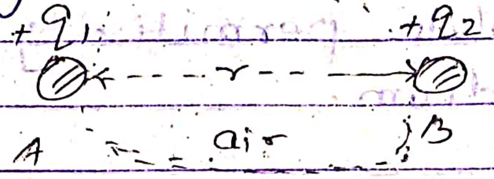
* Coulomb's law

Electric force between two statics point charge varies as the product of the magnitude of the charges and inversly as the square of the distance between the charges.



$$(F_e)_{air} \propto \frac{q_1 q_2}{r^2}$$

$$(F_e)_{air} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$$

Where $\frac{1}{4\pi\varepsilon_0}$ = constant

whose value is $9 \times 10^9 \, Nm^2/C^2$

i.e, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \, Nm^2/C^2$

When the medium between the charge particles is some di-electric.

So, $(F_e)_m = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \times \frac{q_1 q_2}{r^2}$

$= \frac{(F_e)_{air}}{\varepsilon_r}$

Where, ε_r = Relative permitivity of dielectric medium

ε_0 = Absolute permitivity of air or vaccum

$\varepsilon_0 \cdot \varepsilon_r = \varepsilon$ = Absolute permitivity of dielectric medium.

$$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/Nm^2$$

$\varepsilon_r(k)$ is also called dielectric constant of the dielectric medium.

$$(F_e)_m = \frac{1}{4\pi \varepsilon_0 k} \cdot \frac{q_1 q_2}{r^2}$$

Note:

*/ What is dielectric medium?

Ans:- It is an insulating material which gi some induced charges on applying rela high electric field.

Dielectric constant 'k'.

K = electric force b/w two point changes when medium is air or vaccum (Fair) divided by electric force b/w two point changes when medium is some dielectric medium.

$$\therefore \quad K = \frac{(F_e)_{air}}{(F_e)_m}$$

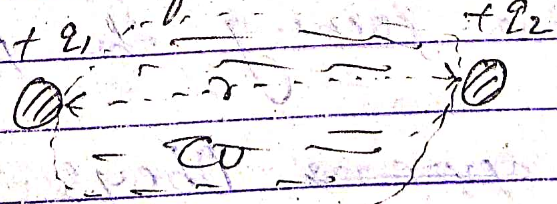
It has no unit.

For air or a vacuum, $K \simeq 1$

For any dielectric medium, $K > 1$
For water $K = 80$ & Mica, $K = 6$

For a conducting material.
$K \to \infty$

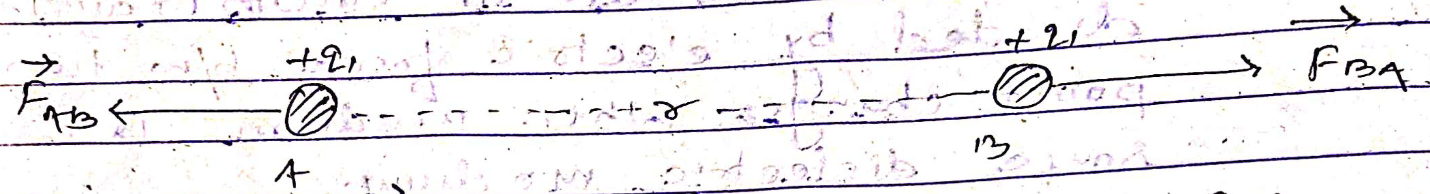
Ques:- What is the force b/w given two changes



$$F = \frac{1}{4\pi \epsilon_0 K} \cdot \frac{q_1 q_2}{r^2}$$

$$F = 0 \quad [\because K = \epsilon_r = \infty]$$

⇒ Coulomb's law in Vector forms

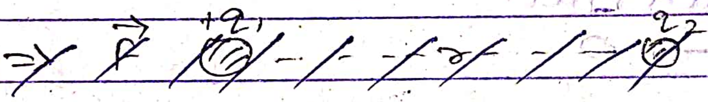


$$|\vec{F}_{BA}| = |\vec{F}_{AB}| = F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\vec{F}_{BA} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|^3} \vec{r}$$

$$\vec{F}_{AB} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|^3} \cdot \vec{r}$$

$$\Rightarrow \vec{F}_{BA} = -\vec{F}_{AB}$$

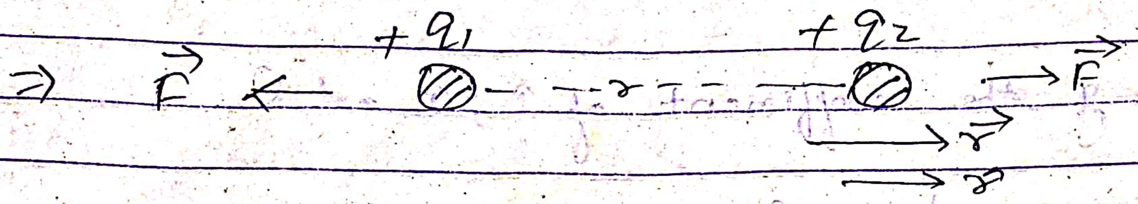


⇒ Forces are equal and opposite and acts along the line joining the point changes

⇒ Coulomb's law ~~appears~~ Obeys Newton's third law of motion and it is central in nature.

