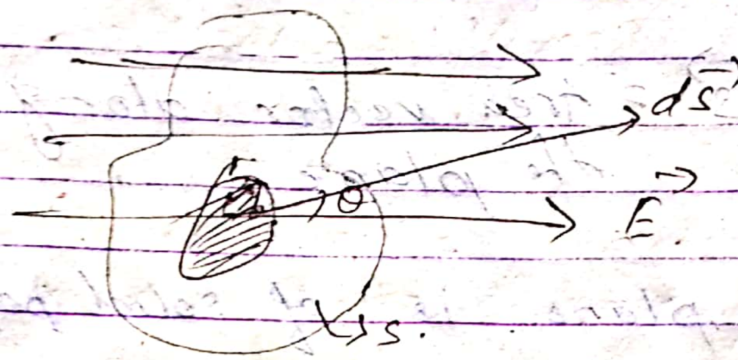
Electric Flux

It is the no. of electric field line passing or cutting normally to the surface held in an electric field.

It is denoted by ϕ_E.

Relation b/w electric field and electric flux.



Electric flux linked with elementary area ds.

$$d\phi_E = \vec{E}\cdot\vec{ds} = Edcos\theta$$

Total electric flux.

$$\phi_E = \int_S \vec{E}\cdot\vec{ds} = \int_S E\cdot ds\cdot\cos\theta$$

(For Plane Surface)

i.e, Electric flux over a plane surface is the surface integral of electric field over the surface.

For close surface.

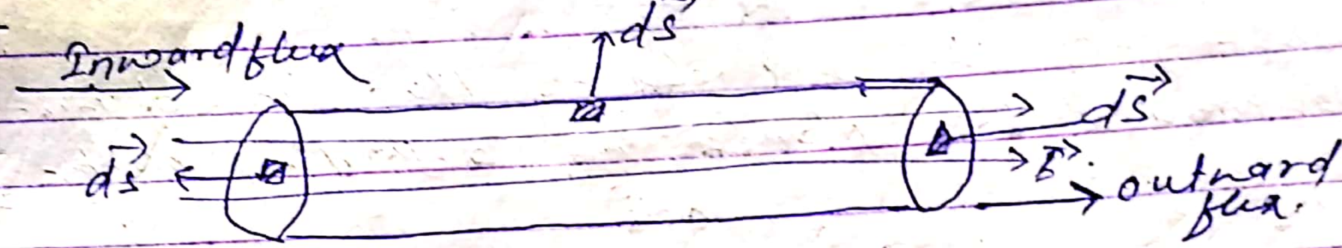
$$\phi_E = \oint_S \vec{E}\cdot\vec{ds}$$

i.e, surface integral of electric field over a closed surface.

SI unit : $NC^{-1}m^2$ or Weber (Wb)

ques:- Find the total Electric flux linked with a hollow cylinder placed in a uniform electric field.

Sol:-



Electric flux linked with the hollow cylinder

$$\phi_E = \oint_S \vec{E}\cdot\vec{ds} = \int_{\substack{\text{left}\\ \text{plane Surface}}} \vec{E}\cdot\vec{ds} + \int_{\substack{\text{curved}\\ \text{Surface}}} \vec{E}\cdot\vec{ds} + \int_{\substack{\text{right plane}\\ \text{Surface}}} \vec{E}\cdot\vec{ds}$$

$$= \int_{\text{left}} E\cdot ds\cdot\cos\pi + \int_{\text{curve}} E\cdot ds\cdot\cos\pi/2 + \int_{\text{Right}} E\cdot ds\cos 0^\circ$$

$$= -\int E ds + 0 + \int E\cdot ds$$

$$= 0.$$

Alternative Method

Total Electric Flux:-

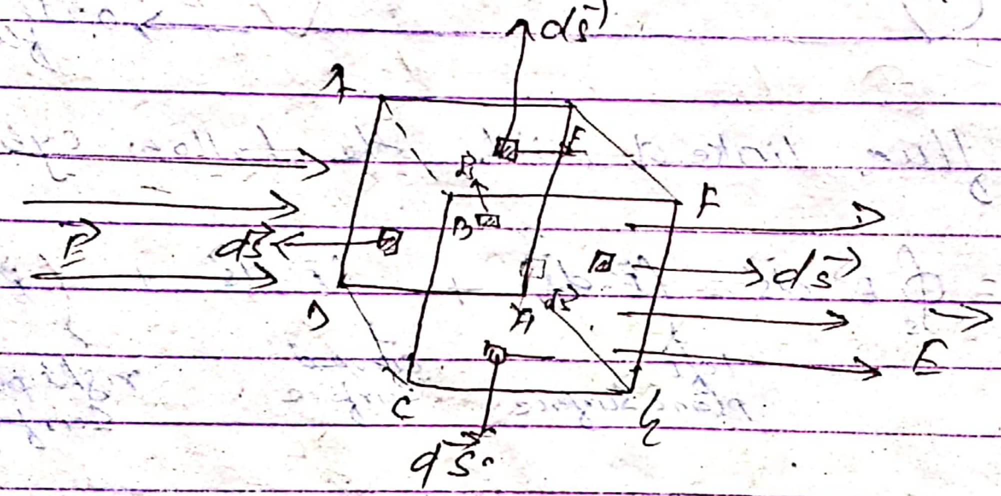
ϕ_E = Inward flux + outward flux.

