BUFN403 Project Report

Sumit Nawathe **Ravi Panguluri** **James Zhang** **Sashwat Venkatesh**
Computational Finance Minor Capstone
University of Maryland College Park
{snawathe, rpangulu, jzhang72, sashvenk}@terpmail.umd.edu

Abstract

We aim to create a reinforcement learning agent that utilizes historical price data as well as sentiment and topical embeddings of news articles to optimally trade on S&P100 stocks. After implementing multiple papers on financial reinforcement learning for equity trading, our contribution will be in experimenting and synthesizing an improved combination of the state space and reward function for our environment. We compare our best strategy against other standard equity portfolio benchmarks.

Preprint. Under review.

Contents

Chapter 1

Project Overview

1.1 Introduction

Our group is seeking to develop a reinforcement learning agent to support portfolio management and optimization. Utilizing both empirical stock pricing data along with alternative data, we look to create a more well-informed portfolio optimization tool.

Our primary motivations for pursuing a reinforcement learning-based approach are as follows:

1. Reinforcement learning lends itself well to learning/opening in an online environment. The agent can interact with its environment, providing real-time feedback/ responsiveness to allow for better results.
2. Our approach involves incorporating alternative data to support the agent's decision making process. Encoding this alt-data into the states matrix of the agent allows for the agent to make better decisions when it comes to adjusting portfolio weights.
3. Given that a reinforcement learning agent's decisions are modeled by a Markov Decision Process, we can easily provide different reward functions to account for a variety of investor preferences or restrictions.

1.2 Dataset Creation

Creating a multimodal dataset of tabular and language data to train a robust reinforcement learning agent will require pulling data from a wide variety of sources. We aim to use three primary types of data: historical stock data, news headlines, and SEC filings.

1.2.1 Stock Data

For historical price data, we will use data from from the Wharton Research Data Services (WRDS). Specifically, we aim to use data from the Center for Research in Security Prices (CRSP), which has security price, return, and volume data for the NYSE, AMEX and NASDAQ stock markets. We will begin by creating a universe with just the S&P 100 stocks. If time permits, we hope to expand to the S&P 500 and/or stocks on all publicly-listed exchanges available through WRDS.

1.2.2 News Data

The inclusion of news data into the reinforcement learning agent's state is important to integrate an external understanding of how well a given stock is performing at a given time and how it is exposed to certain market or sector risks. This can provide our agent a better view of the environment and knowledge of our ticker companies in context, which can help it make better decisions to maximize our chosen reward function.

Scraping Financial News Sources

While attempts have been made to scrape news data from financial sources including Yfinance and Benzinga, we have primarily been rate limited/and or IP blocked for scraping at too high of a frequency. From Kaggle, we have found a source of pre-scraped news data from Benzinga — a financial news source — with daily news headline data spanning from 2014-2020 for over 6000 companies. From this dataset, we plan to start with using headline data companies in the S&P 100. However, we could expand the dataset if time and data storage capacity permits.

Utilizing Paid News APIs

Seeing a gap in real-time and historical data pipelines for business and professional use, many companies have created paid news APIs that allow institutions to query a wide range of news sites for current event information including financial news. Many of these APIs such as Event Registry, newsapi.ai, and Alpha Vantage provide this data. However, in examining most of these APIs there are a few significant issues. Many of them require a significant payment to get data at a velocity that we would need, and historical data is often even more expensive.

However, within the free tier of these services we can make some API calls to query information for some tickers that could be additive to our analysis. It would be difficult to rely on this as a long term solution for data though.

SEC Filings

To enrich our dataset, we are considering utilizing SEC (10-K and 10-Q) filings data for all NYSE/-NASDAQ companies (available through WRDS) that we plan to pull data for. These filings contain essential information regarding a company's financial performance, governance, and compliance that could enhance our measure of company outlook. Specifically we aim to use data from 10-K (annual) and 10-Q (quarterly) filings. 10-K's are an annual report on a company's performance, and include information. For our project, the textual data that is most likely to capture a company's sentiment is Item 1A, Risk Factors. This item is a company issued statement on the risk factors that could affect the operations of the business for the next fiscal year. We will extract this data from 10-Q statements for every company in our trading universe to create sentiment indicators for our RL agent to use.

1.2.3 Sentiment Analysis

Using the news sources and SEC filings data described above, we wish to generate embeddings from which we can extract sentiment related features to provide to our reinforcement learning agent. Our initial approach, which we have conducted some basic testing on, is to utilize the pre-trained FinBERT model, fine-tuned to recognize the sentiment of financial text to create embeddings for us [**?**].

FinBERT Sentiment Scores

We query article headlines for all stocks in our universe over our entire dataset time period. Note that if we use a source like Benzinga, articles are already tagged with relevant stock tickers. We will feed headline data to pre-trained FinBERT. The model then will preprocess the text and generate probabilities of the content being positive, negative, or neutral. From there, we can assign each headline a numerical score based on its maximum probability class. The numerical map could look something like the following: positive: 1, neutral: 0, negative: -1 Over a trading period, we can take some aggregate of these class labels for each stock and feed these class labels to our agent's states matrix; at each time step, this value will be appended to the row corresponding to the stocks' price data. (The state matrix will be defined in greater detail in the Algorithmic and Analytical Challenges section.) We will experiment to find an optimal aggregate function; potential options include providing all logits, taking the mean, or designing a custom function to extract value heuristically. One such function could be

$$\texttt{Value}_{\texttt{Embedding}} = \tanh\left(\frac{\frac{\texttt{positive sentiment probability}}{\texttt{negative sentiment probability}}}{\texttt{neutral sentiment probability}}\right)$$

This approach combines the "log likelihood" (ratio of probabilities of positive and negative sentiment) along with a penalty for high neutral sentiment (a measure of uncertainty), using the tanh for normalization. This approach would allow us to adequately detect strong positive/negative sentiment. We will compare this against other aggregate functions in training experiments.

We will test the exact same preprocessing pipeline on the SEC 10-K and 10-Q filings for each company in our universe and integrate them into our states matrix. We will experiment with the same options mentioned previously for the optimal aggregation function. An issue that incorporates SEC filings is that they are recorded on a relatively infrequent basis compared to news and price data. Preliminarily, we will assume that there is no decay in the sentiment between reports. That means, the sentiment embeddings for each company will only be updated on dates where a new filing was reported. On non-reporting dates, the embeddings will be filled forward from the last filing date.

Alternatively, we intend to use a logistic decay function to generate a multiplier that represents the decaying effect of sentiment over time from a document like a 10-K or 10-Q, which is not reported every day. The function we've designed is:

$$Multiplier = 2\left(\frac{-1}{1+e^{-0.25x}}+1\right)$$

Generating a multiplier, then combining this multiplier with the sentiment score generated by FinBERT will allow us to show that some x days after a 10-K or 10-Q filing is published, the actual effective sentiment wanes as the news becomes older. At 12 days, the sentiment is modeled to be roughly 10 percent as "effective" as it was on day 0. At day 0, the multiplier will always be 1, to indicate that news is most impactful on the day that it is published.

Topic Modeling

In a similar manner to the Financial Statement Analysis assignment from Andy Chakraborty's lecture, we can also utilize FinBERT to categorize news headlines and content into financial-related topics. As demonstrated in that assignment, the correlations between such scores and the performance of companies can be useful to our RL agent by similarly incorporating such embeddings as a tensor. For every news headline and SEC filing related to each stock ticker, we will use the same pre-trained topic classification model as we did in our assignment to give us a numeric mapping of each text's most probable topic. Using this, we will append a column of topic embeddings for each ticker on each day to the states matrix.

1.3 Algorithmic and Analytical Challenge

Our primary algorithmic technique is deep reinforcement learning, which uses deep neural networks to learn an optimal policy to interact with an environment and optimize performance towards a goal. Formally, a reinforcement learning problem is an instance of a Markov Decision Process, which is a 4-tuple (S, A, T, R): S the state space (matrix of selected historical stock price and news data available to our model at a given time; see Methodology section), A the action space (portfolio weights produced by our model, under appropriate constraints), T the transition function (how the state changes over time, modeled by our dataset), and R (the reward function). The goal is to find a policy (function from $S \rightarrow A$) that maximizes future expected rewards. Most reinforcement learning research is spent on providing good information in S to the model, defining a good reward function R, and deciding on a deep learning model training system to optimize rewards.

