DESIGNING AND NUMERICAL ANALYSIS OF SUPERSONIC CONVERGING-DIVERGING (CD) NOZZLE

A PROJECT REPORT

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Submitted by

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CANDIDATE'S DECLARATION

We, **Sumit Kumar,** having **Roll No. 2K17/ME/234** is the students of **B.Tech.** in **Delhi Technological University**, herby declare that the project Dissertation titled **"Designing and Numerical analysis of supersonic Converging-Diverging (CD) nozzle** " which is submitted by us to the Department of Mechanical Engineering, Delhi Technological University, Delhi in partial fulfilment of the requirement for the award of the degree of Bachelor of Technology, is original and not copied from any source without proper citation. This work has not previously formed the for the award of any Degree, Diploma Associateship, Fellowship or other similar title or recognition.

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CERTIFICATE

I hereby certify that, the project Dissertation titled "**Designing and Numerical analysis of supersonic Converging-Diverging (CD) nozzle** " which is submitted by **Sumit Kumar having Roll No. 2K17/ME/234** is the student **Department of Mechanical Engineering, Delhi Technological University**, Delhi in partial fulfilment of the requirement of the award of the degree of Bachelor of Technology, in a record of the project work carried out by the students under my supervision. To the best of my knowledge this work has not been submitted in the part or full for the any Degree or Diploma to this University or elsewhere.

Place: Delhi

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ABSTRACT

Supersonic nozzle has applications in a various of comercial industires, like aviation, aerospace, refrigeration industries, etc. The importance of more efficient nozzle with very compact size is keeps on increasing spacially in aviation and aerospace industires where thrust per weight is very signicant and even a small design improvent may increase overall performance of engine significantly.

Our objective in this paper is to design a supersonic minimum length nozzle by using the method of characteristics such that we have a parellel and shock free flow at the exit of the nozzle. The profile points for the divergent section of nozzle is created using a a open source MATLAB code. The wall curve points then inserted into the SOLIDWORKS to generate divergent wall curve profile of the supersonic CD-nozzle. The simulation of the 2D- geomerty is performed in the ANSYS, since the fluid that is, air is compressible, density based solution and k-elipson turbulence model is used in the simulation and the exit post processing process has been performed in EXCEL to calculated Mach number, mass flow rate and Thrust at the exit of the nozzle. An analytical values for the exit mach number, mass flow rate and thrust is calculated by assuming isentropic flow. And finally the result of the simulation is validated with the value of the analytical solution.

ACKNOWLEDGEMENT

In the vying world, there is race of existence, in which those who have will to come forward succeed. Project is like a viaduct between theoretical and practical knowledge We are heartedly thankful to DTU for giving us golden opportunity to use their assets and were able to work in such a challenging environment.

Successful completion of any type of project requires help from a number of persons. We have also taken help from different people for the preparation of this report. Now, there is a little effort to show our deep gratitude to that helpful person.

We convey our sincere gratitude to my academic supervisor **Prof. Raj Kumar Singh**, Professor, Department of Mechanical Engineering, Delhi Technological University for enabling us to complete this report on "**Designing and Numerical analysis of supersonic Converging-Diverging (CD) nozzle ".** Without his guidance and valuable advice, this study would have been a little success. We would also like to thank our parents, friends and the Almighty who are always there as a strong support in these tough times for everyone

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

A converging-diverging nozzle also known as de Laval nozzle or co-di-nozzle is a varying cross-sectional area, the area decreases gradually till pinched section in the middle known as throat and then increases gradually. The CD nozzle is used to accelerate a compressible fluid to a supersonic velocity jet, the application of these supersonic is in industries like aerospace, aviation, etc. Where pressurized hot gases pass through the nozzle and get accelerated to a supersonic velocity. The nozzle works as an energy converter which coverts high enthalpy of a pressurized hot gas flowing through the nozzle into the high kinetic of the fluid having speed even greater than the speed of sound. This supersonic jet is used to provide a thrust to vehicle that may be aeroplane, a rocket or a satellite.

FIGURE 1. ISRO rocket heading toward space

1.2 LITERATURE REVIEW

Hussain et al. Performed CFD investigation of the flow through a divergent nozzle of three distinct shapes i.e., cone shape, bell and triangular shape nozzles. The geometry of nozzles was made in CATIA V5 configuration packages. The meshing is generated through the application of automatic method mapped body meshing with edged sizing of element of 10mm in the ANSYS workbench. The total temperature remains constant throughout the walls of the nozzle in case of conical nozzle, remains constant in convergent section till throat in case of bell shaped and varies lineally at lower rate in convergent section till throat, varies at high rate in divergent section in case of triangular shape nozzle. The turbulent intensity is very low at the inlet, moderate at the throat and high at the outlet in case of bell-shaped nozzle. A triangular nozzle gives best result as it satisfies the boundary conditions of the supersonic nozzle. [1]

Goyal S et al. Performed CFD analysis of supersonic C-D Nozzle for Optimization of Divergent Angle, variation in divergent angle is made from 4.76 to 10 degree and kept all other inputs and parameters constant. The governing equations of mass, momentum, energy, state and standard k-epsilon turbulence model is used for flow simulation. The nozzle shape *is* designed on ICEM and a 2D planer mesh is created using the vertex and edge association method in ICEM. Optimization of the nozzle divergent angle for the maximum output velocity. Divergent angle 9.5 degree found to give maximum output velocity and thrust and even a small 0.01-degree increment in divergent angle leads to reverse flow and high negative pressure is observed in the whole flow region. [2]

Mubarak A.K., Tide P.S. et al. Designed double parabolic supersonic nozzle and performance evaluation by experimental and numerical method and compared with conventional conical and bell nozzle by keeping same throat diameter and divergent length. Design of double parabola nozzle such that, the maximum slope of the divergent curve is one third of the Prandtl Meyer angle. 2D computational domain incorporating density-based solver with RANS equations and SST k-w turbulent model is used in numerical simulation. The nozzle exit pressures of conical and bell nozzles were higher than the ambient and same in the double parabola nozzle because of no internal shockwave generation near the throat. The double parabola nozzle gives relatively higher discharge and thrust values and least shock included angle and shock cell. The design of double parabola nozzle is better for same length gives overall higher performance. [3]

Md Akhtar et al. developed a MATLAB for designing the minimum length nozzle using method of characteristics and established a numerical solution for two dimensional, steady, inviscid, irrotational and supersonic flow. [4]

Naveen Kumar k et al. employed a second throat ejector diffuser system to simulate actual ambient vacuum conditions present in the earth orbit where satellites operate. The CFD analysis of the flow is carried out, the pressure, temperature and velocity variation are studied. Also, the design of the diffuser is optimized by the variation of different parameter for the sable and shock free flow at the exit. [5]

F. Ferdaus et al. designed and optimized a straight nozzle to attain supersonic flow and to achieve maximum thrust without flow separation due to shock waves. They confirmed that, angle of deflection on the divergent portion increases speed and shock at the exit induces the flow separation. [6]

1.3 LITERATURE GAPS AND FINDING

Majority of the previous research is focused on either the nozzle design or optimization of the supersonic CD-nozzle a comprehensive study is needed to be done on both the aspect of the designing as well as the optimization of supersonic CD-nozzle simultaneously. Previous study of the designing of nozzle suggests that the supersonic CD- nozzle can be of different shape size and they all have different advantage and disadvantages. But, the most study suggest that the best design of supersonic CD-nozzle can be achieved by using the Method of Characteristics, which is used by even NASA and SpaceX for the nozzle. The numerical simulation is one of the cheapest and easy way to optimize the CD-nozzle than every time gone for practical testing and result of the simulation can be validate by the result obtained by the hand calculation assuming isentropic flow for a shock free flow throughout the nozzle.

1.4 OBJECTIVE

Our objective in this study is to design a supersonic CD-nozzle using method of characteristics by employing an open-source MATLAB code to generate the diverging section wall profile points coordinates. Then we will insert these points in SOLIDWORKS to generate diverging wall profile. We will do simulation of nozzle in ANSYS (FLUENT) and will calculate the exit Mach number, mass flow rate and Thrust in the post processing process using EXCEL. We will also do analytical solution for the same nozzle to calculate the parameter at the exit by assuming the slow is isentropic. And finally, we will validate the numerical simulation result with the analytical solution.

CHAPTER 2: NOZZLE DESIGN

Supersonic CD-nozzle that is used to generate high thrust in the rocket and jet propulsion system. A nozzle is a simple device with varying cross-section, very simple in shape, but it has huge importance in producing the thrust in a vehicle the design of the should be such that it produces maximum thrust with minimum size for the minimum weight and should have shock free flow through the nozzle to avoid thrust loss and increase nozzle life. For the designing of nozzle, we will use method of characteristics which is used by even space giant like NASA and SpaceX. A CD-nozzle has two parts one before the throat known as converging section and after the throat known as diverging nozzle and the flow in these two regions is independent and can be studied separately once the system is chocked. The design of the convergent section can be takes anything, since ones the system is chocked the flow of the converging section doesn't alter by the flow condition or parameter in the diverging section and over main focus here will be to design the contour of diverging section of the of supersonic CD-nozzle only.

2.1 METHOD OF CHARACTERISTICS

The method of characteristics is a technique which is used to design supersonic nozzle such that the we get a shock free isentropic flow. The designed supersonic nozzle can be employing two types of diverging contour, and based on which the supersonic nozzle can be divide into gradual-expansion nozzle, which gives very high-quality flow at desired condition but because of their high length they cannot be used for the nozzle use in the rocket and jet aircraft because high length result in high weight makes it unrealistic in use for this purpose, however, since gradual-expansion nozzle has high quality flow it is used in the wind-tunnel application. The other type of nozzle minimum length nozzle which relatively low-quality flow. In the gradual-expansion nozzle the expansion waves generated over a curve and in case of minimum length curve the is reduced to zero that means a sharp corner point is used to initialized the expansion waves as shown below in fig.