Since, Inward flux = −outward flux.

⇒ Inward flux + outward flux = 0

ques:- Find the net electric flux linked with a cubical box placed in a uniform electric field.

Sol:-



∵ out of 6 faces 4 faces of cube make the angle b/w elementary faces $\vec{ds}$ and $\vec{E}$, is 90°. ∴ Electric flux = 0 in these faces.

So, $\phi_E = \oint_S \vec{E}\cdot\vec{ds} = \int_{ABCD} \vec{E}\cdot\vec{ds} + \int_{EFGH} \vec{E}\cdot\vec{ds}$

$= +\int E.ds\cos\pi + \int E.ds\cos 0°$

$= -\int E.ds + \int E.ds$

$= 0.$

* State And prove Gauss's theorem

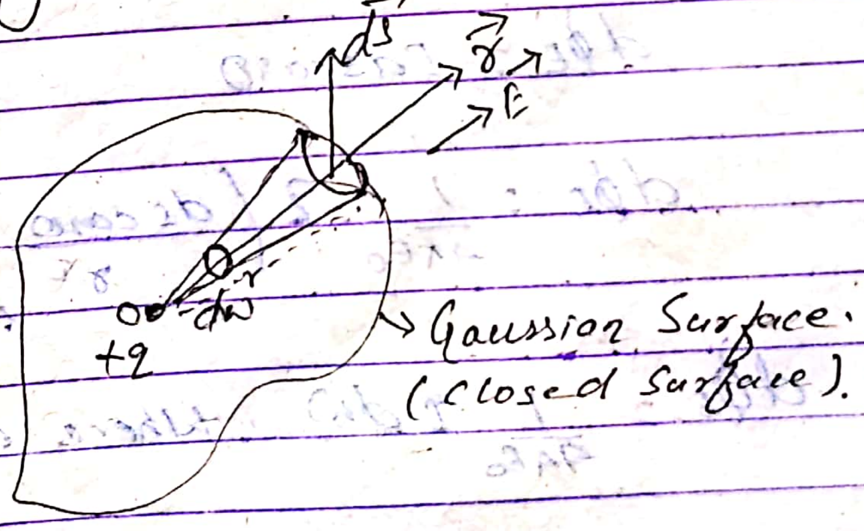
Total electric flux linked with a closed surface (Gaussian surface) is equal to the $\frac{1}{\epsilon_0}$ times the total charge inclosed by that surface.

Mathematically it can expressed as follows:-

$$\phi_E = \oint_S \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0} \times q_{net}.$$

i-e, surface itegral of electric field over a closed surface is equal to $\frac{1}{\epsilon_0}$ times the total charge inclosed by that surface.

proof: By using coulomb's law:-



Consider a point charge $+q$ at a point O, placed inside a gaussion surface as shown in the diagram

From coulomb's law:-

Electric field at a point P lying on the Surface:-

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{——— (1)}$$

Now, consider a patch area ds the surface containing the point 'P'

∴ Electric flux linked with this area.

$$d\phi_E = \vec{E} \cdot \vec{ds}$$

$$d\phi_E = E ds \cos\theta$$

$$d\phi_E = \frac{1}{4\pi\epsilon_0} q \left(\frac{ds \cos\theta}{r^2} \right)$$

$$d\phi_E = \frac{1}{4\pi\epsilon_0} q \, d\omega \quad \text{where } d\omega = \frac{ds \cos\theta}{r^2}$$

solid angle substained by patch area ds at O.

