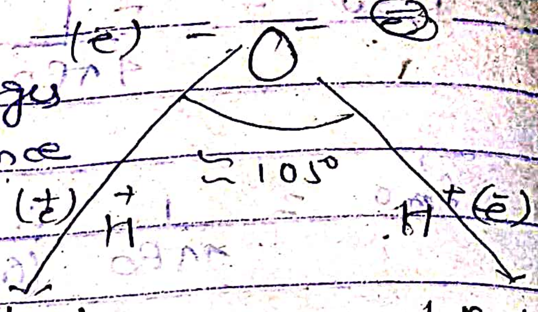
* What is Electric Dipole ?

H_2O



A system of two point charges separated by a finite distance constitute an electric dipole.

$|P_1| = |P_2|$

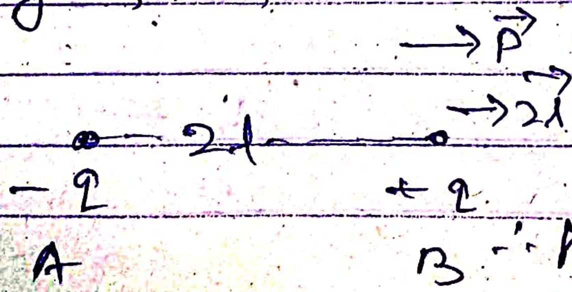
$\sum \vec{P_i} \neq 0$: Molecules is polar

$\sum \vec{P_i} = 0$: Molecules is non-polar

Here, $\vec{P_i}$ = Dipole.

Dipole length (2l) :- It is the separation between the charges. It is a vector quantity, whose direction is form negative to positive charge.

Dipole Moment (p) :- It is the product of dipole length and magnitude of either charge. It is a vector quantity, whose direction is from negative to positive charge, i.e, in the direction of dipole length.



$\therefore p = (2l)q$

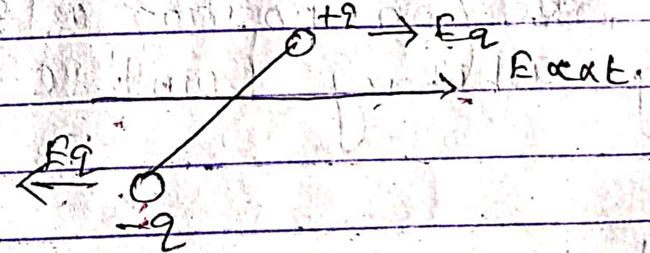
consider an electric dipole AB. Here,
$(2\vec{l})$: dipole length.
$\vec{p}$: $2\vec{l}\cdot q$ dipole moment.

Electric Dipole in an external electric field.

(a) If the Electric field is uniform the dipole experiences equal and opposite forces and hence consistent a couple which tends to rotate a dipole in the direction of electric field.

i.e, tends to a line along the direction of electric field.

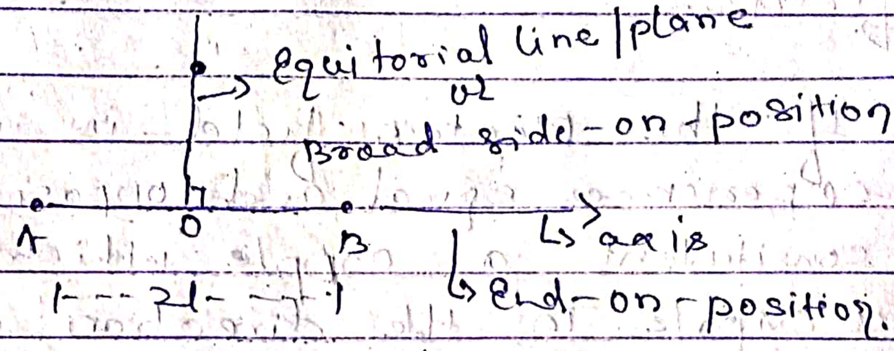
=> Experience only rotational force not any net translational force.



(b) If the electric field is non uniform, the dipole experiences both translation and rotational force and hence the dipole moves along the EF in a helic path. (~~[illegible]~~)

⇒ What is an ideal electric dipole?