1.3.1 Existing Literature

Much of the literature applying RL to portfolio optimization has arisen in the last few years. Some relevant papers are:

- [**?**] Deep Reinforcement Learning Comparison with Mean-Variance Optimization: Using a lookback of recent past returns and a few market indicators (including 20-day volatility and the VIX), this paper implements a simple algorithm for portfolio weight selection to maximize the Differential Sharpe Ratio, a (local stepwise) reward function which approximates

(global) Sharpe Ratio of the final strategy. They compare their model with the standard mean-variance optimization across several metrics.

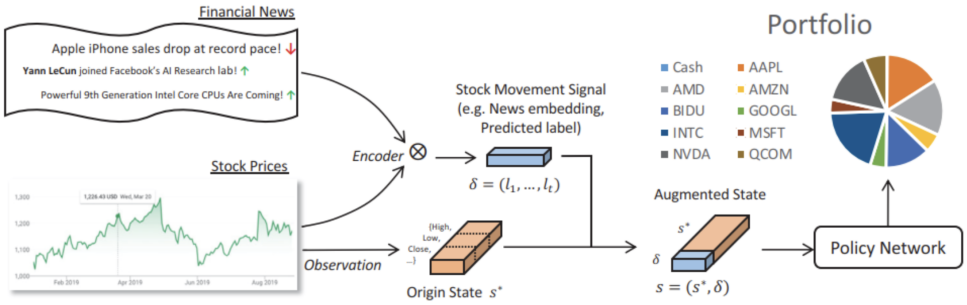
- [**?**] DRL for Stock Portfolio Optimization Connected with Modern Portfolio Theory: This paper applies reinforcement learning methods to tensors of technical indicators and covariance matrices between stocks. After tensor feature extraction using 3D convolutions and tensor decompositions, the DDPG method is used to train the neural network policy, and the algorithm is backtested and compared against related methods.
- [**?**] RL-Based Portfolio Management with Augmented Asset Movement Prediction States: The authors propose a method to augment the state space S of historical price data with embeddings of internal information and alternative data. For all assets at all times, the authors use an LSTM to predict the price movement, which is integrated into S. When news article data is available, different NLP methods are used to embed the news; this embedding is fed into an HAN to predict price movement, which is also integrated into S for state augmentation. The paper applies the DPG policy training method and compares against multiple baseline portfolios on multiple asset classes. It also addresses challenges due to environment uncertainty, sparsity, and news correlations.
- [**?**] A Deep Reinforcement Learning Framework for the Financial Portfolio Management Problem: This paper contains a deep mathematical and algorithmic discussion of how to properly incorporate transaction costs into an RL model. The authors also have a GitHub with implementations of their RL strategy compared with several others.
- [**?**] Stock Portfolio Selection Using Learning-to-Rank Algorithms with News Sentiment: After developing news sentiment indicators including shock and trends, this paper applies multiple learning-to-rank algorithms and constructs an automated trading system with strong performance.
- [**?**] MAPS: Multi-agent Reinforcement Learning-based Portfolio Management System: This paper takes advantage of reinforcement learning with multiple agents by defining a reward function to penalize correlations between agents, thereby producing multiple orthogonal (diverse) high-performing portfolios.

1.3.2 Methodology

We will be implementing, combining, and improving on the methodologies of several of the above papers. Our plan is to develop an RL system that utilizes multiple time periods to achieve strong out-of-sample trading performance. As of this writing, we have partial implementations of papers [**?**], [**?**], and [**?**]. Our final architecture will be most similar to papers [**?**] and [**?**].

Markov Decision Process Problem Formulation

Paper [**?**] includes the following diagram, which is very close to our desired architecture:



An explanation of this diagram: at time t, the origin state S^* is a 3D tensor of dimensions $U \times H \times C$ which contains historical price data. U is the size of our universe (for example, for the S&P100, $U = 100$). H is the size of history we are providing (if we are providing 30 day history, then $H = 30$). C is a categorical value representing the close/high/low price. This format of S^* allows us to store, for example, the last 30 days of stock price data for all companies in the S&P100, for any given day. In addition to this, we have news information δ, obtained from financial news headlines

for that day, processed through a pre-trained encoder. This information is added to S^* to create the full state $S = (S^*, \delta)$

In our architecture, for S^*, we will experiment with the lookback period size and likely reduce it to a 2D array by flattening along the C index, but will otherwise keep S^* largely the same. For δ, we plan to utilize better feature extraction via sentiment scores and topic modeling; we also plan to use different alternative data sources, as described in the Dataset Creation section. In addition, we will extract what company each headline refers to, so our features can be changed over time independently for each company as news articles enter through our environment. The final state S will likely be a 2D matrix, where each row represents a different company (ticker), and along that row we find, concatenated, the following: (1) the past month-or-so of stock price data from S^*, and (2) numerical features extracted from recent news data pertinent to that company (as described in the Dataset section). (The straightforward concatenation of price data and news embeddings did not affect the ability of the neural network-based agent to learn.)

Regarding the reward function R, we plan to experiment with both the profit reward function used in paper [**?**], as well as the Differential Sharpe Ratio developed in paper [**?**]. The former is simply the change in the portfolio value over the last time period based on the weights (action) provided by the agent; the latter attempts to make the cumulative reward approximate the Sharpe ratio over the entire time period.

In all of the papers, the action space A is a length $m + 1$ vector such that the sum of all element is 1, where there are m stocks in our universe (the other weight is for the risk-free asset). Each action represents the agent's desired portfolio for the next time period, and is computed based on the state at the end of the previous time period. We will experiment with short-selling and leverage restrictions (which put lower and upper bounds on the weight components, respectively).

In summary, our project aims to implement and replicate the approach used in [**?**], with some modifications to S and R as previously described. We will conduct experiments alternative data sources, feature extraction methods, and reward functions (both custom and from other papers listed) to find a good combination that allows this approach to work well on S&P100 stocks; this comprises our novel extension/contribution.

Use of Libraries

We will mainly be using the Gymnasium library to implement the reinforcement learning environments. The Stable Baselines 3 library provides several policy learning techniques that we will experiment with, including Proximal Policy Optimization (PPO) and Deep Deterministic Policy Gradients (DDPG). The papers above discuss the advantages and disadvantages of multiple reward functions and constraints, which we will make improvements upon and provide as options to the user, if applicable.

Strategy Benchmarking

Our final model architecture will be compared against several benchmark financial portfolio selection models. Among these will be the CAPM, an exponential moving average strategy, linear factor models such as the Fama French 3/5-factor models, and the QMJ model. We will compare our returns in-sample and out-of-sample plots, as well as our relative performance on portfolio statistics including cumulative return, Sharpe Ratio, Sortino Ratio, drawdown, etc. The experiment sections in the above papers provide a strong reference for our methodological comparison.

1.4 User Interface and Visual Analytics

As our project is primarily oriented towards data curation and algorithms, we will not directly have a user interface for a final end-user. However, in order to experiment with and visualize the performance of our various models, we will create a small suite of visualizations in an interactive manner that allows a researcher to choose methods and hyperparameters for our architecture. We will also create a notebook to demonstrate the infrastructure and usage of our deep reinforcement learning model, similar to those on TensorFlow and Huggingface.

Chapter 2

Literature Notes and Commentary

2.1 Reinforcement Learning Overview

The reader may not be readily familiar with reinforcement learning (RL). Thus, we here provide a brief overview of the terminology, basic definition, essential results, and common algorithms in RL theory.

2.1.1 Markov Decision Process

A Markov Decision Process (MDP) problem is a framework for modeling sequential decision-making by an agent in an environment. A problem is formally defined as a 4-tuple (S, A, T, R).

- S is the state space, which is the set of all possible states of the environment.
- A is the action space, which contains all possible actions that the agent can take (across all possible states).
- $T : S \times A \times S \to [0, 1]$ is the transition function in a stochastic environment. When the environment is in state s and the agent takes action a, then $T(s, a, s')$ is the probability that the environment transitions to state s' as a result. (In a deterministic environment, this function may not be necessary, as there may be only one possible state due to the taken action.)
- $R : S \times A \times S \to \mathbb{R}$ is the reward function. When the environment changes state from s to s' due to action a, the agent recieves reward $R(s, a, s')$.

The environment is Markov, which means that the distribution of the next state s' conditioned on the current state s and action a is independent from the time step.