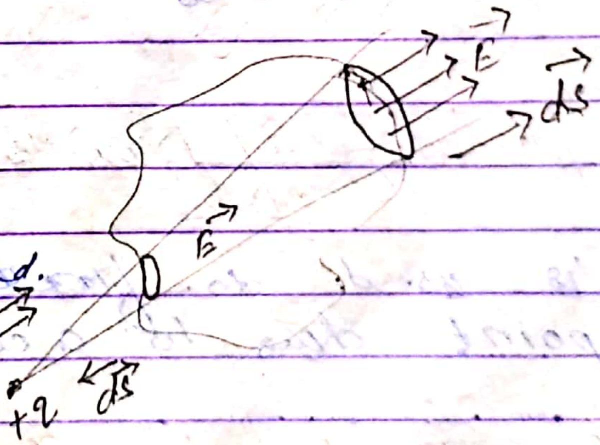
∴ Total electric flux linked with Gaussian surface.

$$\phi_E = \oint_S \phi_E = \frac{1}{4\pi\varepsilon_0} q \oint_S d\omega.$$

$$= \frac{1}{4\pi\varepsilon_0} q\, 4\pi$$

$$\boxed{\phi_E = \frac{q}{\varepsilon_0}}$$

Case 2: When the point charge lies outside the gaussian surface.



Total electric flux.

$$\phi_E = (\phi_E)_{inward} + (\phi_E)_{outward}.$$

Now, $(\phi_E)_{inward} = \frac{-1}{4\pi\varepsilon_0} q \oint_C d\omega.$

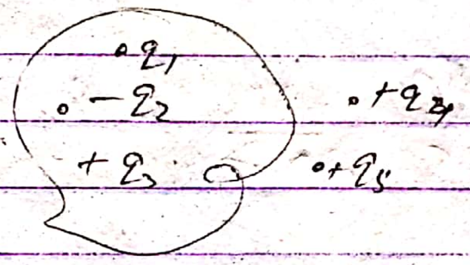
$$(\phi_E)_{outside} = \frac{1}{4\pi\varepsilon_0} q \cdot \oint_S d\omega.$$

$\therefore$ Total Electric flux = 0.

$$\Rightarrow \oint_E = 0.$$

$\Rightarrow$ Net electric flux linked with the gaussian surface is zero., when the point lies outside the gaussian surface.

(ii).

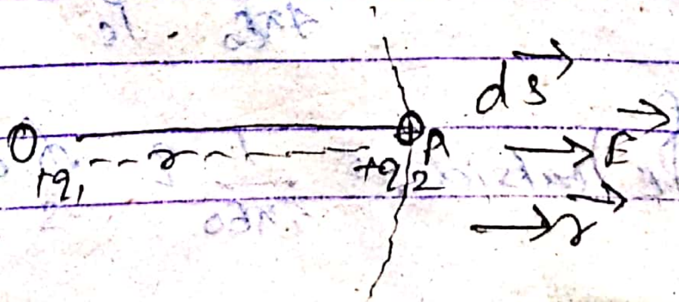


$$\therefore \oint_E = \frac{1}{\varepsilon_0}(q_1 - q_2 + q_3)$$

$\Rightarrow$ Application of Gauss's theorem

It is used to find electric field at a point due to a complicated charged body.

(i) Coulomb's law from Gauss's theorem



consider two point charges $+q_1$ and $+q_2$ placed at point O and A respectively required to find electric force b/w the charges.

let E_1 be the Electric field at A due to charge $+q_1$.

consider a spherical gaussian sphere of radius r.

$\therefore$ Electric flux linked with the patch area ds.

$$\phi_E = \oint_S \vec{E_1} \cdot \vec{ds}$$

$$= E_1 ds = E_1 \oint_S ds$$

$$= E_1 \times 4\pi r^2 \quad \left[\because \oint_S ds = 4\pi r^2\right] \quad \text{— (I)}$$

A/c to gauss's theorem.

$$\phi_E = \frac{1}{\epsilon_0} q \quad \text{— (II)}$$

from (I) and (II)

$$E_1 \times 4\pi r^2 = \frac{1}{\varepsilon_0} q$$

$$\therefore \boxed{E_1 = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}}$$

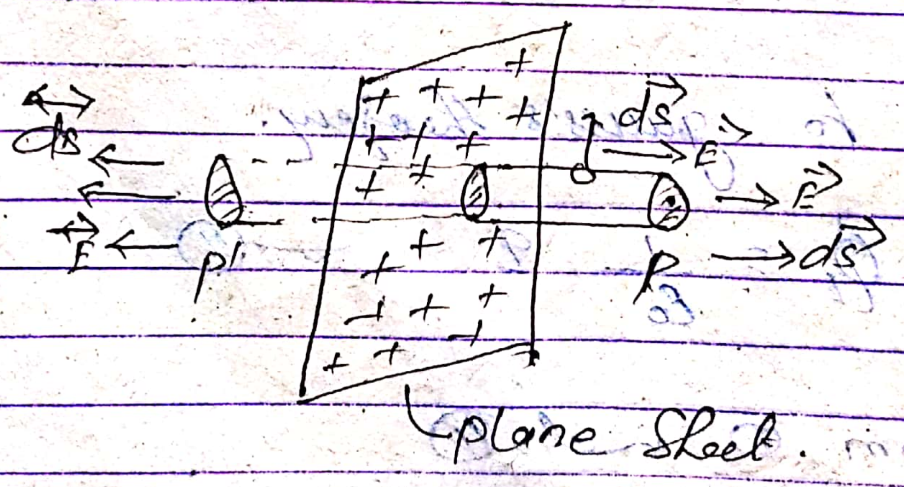
$\therefore$ electric force of $(+q_2)$ due $(+q_1)$

$$F = Eq_2$$

$$\boxed{\therefore F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$$ ~~proved~~.

