.9048	.8958	.8860	.8781	.8694	.8607	.8521	.8437	.8353	.8270
0.2	0.8187	.8106	.8025	.7945	.7866	.7788	.7711	.7634	.7558	.7483
0.3	0.7408	.7334	.7261	.7189	.7118	.7047	.6977	.6907	.6839	.6771
0.4	0.6703	.6636	.6570	.6505	.6440	.6376	.6313	.6250	.6188	.6126
0.5	0.6065	.6005	.5945	.5886	.5827	.5770	.5712	.5655	.5599	.5543
0.6	0.5488	.5434	.5379	.5326	.5278	.5220	.5160	.5117	.5066	.5016
0.7	0.4966	.4916	.4868	.4810	.4771	.4724	.4670	.4630	.4584	.4538
0.8	0.4493	.4449	.4404	.4360	.4317	.4274	.4232	.4190	.4148	.4107
0.9	0.4066	.4025	.3985	.3986	.3906	.3867	.3839	.3791	.3753	.3716

(m : 1, 2, 3, ..., 10)

m	1	2	3	4	5	6	7	8	9	10
e^{-m}	.36788	.13534	.04979	.01832	.00698	.00279	.00092	.000395	.000123	.000045

Note: To obtain values of e^{-m} for other values of m, use of laws of exponents.

Example. $e^{-2.35} = (e^{-2.00})(e^{-0.35}) = (.13534)(.7047) = .095374$.