So, $|\vec{F}_{AB}| = |\vec{F}_{BA}|$

$$\vec{F}_{AB} = -\vec{F}_{BA}$$



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|^3} \vec{r}$$

or

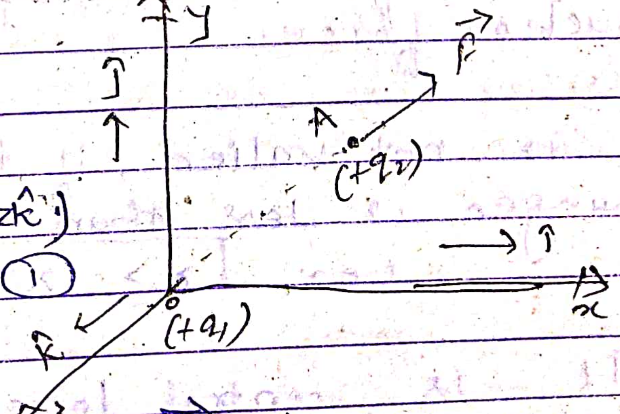
$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r}$$

Where, $\hat{r}$ is a unit vector along $\vec{r}$.

✱ Coulomb's law in terms of position Vectors

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|^3} \vec{r}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(x^2+y^2+z^2)^{3/2}} (x\hat{i}+y\hat{j}+z\hat{k}) \longleftarrow (1)$$



$$OA = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{x}| = \sqrt{x^2+y^2+z^2}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \longleftarrow (2)$$

comparing the coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$

In terms of components

$$F_x = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(x^2+y^2+z^2)^{3/2}} \cdot x$$

$$F_y = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(x^2+y^2+z^2)^{3/2}} \cdot y$$

$$F_z = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(x^2+y^2+z^2)^{3/2}} \cdot z$$

⇒ Properties of Coulomb's Law

1. It is valied for static point charges
2. Coulombic force is a long range unlike nuclear force.
3. It is not valied, if the seperation b/w the changes is less than 1×10^{-15} m.

   i.e $\boxed{r \geq 1 \times 10^{-15} m}$

4. It is central force.

It is 1×10^{38} times greater than the gravitational force of attraction b/w the point charges.

6. Coulombic force b/w two point charge q_1 and q_2 may be attractive or repulsive;
⇒ if q_1 and $q_2 > 0$, force is repulsive
⇒ if q_1 and $q_2 < 0$, force is attractive.

eg :-

$(+q_1) \times (+q_2) = q_1 q_2 > 0$ repulsive
$(-q_1) \times (-q_2) = q_1 q_2 > 0$ "

$(-q_1) \times (q_2) = -q_1 q_2 < 0$ attractive.
$(q_1) \times (-q_2) = -q_1 q_2 < 0$ ".

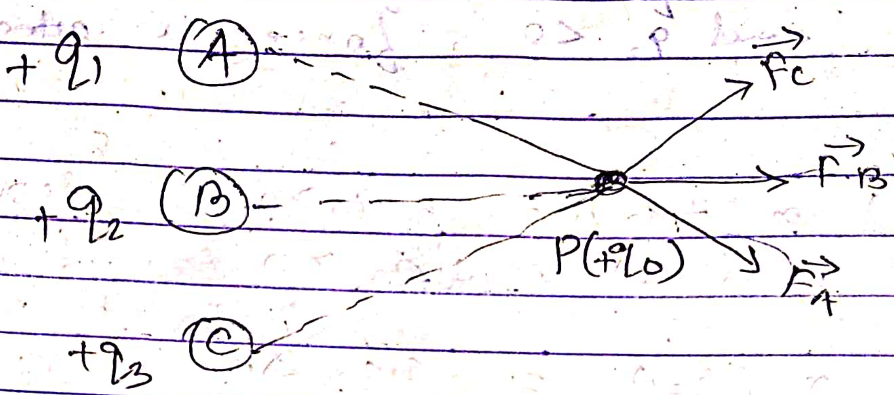
⇒ What are the limitations of coulomb's law?