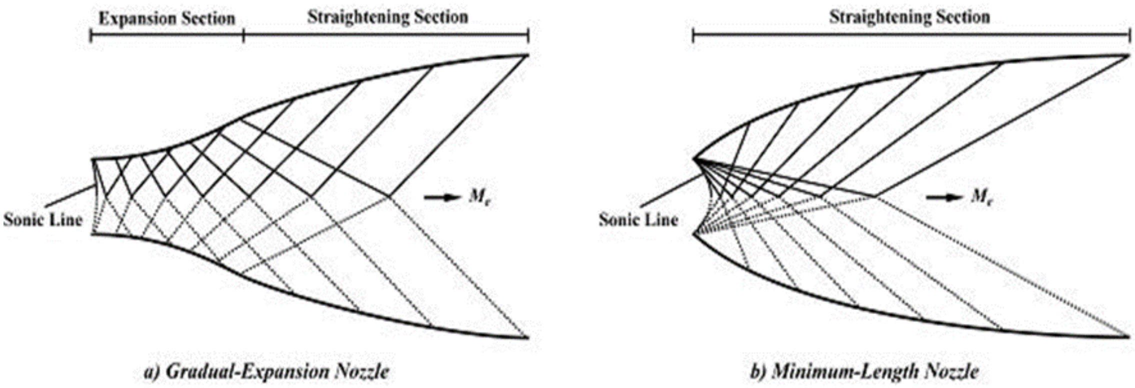

FIGURE 2. TYPES OF NOZZLE DESIGNED USING METHOD OF CHARACTERISTICS

In a supersonic flow curves are exits known as characteristics line, orientated in a specific direction the pressure waves or disturbance moves along this characteristics line. The properties of these characteristics line are;

1. It can be a curve or line along which the disturbance or pressure waves propagates.

2. Properties of the flow across characteristics line are continuous, but their first derivatives may be discontinuous and the derivatives along these lines is indeterminate.

3. The partial differential equations can be manipulated to ordinary differential equation along characteristics lines

2.2 THEORY AND EQUATIONS THAT GOVERNS THE METHOD OF CHARACTERISTICS

Prandtl-Meyer expansion waves are the characteristics line across which the flow turns into itself through an infinitesimal angle and the flow expands, Prandtl-Meyer waves are just opposite of the oblique shock waves and govern by the Prandtl-Meyer function:

$$d\theta = \pm\sqrt{M^2 - 1}\,\frac{du}{u}$$

where, dθ is the small deflection in the flow direction and,

$$v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

where, ν (M) Prandtl-Meyer angle corresponding to Mach Number M

$$\alpha = \sin^{-1}\frac{1}{M}$$

where, α is Mach wave angle

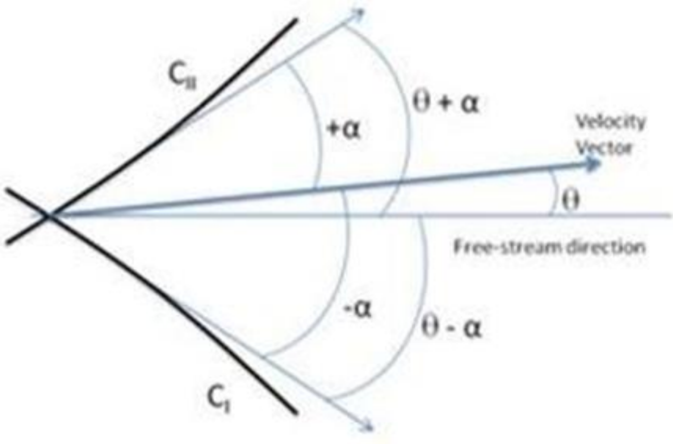

FIGURE 3. MACH WAVES

Slope of characteristics

$$\frac{dy}{dx}_I = tan(\theta - \alpha)$$

$$\frac{dy}{dx}_{II} = tan(\theta + \alpha)$$

Characteristics compatibility equation

$\theta + \nu(M) = constant = K_-$ along C_- characteristics

$\theta - \nu(M) = constant = K_+$ along C_+ characteristics

K_- and K_+ are Riemann invariant and are constant along their respective characteristics curve

Where

$$\theta = \frac{1}{2}(K_- + K_+)$$

and

$$\nu = \frac{1}{2}(K_- - K_+)$$

When two A and B intersects a point P

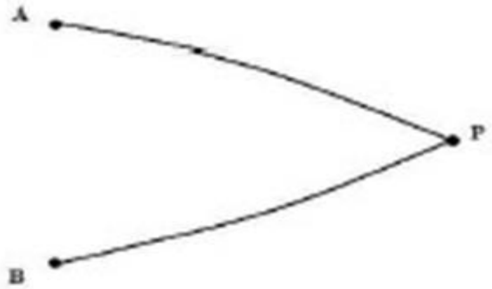

FIGURE 4. INTERSECTION OF TWO CHARACTERISTICS LINES

Then

$$m_I = \tan\left(\frac{(\theta-\alpha)_A+(\theta-\alpha)p}{2}\right)$$

$$m_{II} = \tan\left(\frac{(\theta-\alpha)_B+(\theta-\alpha)p}{2}\right)$$

and

$$y_p = y_A + m_1(x_P - x_A)$$

$$y_p = y_B + m_{11}(x_P - x_B)$$

and

$$x_P = \frac{y_1 - y_B + m_{11}x_B - m_1 x_A}{m_{11} - m_1}$$

These are the equation which governs the method of characteristics and will help in developing the supersonic contour.

The flow regime inside the supersonic nozzle is characterized by the reflection and intersection and the nozzle is design such that after crossing last expansion wave the flow must be straight in the nozzle. Our focus in this study is one the minimum-length nozzle, in which the expansion curve is reduced to a point, a sharp corner point at the throat. The initial slope of the nozzle contour that governs the nozzle design is gives by

$$\theta_{W_{max},\ M_L} = \frac{\nu(M)}{2}$$

where, $\nu(M)$ is the Prandtl-Meyer function for particular exit Mach number

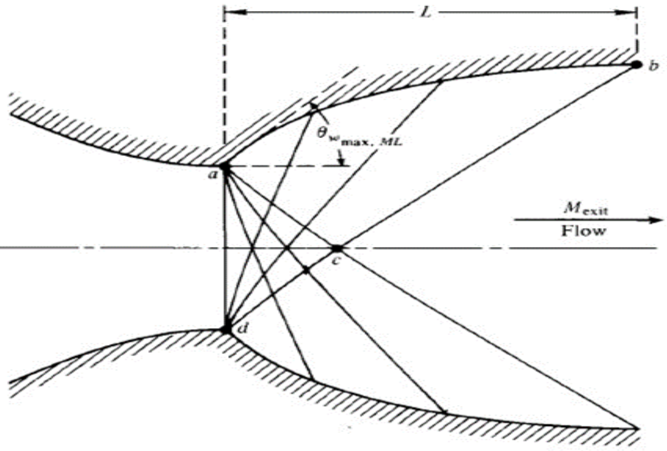

FIGURE 5. MINIMUM LENGTH NOZZLE

above equation demonstrate the expansion angle at the downstream of the throat for the minimum length nozzle and its value is correspond to the exit Mach number at a design value. For different design Mach number, we have different nozzle length as shown in fig. below

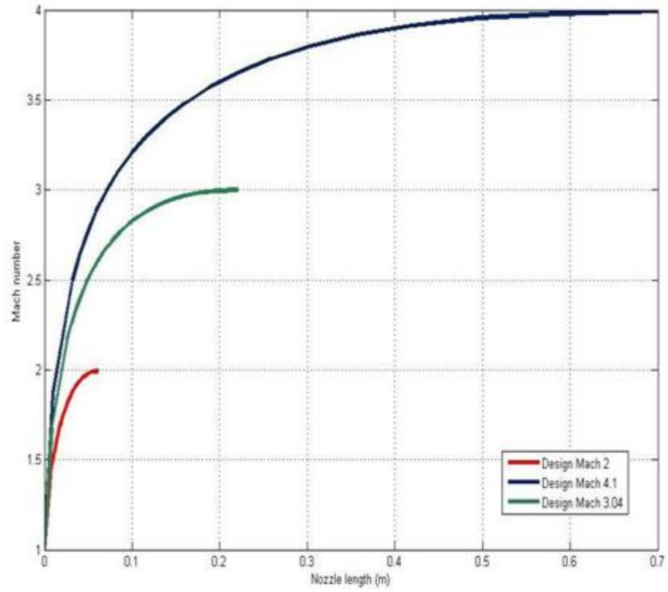

FIGURE 6. VARIATION OF NOZZLE LENGTH WITH MACH NUMBER

2.3 METHODOLOGY USED IN DESIGNING OF THE SUPERSONIC NOZZLE

as we mention earlier that, our main focus will be design a diverging section of the nozzle only because the once the flow is chocked it doesn't alter the upstream condition or flow of the nozzle. To design the diverging contour of the used MATLAB and an open-source MATLAB code based upon the method of characteristics to find out the contour points of the diverging section of the nozzle. Thereafter, we insert these points in the SOLIDWORKS to get a complete curve of the diverging section.

2.4 IMPLEMATION AND RESULT

The first step in designing of the nozzle the designing condition is to calculate the exit Mach number of the nozzle. The value of the exit Mach number is can be calculated using the isentropic flow relation for the specify design condition, which are given below in table;

PARAMETERS	VALUE
Chamber pressure	2268000 Pa
Chamber temperature	1200K
Thrust (N)	4000
Mass flow rate	NA (function of thrust)
Altitude (m)	7500
Coefficient of heat (γ)	1.4
Gas characteristic constant for air (R)	*287* KJ/KgK
Throat radius	35mm

TABLE 1. PROBLEM PARAMETER FOR THE DESIGN OF THE NOZZLE WALL

Using MATLAB

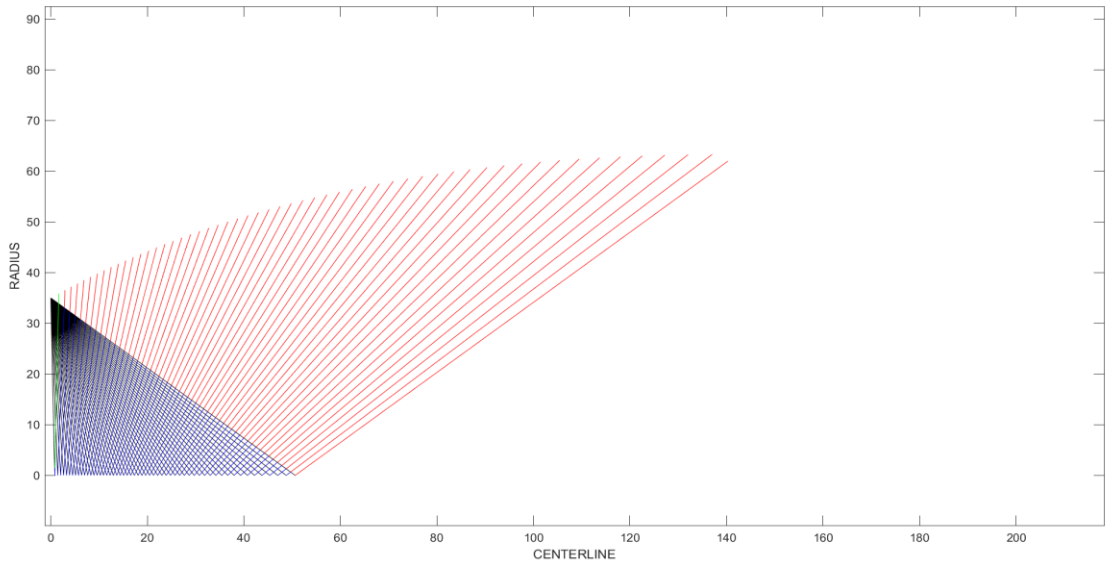

FIGURE 7. EXPANSION WAVE CURVES AND NOZZLE WALL CONTOUR (MATLAB)