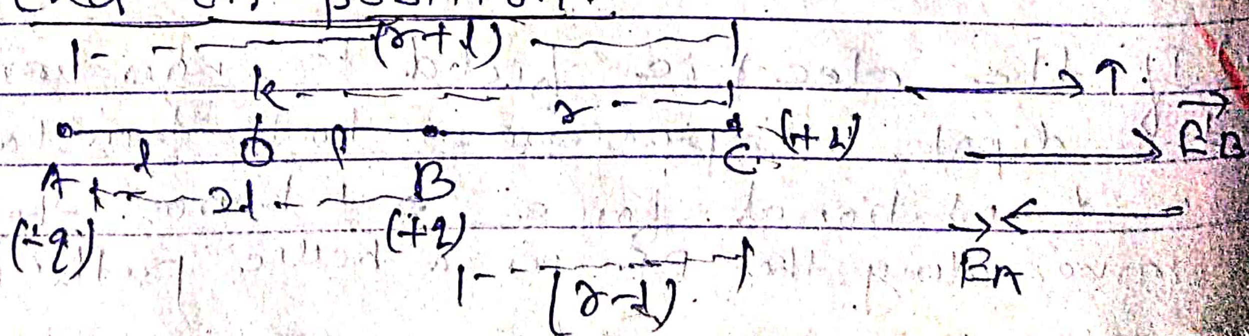
An electric dipole is said to be ideal, if dipole length is very short i.e, magnitude of the charges constituenting the dipole should be affectively larged.



Ques:- Find out an expression for Electric field due to an electric dipole having dipole moment $p = 2lq$.

(i) End-on-position or axial point.
(ii) Broad side-on-position or Equitorial plane
(iii) At any arbitary point $C(r, \theta)$ [C (dis, dir)]

Ans:-
(i) End-on-position.



considera an electric dipole AB having dipole moment $p = 2lq$ — ①

Where, $2l$ = dipole length
q = charge

required to find electric field at an axial point C which lies at a distance r from centre O of the dipole.

$$\vec{E_B} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-l)^2}(\hat{i})$$

$$\vec{E_A} = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+l)^2}(-\hat{i})$$

Net, electric field at C

$$\vec{E}_{axis} = \frac{1}{4\pi\epsilon_0} q\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]\hat{i}$$

$$|\vec{E}_{axis}| = \frac{1}{4\pi\epsilon_0} q\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$= \frac{1}{4\pi\epsilon_0} q\left[\frac{(r+l)^2 - (r-l)^2}{(r^2-l^2)^2}\right]$$

$$= \frac{1}{4\pi\epsilon_0} q\left[\frac{(2r)(2l)}{(r^2-l^2)^2}\right]$$

$$\therefore \boxed{E_{axis} = \frac{1}{4\pi\varepsilon_0} \frac{2pr}{(r^2-l^2)^2}}$$

Direction of EF at an axial point is along the dipole moment or dipole length.

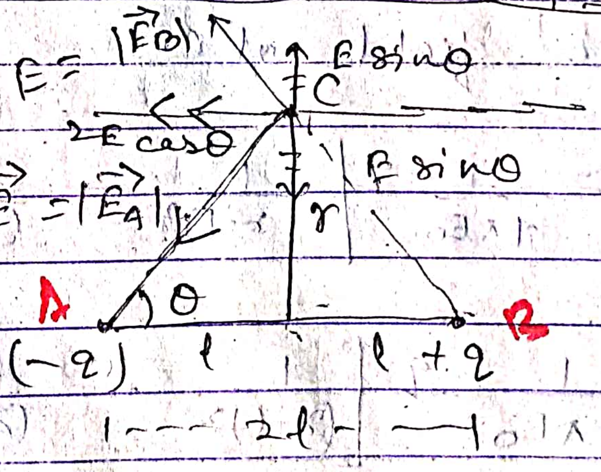
In case of an ideal dipole

i.e., $r >> l$

$\Rightarrow (r^2-l^2)^2 \approx r^4$

$$\therefore \boxed{E_{axis} = \frac{1}{4\pi\varepsilon_0} \frac{2p}{r^3}}$$

(ii) On Broad-side-on-position



Here, $AC = BC = \sqrt{r^2+l^2}$

$$|\vec{E_A}| = |\vec{E_B}| = E = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2+l^2)}$$