• Following are the limitations :-

1. Charges should be point and static.

2. Separation b/w the point charges ~~comes to the nuclear~~ should not be less than 1×10^{-15} m, if it happen ~~show~~ so, the point charge comes in the nuclear region and hence does not obey Coulomb's law.

⇒ Principle of Superposition of Coulombic force



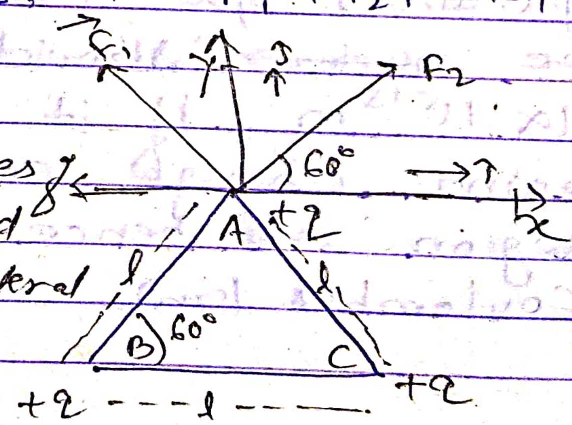
$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

or $\vec{F} = \sum_{i=1}^{n} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n$

This principle helps us to determine electric force at a point due to a group of point charges it is vector sum of electric force due to each individual charge i.e. $\vec{F} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$.

NB:

Ques:- Three point charges of equal magnitude (+q) placed at the vertices of an equilateral triangle. Find the E.F at (+q) A.



Sol:— $\vec{F_A} = |\vec{F_1}| = |\vec{F_2}| = F.$

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}$$

$$\vec{F_2} = |\vec{F_2}|\cos 60\,(\hat{i}) + |\vec{F_2}|\sin 60^\circ\,\hat{j}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\left(\frac{\hat{i}}{2} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

$$\vec{F_1} = |\vec{F_1}|\cos 60\,(-\hat{i}) + |\vec{F_1}|\sin 60\,\hat{j}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\left(\frac{-\hat{i}}{2} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

$$\therefore \vec{F_A} = \vec{F_1} + \vec{F_2}$$

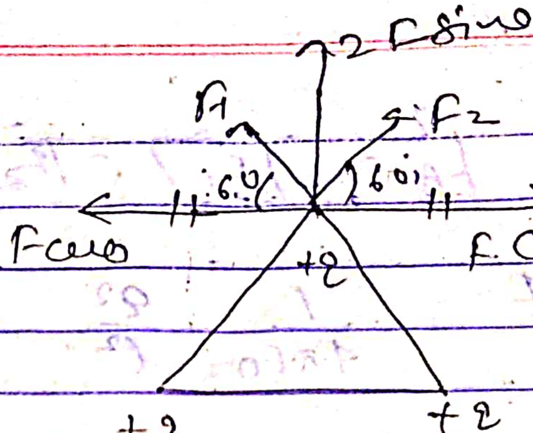
$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\left(\frac{\sqrt{3}}{2}\hat{j} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2} \times \frac{2\sqrt{3}}{2}\,\hat{j}$$

$$\vec{F_A} = \frac{\sqrt{3}}{4\pi\epsilon_0}\frac{q^2}{l^2}$$ Ans.

5

__Alternative method__

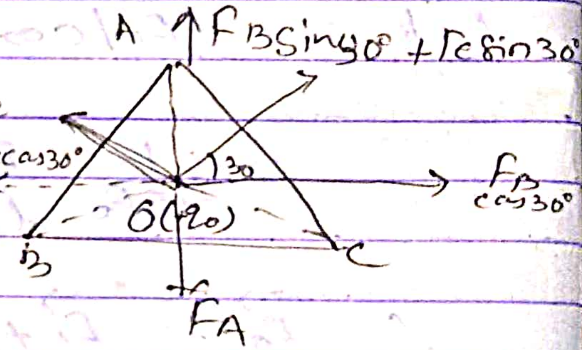


$\vec{F} = 2F\sin\theta$

$= \cancel{2} \times \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2}\frac{\sqrt{3}}{\cancel{2}}$

$\vec{F} = \frac{\sqrt{3}}{4\pi\epsilon_0}\frac{q^2}{l^2}$ Ans

Ques:- Find the electric force at the centroid O of an equilateral triangle. Assuming a test charge $(+q_0)$ is placed at O.