A policy is a function $\pi : S \to A$ that dictates actions to take at a given state. A solution to an MDP problem is an optimal policy π^* that maximizes the agent's utility, however that is defined.

2.1.2 RL Terminology

In RL, the agent's utility is generally defined as the total expected discounted reward. Let $\gamma \in [0, 1]$ be a constant discount factor. The utility from a sequence of reward $\{r_t\}_{t=0}^{\infty}$ is thus commonly defined as $U([r_0, r_1, \ldots]) = \sum_{t=0}^{\infty} \gamma^t r_t \leq \frac{\sup_t r_t}{1-\gamma}$. The benefit of this formulation is that (1) utility is bounded if the rewards are bounded, and (2) there is a balance between small immediate rewards and large long-term rewards. (The use of the discount factor depends on the actual reward function. For custom reward functions, it may not be necessary or even desireable; we include it because it is common in RL literature.)

Given a policy $\pi : S \to A$, we define the value function $V^\pi : S \to \mathbb{R}$ and the Q-function $Q^\pi : S \times A \to \mathbb{R}$ as

$$V^\pi(s) = \mathop{\mathbb{E}}_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R_t \;\middle|\; s_0 = s\right] \qquad Q^\pi(s,a) = \mathop{\mathbb{E}}_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R_t \;\middle|\; s_0 = s, a_0 = a\right]$$

$V^\pi(s)$ is the expected utility from starting at s and following policy π, and $Q^\pi(s,a)$ is the expected utility from starting at s, taking action a, and then following policy π thereafter. The goal of RL is to find the optimal policy π^*, from which we have the optimal value function V^* and optimal Q-function Q^*. These optimal values can further be defined as follows:

$$Q^*(s,a) = \mathop{\mathbb{E}}_{s'}[R(s,a,s') + \gamma V^*(s')] = \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

$$V^*(s) = \max_a Q^*(s,a) = \max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

The second equation is known as the Bellman equation.

There are two main branches of RL: model-based and model-free. In model-based RL, the agent attempt to build a model of the environment transition function T and reward function R. Based on these model, it then attempts to directly maximize the total expected reward. In model-free RL, the agent does not attempt to model the environment, but instead attempts to learn either the value function or Q-function. Once it has one of these, it can derive an optimal policy from it as:

$$\pi^*(s) = \arg\max_a Q^*(s,a) = \arg\max_a \sum_{s'} T(s,a,s')\left[R(s,a) + \gamma V^*(s')\right]$$

We proceed with model-free RL in this project, specifically policy gradient methods.

2.1.3 Policy Gradient Methods

Policy gradient methods optimize a performance objective by finding a good policy — typically a neural network parameterized policy [**?**].

Deep Deterministic Policy Gradients

[**?**] aims to capture the ideas and successes of Q-Learning [**?**] in low dimensional, discrete action spaces and extend them to the high-dimensional, continuous action spaces. One approach is to discretize action spaces. However, the most notable issue with this is the curse of dimensionality, which only exasperates the exploration versus exploitation dilemma that agents encounter. Large action spaces are difficult to sufficiently explore and thus intractable to train. Consequently, the authors present a model-free, actor-critic algorithm using deep function approximators that can learn in high-dimensional continuous action spaces.

The following equations, which follow standard MDP notation, set up the author's motivations for extending Q-Learning. Recall the Q-function, denoted as

$$Q^\pi(s_t, a_t) = \mathop{\mathbb{E}}_{r_{i\geq t}, s_{i\geq t} \sim E, a_{i>t} \sim \pi}[R_t \mid s_t, a_t]$$

Applying the recursive Bellman Equation,

$$Q^\pi(s_t, a_t) = \mathop{\mathbb{E}}_{r_t, s_{t+1} \sim E}\left[r(s_t, a_t) + \gamma \mathop{\mathbb{E}}_{a_{t+1} \sim \pi}[Q^\pi(s_{t+1}, a_{t+1})]\right]$$

If the policy is deterministic, then define $\mu : \mathcal{S} \to \mathcal{A}$ and the inner expectation is alleviated such that

$$Q^\mu(s_t, a_t) = \mathop{\mathbb{E}}_{r_t, s_{t+1} \sim E}[r(s_t, a_t) + \gamma Q^\mu(s_{t+1}, a_{t+1})]$$

Since the expectation only depends on the environment E, it is possible to learn Q^μ off-policy using transitions generated by a seprate stochastic policy β. [**?**] selects the greedy policy $\mu(s) =$

$\text{argmax}_a Q(s,a)$. Consider function approximators parameterized as θ^Q. Mean squared error loss can be minimized as

$$L(\theta^Q) = \mathop{\mathbb{E}}_{s_t \sim \rho^\beta, a_t \sim \beta, r_t \sim E} \left[\left(Q(s_t, a_t \mid \theta^Q) - y_t \right)^2 \right]$$
$$y_t = r(s_t, a_t) + \gamma Q(s_{t+1}, \mu(s_{t+1}) \mid \theta^Q)$$

where ρ^β is the discounted state visitation distribution for a policy β. However, greedy policies in continuous action spaces requires nonstop differentiation of a_t at every timestep, which is far too computationally expensive for nontrivial action spaces. The method of Deterministic Policy Gradients [**?**] builds on [**?**] by implementing an actor-critic approach, which we introduce below. The parameterized actor function $\mu(s \mid \theta^\mu)$ maps states to actions and resembles the agent's current policy. The critic $Q(s,a)$ is learned from the Bellman equation above. Let the expected return from the start distribution be denoted as $J = \mathbb{E}_{r_i, s_i \sim E, a_i \sim \mu}[R_1]$. Then differentiating J with respect to the actor's weights and following the chain rule,

$$\nabla_{\theta^\mu} J = \mathop{\mathbb{E}}_{s_t \sim \rho^\beta} \left[\nabla_a Q(s, a \mid \theta^Q) \mid_{s=s_t, a=\mu(s_t)} * \nabla_{\theta^\mu} \mu(s \mid \theta^\mu) \mid_{s=s_t} \right]$$

The contribution of [**?**] that expends [**?**] and [**?**] is allowing function approximators to be deep neural networks, hence the name Deep Deterministic Policy Gradients (DDPG).

Trust Region Policy Optimization

The novelty of DDPG is using neural networks as function approximators for policies, but optimizing such large neural networks nevertheless nontrivial. DDPG is also not designed for stochastic policies. Trust Region Policy Optimization (TRPO) [**?**] is an iterative method similar to natural policy gradient methods used to optimize nonlinear neural network policies and and guarantee monotonic improvement with little hyperparameter tuning.

Once more, consider standard MDP notation, let $\rho_0 : \mathcal{S} \to \mathbb{R}$ be the distribution of the initial state s_0, and let $\pi : \mathcal{S} \times \mathcal{A} \to [0,1]$ be a stochastic policy. Then we denote the expected discount reward

$$\eta(\pi) = \mathbb{E}_{s_0 \sim \rho_0(s_0), a_t \sim \pi(a_t \mid s_t), s_{t+1} \sim P(s_{t+1} \mid s_t, a_t)} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t) \right]$$

Note that this paper defines $\eta(\pi)$ such that t starts from 0 and the value function $V^\pi(s)$ such that t can start from any arbitrary natural number. In a similar manner to the definition of the Advantage function $A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$, the following identity expresses the advantage of some policy $\tilde{\pi}$ over π

$$\eta(\tilde{\pi}) = \eta(\pi) + \mathbb{E}_{s_0, a_0, \ldots, \tilde{\pi}} \left[\sum_{i=1}^{n} \gamma^t A^\pi(s_t, a_t) \right]$$

where the subscript $s_0, a_0, \ldots, \tilde{\pi}$ illustrates that actions are sampled from the $\tilde{\pi}$ policy. Now let $\rho_\pi(s)$ be discounted visitation frequencies $\rho_\pi(s) = P(s_0 = s) + \gamma P(s_1 = s) + \gamma^2 P(s_2 = s) + \cdots$ Equipped with this representation, we can rewrite the policy advantage function as a sum over states instead of as a sum over timesteps.

$$\eta(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_{\tilde{\pi}}(s) \sum_a \tilde{\pi}(a \mid s) A^\pi(s,a)$$