(ii) Due to an infinite plane sheet of charge.

(a) Non-conducting infinite plane sheet of charge $(+\sigma)$ (surface charge density).



It is required to find Electric field at P due to a finite plane sheet for this we consider a cylindrical gaussian surface containing a point 'p' as shown in fig.

Electric flux linked with this consider gaussian surface.

$$\phi_E = \oint_E \vec{E}\,\vec{ds}$$

$$= \int_{left} E\,ds + \int_{righ} E\,ds + \cancel{\int_{curve} E\,ds} \text{ das } 90^\circ$$

$$= E\int_{left} ds + E\int_{righ} ds$$

$$= 2EA \quad \left[A = \int ds = \text{Area of cross section of cylindrical gaussian surface}\right]$$

$$\phi_E = 2EA \quad \text{---} (1)$$

A/c to gauss's theorem.

$$\phi_E = \frac{1}{\varepsilon_0}(\sigma A) \quad \text{---} (11)$$

from (I) and (II)

$$\frac{1}{\varepsilon_0}\sigma A = 2AE$$

$$\boxed{E = \frac{1}{2\varepsilon_0}\sigma}$$

dirⁿ :– Along normal to the plane of the surface.

In Vector form.

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}}$$

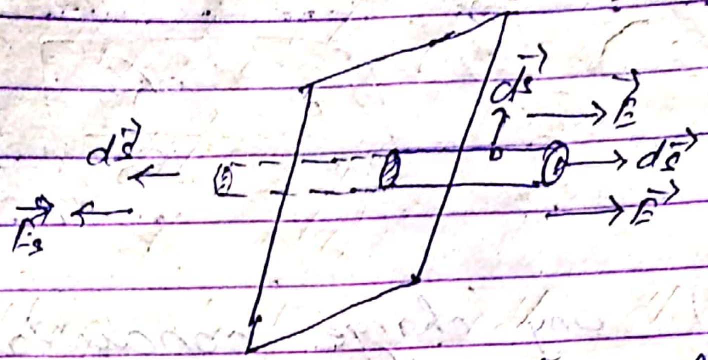
Where, $\hat{n}$ = unit vector along normal to the plane of the surface.

Also, it is independent of position of point.

i.e, Electric field remains same at all close outside point.

(b) When the plane sheet is conducting.

When the plane sheet is conducting i.e, charge resides only on the surfaces i.e, inside the conductor no charge exists.



Now, electric flux linked with this consider gaussian surface.

$$\phi_E = \oint_S \vec{E} \cdot \vec{ds}$$

$= 2EA$ [We find it from pervious section] — (I)

Where A = area of cross section of cylinderical gaussion surface.

So, Acc to gaussian theorem,

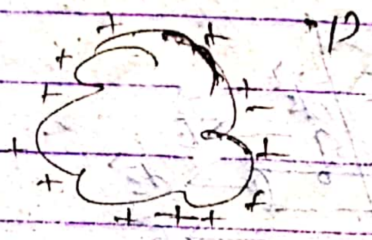
$$\phi_E = \frac{1}{\varepsilon_0} \sigma A \times 2$$ [2 for two charged faces] — (II)

from (I) and (II)

$$2EA = \frac{\sigma A \times 2}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Note!

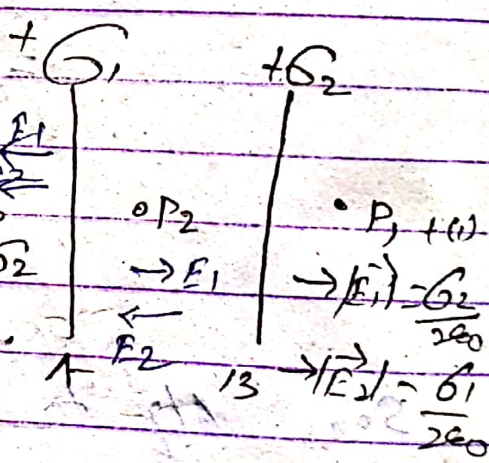


$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$

In case of any charge conductor electric field at a point near the body.

$$\boxed{E = \frac{\sigma}{\epsilon_0}}$$ Where, σ = local surface density of charge.