The designed nozzle wall curve coordinates points (in mm units) are:

S.NO.	X	Y	Z	S.NO.	X	Y	Z
1	0.00000	35.00000	0	33	57.22977	55.36429	0
2	1.66653	35.87320	0	34	59.82966	55.91928	0
3	2.93148	36.52372	0	35	62.50249	56.46391	0
4	4.22097	37.17434	0	36	65.25113	56.99731	0
5	5.53584	37.82502	0	37	68.07861	57.51858	0
6	6.87695	38.47566	0	38	70.98809	58.02674	0
7	8.24518	39.12619	0	39	73.98292	58.52074	0
8	9.64146	39.77650	0	40	77.06657	58.99947	0
9	11.06675	40.42649	0	41	80.24275	59.46175	0
10	12.52204	41.07605	0	42	83.51531	59.90631	0
11	14.00835	41.72503	0	43	86.88835	60.33178	0
12	15.52674	42.37328	0	44	90.36617	60.73673	0
13	17.07833	43.02065	0	45	93.95329	61.11960	0
14	18.66426	43.66696	0	46	97.65452	61.47873	0
15	20.28570	44.31202	0	47	101.47492	61.81235	0
16	21.94392	44.95561	0	48	105.41983	62.11858	0
17	23.64017	45.59750	0	49	109.49492	62.39536	0
18	25.37580	46.23746	0	50	113.70619	62.64054	0
19	27.15219	46.87521	0	51	118.06000	62.85176	0
20	28.97078	47.51046	0	52	122.56309	63.02654	0
21	30.83308	48.14291	0	53	127.22263	63.16217	0
22	32.74064	48.77222	0	54	132.04624	63.25578	0
23	34.69509	49.39804	0	55	137.04201	63.30425	0
24	36.69812	50.01997	0				
25	38.75150	50.63762	0				
26	40.85706	51.25053	0				
27	43.01674	51.85824	0				
28	45.23252	52.46024	0				
29	47.50651	53.05598	0				
30	49.84089	53.64490	0				
31	52.23795	54.22637	0				
32	54.70007	54.79973					

TABLE 2. WALL PROFILE CURVE COORDINATES (MATLAB)

Now, after inserting these points in the Solidworks,

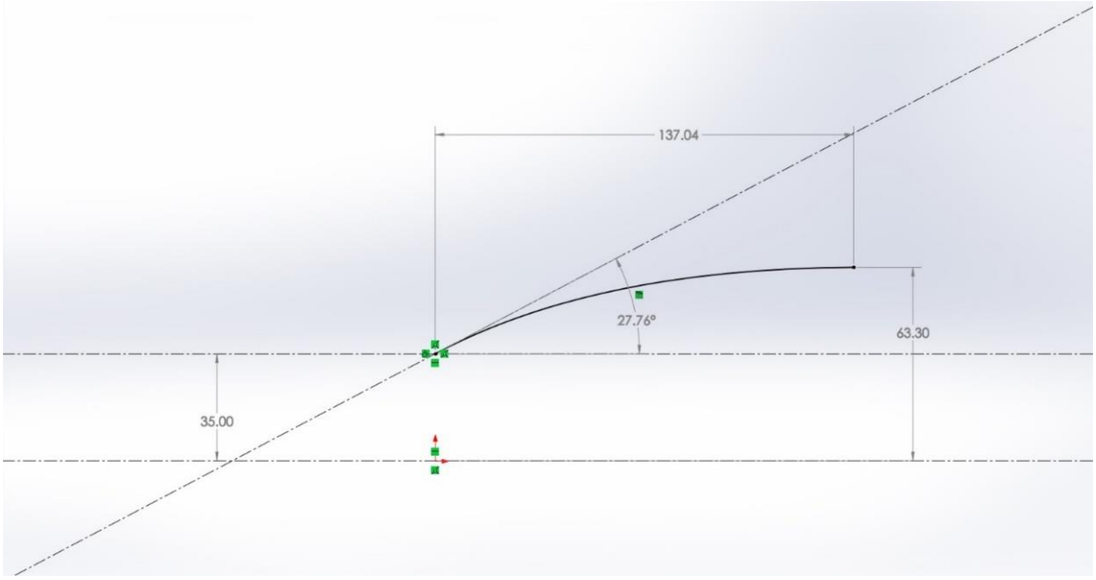

FIGURE 8. NOZZLE WALL PROFILE CURVE (SOLIDWORKS)

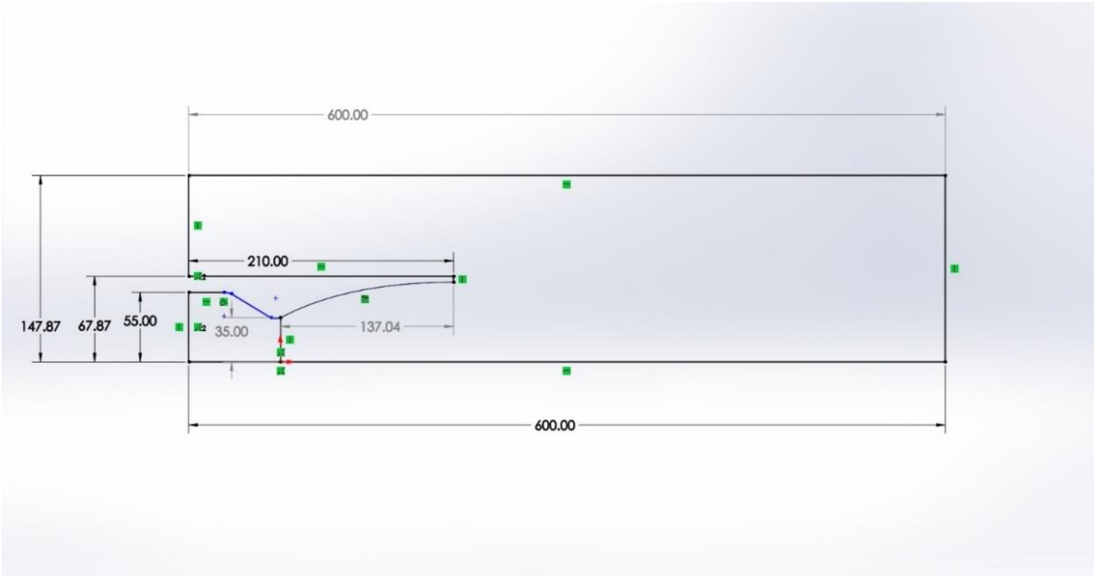

FIGURE 9. 2D NOZZLE DOMAIN (SOLIDWORKS)

The above curve is the final wall profile of the supersonic minimum length nozzle using method of characteristics. Nozzle have throat radius of 35mm, exit radius is 6*7.87*mm, length of nozzle is 137.04mm and having maximum wall angle 27.76 degrees. Pressure exit is 38382.9574Pa.

CHAPTER 3: NOZZLE SIMULATION

As in previous chapter we have designed a minimum length nozzle using method of characteristics the next step will be to perform a Computational Fluid Dynamics (CFD) for this nozzle. But before that, we have to understand the theory involved

3.1 THEORY RELATED TO NOZZLE FLOW

3.1.1 Compressible flow

To get a supersonic flow at the exit of the supersonic nozzle, the first condition that is needed to be satisfy is the fluid must have to be compressible, only a compressible flow can be accelerated to a supersonic velocity as can be seen in the equation (1), the sound of velocity is a depends upon the change in the density and for incompressible fluid the velocity of sound will be infinite, hence flow can be reach that infinite value of velocity. Therefore, the flowing fluid must be compressible to get supersonic flow.

$$C=\sqrt{\frac{dp}{d\rho}}\ at\ constant\ entropy$$

C = velocity of sound

Now, almost very fluid show compressibility to some extent but all of them cannot be considered as the compressible fluid e.g., water, now the fluid can be considered as a compressible fluid only if the change in the density is greater than 10% or the Mach number M of the flow must be greater than 0.3.

3.1.2 Types of Flow through Nozzle

Mach no. (M) is defined as the ratio of velocity of the fluid in the flow to the velocity of sound in the same flow, now

If $M < 1$ subsonic flow

$M = 1$ sonic flow

$M > 1$ supersonic flow

The velocity u will increase or decrease is depending upon the nozzle profile and related with the relation given as;

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u}$$

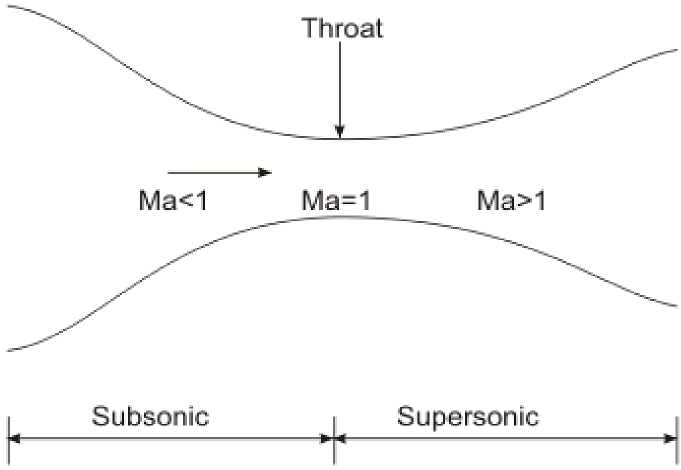

FIGURE 10. MACH NO. VARIATION ALONG THE LENGTH OF CD-NOZZLE

In the supersonic the Mach number at the throat is always equal to 1 and this condition is known as chocking, that means supersonic flow in the CD nozzle exist only if chocking occurs in the nozzle and chocking always occur at the throat i.e., at the minimum area of the nozzle.