This equation implies that a policy update from $\pi \to \tilde{\pi}$ improves or at least maintains policy performance if the expected policy advantage at every state s is positive, that is $\rho_{\tilde{\pi}}(s) \sum_a \tilde{\pi}(a|s) A^\pi(s,a) \geq 0 \; \forall \; s$. However, the dependency of $\rho_{\tilde{\pi}}(s)$ on $\tilde{\pi}$ makes the above equation difficult to optimize. Consider the first order approximation where π is a differentiable policy with respect to policy parameters θ

$$L_\pi(\tilde{\pi}) = \eta(\pi) + \sum_s \rho_\pi(s) \sum_a \tilde{\pi}(a \mid s) A^\pi(s,a)$$

It can be shown that $L_{\pi_{\theta_0}}(\tilde{\pi}) = \eta(\tilde{\pi_{\theta_0}})$ and $\nabla_\theta L_{\pi_{\theta_0}}(\tilde{\pi})|_{\theta=\theta_0} = \nabla_\theta \eta(\tilde{\pi_{\theta_0}})|_{\theta=\theta_0}$

Proximal Policy Optimization

[**?**]

2.1.4 Specialization to Our Application

In the portfolio optimization setting, our RL agent seeks to produce an optimal set of portfolio weights given all the information it knows. Assume that there are n tradeable stocks in our universe, and one risk-free asset. The action space $A = \left\{a \in \mathbb{R}^{n+1} \mid \sum_i a_i = 1\right\}$ is the set of all possible portfolio weights.

The state space S encompasses all information available to the agent when it is asked to make a portfolio allocation decision at a given time. Depending on what information is provided, this could include past performance of the strategy, historical stock prices, encoded news information for select/all tickers in the universe, or some combination of these. For most of the scenarios we consider, $S \in \mathbb{R}^{n \times N}$ is a matrix, where each row corresponds to a different stock ticker, and along that row we find the past few weeks of historical price data as well as some aggregate of news sentiment indicators/scores.

The transition function T is a delta function since state transitions are deterministic. The environment uses the weights provided by the agent to reallocate the portfolio, computes the new portfolio value, and reads in new historical stock data and news datapoints to form the next state (for the next time period) which is provided to the agent. (The exact for of T is not needed; it is implicitly defined by the deterministic environment udpates.)

The reward function R should be such that it encourages the agent to produce good portfolio weights. One simple reward function is pure profit: $R(s_t, a_t)$ is how much profit is gained to portfolio allocation a_t during time interval $[t, t+1)$. Another possible reward function is the Differential Sharpe ratio (as described in section **??**), which urges the agent to make portfolio allocations to maximize its total Sharpe ratio.

2.2 Differential Sharpe Ratio

[**?**] utilizes the Differential Sharpe Ratio to implement and evaluate a reinforcement learning agent. The Differential Sharpe Ratio is based on Portfolio Management Theory, and is developed in the author' previous works [**?**] and [**?**]. We briefly review the theory developed in both sources.

The traditional definition of the Sharpe Ratio is the ratio of expected excess returns to volatility. If R_t is the return of the portfolio at time t, and r_f is the risk-free rate then

$$\mathcal{S} = \frac{\mathbb{E}_t[R_t] - r_f}{\sqrt{\mathrm{Var}_t[R_t]}}$$

This works well to analyze a strategy once all data is collected. The goal of traditional portfolio theory is to maximize the Sharpe Ratio over the given time period (equivalently, to maximize the mean-variance utility function).

Unfortunately, this will not work for a reinforcement learning agent. The agent must be given a reward after every time step, but the traditional Sharpe ratio is only calculated at the end.

The Differential Sharpe Ratio attempts to remedy this by approximating a change in the total Sharpe ratio up to that point. By summing together many of these incremental changes (though approximate), the cumulative rewards is an approximation of the total Sharpe ratio over the complete time period.

The approximation works by updating moment-based estimators of the expectation and variance in the Sharpe Ratio formula. Let A_t and B_t be estimates of the first and second moments of the return R_t up to time t. After time step t, having obtained R_t, we perform the following updates:

$$\Delta A_t = R_t - A_{t-1} \qquad A_t = A_{t-1} + \eta \Delta A_t$$
$$\Delta B_t = R_t - B_{t-1} \qquad B_t = B_{t-1} + \eta \Delta B_t$$

where $A_0 = B_0 = 0$ and $\eta \sim 1/T$ is an update parameter, where there are T total time periods. These updates are essentially exponential moving averages.

Let S_t be an approximation of the Sharpe Ratio up to time t based on estimates A and B. That is,

$$S_t = \frac{A_t}{\sqrt{B_t - A_t^2}}$$

The definition here ignores the risk-free rate term. K_η is a normalization constant to ensure an unbiased estimator.

Pretend that at the update for time t, A_{t-1} and B_{t-1} are constants, and R_t is also a known constant. Then the updates to A_t and B_t really only depend on the time step parameter η. Indeed, if $\eta = 0$, then $A_t = A_{t-1}$ and $B_t = B_{t-1}$, so $S_t = S_{t-1}$. Now consider varying η; expanding the Sharpe ratio estimator formula in Taylor series gives

$$S_t \approx S_{t-1} + \eta \frac{\mathrm{d}S_t}{\mathrm{d}\eta}\Big|_{\eta=0} + o(\eta^2)$$

If η is small, the final term is negligible, so this formula gives us an exponential-moving-average update for S_t. The Differential Sharpe Ratio is defined to be proportional derivative in that expression. With some tedious calculus, we find that

$$\begin{aligned} D_t = \frac{\mathrm{d}S_t}{\mathrm{d}\eta} = \frac{\mathrm{d}}{\mathrm{d}\eta}\left[\frac{A_t}{\sqrt{B_t - A_t^2}}\right] &= \frac{\frac{\mathrm{d}A_t}{\mathrm{d}\eta}\sqrt{B_t - A_t^2} - A_t \frac{\frac{\mathrm{d}B_t}{\mathrm{d}\eta} - 2A_t\frac{\mathrm{d}A_t}{\mathrm{d}\eta}}{2\sqrt{B_t - A_t^2}}}{B_t - A_t^2} \\ &= \frac{\Delta A_t \sqrt{B_t - A_t^2} - A_t \frac{\Delta B_t - 2A_t \Delta A_t}{2\sqrt{B_t - A_t^2}}}{B_t - A_t^2} = \frac{B_t \Delta A_t - \frac{1}{2} A_t \Delta B_t}{(B_t - A_t^2)^{3/2}} \end{aligned}$$

This reward function is simple to implement in an environment. The authors of the original papers provide experimental support for the value of this reward function in a reinforcement learning setting.

2.3 Transaction Costs

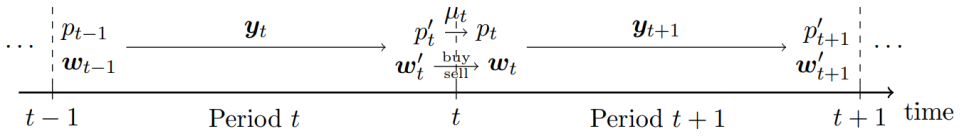
[**?**] contains an excellent walkthrough of the mathematics for modeling transaction costs in RL environment updates. We provide a shortened version here.

Suppose we have m tradeable assets and the risk-free asset. Let $\mathbf{v}_t = (1, v_{1,t}, v_{2,t}, \dots v_{m,t}) \in \mathbb{R}^{m+1}$ be the prices of the assets at time t (the first entry is the risk-free asset). The raw return vector is defined as $\mathbf{y}_t = \mathbf{v}_t \oslash \mathbf{v}_{t-1} \in \mathbb{R}^{m+1}$, where division is element-wise. Suppose the portfolio weight vector during time period t is $\mathbf{w}_t \in \mathbb{R}^{m+1}$, and let the value of the portfolio value at time t be p_t. If we were not considering transaction costs, then the portfolio return would be $\frac{p_t}{p_{t-1}} = \mathbf{y}_t \cdot \mathbf{w}_t$.

