* Electric field

Electric field due to a charged body may be defined as the space around it in which a ~~element~~ unit positive charge experience a electric force without disturbing the source charge.

⇒ Expression for electric field due to a charged body :-

Suppose +q be the charge on a body, it is required to find electric field at a point for this we place a positive test charge (+q₀) at that point.

Let Fe be the electric force between the charges hence, electric field at that point.

$$\boxed{E = \frac{F_e}{q_0}}$$

Mathematically it can be stated as follow:-

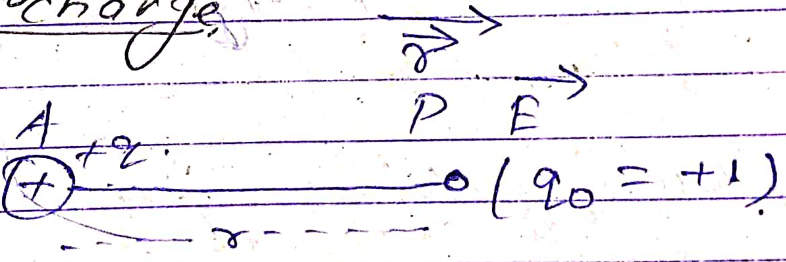
$$E = \lim_{q_0 \to 0} \left(\frac{F_e}{q_0}\right)$$

"Electric Field Intensity" or Intensity of Electric field.

Intensity of electric field at a point due to a charge body may be defined as the electric force experienced by unit positive test charge placed at that point without ~~distribi~~ disturbing the source charge.

⇒ ~~Electric field~~

⇒ Expression for Electric field due to a point charge



$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}|^3}\vec{r}}$$

In vector form

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\vec{r}|^3}\vec{r}$$

or $$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(|\vec{r}|)^2}\hat{r}.$$

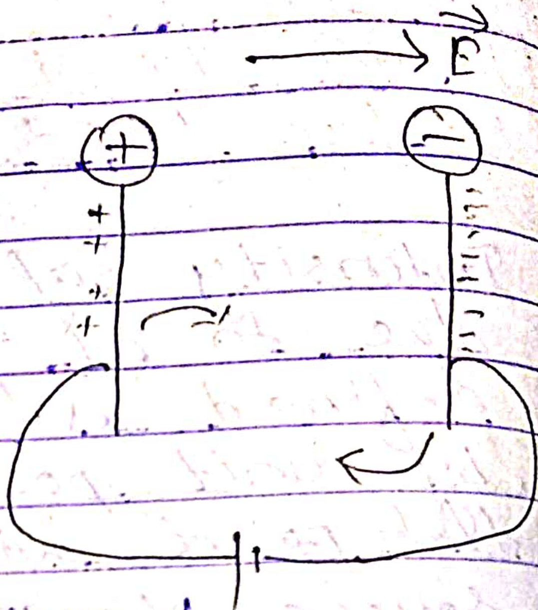
Note :-

$\longrightarrow \vec{F}$

$\oplus \longrightarrow \vec{F} = \vec{E}q$

$+q$

$\vec{F} = -\vec{E}q \quad \longleftarrow \ominus$



Positive
⇒ ~~The~~ charge exprences electric force in the direction of Electric field, while the negative charge opposite to the direction of electric field.

i.e, $\vec{F} = \vec{E}q$ (for +ve)
$\vec{F} = -\vec{E}q$ (for -ve)

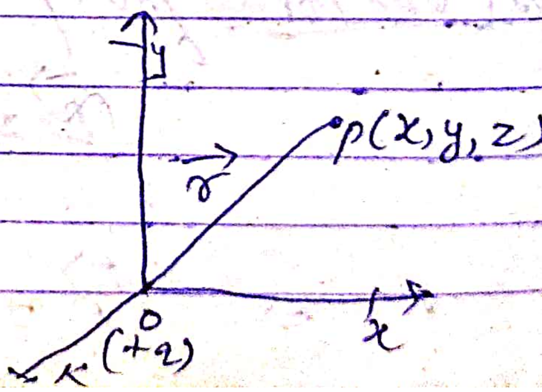
⇒ Electric field in terms of components

Electric field due to a point charge at a point.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{r}|^3} \vec{r}.$$