3.1.4 Effect of Back Pressure or Ambient Pressure on flow

The chocking of the system depends upon another parameter that is, back pressure Pb or the ambient pressure at the exit of the nozzle. A diagram is shown below which shows the pressure variation throughout the nozzle in terms pressure ratio P/Pt where P is the pressure at any point along the length and Pt is the pressure of the chamber or reservoir, since the velocity is zero in the chamber pressure Pt is also known as stagnation pressure.

Now, let’s try to understand the flow and pressure in the nozzle at different back pressure

1. When the back pressure Pb = Pt, no-flow condition is occurring and the pressure throughout the nozzle is same.

2. When the back pressure Pb is reduced slightly a flow is setup in the nozzle pressure reduces till the throat and minimum at the throat then again start increasing and at the exit it is equal to Pb or Pambient. In this condition flow in

both converging and diverging part is subsonic M>1, because back pressure Pb is still high enough, which prevents the chocking of the nozzle.

3. On again decreasing a value of the back pressure Pb, the flow finally chocked at the throat and pressure at the throat is P_{th}^{*} , * indicates the chocking, since the pressure is still high the pressure again starts to increase in the diverging part and equal to $P_{ambient}$. The flow at the throat is sonic (M=1) and subsonic in both converging and diverging section of the nozzle

4. Further decrease in the pressure Pb result in the decrease in the pressure even in the diverging part of the nozzle and flow starts to accelerate to supersonic value, but after some distance a small Normal Shock occurs in the diverging section and flow becomes subsonic again and pressure at exit is equal to $P_{ambient}$, the flow is non-isentropic regime. the pressure at the throat is P_{th}^{*} ,which remain constant once the nozzle is chocked and doesn't alter by the back pressure condition.

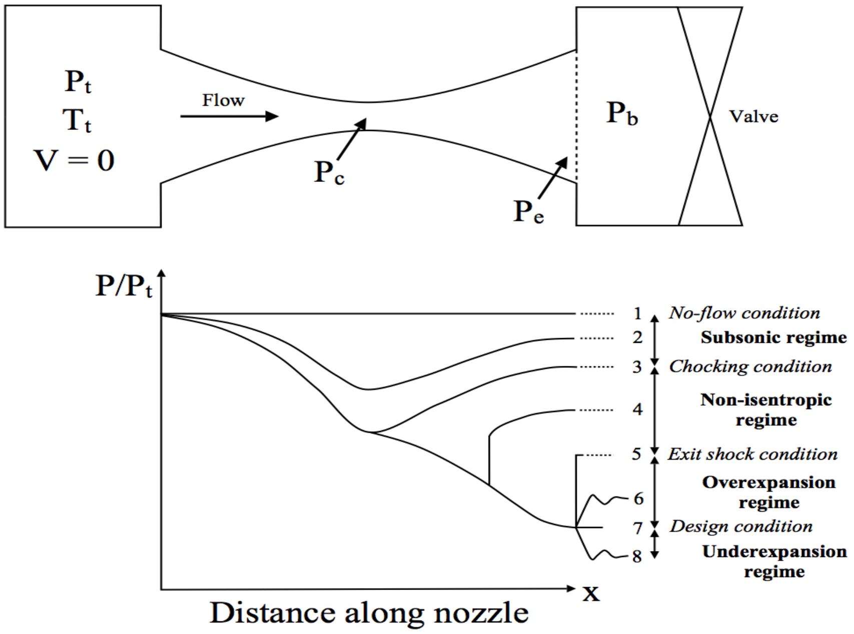

FIGURE 11. EFFECT OF BACK PRESSURE ON FLOW CONDITION

5. On further decreasing the back pressure the flow expands to a greater extend and pressure is decreasing in this time a strong Normal Shock generates and also its position is also move toward the exit. The flow in converging section and after the Normal Shock is subsonic and supersonic between throat and Normal Shock. The pressure at the throat is P_{th}^{*} and at the exit it is $P_{ambient}$ since, the shock is generated at the exit it is also known as the Exit shock condition

6. On further decrease in the back pressure Pb, the flow expands in the diverging part to even a pressure value less than the back pressure and an Oblique Shock is generating at the exit of the nozzle which converges the flow to increase the pressure to back pressure value. The flow in the diverging section is fully supersonic and the flow at the exit is overexpanded known as overexpanded regime.

7. Finally reducing back pressure Pb to a value P_{design}, the expands throughout the nozzle perfectly without and Shock Waves. The flow in the diverging section is supersonic and profile of the jet at the exit is straight, because the flow expands to the exactly back pressure value Pb = P_{design} and this condition of flow is known as Design condition.

8. On further reducing the value of the back pressure below the P_{design}, the flow inside the nozzle is now supersonic in throughout the diverging section, the regime of the flow remain same doesn't alter with reducing in the back pressure below deign pressure. The pressure at exit Pe is now greater than the back pressure and the flow at exit is needed to expand so, Expansion Waves are generated at the exit which help flow to expand the to $P_{ambient}$ and expanding flow profile is form at the exit of the nozzle and this flow regime is known as Under expansion regime.

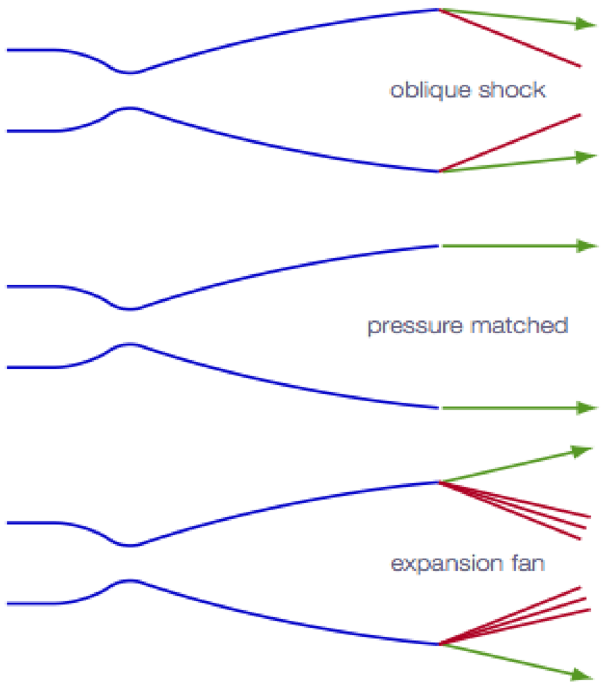

FIGURE 12. DIFFERENT FLOW PROFILE AT THE EXIT OF NOZZLE

As shown in the fig above, we can have three different flow profile at the exit of the nozzle, first when pressure at the exit of the nozzle is greater than the ambient pressure the flow needed to converge in order to match the ambient and pressure so, Oblique shocks are generated at the exit of the nozzle converges the flow and the flow in this condition is overexpanded. Secondly, the when the pressure at the exit is equal to the ambient pressure and the flow profile is perfectly expanded. Thirdly pressure at the exit match the ambient pressure so, Expansion Waves are generated which helps in expansion of flow and a diverging flow profile is generated.

3.2 COMPUATIONAL METHODOLOGY

For the computational simulation ANSYS software has been used, which is used by all over the world for academic purpose. The state of the fluid flowing in the nozzle is describe by three parameters like velocity, pressure, and temperature. The fluid flowing through the nozzle must have to satisfy the governing equations that describes the flow through the nozzle, like mass conservation or continuity equation, momentum conservation equation, energy equation and gas state equation. For the analysis of the compressible fluid, we used density-based solver and the model is used is the standard k-epsilon turbulence model is used which has been used by many of the researcher in their study. The fluid is air as Ideal gas and Sutherland model has been used for the viscosity.

3.2.1 Governing equations involved

1. Mass Conservation Equation or Continuity Equation

$$\frac{d\rho}{\rho}+\frac{dA}{A}+\frac{du}{u}=0$$

2. Conservation of Momentum

$$\frac{dP}{\rho}+ udu=0$$

3. Energy Equation

$$dh+udu=0$$

4. Equation of state

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

5. k-ε Equation

$$\frac{\partial \rho \kappa}{\partial t} + div(\rho u \kappa) = div\left[\left(\mu_t + \frac{\rho \mu_t}{\sigma_\kappa}\right) grad\kappa\right] + \rho \mu_t G - \rho \varepsilon$$

$$\frac{\partial \rho \varepsilon}{\partial t} + div(\rho u \varepsilon) = div\left[\left(\mu_t + \frac{\rho \mu_t}{\sigma_\varepsilon}\right) grad\varepsilon\right] + C_{1\varepsilon} \rho \mu_t \left(\frac{\varepsilon}{\kappa}\right) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{\kappa}$$