Sol- Net force $= (F_B + F_C)\sin 30 - F_A$

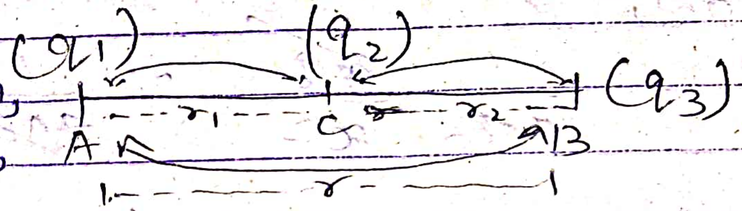
$= \left(\frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} + \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}\right) \times \frac{1}{2} - \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$

$= \frac{1}{4\pi\epsilon_0}\cancel{\frac{qq_0}{r^2}} - \frac{1}{4\pi\epsilon_0}\cancel{\frac{qq_0}{r^2}}$

$= 0$ Ans

* Case of Equilibrium of a system of a charges :-

The system of charges will be in equilibrium, if the net force at A, B and C should be zero.



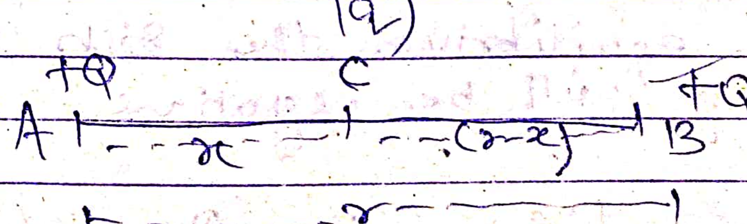
The condition for the charges to be equilibrium i.e, $F_{AC} = F_{CB} = F_{AB}$.

$$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_1^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{r_2^2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_3^2}$$

Ques :- Two point charges each of +Q units are placed along a line at third charge q is placed in b/w them.
(i) What will be the position of the charge q?
(ii) What will be the value of q in terms of Q.
(iii) What will be the sign of q. ?

Sol :- $F_{AC} = F_{CB} = F_{AB}$

Suppose the charged particle 'q' is at a dis. x from A, if the charge are in equilibrium, then,



$$-\cancel{r^2} = -\cancel{2}rx$$

$$\frac{1}{4\pi\epsilon_0}\frac{Qq}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{(r-x)^2} = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{r^2}$$

Now,

(1) $$\frac{1}{\cancel{4\pi\epsilon_0}}\frac{\cancel{Qq}}{x^2} = \frac{1}{\cancel{4\pi\epsilon_0}}\frac{\cancel{Qq}}{(r-x)^2}$$

$$\cancel{x^2} = \cancel{x^2} + r^2 - 2rx$$

$$\cancel{r^2} = 2\cancel{r}x$$

$$\therefore x = \frac{r}{2}$$

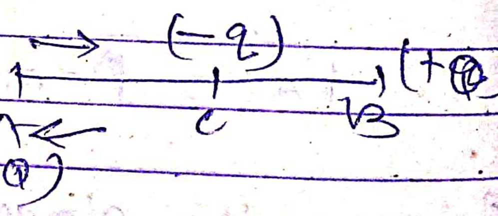
⇒ The charge particle q should be at the mid-pnt on the line joining +Q.

(ii) $$\frac{1}{\cancel{4\pi\epsilon_0}}\frac{\cancel{Q}q}{x^2} = \frac{1}{\cancel{4\pi\epsilon_0}}\frac{Q^{\cancel{2}}}{r^2}$$

$$\frac{4q}{\cancel{r^2}} = \frac{Q}{\cancel{r^2}} \quad \left[\because x = \frac{r}{2}\right]$$

$$\therefore q = \frac{Q}{4}$$

(iii) In ordered to be equilibrium the sign of q will be negative



i.e, $q = -\frac{Q}{4}$

Ques:- What will be no. of electron transfered, when a charged body gets 1C of charge.

Sol:- $q = \pm ne$

$n = q/e = \frac{1}{1.6 \times 10^{-19}} = \pm 6.25 \times 10^{18}$ elec

Ans.

Ques:- Calculate total +ve or -ve charges on 3.11 gm of Cu penny.

Sol:- Given: At. No. of Cu = 29

At wt of Cu = 63.5 : $N_A = 6 \times 10^{23}$

and condition 1 atom of Cu gives 1 electron.

No. of atoms $= \frac{W}{M} \times N_A = \frac{3.11}{63.5} \times 6 \times 10^{23}$

$= 2.9 \times 10^{22} = N$

∴ Since, each Cu atom gives one electron.

∴ Total no. of electron = N

∴ $q = \pm ne$ Total charge = Ne.

∴ $n = q/e$ =

$= 2.9 \times 10^{22} \times 1.6 \times 10^{-19} = 4.64 \times 10^{3}$ Ans.