Static Optimization Problem with Inequality Constraints

Sumit Pokhrel

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1 Introduction

A simple Static Optimization problem was solved. Here an objective function was minimized under four inequality constraints. Please refer to problem statement. Two methods were employed. The first one was using Matlab's proprietary FMINCON Nonlinear programming solver. The second one was using the classical bracket-multiplier function approach. Newton's method was used to perform the unconstrained minimization.

2 Discussion

Figure in the next page shows the constraints and the optimized minimum value.

First I used the initial guess of X = [-0.5, 0.5] as instructed,
The FMINCON function converges after 5 iterations at
X_1 = -0.3769 X_2 = 0.1558 and Fmin = 1.9147
The only active constrain is λ_3 = 0.6854 (Using Karush Kuhn Tucker (KKT) conditions)and rest of the constraints are zeros.
The four inequality constraints are [0, 0, 0.6854, 0].

With the same initial guess, it took 8 iterations (M = 8 and N =4) for bracket multiplier penalty function to converge to the same solution,
X_1 = -0.3769 X_2 = 0.1558 and Fmin = 1.9147.
The four inequality constraints are [0, 0, -0.6854, 0].The negative sign is present because the constraints are implemented in the form of Ax $\geq$ b while in FMINCON function they are implemented in the form of Ax $\leq$ b form. So KKT conditions for Ax $\geq$ b just reverses. Therefore, the only active constrain is λ_3 = -0.6861.

I played with the initial guesses and found an *optimal* one. With the initial guess of X = [0.5, 0.5],
The FMINCON function converges after 9 iterations at
X_1 = 0.7813 X_2 = 0.6094 and Fmin = 0.0479
The only active constrain is λ_4 = .2117.

With the same initial guess, it took 10 iterations (M = 10 and N =4) for bracket multiplier penalty function to converge to the same solution,
X_1 = .7813 X_2 = 0.6094 and Fmin = 0.0479.
The only active constrain is λ_3 = -0.2121.

3 Conclusion

We obtain the minimum only with the initial guess of X = [0.5, 0.5] and not at any other points at LHP. A number of other initial guesses at RHP were used to confirm the Fmin = 0.0479 value is the global minimum. FMINCON has much better performance than the bracket multiplier approach. Since both

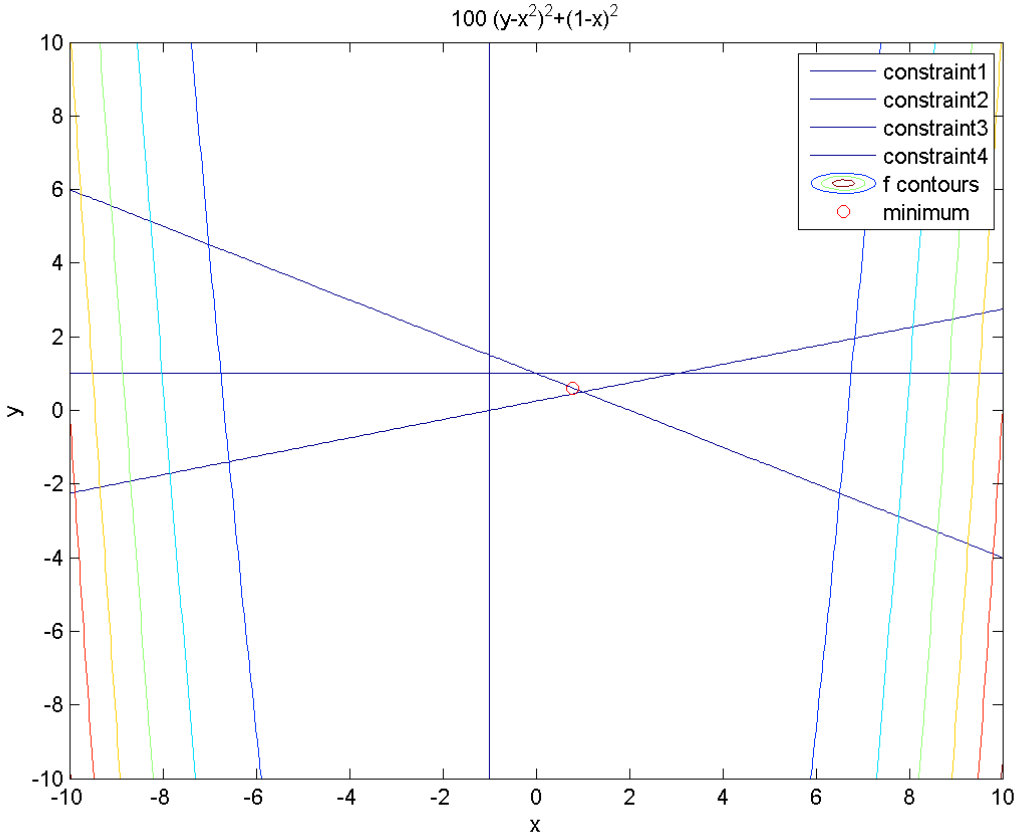

Figure 1: Notice the constraints' contours and objective function's minimum value

methods were given the same starting guess, this higher performance is most likely the result of more sophisticated searching and updating algorithms.