3.2.2 Meshing

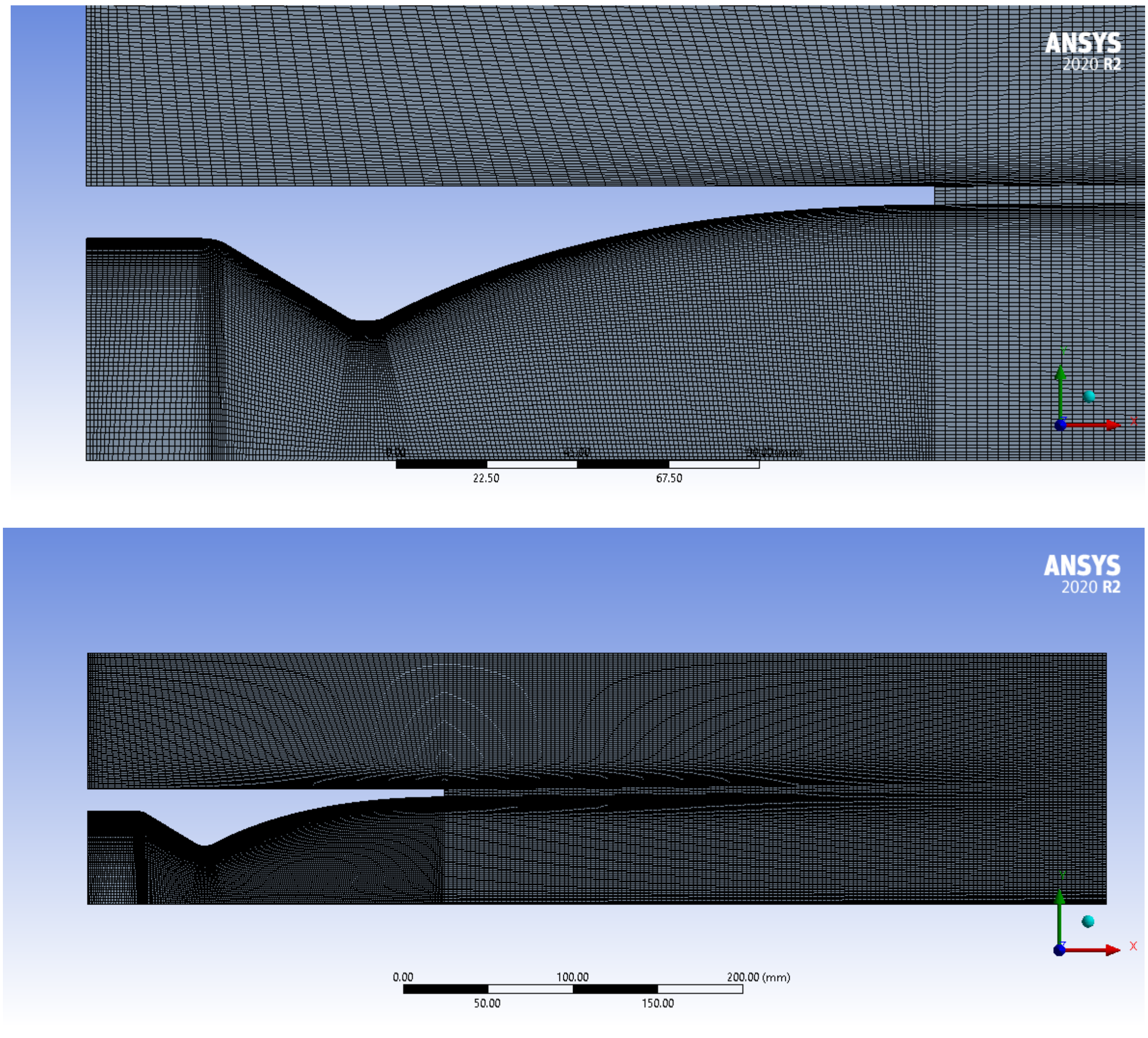

FIGURE 13. CD-NOZZLE DOMAIN MESHING IN ANSYS (FLUENT)

Nodes = 45064

Elements = 44484

3.2.3 Setup and Solution

Parameter	Value
Inlet Pressure	2268000 Pa
Inlet Temperature	1200 K
Outlet pressure	39365 Pa
Outlet Temperature	243K

TABLE 3. LIST OF INPUT PARAMETER IN NUMERICAL SIMULATION

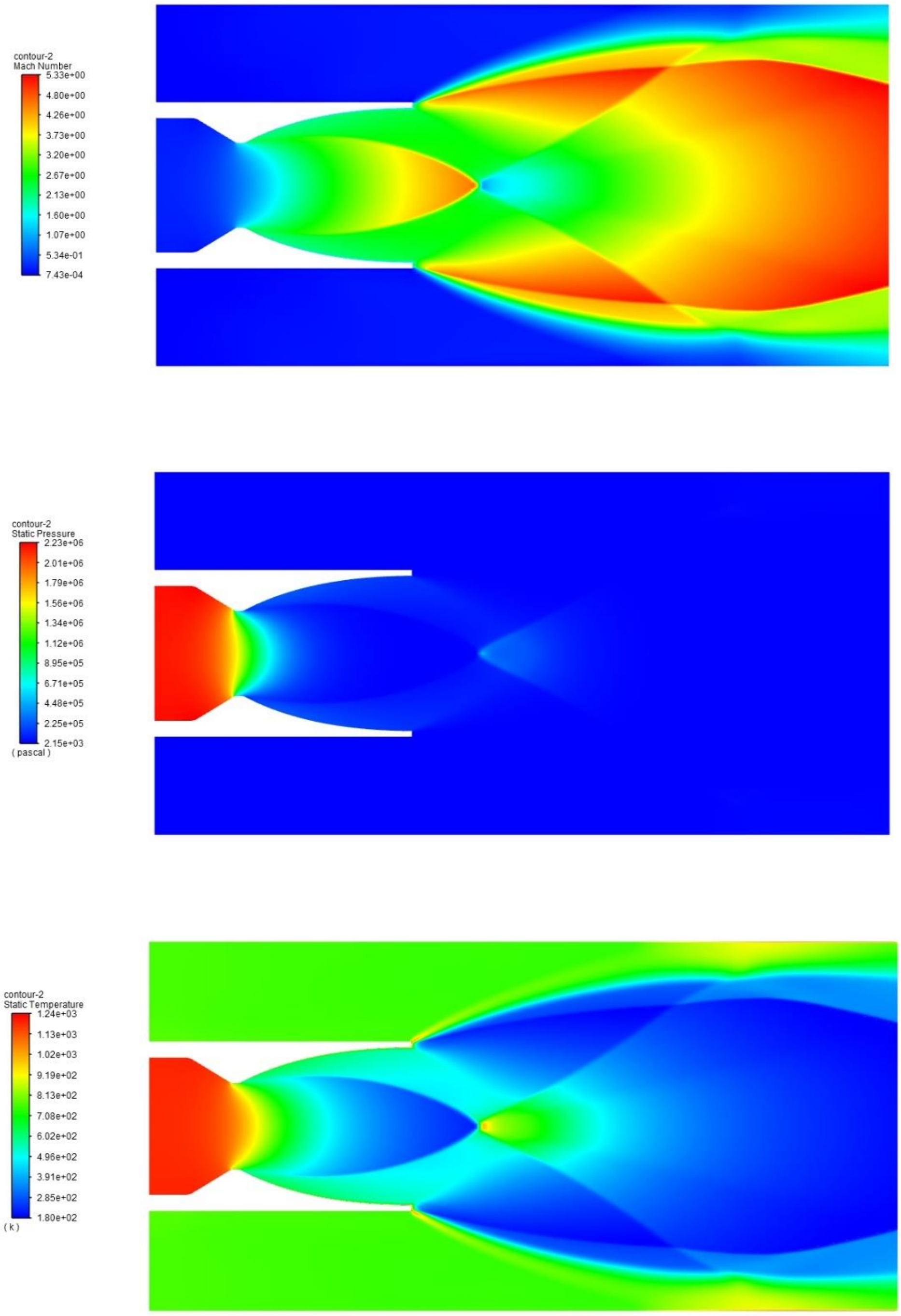

FIGURE 14. VARIATION OF DIFFERENT PARAMETERS IN THE COMPLETE SIMULATION DOMAIN I.E., MACH NUMBER, STATIC PRESSURE AND STATIC TEMPERATURE RESPECTIVELY (ANSYS)

3.2.4 Result

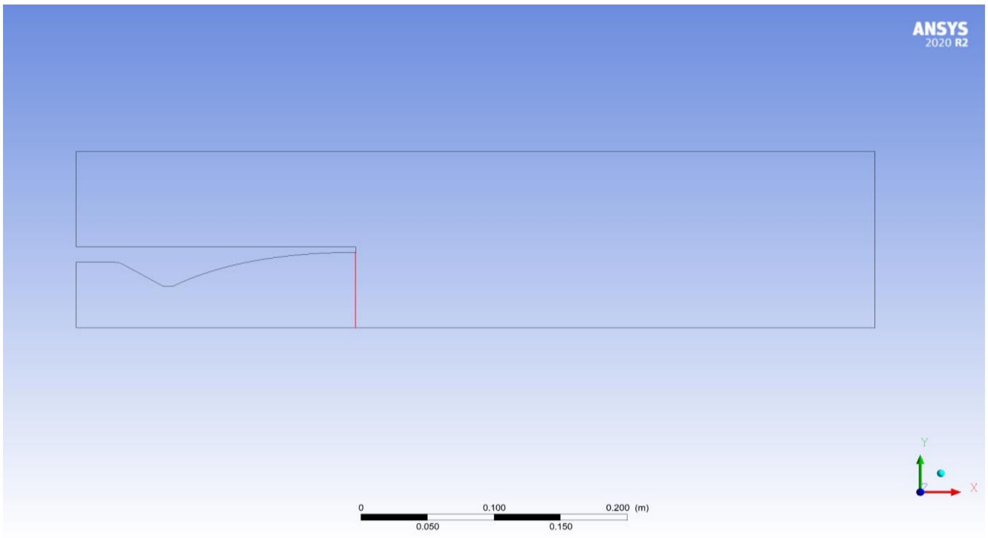

FIGURE 15. CALCULATION AT THE EXIT OF NOZZLE (ANSYS)

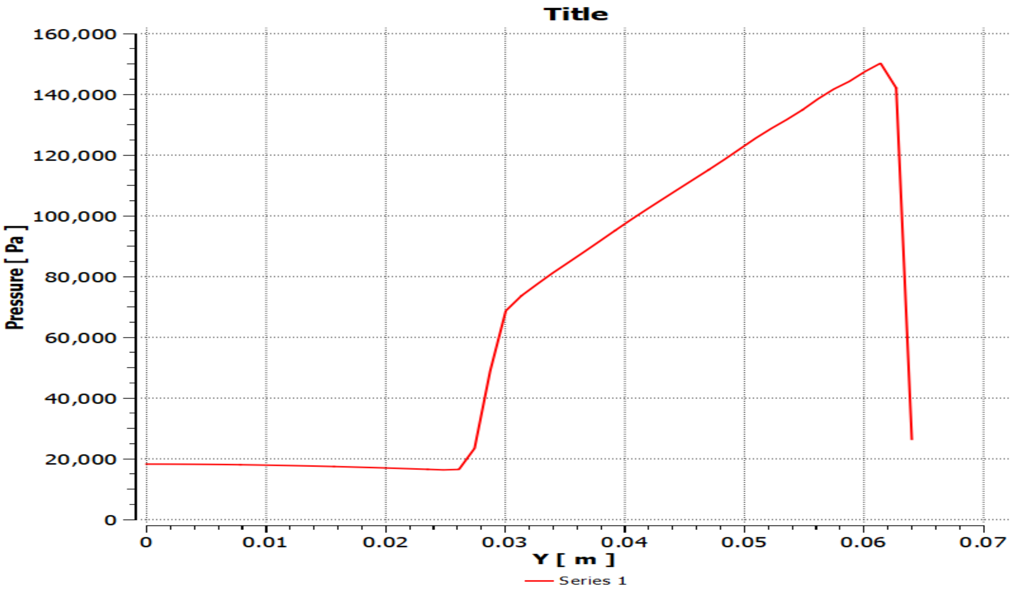

FIGURE 16. PRESSURE VARIATION FROM CENTRELINE TO NOZZLE WALL AT EXIT (ANSYS)