$\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(x^2+y^2+z^2)^{3/2}} (x\hat{i} + y\hat{j} + z\hat{k})$$

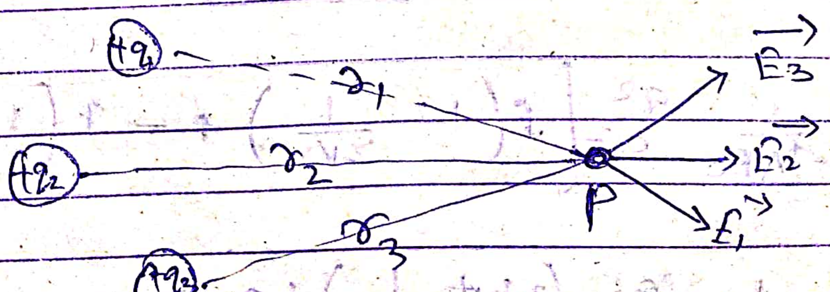
$$\therefore \vec{E_x} = \frac{1}{4\pi\varepsilon_0} \frac{qx}{(x^2+y^2+z^2)^{3/2}}$$

$$\vec{E_y} = \frac{1}{4\pi\varepsilon_0} \frac{qy}{(x^2+y^2+z^2)^{3/2}}$$

$$\vec{E_z} = \frac{1}{4\pi\varepsilon_0} \frac{qz}{(x^2+y^2+z^2)^{3/2}}$$

⇒ Principle of superposition of a electric field

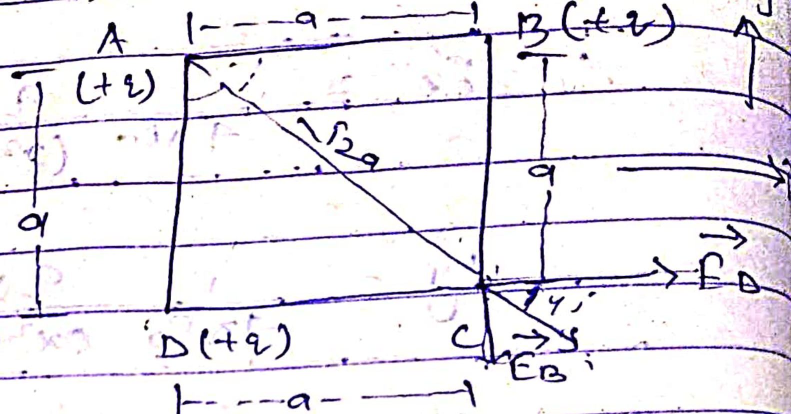
Electric field at a point due to a group of point charges is the vector sum of electric fields at the point due to the each indiviusal point charge.



$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3}$$

$$\vec{E} = \sum_{i=1}^{n} \vec{E_i}$$

ques:- Find the electric fied at C of the given fig^n.



Sol:-

$$\vec{E_C} = \vec{E_A} + \vec{E_B} + \vec{E_D} \quad \text{---(1)}$$

(+q) = (+1).

Now, $\vec{E_D} = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \hat{i}$

$$\vec{E_B} = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2} \, -\hat{j}$$

$$\vec{E_A} = \frac{1}{4\pi\epsilon_0} \frac{q}{2a^2} (\hat{i}\cos 45^\circ - \hat{j}\sin 45^\circ)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{2\sqrt{2}a^2} (\hat{i} - \hat{j})$$

$$\vec{E} = \vec{E_A} + \vec{E_B} + \vec{E_D}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \left[\hat{i}\left(1 + \frac{1}{2\sqrt{2}}\right) + -\hat{j}\left(1 + \frac{1}{2\sqrt{2}}\right) \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q^2}{a^2} \left(2\sqrt{2} + \frac{1}{2\sqrt{2}}\right)(\hat{i} - \hat{j})$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{q}{2\cancel{\sqrt{2}}a^2}(1+2\sqrt{2})(\cancel{\sqrt{2}})$$