Q:- Two infinite plane sheet of charge A and B having surface density of charge $+\sigma_1$, $+\sigma_2$. Find the electric field at P_1, P_2, P_3.



Sol:- (i) At P_1 = $E_{P_1} = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{\sigma_1 + \sigma_2}{2\epsilon_0}$

if $\sigma_1 = \sigma_2 = \sigma$

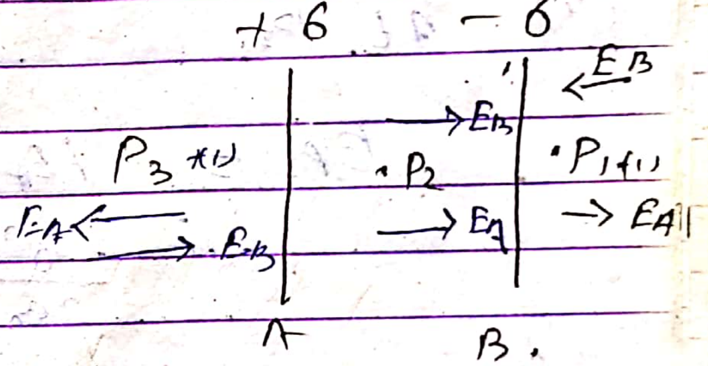
then $E_{P_1} = \frac{2\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$

(ii) $E P_2 = |E_1 - E_2| = \frac{\sigma_1 - \sigma_2}{2\varepsilon_0}$

if $\sigma_1 = \sigma_2$
then $E P_2 = 0$.

(iii) $|\vec{EP_3}| = E_1 + E_2 = \frac{\sigma_1 + \sigma_2}{2\varepsilon_0}$

ques:- Find electric field at P_1, P_2 and P_3.



(i) At P_1

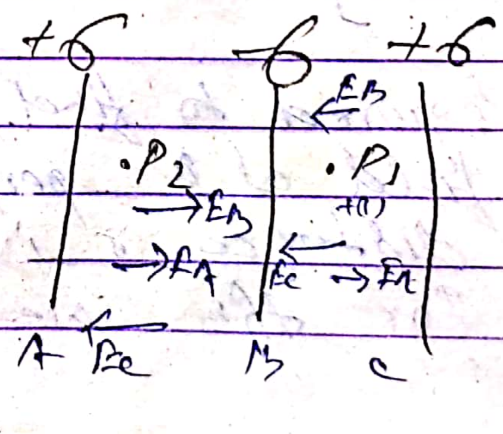
$EP_1 = |\vec{EB} + \vec{EA}| = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$.

(ii) At P_2

$EP_2 = |\vec{EA} + \vec{EB}| = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

(iii) At P_3 $\quad EP_3 = |E_A - E_B| = 0$

ques:- Find electric field at P_1 and P_2.



Ans:- (i) At P_1

$$E_{P_1} = |E_A + E_B - E_A|$$

$$= \frac{\sigma}{2\epsilon_0} + \cancel{\frac{\sigma}{2\epsilon_0}} - \cancel{\frac{\sigma}{2\epsilon_0}} = \frac{\sigma}{2\epsilon_0}.$$

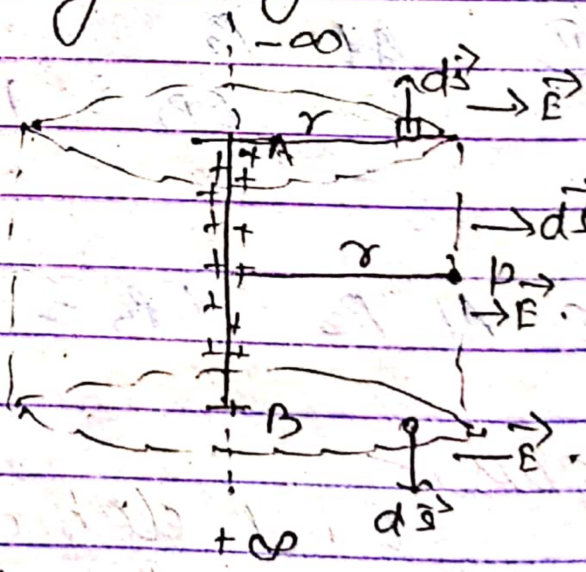
(ii) At P_2.

$$E_{P_2} = |E_A + E_B - E_C|$$

$$= \frac{\sigma}{2\epsilon_0} + \cancel{\frac{\sigma}{2\epsilon_0}} - \cancel{\frac{\sigma}{2\epsilon_0}} = \frac{\sigma}{2\epsilon_0}.$$

Application:-

(1) Due to an infinite long charged wire.