Unfortunately, buying and selling assets incurs transaction costs. Let $\mathbf{w}'_t \in \mathbb{R}^{m+1}$ be the effective portfoliio weights at the end of time t (it has changed from $\mathbf{w}_t$ due to the changes in price). We have

$$\mathbf{w}'_t = \frac{\mathbf{y}_t \odot \mathbf{w}_t}{\mathbf{y}_t \cdot \mathbf{w}_t}$$

where $\odot$ is element-wise multiplication. Between time $t-1$ and time t, the portfolio value is also adjusted from $p_{t-1} \in \mathbb{R}$ to $p'_t = p_{t-1}\mathbf{y}_t \cdot \mathbf{w}_{t-1}$. Let p_t be the value of the portfolio after transaction costs, and let $\mu_t \in \mathbb{R}$ be the transaction cost factor, such that $p_t = \mu_t p'_t$. We can keep track of the relevant time of each variable with the following diagram:



In this paradigm, the final portfolio value at time T is

$$p_T = p_0 \prod_{t=1}^{T} \frac{p_t}{p_{t-1}} = p_0 \prod_{t=1}^{T} \mu_t \mathbf{y}_t \cdot \mathbf{w}_{t-1}$$

The main difficulty is in determining the factor μ_t, since it is an aggregate of all the transaction cost penalties.

Let $c_s \in [0, 1)$ be the commission rate for selling. We need to sell some amount of asset i if there is more of asset i in $\mathbf{w}'_t$ than in $\mathbf{w}_t$ by dollar value. Mathematically, this condition is $p'_t w'_{i,t} > p_t w_{t,i}$, which is equivalent to $w'_{i,t} > \mu_t w_{i,t}$. Thus, the total amount of money raised from selling assets is

$$(1 - c_s)p'_t \sum_{i=1}^{m} (w'_{i,t} - \mu_t w_{i,t})^+$$

where $(\cdot)^+ = \max\{0, \cdot\} = \mathrm{ReLU}(\cdot)$. This money, as well as the money from adjusting the cash reserve from $p'_t w'_{0,t}$ to $p_t w_{0,t}$, is used to purchase assets according to the opposite condition. Let $c_p \in [0, 1)$ be the commision rate for purchasing. Equating the amount of money available from selling/cash and the amount of money used for purchasing assets yields

$$(1 - c_p)\left[w'_{0,t} - \mu_t w_{0,t} + (1 - c_s)p'_t \sum_{i=1}^{m} (w'_{i,t} - \mu_t w_{i,t})^+\right] = p'_t \sum_{i=1}^{m} (\mu_t w_{i,t} - w'_{i,t}) +$$

Moving terms around and simplifying the ReLU expressions, we find that μ_t is a fixed-point of the function f defined as:

$$\mu_t = f(\mu_t) = \frac{1}{1 - c_p w_{0,t}}\left[1 - c_p w'_{0,t} - (c_s + c_p - c_s c_p)\sum_{i=1}^{m} (w'_{i,t} - \mu_t w_{i,t})^+\right]$$

The function f is a nonlinear. However, for reasonable values of c_s and c_p, f is both monotone increasing and a contraction, so its unique fixed point «««< HEAD can be found by iteratively computing values of f. This procedure is fairly efficient and easy to implement. ======= can be found by iteratively computing values of f. This procedure is fairly efficient and easy to implement.

2.4 EIIE Policies

The second main contribution of [**?**] is to create the framework of Ensemble of Identical Independent Evaluators (EIIE) for a policy. The principle is to have a single evaluation function that, given the price history and other data for a single asset, produces a scores representing potential growth for the immediate future. This same function is applied to all assets independently, and the softmax of the resulting scores become the weights of the portfolio.

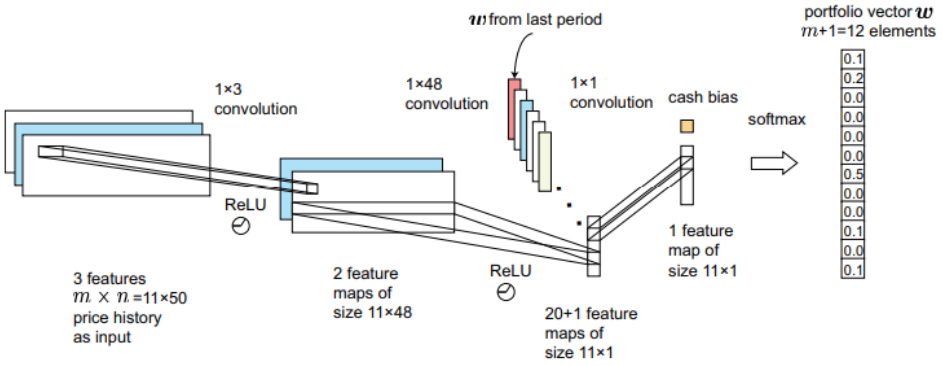
Written mathematially: let $X_{i,t}$ be the historical price information for asset i at time t, and let $w_{i,t}$ represent the portfolio weights for asset i at time t. Let α, β, and γ be trainable parameters. First, we define a function f_α to extract features from each asset's price data $X_{\times,t}$. This function is applied to each asset individually. The agent also needs to incorporate the previous portfolio weights into its action, in order to deal with the transaction costs as in section **??**. We define a second function g_β that takes the features produced by f_{θ_1}, as well as the previous portfolio weights, to produce new weights. Then the portfolio weights for the next time period are

$$w_{t+1} = \mathrm{Softmax}(g_\beta(f_\alpha(X_{1,t}), w_{1,t}), \ldots, g_\beta(f_\alpha(X_{m,t}), w_{m,t}), g_\beta(\gamma, w_{m+1,t}))$$

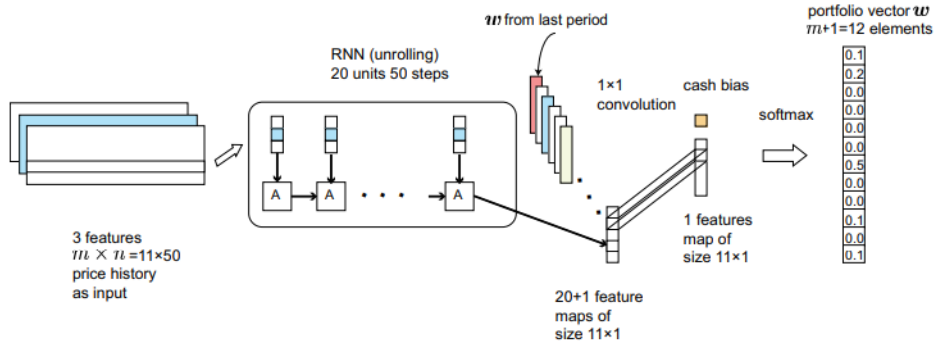
(Note that there are m tradeable assets in our universe, and that $w_{m+1,t}$ is the weight for the risk-free asset).

The form of g_β is fairly arbitrary; the authors take it to be an MLP neural network. The form of f_α is more interesting. The authors of [**?**] suggest two forms: a CNN and an RNN/LSTM.

For the CNN they provide the following diagram of the EIIE system:



For the RNN they provide a similar diagram:



The three channels in the state matrix on the left are the high, low, and closing price for each asset on each day in the historical sliding window.

The authors claim that this framework beats all of their benchmark strategies in the cryptocurrency market. The architecture remains essentially the same in our implementation. Due to the use of multiple channels in the state tensor, it makes it easy to add additional channels for alternative data sources, such as news sentiment.

Chapter 3

Documentation

3.1 Gym Environments

Gymnasium is an open-source reinforcement learning environment. Its goal is to standardize the interface between reinforcement learning algorithms and their environments. Much of the information for this section is taken from their documentation: https://gymnasium.farama.org/

The extendable base class for the gym environment is `gym.Env`. A subclass must define the following two methods:

- `reset()` initializes the environment. Its return type is `tuple[ObsType, dist[str, Any]]`. The first element is the initial state, and the dictionary is for intitial logging information.
- `step(a: ActType)` performs one update of the environment where the agent takes action `a`. Its return type is `tuple[ObsType, float, bool, bool, dist[str, Any]]`. In order, the elements are: (1) the next state provided to the agent, (2) the reward value for that step, (3) whether the environment has terminated, (4) whether the environment has truncated, and (5) a dictionary of logging information.

`ObsType` and `ActType` are the types of the observation (state) space and action spaces, respectively, which must set within the environment initialization. There are a few standard types of spaces supported by Gymnasium:

- `gym.spaces.Discrete(n)`: A set of `n` distinct discrete values.
- `gym.spaces.Box(low, high, shape, dtype)`: A generalized rectangle. The dimensions are specified by the `shape` parameter, and all entries have `dtype` type (generally `np.float64`). Each component can have its own upper and lower bound if they are tuples (if a single float is provided, that bound is used for all components).
- `gym.spaces.Tuple(spaces)`: A space that is a tuple (cartesian product) of other spaces.