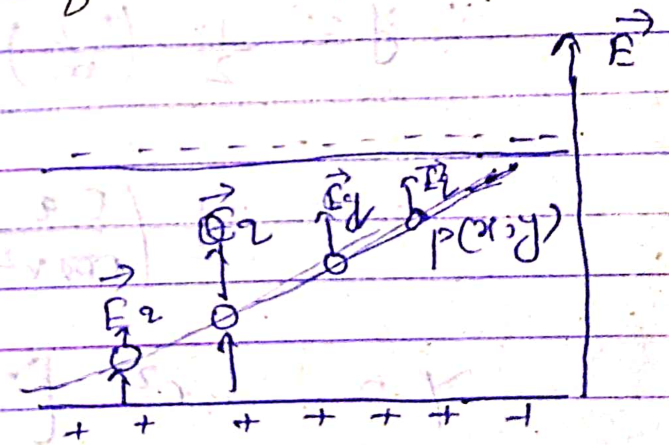
$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0} \frac{q}{2a^2}(1+2\sqrt{2})$$ = Ans.

ques:- Prove that the trajectory of a charged particle (+q) is parabolic, when it is projected horizontally with a speed 'v' in a region having uniform electric field $\vec{E}$ in vertical direction.

Sol:-

$v_x = v$

$v_y = 0$



Suppose the change particle is at a position 'p' (x,y) at any instant 't' in the region having uniform electric field $\vec{E}$.

For horizontal displacement

$x = v_x$ ($a_x = 0$)

$x = vt$ $\quad \therefore t = \frac{x}{v}$ — ①

For vertical displacement

$$y = \cancel{v_y \cdot t} + \frac{1}{2} a_y t^2 \qquad [\because v_y = 0]$$

$$y = \frac{1}{2} a_y \cdot t^2$$

Since, $E\cancel{q} = m.a_y$

$$\therefore a_y = \frac{Eq}{m} \quad \& \ t^2 = \frac{x^2}{v^2}$$

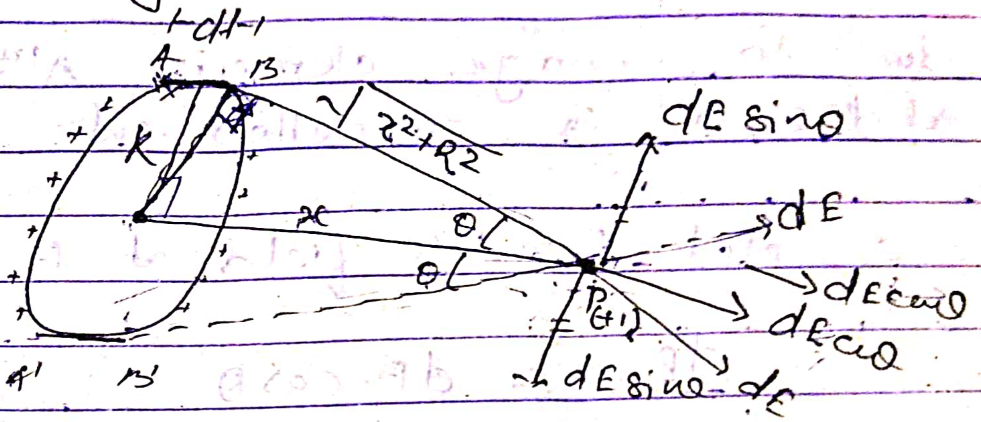
$$\therefore y = \frac{1}{2}\left(\frac{Eq}{m}\right)\left(\frac{x^2}{v^2}\right)$$

$$= \frac{1}{2}\left(\frac{Eq}{mv^2}\right) x^2$$

$$y = bx^2 \quad \left[\because b = \frac{Eq}{2mv^2}\right] = \text{constant.}$$

Which is a parabola, hence, the trajectory of the charged particle will be parabolic.

* Electric field at a point due to charge Ring.



Considered a charged ring having charge (+q) and radius R. It is required to find Eelectric field at the axial point 'P' lying at a distance x from the centre of the ring, for this we consider AB = dl in the ring.

∴ Charge on the element

$$AB = dl.$$

$$dq = \frac{q}{2\pi r} \cdot dl.$$

∴ Electric field at P due to element AB.

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{(R^2+x^2)}$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{q}{2\pi R} \frac{dl}{(R^2+x^2)}$$

due to image element A'B' vertical component ~~at~~ dE sin θ is cancelled out.