Consider an infinite long charged wire having linear density charge $+\lambda$.

Required to find electric field at a perpendicular distance r i.e. at P.

For this we consider a cylindrical

gaussian surface of length 'l' as shown above.

Total electric flux linked with the gaussian surface.

$$\phi_E = \oint_E \vec{E}.\vec{ds}.$$

$$= \cancel{\int_{\text{upper plane}} \vec{E}.\vec{ds}} + \cancel{\int_{\text{lower plane}} \vec{E}.\vec{ds}} + \int_{\text{curve plane}} \vec{E}.\vec{ds}$$

$\because \theta = 90°$

$$\therefore \phi_E = \int_{\text{curved}} \vec{E}.\vec{ds}$$

$$= E \cdot 2\pi r l \quad \text{—(I)}$$

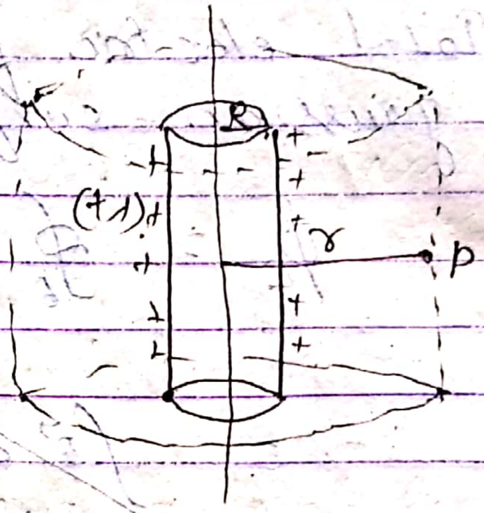
A/c to gauss's theorem

$$\phi_E = \frac{1}{\varepsilon_0} \times q = \frac{1}{\varepsilon_0} \times \lambda l \quad \text{—(II)}$$

from (I) and (II)

$$E = \frac{\lambda l}{\varepsilon_0 2\pi r l} \qquad \therefore \boxed{E = \frac{\lambda}{\varepsilon_0 2\pi r}}$$

(IV) Due to a charged cylindrical shell (conducting or non-conducting) and having linear charge density λ.



$$\phi_E = E \times 2\pi r l \quad \text{--- (I)}$$

and a/c to gauss's theorem,

$$\phi_E = \frac{1}{\epsilon_0} \times \lambda l \quad \text{--- (II)}$$

from (I) and (II)

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

Case I When the point lies on the surface of the shell. i.e, $r=R$. then,

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 R}}$$

Case II When the point lies inside the shell.

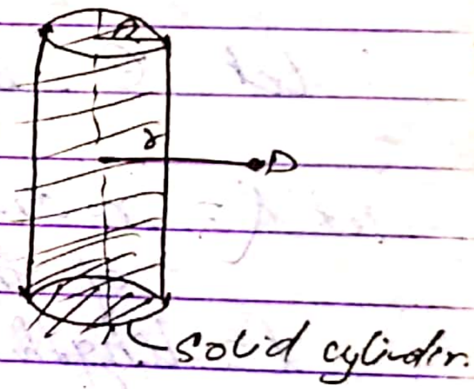
Since, charge resides only on the surface of the shell i.e., no charge exists ~~its~~ inside the shell.

∴ Electric field will be 0

(V) Due to a charged Solid cylinder.

Case I When the point lies outside the surface of the cylinder.

i.e., $r > R$.



For an outside point electric field $= \frac{\lambda}{2\pi\varepsilon_0 r}$.

~~So~~, (For both conducting and non-conducting).

Case II When the point lies on the surface of the cylinder. i.e., $r = R$.

then $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$ (For both conducting and non-conducting).

Case III When the point lies inside the cylinder.

(a) For conducting solid cylinder.

∵ No charge exists inside the cylinder

∴ $E=0$

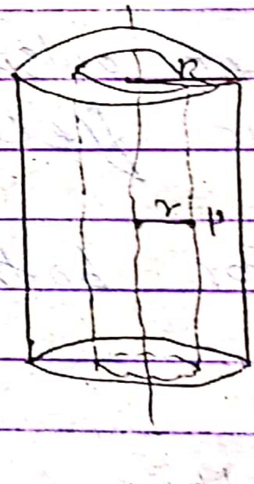
(b) For non-conducting solid cylinder.

In case of a non-conducting material charge resides throughout the material.

∴ Let ρ be the volume density of charge.

Let R = radius of cross section of the cylinder

l = length of the cylinder.