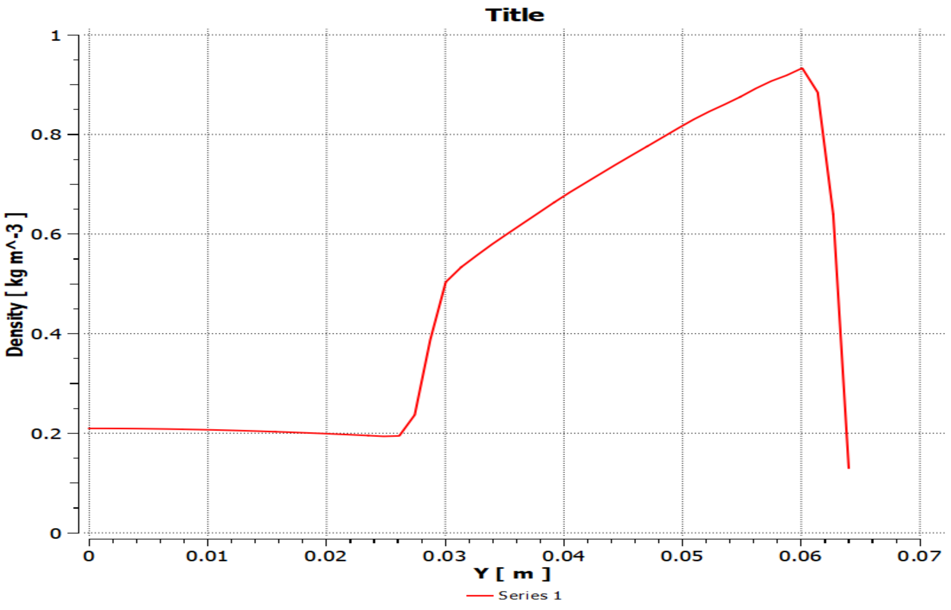

FIGURE 17. DENSITY VARIATION FROM CENTRELINE TO NOZZLE WALL AT EXIT (ANSYS)

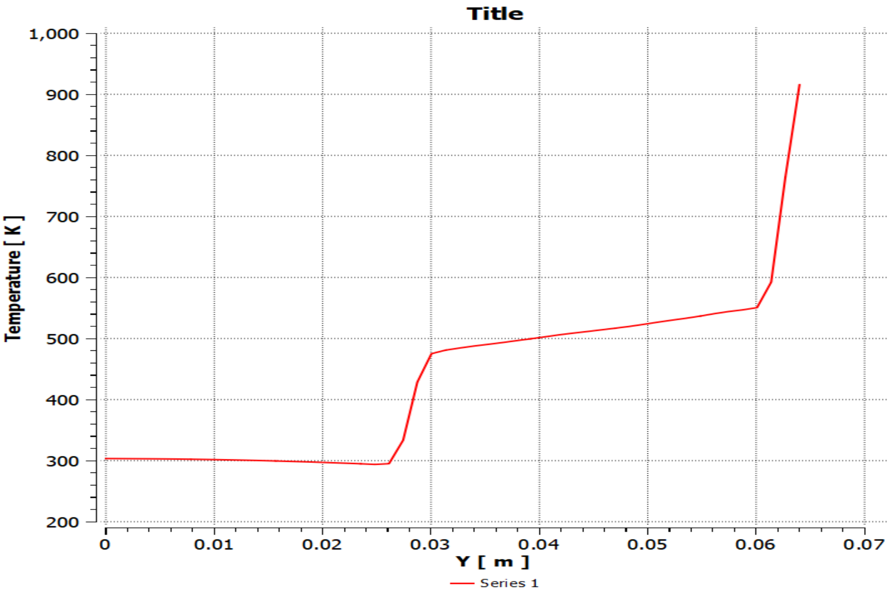

FIGURE 18. TEMPERATURE VARIATION FROM CENTRELINE TO NOZZLE WALL AT EXIT (ANSYS)

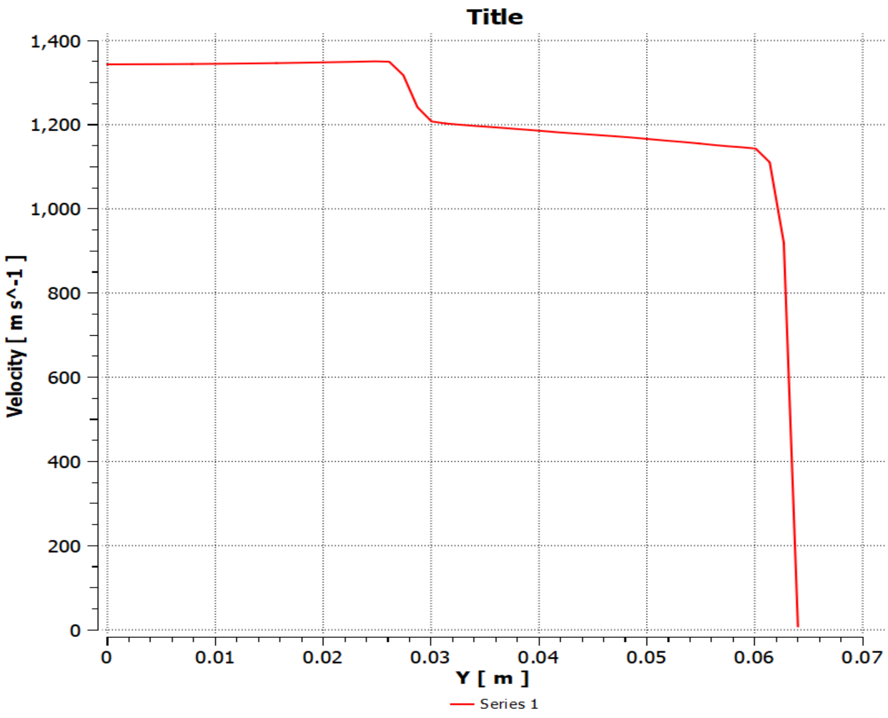

FIGURE 19. VELOCITY VARIATION FROM CENTRELINE TO NOZZLE WALL AT EXIT (ANSYS)

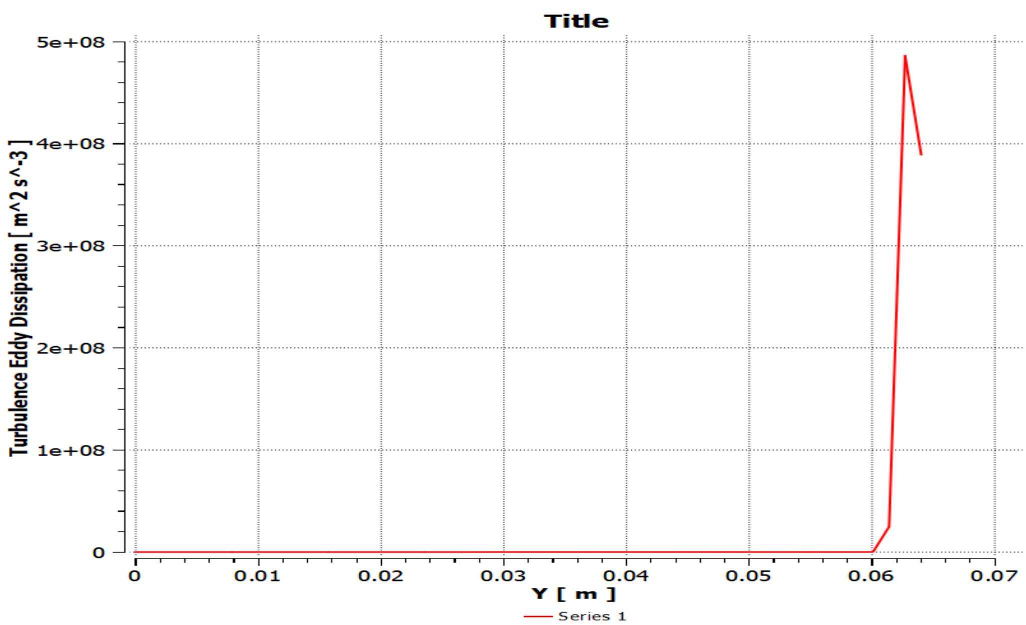

FIGURE 20. TURBULENCE EDDY DISSIPATION FROM CENTRELINE TO NOZZLE WALL AT EXIT (ANSYS)

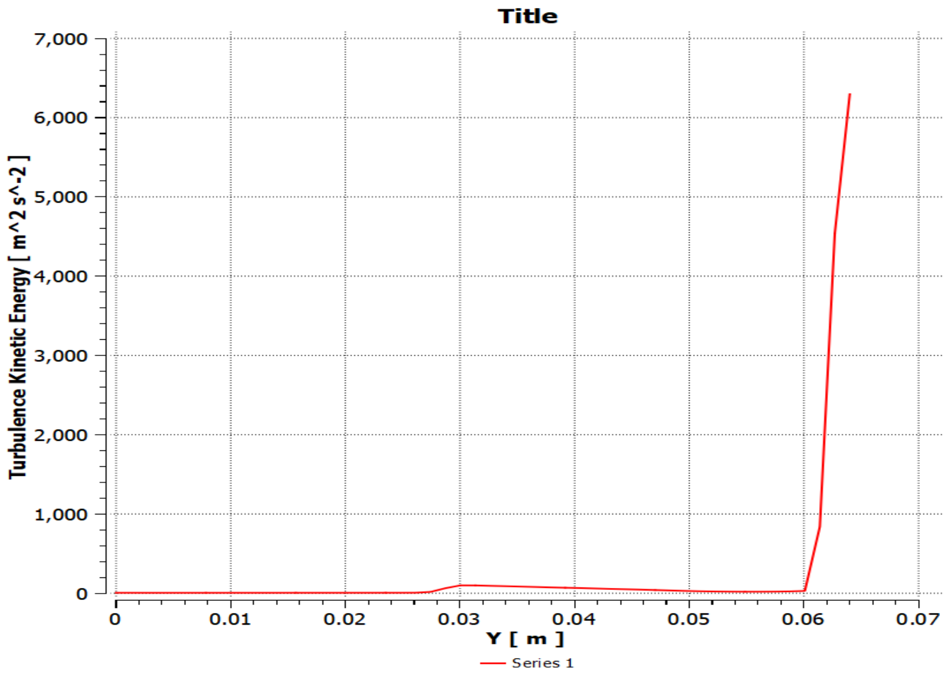

FIGURE 21. TURBULENCE KINETIC ENERGY VARIATION FROM CENTRELINE TO NOZZLE WALL AT EXIT (ANSYS)

3.2.5 Post processing to calculate Mach no, Mass flow and Thrust

Calculation of the thrust value based on the data from simulation, the values for the constant parameter is given below in table.