∴ net Eelectric field at P due to element AB

$$dE_{net} = dE \cdot \cos\theta$$

$$dE_{net} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{2\pi R} \frac{dl}{(R^2+x^2)} \times \frac{x}{(R^2+x^2)^{1/2}} \quad \left[\because \cos\theta = \frac{x}{(R^2+x^2)} \right.$$

$$= \frac{1}{4\pi\epsilon_0} \frac{x\, q}{2\pi R} \frac{dl}{(R^2+x^2)^{3/2}}.$$

Interegating, we get.

$$E = \frac{1}{4\pi\epsilon_0} \frac{x \cdot q}{2\pi R (R^2+x^2)^{3/2}} \int_0^{2\pi R} dl.$$

$$= \frac{1}{4\pi\epsilon_0} \frac{x\, q}{2\cancel{\pi R}(R^2+x^2)^{3/2}} \times 2\cancel{\pi R}$$

$$\therefore \boxed{E_{net} = \frac{1}{4\pi\epsilon_0} \frac{x \cdot q}{(R^2+x^2)^{3/2}}}$$

Case I

When the point lies at the centre of the ring.

i.e., $x = 0$

$$\therefore \boxed{E_{net} = 0}$$

Case II When $x >> R$ [$\because R = 0$]

i.e., $(x^2 + R^2)^{3/2} \simeq x^3$.

$$\therefore \boxed{E_{net} = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}}$$

$\Rightarrow$ For a distant point the charge ring behaves as a point charge centred at its centre.

Case III

When $R >> x$ [$\because x \simeq 0$]

i.e, Point P is very close to the ring.

$\Rightarrow \therefore (R^2 + x^2)^{3/2} \simeq R^3$

$$\therefore \boxed{E_{net} = \frac{1}{4\pi\epsilon_0} \frac{x \cdot q}{R^3}}$$

Ques :- Find the condition, when the electric field at an axial point due to a charge ring is maximum?

Sol :- We have,

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{q \cdot x}{(R^2+x^2)^{3/2}}$$

Where, q = charge on the Ring
R = Radius of the ring.

For E_x = Maximum.

$$\frac{dE_x}{dx} = 0$$

$$\therefore \cancel{\frac{1}{4\pi\epsilon_0} q} \cdot \frac{d\, x (R^2+x^2)^{-3/2} \cdot dx}{dx} = 0$$

$$\therefore \frac{d(x)}{dx} (R^2+x^2)^{-3/2} \cdot dx = 0.$$

$$(R^2+x^2)^{-3/2} + x \cdot \frac{d(R^2+x^2)^{-3/2}}{d(R^2+x^2)} \cdot \frac{d(R^2+x^2)\, dx}{dx} = 0$$

$$(R^2+x^2)^{-3/2} + x \cdot \left(\frac{-3}{\cancel{2}}\right) \cdot (R^2+x^2)^{-5/2} \times \cancel{2}x = 0$$

$$(R^2+x^2)^{-1}\left(1-\frac{3}{2}x^2\right)=0$$

$$\frac{1}{(R^2+x^2)}\times\left(1-\frac{3x^2}{2}\right)=0$$

$$\frac{1}{1}-\frac{3x^2}{2(R^2+x^2)}=0$$

$$R^2+x^2-3x^2=0$$

$$R^2-2x^2=0$$

$$R^2=2x^2$$

$$\therefore \boxed{x=\frac{R}{\sqrt{2}}}$$

$$(R^2+x^2)^{-5/2}\left[R^2+x^2-3x^2\right]=0$$

$$\Rightarrow R^2+x^2-3x^2=0$$

$$R^2-2x^2=0$$

$$R^2=2x^2$$

$$x=\sqrt{\frac{R^2}{2}}$$

$$x=\frac{R}{\sqrt{2}}$$

$$\therefore E_{max}:=\frac{1}{4\pi\varepsilon_0}\frac{q\cdot x}{(R^2+x^2)^{3/2}}$$

putting $x=\frac{R}{\sqrt{2}}$

$$E_{max}=\frac{1}{4\pi\varepsilon_0}\frac{q\cdot R}{\left(R^2+\frac{R^2}{2}\right)^{3/2}\cdot\sqrt{2}}$$

$$=\frac{1}{4\pi\varepsilon_0\cancel{\sqrt{2}}}\frac{q\cdot R}{\left(\frac{3R^2}{\sqrt{2}\cancel{2}}\right)^{3/2}}.$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2q \cdot R}{(3)^{3/2} \cdot R^{3}}$$

$$\boxed{F_{max} = \frac{1}{4\pi\epsilon_0} \frac{2q}{(3)^{3/2} R^2}}$$

Ques:- An electron is released along a axis of a charged ring. Prove that the electron executes simple harmonical when it is very close to the centre of the ring.