Required to find electric field at an internal point P

∴ electric flux linked with a gaussian surface having radius of cross-section 'r'.

$$\phi_E = E \times 2\pi r l \quad \text{---(1)}$$

A/c to gauss's theorem

$$\phi_E = \frac{1}{\varepsilon_0} \times q' \quad \text{—— (2)}$$

where, q' = charge contained by the gaussian surface.

$q' = \rho . \pi r^2 l$ [from linear charge density]

$$\therefore \phi_E = \frac{1}{\varepsilon_0} \rho . \pi r^2 l \quad \text{—— (3)}$$

from (1) and (3)

$$E \times 2\cancel{\pi}\cancel{r}\cancel{l} = \frac{1}{\varepsilon_0} \rho \cancel{\pi} r^{\cancel{2}} \cancel{l}$$

$$\therefore \boxed{E = \frac{\rho r}{2\varepsilon_0}}$$

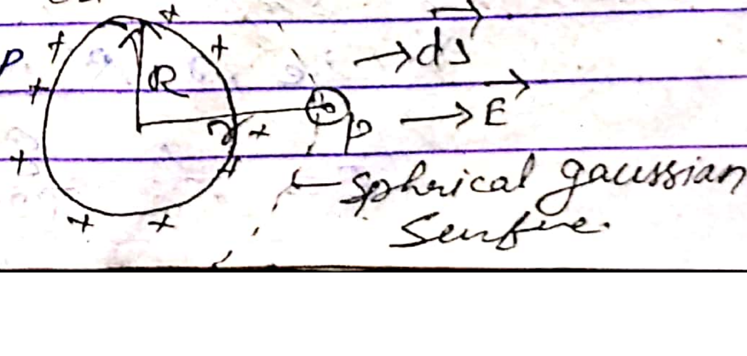
Case I At the axis of the cylinder

i.e $r = 0$

$\therefore E = 0$

(VI) Due to spherical shell (conducting or Non-conducting).

required to find electric field at an external point P.



let the ~~gav~~ spherical gaussian surface.
electric ~~flux~~ linked with it.

$$\phi_E = E \times 4\pi r^2 \quad \text{—} \ (1)$$

A/c to gauss's theorem;

$$\phi_E = \frac{1}{\varepsilon_0} q \quad \text{—} \ (2)$$

from (1) and (2)

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0} q$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}}$$

=> For an outside point a charge spherical shell behaves as a point charge assuming that the total charge is concentrated at its centre.

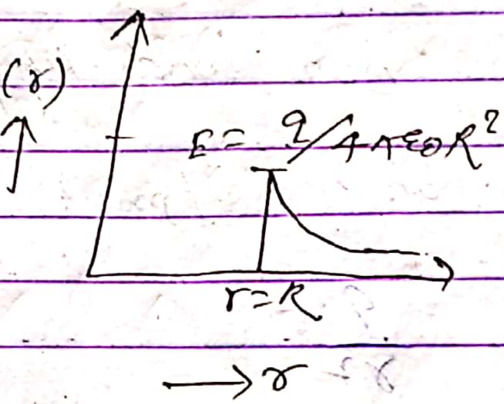
Case I. on the surface of the shell
i.e, $r = R$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}}$$

Case II: At the internal point of the shell
i.e., $r < R$.
So, ∵ It is conductor.
No, charge resides inside the shell.

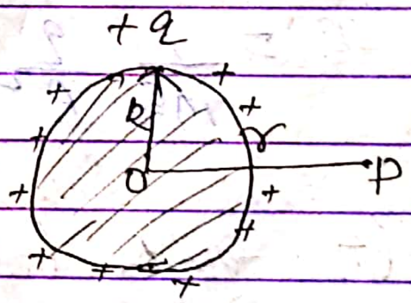
$E = 0$.

graph :-



(VII) Due to charged solid sphere

~~(a)~~ conducting.



(a) At P. (as external point).

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

(b) on the surface.
i.e., $r = R$.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

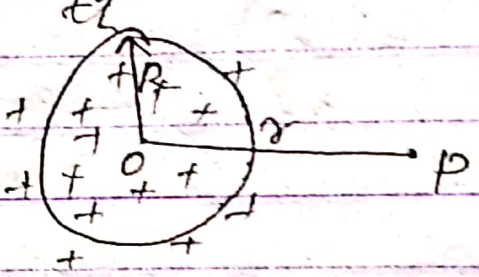
(c) At an inside point

$E = 0.$

i.e) charged spherical shell & charge solid sphere.

⇒ due to a non-conducting charged solid sphere

(a) At P (as an external point).