Net Electric field at C

$$E_{equi} = 2E\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\,2\,\frac{q}{(r^2+l^2)}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{2ql}{(r^2+l^2)^{3/2}} \quad \left[\because \cos\theta = \frac{l}{\sqrt{r^2+l^2}}\right]$$

$$\therefore \boxed{E_{eq} = \frac{1}{4\pi\varepsilon_0}\,\frac{p}{(r^2+l^2)^{3/2}}}$$

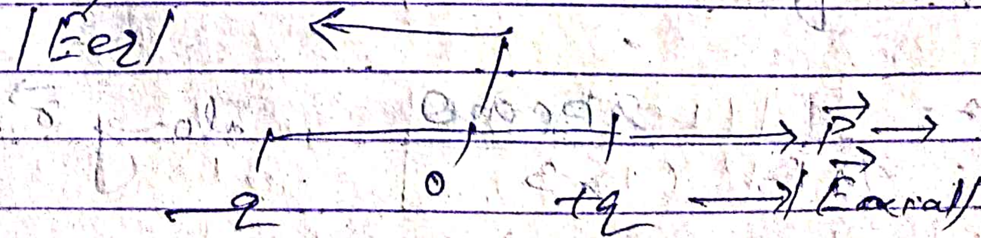
In case of an ideal Electric dipole

$r \gg l$

$\therefore (l^2+r^2)^{3/2} \approx r^3$

$$\therefore \boxed{E_{eq} = \frac{1}{4\pi\varepsilon_0}\,\frac{p}{r^3}}$$

Its point opposite to the direction of dipole moment.



(iii) At an arbitary point $C(r,\theta)$

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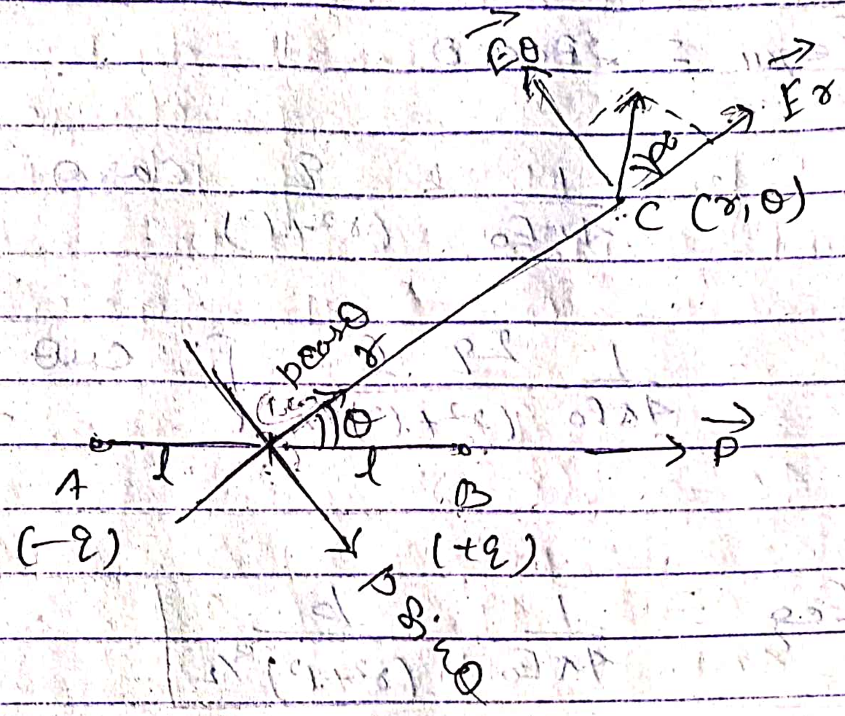


Consider an electric ideal dipole having dipole moment 'p', it is required to find electric field at $C(r,\theta)$ due to their dipole, for this we $\vec{p}$ along $\vec{r}$ and normal to $\vec{r}$.

let it $p\cos\theta$ and $p\sin\theta$

For dipole $p\cos\theta$; $C(r,\theta)$ is on axial point

∴ electric field at $C(r,\theta)$

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}, \text{ along } \vec{r}.$$

Similarly, electric field at $C(r, \theta)$ due to $p \sin\theta$:

$$E_\theta = \frac{1}{4\pi\epsilon_0} \frac{p \sin\theta}{r^3} \quad , \text{ along } \perp \text{ to } \vec{r}.$$

from, law of 11gm.