Sol:- Given:- $(+q)$:- Charge on the ring.

R :- Radius of the ring.

m :- Mass of the electron.

<u>Electric field at a point close to centre</u>

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{qx}{R^3}$$

∴ restoring force on electron.

$$F_e = -E_x \cdot e.$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{q \cdot e}{R^3} \cdot x$$

Since, $F_e = m \cdot \frac{d^2x}{dt^2}$;

$$m \cdot \frac{d^2x}{dt^2} = -\frac{q \cdot e}{4\pi\epsilon_0 \cdot R^3} \cdot x$$

$$\frac{d^2x}{dt^2} = -\frac{q \cdot e}{4\pi\epsilon_0 m \cdot R^3} \cdot x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \text{SHM.}$$

Where, $\omega^2 = \frac{q \cdot e}{4\pi\epsilon_0 m R^3} = \text{constant}$.

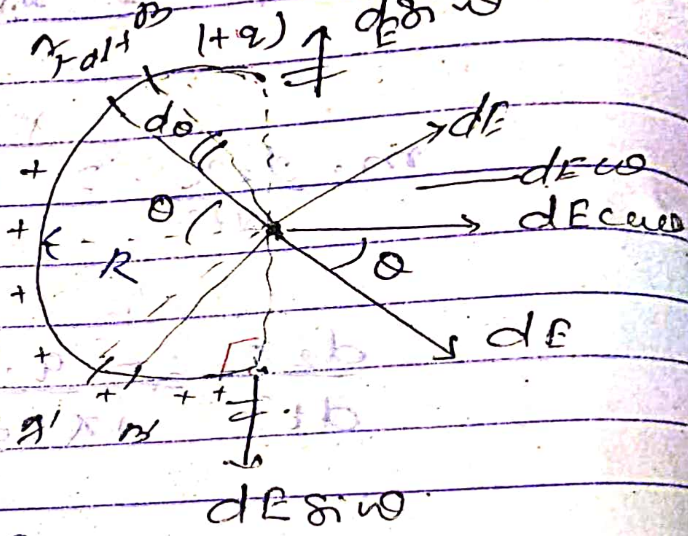
⇒ Motion of electron in S.H.M.

$$T = \frac{2\pi}{\omega}.$$

$$\boxed{T = 2\pi\sqrt{\frac{4\pi\epsilon_0 m R^3}{q \cdot e}}}$$

✓ OK

⇒ Electric field at the centre due to a semi-circular charged ring.



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}$$

$$dq = \left(\frac{q}{\pi R}\right)\cdot dl$$

(from linear charge density)

$$dE_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi R^3}\,dl\cdot\cos\theta$$

$$dl = R\cdot d\theta$$

$$\therefore dE_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi R^{\cancel{3}\,2}}\times \cancel{R}\cdot d\theta\cdot\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi R^2}\cos\theta\cdot d\theta$$

$$\int_0^{E_{net}} dE_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi R^2}\int_{-\pi/2}^{\pi/2}\cos\theta\cdot d\theta$$