Constant parameters	Values
Ambient pressure	39365 Pa
Exit area	0.01447 m^2
Throat area	0.00384845 m^2
Area ratio	3.76027

TABLE 4. CONSTANT PARAMETER IN POST PROCESSING PROCESS

Length Y [m]	Density [kg m^-3]	Velocity [m s^-1]	Pressure [Pa]	Temp. [K]	Mach No.	Mass Kg s^-1	Thrust [N]
0.00E+00	2.10E-01	1.34E+03	1.83E+04	3.03E+02	3.85E+00	4.0768	5171.37086
1.31E-03	2.10E-01	1.34E+03	1.83E+04	3.03E+02	3.85E+00	4.0765	5171.17928
2.61E-03	2.10E-01	1.34E+03	1.82E+04	3.03E+02	3.85E+00	4.07443	5168.57381
3.92E-03	2.09E-01	1.34E+03	1.82E+04	3.03E+02	3.85E+00	4.0699	5162.48939
5.22E-03	2.09E-01	1.34E+03	1.82E+04	3.03E+02	3.85E+00	4.06332	5153.53872
6.53E-03	2.09E-01	1.34E+03	1.81E+04	3.03E+02	3.85E+00	4.05541	5142.86466
7.84E-03	2.08E-01	1.34E+03	1.81E+04	3.02E+02	3.86E+00	4.04578	5129.85136
9.14E-03	2.07E-01	1.34E+03	1.80E+04	3.02E+02	3.86E+00	4.03477	5115.03306
1.04E-02	2.07E-01	1.34E+03	1.79E+04	3.02E+02	3.86E+00	4.02232	5098.28726
1.18E-02	2.06E-01	1.34E+03	1.78E+04	3.01E+02	3.87E+00	4.00819	5079.27629
1.31E-02	2.05E-01	1.34E+03	1.77E+04	3.01E+02	3.87E+00	3.9925	5058.14826

1.44E-02	2.04E-01	1.34E+03	1.76E+04	3.00E+02	3.88E+00	3.97536	5035.07974
1.57E-02	2.03E-01	1.34E+03	1.75E+04	2.99E+02	3.88E+00	3.95679	5010.09588
1.70E-02	2.02E-01	1.34E+03	1.73E+04	2.99E+02	3.89E+00	3.9368	4983.18681
1.83E-02	2.01E-01	1.34E+03	1.72E+04	2.98E+02	3.89E+00	3.91521	4954.08057
1.96E-02	2.00E-01	1.34E+03	1.70E+04	2.97E+02	3.90E+00	3.89229	4923.18521
2.09E-02	1.98E-01	1.33E+03	1.69E+04	2.97E+02	3.91E+00	3.86807	4890.5435
2.22E-02	1.97E-01	1.33E+03	1.67E+04	2.96E+02	3.91E+00	3.8426	4856.20768
2.35E-02	1.95E-01	1.33E+03	1.65E+04	2.95E+02	3.92E+00	3.81574	4819.95072
2.48E-02	1.94E-01	1.33E+03	1.63E+04	2.94E+02	3.93E+00	3.78668	4780.74144
2.61E-02	1.95E-01	1.33E+03	1.65E+04	2.95E+02	3.92E+00	3.80142	4799.39341
2.74E-02	2.37E-01	1.30E+03	2.35E+04	3.34E+02	3.60E+00	4.51736	5721.66673
2.87E-02	3.87E-01	1.24E+03	4.89E+04	4.29E+02	2.99E+00	6.96097	8780.46481
3.00E-02	5.04E-01	1.20E+03	6.87E+04	4.75E+02	2.76E+00	8.80036	11053.2684
3.13E-02	5.34E-01	1.20E+03	7.37E+04	4.81E+02	2.74E+00	9.28895	11670.3879
3.27E-02	5.57E-01	1.20E+03	7.75E+04	4.85E+02	2.72E+00	9.67293	12158.7855
3.40E-02	5.80E-01	1.19E+03	8.12E+04	4.88E+02	2.70E+00	10.0451	12631.772
3.53E-02	6.01E-01	1.19E+03	8.46E+04	4.91E+02	2.69E+00	10.3925	13073.4854
3.66E-02	6.22E-01	1.19E+03	8.81E+04	4.93E+02	2.68E+00	10.7327	13504.0579
3.79E-02	6.43E-01	1.19E+03	9.16E+04	4.97E+02	2.66E+00	11.068	13925.8418
3.92E-02	6.64E-01	1.18E+03	9.52E+04	5.00E+02	2.65E+00	11.405	14349.6766
4.05E-02	6.84E-01	1.18E+03	9.87E+04	5.03E+02	2.64E+00	11.7217	14744.4462
4.18E-02	7.03E-01	1.18E+03	1.02E+05	5.06E+02	2.62E+00	12.0153	15107.4318
4.31E-02	7.21E-01	1.18E+03	1.05E+05	5.09E+02	2.61E+00	12.3114	15475.906
4.44E-02	7.40E-01	1.18E+03	1.09E+05	5.11E+02	2.60E+00	12.6067	15843.7059
4.57E-02	7.58E-01	1.17E+03	1.12E+05	5.14E+02	2.58E+00	12.8909	16195.1311
4.70E-02	7.76E-01	1.17E+03	1.15E+05	5.17E+02	2.57E+00	13.1702	16539.7824
4.83E-02	7.94E-01	1.17E+03	1.19E+05	5.20E+02	2.56E+00	13.4497	16882.6951
4.96E-02	8.13E-01	1.17E+03	1.22E+05	5.23E+02	2.55E+00	13.725	17215.8355
5.09E-02	8.30E-01	1.16E+03	1.26E+05	5.27E+02	2.53E+00	13.9858	17527.4396
5.22E-02	8.46E-01	1.16E+03	1.29E+05	5.30E+02	2.52E+00	14.2168	17801.7497
5.36E-02	8.60E-01	1.16E+03	1.32E+05	5.33E+02	2.50E+00	14.4233	18044.7849
5.49E-02	8.75E-01	1.16E+03	1.35E+05	5.37E+02	2.49E+00	14.6364	18292.7413
5.62E-02	8.92E-01	1.15E+03	1.39E+05	5.41E+02	2.47E+00	14.8737	18567.5214
5.75E-02	9.07E-01	1.15E+03	1.42E+05	5.44E+02	2.46E+00	15.0806	18806.3165
5.88E-02	9.19E-01	1.15E+03	1.44E+05	5.47E+02	2.45E+00	15.2426	18992.3211
6.01E-02	9.33E-01	1.14E+03	1.48E+05	5.51E+02	2.43E+00	15.4406	19217.8628
6.14E-02	8.84E-01	1.11E+03	1.50E+05	5.92E+02	2.28E+00	14.2126	17389.2793
6.27E-02	6.40E-01	9.18E+02	1.42E+05	7.64E+02	1.66E+00	8.50635	9305.1682
6.40E-02	1.30E-01	6.62E+00	2.64E+04	9.16E+02	1.37E-02	0.01563	-187.07234

TABLE 5. POST PROCESSING TABLE FOR THE CALCULATION OF MACH, MASS FLOW RATE AND THRUST AT EXIT OF NOZZLE (EXCEL)

FIGURE 22. VARIATION OF MACH NUMBER FROM CENTRELINE TO WALL AT THE EXIT (EXCEL)

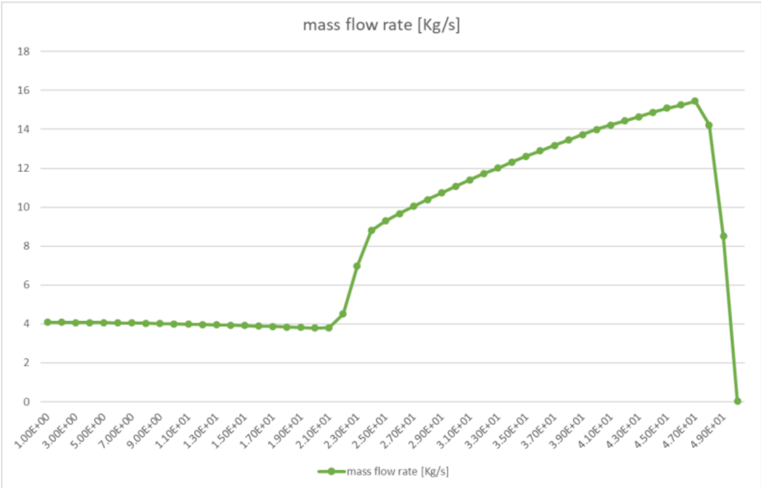

FIGURE 23. VARIATION OF MASS FLOW RATE FROM CENTRELINE TO WALL AT THE EXIT (EXCEL)

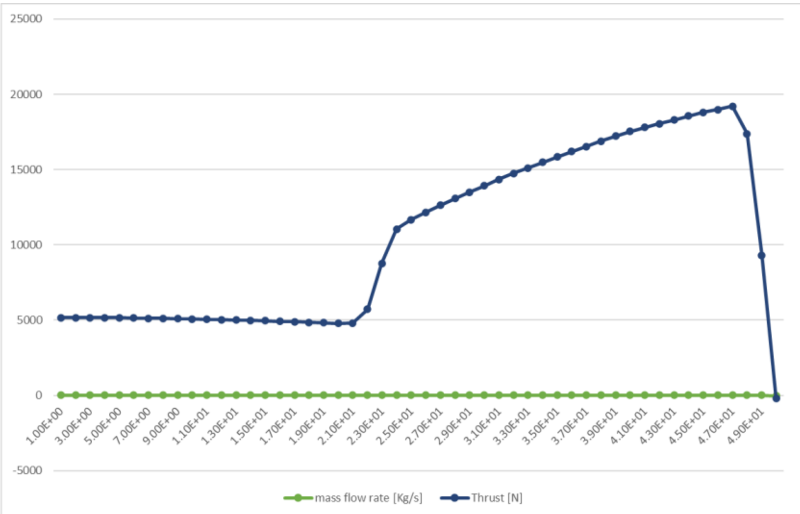

FIGURE 24. VARIATION OF THRUST FROM CENTRELINE TO WALL AT THE EXIT (EXCEL)

In numerical simulation, we have to ignore the beginning and the ending values of the curve, because of the shock at the boundary and at the centreline, the average value from table is taken at the vertical distance from the centreline is .035m. the corresponding **average** values of the **Mach number is 2.69**, **mass flow rate is 10.3925Kg/s** and the **average thrust is 13073.4854 N**

3.3 ANALYTICAL METHODOLOGY

For the validation of the result obtained from the computational methodology, we will calculate the thrust value analytical assuming isentropic flow.