It is possible to define custom spaces, but standard RL training algorithms and models generally only support select compositions of the standard spaces.

3.2 RL Framework

In order to ensure consistency and modularity, we have created a framework of classes that allows us to easily swap components of our RL environment to test various data and reward strategies.

3.2.1 AbstractRewardManager

This abstract class manages the calculation of rewards for the RL agent. Its interface is minimal. Subclasses can optionally have a constructor.

```python
from abc import ABC, abstractmethod

class AbstractRewardManager(ABC):
  @abstractmethod
  def initialize_reward(self):
    """
    Initializes all constants used in the reward calculation.
    Must be called at least once before compute_reward() is called.
    Should be called in the environment's reset() method.
    """
    pass

  @abstractmethod
  def compute_reward(self, old_pval: float, new_pval: float) -> float:
    """
    Computes the reward based on the old and new portfolio values.
    Should be called in the environment's step() method.
    """
    pass
```

3.2.2 AbstractDataManager

This abstract class manages reading and iterating through the state and price data for the environment.

```python
from abc import ABC, abstractmethod
import numpy as np
import numpy.typing as npt
import gymnasium as gym

class AbstractDataManager(ABC):
  @abstractmethod
  def get_obs_space(self) -> gym.spaces.Box:
    """Result is assigned to the environment's observation space"""
    pass

  @abstractmethod
  def get_data(self) -> tuple[int, int]:
    """
    This function loads/fetches state data from files and stores it.
    Should be called in the environment's reset() method.
    The properties assigned here should be accessed in get_state().
    Note that the data should provide for one more than the number
      of time periods desired (for the initial state).
    Returns: (number of time periods, number of stock tickers)
    """
    pass

  @abstractmethod
  def get_state(
    self,
    t: int,
    w: npt.NDArray[np.float64],
    port_val: np.float64
  ) -> npt.NDArray[np.float64]:
    """
    Computes and returns the new state at time t.
    State can include the current portfolio weight and value,
      provided as additional parameters.
    This state will be used by the agent for calculating weights
```

```
        at the start time time period t+1.
      When t=0, it should output the initial state.
      """
      pass

  @abstractmethod
  def get_prices(self, t: int) -> npt.NDArray[np.float64]:
      """
      Returns the security prices at time t (at the beginning
        of time period t+1).
      When t=0, it should output the initial prices.
      """
      pass
```

3.2.3 PortfolioEnvWithTCost

This class is the main gym environment. It iterates through the data and rewards using the `AbstractDataManager` and `AbstractRewardManager` provided as arguments. To compute the portfolio updates, it employs the transaction costs-cognizant approach described in section **??**.

Let `w` be the portfolio weights. We use the convention that `w[-1]` is the weight for the risk-free asset, and `w[:-1]` are the weights for the risky assets. (Note that this is a different convention than the one used in [**?**].)

We omit much of the actual code and instead provide pseudocode or equations where relevant.

```
from abc import ABC, abstractmethod
import numpy as np
import numpy.typing as npt
import gymnasium as gym

class PortfolioEnvWithTCost(gym.Env):
  def __init__(
    self,
    dm: AbstractDataManager,
    rm: AbstractRewardManager,
    w_lb=0.0, w_ub=1.0,
    cp=0.0, cs=0.0,
    logging=True
  ):
    """
    Initializes the environment with the given managers and constants.
    w_lb and w_ub are the lower and upper bounds for the portfolio weights.
    cp and cs are commision weights for purchasing and selling assets.
    If logging is enabled, step() will return the portfolio value.
    """
    # register managers
    # set constants from given arguments

    # get data from manager
    self.num_time_periods, self.universe_size = self.dm.get_data()

    # set environment observation and action spaces
    assert w_lb <= w_ub
    self.observation_space = self.dm.get_obs_space()
    self.action_space = gym.spaces.Box(
      low=w_lb,
      high=w_ub,
      shape=(self.universe_size + 1,),
      dtype=np.float64
```

```
    )

  def find_mu(
    self,
    w_old: npt.NDArray[np.float64],
    w_new = npt.NDArray[np.float64]
  ) -> float:
    """
    Uses the iterative technique to find mu for the given old weights
    (after returns), new weights, and the transaction cost commision rates.
    """
    pass

def step(self, action: npt.NDArray[np.float64]) -> tuple:
  """
  Performs one time step update using the agent's action.
  Returns: a tuple of
    - copy of the new state
    - reward for the action
    - whether the environment has terminated (reached end of data)
    - whether the environment has truncated (always False)
    - information dictionary (possibly containing portfolio value)
  """
  # check that the action is normalized
  # find the value of mu using the helper function
  # perform the weight and portfolio value updates
  # obtain the reward uding the reward manager
  # obtain the new state using the data manager
  # update all instance variables
  # if logging enabled, return the portfolio value in addition to
  pass

def reset(self, *args, **kwargs) -> tuple[np.ndarray, dict]:
  """
  Resets the environment to an initial state.
  Returns: the initial state and an empty dictionary (required by gym).
  """
  # reset portfolio weights
  # initialize reward manager
  # obtain initial state and prices from data manager
  pass
```

3.2.4 BasicPortfolioWithTCost

[**?**] provides a simple yet effective state space setup augmented with volatility indicators that characterize market trends. We provide a brief description here.

Let asset i's price at time t be denoted $P_{i,t}$. Then the one-period simple return of the asset is $R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$ and the one-period gross return is $\frac{P_{i,t}}{P_{i,t-1}} = R_{i,t} + 1$. Thus, one-period log returns can be denoted $r_{i,t} = \log(R_{i,t} + 1)$. Given a universe of n assets and cash denoted by c, the authors form the subsequent $[(n+1) \times T]$ state matrix:

$$S_t = \begin{bmatrix} w_1 & r_{1,t-1} & \cdots & r_{1,t-T+1} \\ w_2 & r_{2,t-1} & \cdots & r_{2,t-T+1} \\ \vdots & \vdots & \ddots & \vdots \\ w_n & r_{n,t-1} & \cdots & r_{n,t-T+1} \\ w_c & vol_{20} & \frac{vol_{20}}{vol_{60}} & VIX_t \ldots \end{bmatrix}$$

where the first column is the portfolio weights of each asset at the beginning of time t, which can differ slightly from the portfolio weights at the end of time $t-1$. The last row features three market

volatility indicators — vol_{20}, $\frac{vol_{20}}{vol_{60}}$, and VIX_t — which are particularly important given the authors' imposed long-only and no-leverage constraints.

$$w_c \geq 0,\ w_i \geq 0 \,|\, \forall\, i \qquad\qquad w_c + \sum_{i=1}^{n} w_i = 1$$