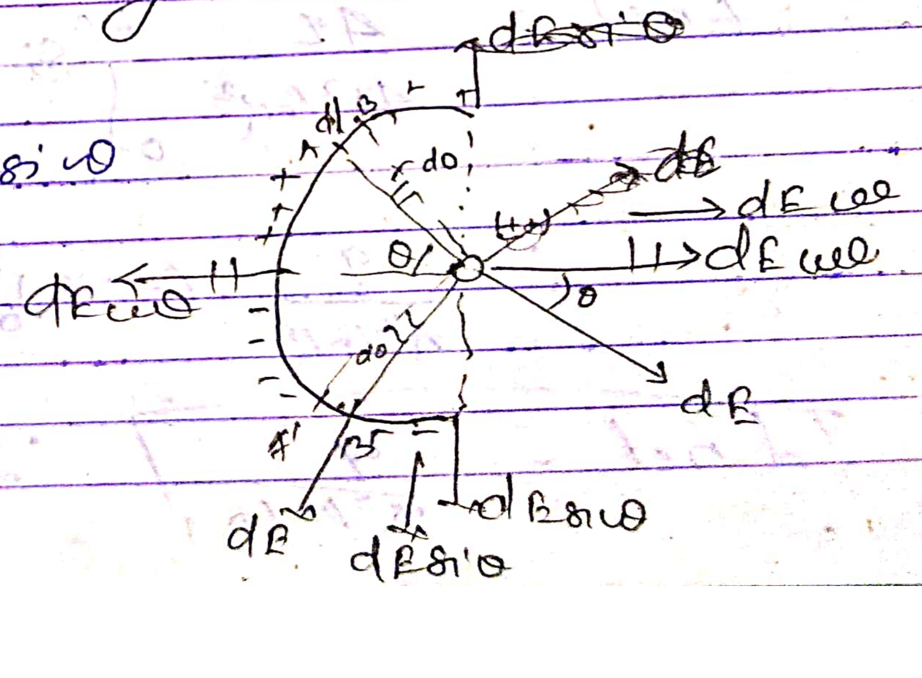
$$E_{net} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q}{\pi R^2} \int_0^{\pi/2} \cos\theta \cdot d\theta$$

$$E_{net} = \frac{1}{4\pi\epsilon_0} \frac{2q}{\pi R^2} (\sin\theta)_0^{\pi/2}$$

$$E_{net} = \frac{2q}{4\pi^2 R^2 \epsilon_0}$$

$$\therefore \boxed{E_{net} = \frac{q}{2\pi^2 R^2 \epsilon_0}}$$

ques :- Find the Electric field at the centre due to semi-circular charged ring having half-portion of positive and half of negative charges



Sol :- $dE_{net} = dE \sin\theta$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2} \sin\theta$$

$$\therefore dq = \frac{q}{\pi R/2} = \left(\frac{2q}{\pi R}\right) \cdot dl$$

[from linear charge density]

$$dE = \frac{1}{4\pi \varepsilon_0} \frac{2q}{\pi R^3} dl \cdot \sin\theta$$

$$dl = R \cdot d\theta$$

$$\therefore dE_{net} = \frac{2q}{4\pi^2 \varepsilon_0 R^{\cancel{3}2}} \cancel{R} \cdot \sin\theta \cdot d\theta$$

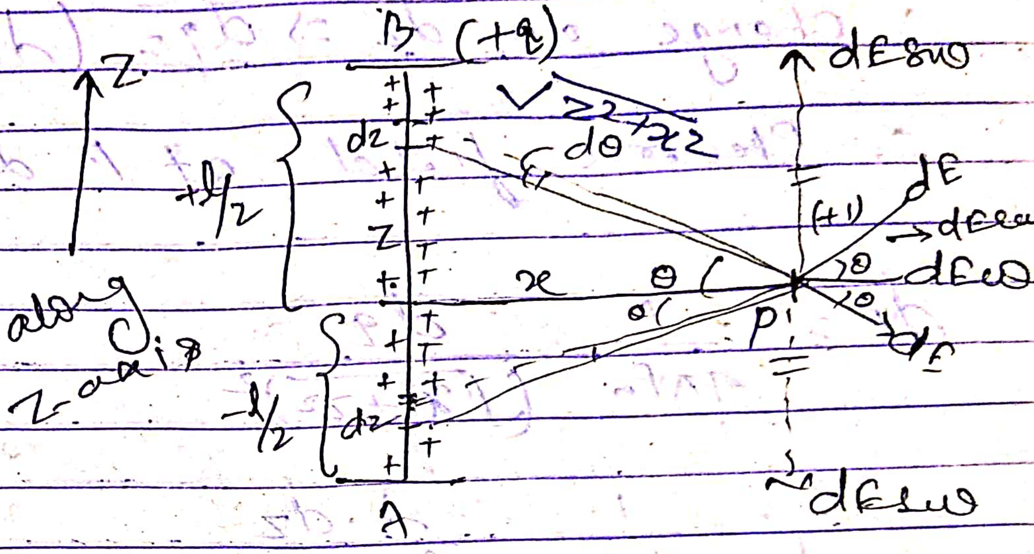
$$\int_0^{E_{net}} dE_{net} = \frac{2q}{4\pi^2 \varepsilon_0 R^2} \int_{-\pi/2}^{\pi/2} \sin\theta \cdot d\theta$$