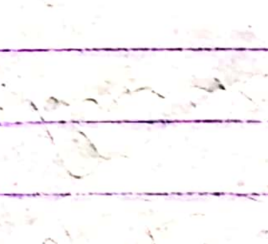
$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

(b) At R=r (on the surface)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

(c) At r<R (if P lies inside the shell).

electric flux linked with the spherical gaussian surface of radius r:



$$\phi_E = E \times 4\pi r^2 \quad - (1)$$

A/c to gauss's theorem.

$$\phi_E = \frac{1}{\varepsilon_0} q'$$

where q' = charge on the part of the sphere having radius r.

$$q' = \frac{q}{\left(\frac{4}{3}\pi R^3\right)} \times \frac{4}{3}\pi r^3 \left[\begin{array}{c}\text{charge per unit} \times \text{area} \\ \text{having radius } r\end{array}\right].$$

$$\therefore q' = \frac{q r^3}{R^3}$$

$$\therefore \phi_E = \frac{1}{\varepsilon_0} \frac{q r^3}{R^3} \quad \text{—(2)}$$

from (I) and (II)

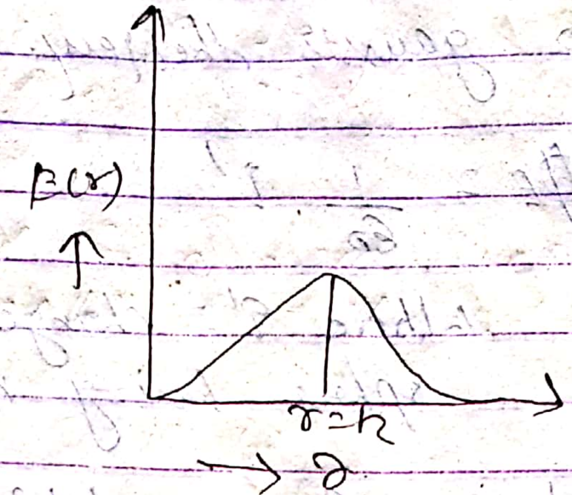
$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \frac{q r^3}{R^3}$$

$$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot r}{R^3}}$$

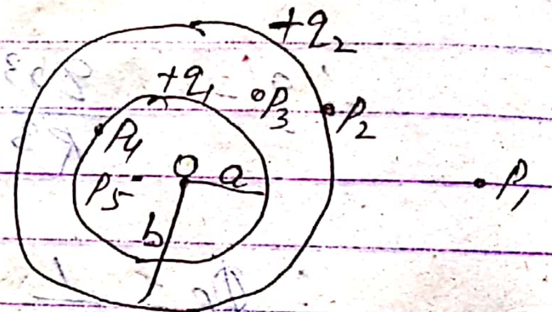
Case I At the centre.
i.e, $r = 0$

$$\therefore E = 0.$$

graph:



Ques:-
Find the electric field at P_1, P_2, P_3, P_4, P_5



$OP = r_1$, $OP_2 = r_2 = b$
$OP_3 = r_3$, $OP_4 = r_4 = a$
$OP_5 = r_5$

Sol:- At P_1

$\rightarrow E_{q_2}$
$\rightarrow E_{q_1}$

$$E_{P_1} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{r_1^2} + \frac{q_1}{r_1^2}\right] = \frac{q_1+q_2}{4\pi\varepsilon_0 r_1^2}$$

At P_2 $$E_{P_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2 + q_1}{b^2}\right)$$

At P_3

$\Rightarrow$ EF due to $q_2 = 0$

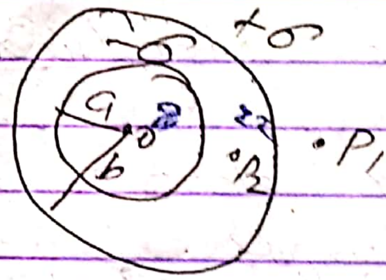
So, $E P_3 = E q_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_3^2}$

At P_4, EF due to $q_2 = 0$

$\therefore E P_4 = E q_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{a^2}$

At P_5, EF due due q_1 and q_2 both zero.

ques:- in



$OP_1 = r_1$

$OP_2 = r_2$

find the electric field at P_1 and P_2

Sol:- At P_1

$\cancel{E P_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{\sigma}{r_1^2} - \frac{\sigma}{r_1^2} \right]} = 0.$

~~At P_2, EF due to $+\sigma$ is 0,~~

$\therefore \cancel{E P_2} =$

P.T.O

Sol:ⁿ $E_{P_1} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 + q_2}{r_1^2}\right]$

$q_1 = \sigma 4\pi a^2$ {from volume
$q_2 = -\sigma 4\pi b^2$ charged density}

$\therefore E_{P_1} = \frac{1}{\varepsilon_0} \sigma(a^2 - b^2)$

$E_{P_2} = \frac{1}{\varepsilon_0} \frac{\sigma a^2}{r_2^2}$