The first indicator, vol_{20}, is the 20-day rolling window standard deviation of daily S&P500 returns. Similarly, vol_{20} is the 60-day rolling window standard deviation of daily S&P500 returns. Thus, their ratio is given by $\frac{vol_{20}}{vol_{60}}$. If $\frac{vol_{20}}{vol_{60}} < 1$, then the last 20 days have been less volatile than the last 60 days, which suggests a shift from a high volatility market trend to a lower volatility market trend. Finally, the last indicator VIX is a measure of the stock market's expected volatility.

```
import numpy as np
import pandas as pd
import gymnasium as gym
from portfolio_env_with_tcost import PortfolioEnvWithTCost
import numpy.typing as npt
import gymnasium as gym

class PortfolioEnvWithTCost(PortfolioEnvWithTCost):
  def get_obs_space(self) -> gym.spaces.Box:
      return gym.spaces.Box(low=-np.inf, high=np.inf, shape=(self.universe_size+1,
      100+1), dtype=np.float32)

  def get_data(self) -> tuple[int, int]:
      # read SNP data
      df = pd.read_csv('crsp_snp100_2010_to_2024.csv', dtype='string')

      # convert datatypes
      df = df[['date', 'TICKER', 'PRC', 'VOL', 'ASKHI', 'BIDLO', 'FACPR']]
      df.date = pd.to_datetime(df.date)
      df.FACPR = df.FACPR.fillna('0.0')
      df.astype({
          'PRC': float,
          'VOL': float,
          'ASKHI': float,
          'BIDLO': float,
          'FACPR': float
      })

      # drop duplicates and nans
      df = df.drop_duplicates(subset=['date', 'TICKER'])
      df.dropna(inplace=True)

      # only include stocks that are present in all dates
      ticker_ok = df.TICKER.value_counts() == df.TICKER.value_counts().max()
      def is_max_val_count(ticker: str) -> bool:
        return ticker_ok[ticker]
      ok = df.apply(lambda row: is_max_val_count(row['TICKER']), axis=1)
      df = df[ok]
      df = df[(df.date.dt.year >= 2010) & (df.date.dt.year <= 2019)]

      # create stock array
      self.stock_df = df.pivot(index='date', columns='TICKER', values='PRC').astype(float)

      # adjust for stock splits
      facpr_df = df.pivot(index='date', columns='TICKER', values='FACPR').astype(float)
      self.stock_df = self.stock_df * (1+facpr_df).cumprod(axis=0)
      # assert np.all(self.stock_df.pct_change().iloc[1:, :] > -1),
```

```
        f"{(self.stock_df.pct_change().iloc[1:, :] <= -1).sum().sum()=},
        {np.any(pd.isna(self.stock_df.pct_change().iloc[1:, :]))}"
        self.ret = np.log(self.stock_df.pct_change().iloc[1:, :] + 1)

        # get times and dickers
        self.times = df.date.unique()[1:]
        self.tickers = df.TICKER.unique()

        # read index data and compute volatilities
        idx_df = pd.read_csv('crsp_snpidx_2010_to_2024.csv', dtype={
          'DATE': 'string',
          'vwretd': float
        })
        idx_df.DATE = pd.to_datetime(idx_df.DATE)
        idx_df['vol_20'] = idx_df.vwretd.rolling(20).std()
        idx_df['vol_60'] = idx_df.vwretd.rolling(60).std()
        idx_df.set_index('DATE', inplace=True)
        self.vol_20 = idx_df.vol_20
        self.vol_60 = idx_df.vol_60

        # get vix data
        vix_df = pd.read_csv('crsp_vix_2010_to_2024.csv', dtype={
          'Date': 'string',
          'vix': float
        })
        vix_df.Date = pd.to_datetime(vix_df.Date)
        vix_df.set_index('Date', inplace=True)
        self.vix_df = vix_df.vix

        return len(self.times)-100-1, len(self.tickers)

    def get_state(self) -> npt.NDArray[np.float64]:
        # today is self.times[self.t+100]
        s = np.zeros((self.universe_size+1, 100+1))
        s[:, 0] = self.w
        # s[1:, :-1] = self.ret[self.t:self.t+100, :].T
        # 100 past returns, up to yesterday
        s[1:, :-1] = self.ret.loc[self.times[self.t:self.t+100], :].to_numpy().T
        s[-1, 1] = self.vol_20[self.times[self.t+100-1]] # yesterday's vol_20
        s[-1, 2] = self.vol_20[self.times[self.t+100-1]] /
                    self.vol_60[self.times[self.t+100-1]] # yesterday's vol ratio
        s[-1, 3] = self.vix_df[self.times[self.t+100-1]] # yesterday's vix
        return s

    def get_prices(self) -> npt.NDArray[np.float64]:
        # today is self.times[self.t+100]
        return np.append(self.stock_df.loc[self.times[self.t+100], :].to_numpy()
        .flatten(), 1.0)
```

3.2.5 MPTWithTCost

Covariance and correlation matrices of historical stocks returns are the core of modern portfolio theory; indeed the investor who incorporates the covariance matrix can effectively minimize portfolio variance. [**?**] integrates correlation matrices and price-based technical indicators into an RL-setting, effectly combining RL with modern portfolio theory. We provide a brief overview of their methodology and infrastructure.

Technical indicators are pattern-based signals used to describe the current state of an asset's price based on previous trends. The authors use asset price and three technical indicators to construct

tensors: moving average (MA), relative strength index (RSI), and moving avarage convergence divergence (MACD).

MA computes the average price over a rolling window. Let $P_{i,t}$ be the close price for asset i at time t and let $w = 28$ be the window.

$$MA_w(t) = \frac{\sum_{t=0}^{w-1} P_{i,t}}{w}$$

RSI describes the upward or downward pressure on an asset's close price. Let $w = 14$ and consider an exponential moving average

$$EMA_w(t) = \alpha P(t) + (1-\alpha) * EMA_w(t-1), \alpha = \frac{2}{1+w}$$

Then RSI is computed as

$$RSI_t(w) = 100 * \left(1 - \frac{1}{1 + \frac{EMA_w(\max(P_t - P_{t-1}, 0))}{EMA_w(\min(P_t - P_{t-1}, 0))}}\right)$$

Finally, MACD describes the relationship between two EMAs. Let $k = 26, d = 12, w = 9$

$$MACD_w(t) = \sum_{i=1}^{n} EMA_k(i) - \sum_{i=1}^{w} EMA_d(i)$$

Using these price and technical indicator matrices, construct tensors

$$V_t = [V_{\text{t, close}}, V_{\text{t, MA}}, V_{\text{t, RSI}}, V_{\text{t, MACD}}] \in \mathbb{R}^{4\times m\times n}$$
$$Cor_t = [Cor_{\text{t, close}}, Cor_{\text{t, MA}}, Cor_{\text{t, RSI}}, Cor_{\text{t, MACD}}] \in \mathbb{R}^{4\times n\times n}$$

For indicator i and asset n, the authors use the following trasnformation to fuse the properties into one tensor.

$$F_{t(i,n)} = V_{t(i,n)} Cor^T_{t(i,n)}, F_{t(i,n)} \in R^{m\times n}$$

Now for all indicators and assets, we can efficiently compute the transformed tensor to be $F_t \in R^{4\times n\times m\times n}$. This transformed tensor then goes through a convolution layer with 32 feature maps and a $(1, 3, 1)$ kernel and then ReLU activation try try and capture spacial features.

$$F'_t = \text{Conv3D}(F_t), F'_t \in \mathbb{R}^{32\times n\times m\times n}$$

where the equal shape of F_t and F'_t can be achieved using "equal" padding. Finally, the Tucker Decomposition is used to decompose F'_t into a core tensor C and a list of factor matrices M. Tucker Decomposition is also recognized as a higher order Singular Value Decomposition.

$$C, M = \text{Tucker Decomposition}(F'_t), C \in \mathbb{R}^{1\times 1\times n\times m} \implies C \in \mathbb{R}^{n\times m}$$