$$\therefore E_{net} = \frac{4q}{4\pi^2 \varepsilon_0 R^2} \int_0^{\pi/2} \sin\theta \cdot d\theta$$ [by property of definite integration]

$$= \frac{q}{\pi^2 \varepsilon_0 R^2} (-\cos\theta)_0^{\pi/2}$$

$$\boxed{E_{net} = \frac{q}{\pi^2 R^2 \varepsilon_0}}$$ $\because [-\cos \pi/2 = 1]$

⇒ Electric field at a point due to a finite charged wire.



considered a finitely charged wire AB.

let AB = l = length of the wire
q = charge on the wire

∴ Linear density of charge,

$$\lambda = \left(\frac{q}{l}\right)$$

Now, required to find electric field at a point P lying at the perpendicular bisector at distance x.

i.e, $OP = x$

for this we consider an element dz

$\therefore$ Charge on $dz \Rightarrow dq = (\lambda \cdot dz)$ $\left[\because \lambda = \frac{dq}{dz}\right]$

$\therefore$ Electric field at P due this element.

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{(\sqrt{x^2+z^2})^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{\lambda \cdot dz}{(z^2+x^2)}$$

due to image element only horizontal component is effective.

$$\therefore dE_{net} = dE \cdot \cos\theta$$

$$dE_{net} = \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dz}{(z^2+x^2)} \cos\theta$$

$$\text{But, } \cos\theta = \frac{x}{\sqrt{z^2+x^2}}$$

$$\therefore dE_{net} = \frac{1}{4\pi\epsilon_0} \frac{\lambda x \cdot dz}{(x^2+z^2)^{3/2}}$$

$$\therefore E_{net} = \int_{-l/2}^{l/2} \frac{1}{4\pi E_0} \cdot \frac{\lambda x \cdot dz}{(x^2+z^2)^{3/2}}$$

$$E_{net} = \frac{2\lambda x}{4\pi E_0} \int_{0}^{l/2} \frac{dz}{(z^2+x^2)^{3/2}} \quad \text{— (1)}$$

let $I = \int \frac{dz}{(z^2+x^2)^{3/2}}$

$\because \tan\theta = \frac{z}{x}$

$\therefore z = x \cdot \tan\theta$

$dz = x \cdot \sec^2\theta \cdot d\theta$

$(z^2+x^2)^{3/2} = x^3 \sec^3\theta$

$$\therefore I = \int \frac{x \cdot \cancel{\sec^2\theta}}{x^3 \sec^3\theta} \cdot d\theta$$

$$= \frac{1}{x^2} \int \cos\theta \cdot d\theta$$

$$= \frac{1}{x^2} \sin\theta = \frac{1}{x^2} \frac{z}{\sqrt{x^2+z^2}}$$

$\because$ from dia

$$\sin\theta = \frac{z}{\sqrt{z^2+x^2}}$$

$$\therefore \theta = \frac{1}{x^2} \frac{z}{\sqrt{z^2+x^2}}$$

from eqn ①

$$E_{net} = \frac{\lambda x}{2\pi\varepsilon_0} \frac{1}{x^2} \left(\frac{z}{\sqrt{z^2+x^2}} \right)_0^{l/2}$$

$$= \frac{\lambda}{2\pi\varepsilon_0 x} \left(\frac{z}{\sqrt{z^2+x^2}} \right)_0^{l/2}$$

$$\therefore E_{net} = \frac{\lambda}{2\pi\varepsilon_0 x} \frac{z}{z} \left(\frac{1}{\sqrt{1+\frac{x^2}{z^2}}} \right)_0^{l/2}$$

$$\boxed{E_{net} = \frac{\lambda}{2\pi\varepsilon_0 x} \frac{1}{\sqrt{1+\frac{4x^2}{l^2}}}} \quad \because z = l/2$$

End.

Case I

Electric field due to infinitely charged wire.

i.e., $l \to \infty$

$$\frac{1}{\sqrt{1+\frac{4x^2}{l^2}}} = 0.$$

So,

$$\boxed{E_{net} = \frac{\lambda}{2\pi\epsilon_0 x}}$$