Parameter	Value
Inlet Pressure	2268000 Pa
Inlet Temperature	1200 K
Outlet pressure	39365 Pa
Outlet Temperature	243K
Throat Radius (throat area)	35mm or .035m (0.00384845 m^2)
Exit Radius (exit area)	67.87mm or 0.06787m (0.0144712 m^2)
Area ratio	10.269277
R	287 KJ/KgK

TABLE 6. DESIGN PARAMETERS

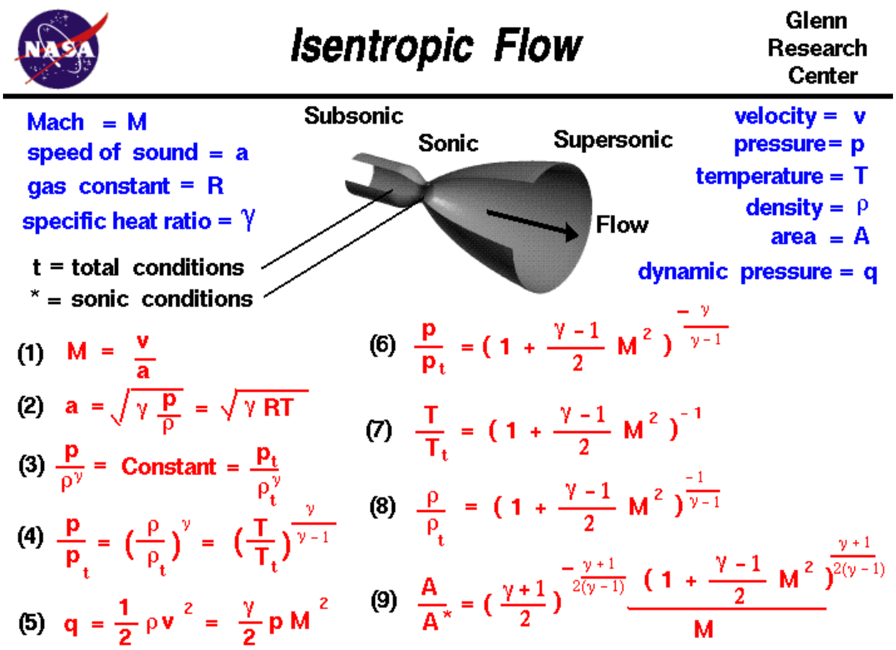

FIGURE 25. FORMULA USED IN ANALYTICAL SOLUTION FOR ISENTROPIC FLOW (NASA)

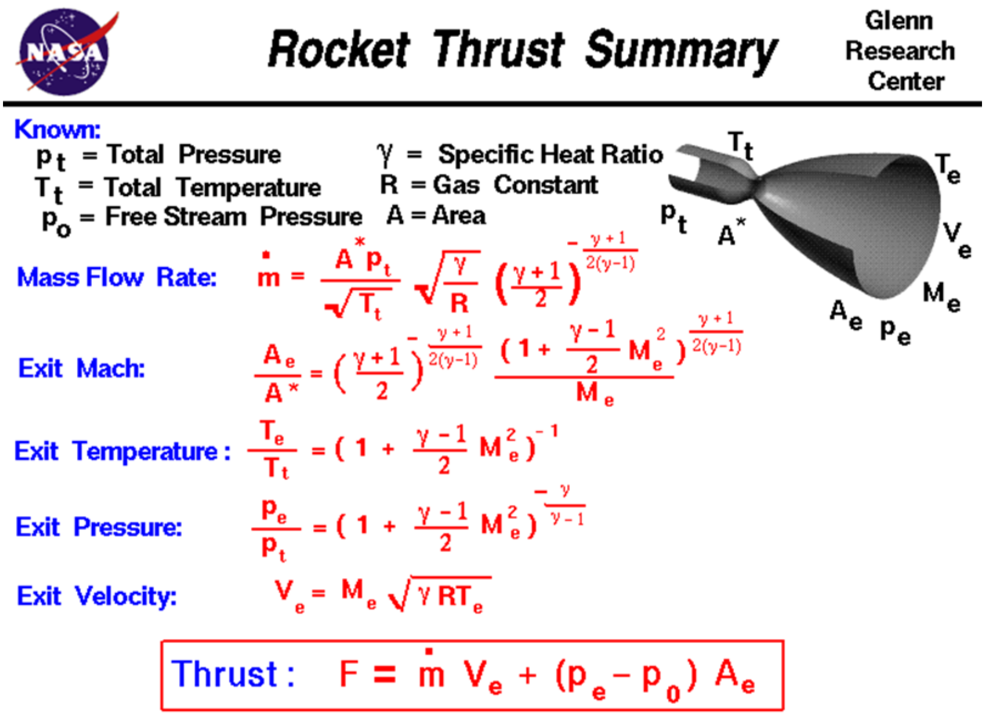

FIGURE 26. FORMULAS USED FOR THE CALCULATION OF THRUST (NASA)

Calculations:

1. Exit Mach no. correspond to area ratio using isentropic table

Mach at exit (Me) = 2.875

2. Stagnation Density (Inside chamber)

$$\boldsymbol{\rho = \frac{P}{RT}} \quad \textbf{= 6.5853 Kg/m^3}$$

3. Throat velocity (M=1) T = chamber stagnation temperature =1200K

$$\boldsymbol{V = \sqrt{\frac{2\gamma.R\,T}{\gamma+1}}} \textbf{ = 633.8769 m/s}$$

4. Throat density (using isentropic flow chart)

Density at throat is = density in chamber*.633938
= 4.17498Kg/m^3

5. Mass flow rate

= (throat density* throat velocity* throat area)

= 4.17498*633.8769*0.00384845

10.1846 Kg/s

6. Throat pressure using isentropic table for Mach no = 1

Pt = pressure in chamber* .5282

= 1197957.6

7. Pressure at exit using isentropic flow relation at Mach No. = 2.88

Pe = pressure chamber *.03263

= **74004.84Pa**

Note: exit pressure is 74004.84 greater than 39365 Pa hence, system is **Underexpanded**

8. Exit temperature using isentropic flow table for Mach No. =2.88

Te = chamber temperature*.3761

= **451.32K**

9. Exit velocity

Ve = Me*(1.4*287*Te)^2

= 1226.4214m/s

Now, we have all value to calculate the thrust at the exit of the nozzle

Thrust = mVe +(Pe-Pamb)Ae

T= 12991.89144 N

CHAPTER 4: RESULT VALIDATION, OBSERVATION AND DISSCUSSION

4.1 SANITY CHECK

As we have performed the numerical simulation for the Supersonic CD-nozzle and we have performed analytical solution for the isentropic flow in same. Now, we will try to validate the numerical solution result with the analytically calculated result.

If we look the flow in fig. 14 , for the underexpansion flow, fluid must to expand in the diverging as it deos so, we have flow boundary, since the flow expands at the exit then we have expansion fans or expansion waves at the outlet of the nozzle, aslo have barrel shock due to viscosity of the fluid, also we have slip lines in the flow. So, from all the above observation our simulation look like accurate.

4.2 COMPARISON OF NUMERIAL AND ANALYTICAL RESULT

1. Mach Number

Numerical simulation calculation of the average Mach number is **2.69**

Analytical calculated value of the Mach Number for an Isentropic flow is **2.88**

2. Mass Flow Rate

Average numerical simulation value of the mass flow rate is **10.3925 Kg/s**

Analytical calculated value for the mass flow rate is **10.1486 Kg/s**

3 Thrust

Average thrsut value for numerical simulation is **13073.4854 N**

Analytical calculated value of thurst is **12991.89144 N**

All the above values are very close to their corrsponding values, hence our simulation is acurate and the result is validated.

4.3 LIMITATIONS INVOLVED

There are some limitation involved in the analysis as mention below;

1. First we have to acknownledge that, even we have high performing computers to do the simulation or designing the nozzle, the accuracy of the real life suitation cannot be met.
2. Our numerical calculation and as well as analytical solution is based on some assumption which makes our calculatation slightly inaccurate. For example we

have different simulation models used in the simulation like k-elipson, k-omega, etc models which will have different result. Also in the case of the analytical solution we have assumed the flow is isentropic for our calculatation but, in actual, there is some turbulence, viscosity, shock waves, etc. which is not considered in the isentropic flow assumption.

3. Also, in the analytical process human error involved which cannot be ignored.

But, above all over analysis gives us a very accurate and useful ideal about the actual simulation and it helps us reducing in the cost of the testing of the real prototype, which may increase our exploration and study cost to a very high.

4.4 FUTURE SCOPE

The study of the nozzle and the improvement of the design useful in the space and aviation industries which are growing at super speed and there is race to make air travel as well as space exploration and travel offordable for all and that’s why have huge business opportunity also invloved. Even a small improvement in the nozzle design or performannce can save huge amount in the cost of operation.

CHAPTER 5 CONCLUSION

Nozzle Design

We have designed a nozzle using the method of the characteristics for the minimum length of the nozzle it has lower quality of flow than the gradual increasing nozzle which used in the supersonic wind tunnel for very high quality flow at the exit. The minimum length is used in space industries because, we want of minimize the weight of the nozzle which result cost of waste in thrsut to lift the weight of the nozzle. We have designed the spersonic CD- nozzle a open source MATLAB code to generate the wall points for a perticular designed Mach number such we have parellel and shock free flow at the exit of the nozzle, which require to improve the performanc and the life of the nozzle. Then we inserted these points in the SOLIDWORKS to generate diverging wall curve. The designed length of the nozzle is 137.04mm exit radius is 67.87mm, inlet radius 35mm (input value) and the maximum wall angle of the nozzle at the downstream is 27.26 degree.

Nozzle simulation

We have performed the numerical simulation of the supersonic CD- nozzle using the ANSYS software a mesh is generated having 45064 nodes and 44484 elements. Then the simulation is performed using the density based solution because the fluid that is air is compressible and k-elipson model is used which is used by most of the researcher in their work. And the simulation is performed, the exit data curve for the exit density, pressure velocity and temperature is generated and then then post processing process is performed in the EXCEL to calculate the exit Mach Number Mass flow rate and Thrust value at the exit and the average values are 2.69, 10.3925Kg/s and 13073.4854 N respectively. We also performed analytical solution by taking assumption the flow is isnetropic and the calculated values of the Mach number is 2.88, mass flow rate is 10.1486Kg/s and thrust is 12991.89144 N. the values from both the methodology is quite similar hence, validate our result.

Future area of study

we have designed the minimum length nozzle, gradual increasing length nozzle also be designed using method of characteristics and simulation can be performed for the same.

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