Setting $C = \mathcal{S}$ the new state space at time, the agent proceeds with the learing process using the DDPG algorithm, which we have aforementioned.

```
import numpy as np
import pandas as pd
import gymnasium as gym
import pandas_ta as ta
from tensorly.decomposition import Tucker
tensorly.set_backend('pytorch')
import torch
from portfolio_env_with_tcost import PortfolioEnvWithTCost
from typing import Tuple, Optional
import numpy.typing as npt

class MPTWithTCost(PortfolioEnvWithTCost):

  def get_obs_space(self) -> gym.spaces.Box:
      self.state_shape = (1, 1, 28, self.universe_size)
      self.t = 0
```

```
        self.get_indicators()
        return gym.spaces.Box(low=-np.inf, high=np.inf, shape=self.state_shape,
        dtype=np.float64)

    def get_data(self) -> Tuple[int, int]:
        # read SNP data
        df = pd.read_csv('crsp_snp100_2010_to_2024.csv', dtype='string')

        # convert datatypes
        df = df[['date', 'TICKER', 'PRC', 'VOL', 'ASKHI', 'BIDLO', 'FACPR']]
        df.date = pd.to_datetime(df.date)
        df.FACPR = df.FACPR.fillna('0.0')
        df.astype({
            'PRC': float,
            'VOL': float,
            'ASKHI': float,
            'BIDLO': float,
            'FACPR': float
        })

        # drop duplicates and nans
        df = df.drop_duplicates(subset=['date', 'TICKER'])
        df.dropna(inplace=True)

        # only include stocks that are present in all dates
        ticker_ok = df.TICKER.value_counts() == df.TICKER.value_counts().max()

        def is_max_val_count(ticker: str) -> bool:
          return ticker_ok[ticker]

        ok = df.apply(lambda row: is_max_val_count(row['TICKER']), axis=1)
        df = df[ok]
        df = df[(df.date.dt.year >= 2010) & (df.date.dt.year <= 2019)]

        # create stock array
        self.stock_df = df.pivot(index='date', columns='TICKER', values='PRC').astype(float)

        idx_df = pd.read_csv('crsp_snpidx_2010_to_2024.csv', dtype={
          'DATE': 'string',
          'vwretd': float
        })
        idx_df.DATE = pd.to_datetime(idx_df.DATE)
        idx_df['vol_20'] = idx_df.vwretd.rolling(20).std()
        idx_df['vol_60'] = idx_df.vwretd.rolling(60).std()
        idx_df.set_index('DATE', inplace=True)
        self.idx_df = idx_df

        # adjust for stock splits
        facpr_df = df.pivot(index='date', columns='TICKER', values='FACPR').astype(float)
        self.stock_df = self.stock_df * (1+facpr_df).cumprod(axis=0)
        self.ret = np.log(self.stock_df.pct_change().iloc[1:, :] + 1)
        self.times = df.date.unique()[1:]
        self.tickers = df.TICKER.unique()

        return len(self.times)-28-1, len(self.tickers)

    def get_indicators(self):
        self.conv3d = torch.nn.Conv3d(in_channels=4, out_channels=32,
        kernel_size=(1, 3, 1), padding="same").to('cuda')
```

```
        self.relu = torch.nn.ReLU()
        self.tucker = Tucker(rank=self.state_shape, init="random")
        self.m = 28
        self.w1, self.w2, self.w3 = 28, 14, 9

        df = (pd.DataFrame(self.stock_df, columns=self.tickers))
        df = df.dropna()
        mp = {ticker: pd.DataFrame(df[ticker]).rename(columns={ticker: "close"})
         for ticker in self.tickers}
        # SMA df
        sma = {ticker: pd.DataFrame(mp[ticker].ta.sma(self.w1)).rename(
          columns={"SMA_28": ticker}) for ticker in self.tickers}
        sma_df = pd.concat(sma.values(), axis=1).fillna(0)
        # RSI df
        rsi = {ticker: pd.DataFrame(mp[ticker].ta.rsi(self.w2)).rename(
          columns={"RSI_14": ticker}) for ticker in self.tickers}
        rsi_df = pd.concat(rsi.values(), axis=1).fillna(0)
        # MACD df
        macd = {ticker: pd.DataFrame(mp[ticker].ta.macd(self.w3, 26, 12)["MACD_9_26_12"])
        .rename(columns={"MACD_9_26_12": ticker}) for ticker in self.tickers}
        macd_df = pd.concat(macd.values(), axis=1).fillna(0)

        # Compute F = V @ Corr
        V = np.array([np.array(x.T) for x in [df, sma_df, rsi_df, macd_df]])
        Corr = np.array([np.corrcoef(x) for x in V])
        self.F = torch.from_numpy(np.einsum('aki,akj->akij', V, Corr)).to('cuda').float()


    def get_state(self) -> npt.NDArray[np.float64]:
        f = self.F[:, :, self.t + 28 - self.m : self.t + 28, :].clone().detach()
        f = torch.unsqueeze(f, dim=0)
        f = self.conv3d(f)
        f = self.relu(f)
        f = torch.squeeze(f, dim=0)
        f = f.cpu()
        core, _ = self.tucker.fit_transform(f)
        return core.detach().numpy()

    def get_prices(self) -> npt.NDArray[np.float64]:
        return np.append(self.stock_df.loc[self.times[self.t+28], :].to_numpy().
        flatten(), 1.0)
```

Bibliography

[] Sandro Gössi, Ziwei Chen, Wonseong Kim, Bernhard Bermeitinger, and Siegfried Handschuh. FinBERT-FOMC: Fine-Tuned FinBERT Model with Sentiment Focus Method for Enhancing Sentiment Analysis of FOMC Minutes. In *Proceedings of the Fourth ACM International Conference on AI in Finance*, ICAIF '23, pages 357–364, New York, NY, USA, November 2023. Association for Computing Machinery. ISBN 9798400702402. doi: 10.1145/3604237.3626843. URL https://dl.acm.org/doi/10.1145/3604237.3626843.

[] Srijan Sood, Kassiani Papasotiriou, Marius Vaiciulis, and Tucker Balch. Deep Reinforcement Learning for Optimal Portfolio Allocation: A Comparative Study with Mean-Variance Optimization.

[] Junkyu Jang and NohYoon Seong. Deep reinforcement learning for stock portfolio optimization by connecting with modern portfolio theory. *Expert Systems with Applications*, 218:119556, May 2023. ISSN 0957-4174. doi: 10.1016/j.eswa.2023.119556. URL https://www.sciencedirect.com/science/article/pii/S095741742300057X.

[] Yunan Ye, Hengzhi Pei, Boxin Wang, Pin-Yu Chen, Yada Zhu, Jun Xiao, and Bo Li. Reinforcement-Learning based Portfolio Management with Augmented Asset Movement Prediction States, February 2020. URL http://arxiv.org/abs/2002.05780. arXiv:2002.05780 [cs, q-fin, stat].

[] Zhengyao Jiang, Dixing Xu, and Jinjun Liang. A Deep Reinforcement Learning Framework for the Financial Portfolio Management Problem, July 2017. URL http://arxiv.org/abs/1706.10059. arXiv:1706.10059 [cs, q-fin] version: 2.

[] Qiang Song, Anqi Liu, and Steve Y. Yang. Stock portfolio selection using learning-to-rank algorithms with news sentiment. *Neurocomputing*, 264:20–28, November 2017. ISSN 0925-2312. doi: 10.1016/j.neucom.2017.02.097. URL https://www.sciencedirect.com/science/article/pii/S0925231217311098.

[] Jinho Lee, Raehyun Kim, Seok-Won Yi, and Jaewoo Kang. MAPS: Multi-agent Reinforcement Learning-based Portfolio Management System. In *Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence*, pages 4520–4526, July 2020. doi: 10.24963/ijcai.2020/623. URL http://arxiv.org/abs/2007.05402. arXiv:2007.05402 [cs].

[] Vincent François-Lavet, Peter Henderson, Riashat Islam, Marc G. Bellemare, and Joelle Pineau. An introduction to deep reinforcement learning. *Foundations and Trends® in Machine Learning*, 11(3–4):219–354, 2018. ISSN 1935-8245. doi: 10.1561/2200000071. URL http://dx.doi.org/10.1561/2200000071.

[] Timothy P. Lillicrap, Jonathan J. Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. Continuous control with deep reinforcement learning, 2019.

[] Christopher J. C. H. Watkins and Peter Dayan. Q-learning. *Machine Learning*, 8(3):279–292, May 1992. ISSN 1573-0565. doi: 10.1007/BF00992698. URL https://doi.org/10.1007/BF00992698.

[] David Silver, Guy Lever, Nicolas Heess, Thomas Degris, Daan Wierstra, and Martin Riedmiller. Deterministic policy gradient algorithms. In Eric P. Xing and Tony Jebara, editors, *Proceedings of the 31st International Conference on Machine Learning*, volume 32 of *Proceedings of Machine Learning Research*, pages 387–395, Bejing, China, 22–24 Jun 2014. PMLR. URL https://proceedings.mlr.press/v32/silver14.html.

[] John Schulman, Sergey Levine, Pieter Abbeel, Michael Jordan, and Philipp Moritz. Trust region policy optimization. In Francis Bach and David Blei, editors, *Proceedings of the 32nd International Conference on Machine Learning*, volume 37 of *Proceedings of Machine Learning Research*, pages 1889–1897, Lille, France, 07–09 Jul 2015. PMLR. URL https://proceedings.mlr.press/v37/schulman15.html.

[] John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms, 2017.

[] J. Moody and Lizhong Wu. Optimization of trading systems and portfolios. In *Proceedings of the IEEE/IAFE 1997 Computational Intelligence for Financial Engineering (CIFEr)*, pages 300–307. doi: 10.1109/CIFER.1997.618952. URL https://ieeexplore.ieee.org/document/618952.

[] John Moody, Matthew Saffell, Yuansong Liao, and Lizhong Wu. Reinforcement learning for trading systems and portfolios: Immediate vs future rewards. In Apostolos-Paul N. Refenes, Andrew N. Burgess, and John E. Moody, editors, *Decision Technologies for Computational Finance: Proceedings of the fifth International Conference Computational Finance*, pages 129–140. Springer US. ISBN 978-1-4615-5625-1. doi: 10.1007/978-1-4615-5625-1_10. URL https://doi.org/10.1007/978-1-4615-5625-1_10.