∴ Resultant of electric field at $C(r, \theta)$.

$$E = \sqrt{(E_r)^2 + (E_\theta)^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \sqrt{\sin^2\theta + 4\cos^2\theta}$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \sqrt{1 + 3\cos^2\theta}}$$

$\vec{r} = x\hat{i} + y\hat{j}$
then $\tan\theta = y/x$

Now, Direction of $\vec{E}$ w.r.t. $\vec{E_r}$

let it be α,

$$\therefore \tan\alpha = \frac{|\vec{E_\theta}|}{|\vec{E_r}|}$$

$$\tan\alpha = \frac{1}{2} \tan\theta$$

$$\therefore \boxed{\alpha = \tan^{-1} \frac{1}{2} \tan\theta}$$

Case I

At an axial point.

i.e, $\theta = 0^\circ$

$\therefore \cos\theta = 1$

$$\therefore \boxed{E_{axis} = \frac{1}{4\pi\epsilon_0} \frac{2p}{r^3}}$$

Case II

At broad side on position.

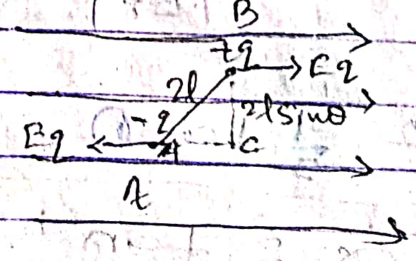
i.e, $\theta = 90^\circ$.

$\therefore \cos\theta = 0$

$$\therefore \boxed{E_{equ.} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}}$$

Electric dipole in a uniform electric field

1. Expression for torque



consider an electric dipole AB having dipole moment $p = 2lq$ —— (1) placed in a uniform electric field. $\vec{E}$

where, $2l$ = length of the dipole
q = charge on " "

Electric forces on the charges having magnitude Eq each are equal and opposite. Hence, constitute a couple which rotates the dipole in the direction of electric field.

$$\tau = (Eq) \times 2l\sin\theta$$

$$\tau = 2lq \cdot E\sin\theta$$

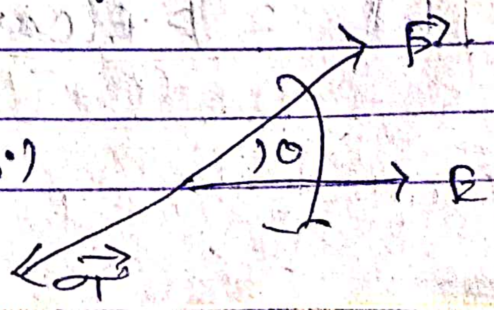
$$\boxed{\tau = (p) \cdot E\sin\theta}$$

$\vec{\tau} = \vec{r} \times \vec{F}$
$= rF\sin\theta$
$\tau = Fd$
where $r\sin\theta = d$

In Vector form

$$\vec{\tau} = \vec{p} \times \vec{E}$$

i.e. Torque diverts in a direction perpendicular to the plane containing $\vec{p}$ and $\vec{E}$.



=> $\tau_{max} = pE$ ($\theta = 90°$)
=> $\tau_{min} = 0$ ($\theta = 0°$ or $180°$)

imp.

2. Expression for potential energy when a dipole is displaced to an angle θ from equilibrium position in an electric field $\vec{E}$.

When an electric dipole $p = 2lq$ placed in a uniform electric field $\vec{E}$ is displaced to angle θ; by an external agent, so, work has to be done against the restoring torque.

$$\tau = p \cdot E \sin\theta$$

$\therefore$ Work done for further angular displacement $d\theta$.

$$dW = \tau \cdot d\theta$$

$$= p \cdot E \sin\theta \cdot d\theta$$

$$\therefore W = p \cdot E \int_{\theta_1}^{\theta_2} \sin\theta \cdot d\theta$$

$$W = p E (-\cos\theta)_{\theta_1}^{\theta_2}$$

$$\boxed{W = p \cdot E(\cos\theta_1 - \cos\theta_2)}$$

This work done is stored as potential energy.

$$U = W = p.E(\cos\theta_1 - \cos\theta_2)$$

$\Rightarrow$ $\theta_1 = 0°$, $\theta_2 = \theta$

$$\therefore U = p.E(1 - \cos\theta)$$

$\Rightarrow$ If $\theta_1 = 90°$, $\theta_2 = \theta$

$$U = -p \cdot E\cos\theta$$

$U = -\vec{p} \cdot \vec{E}$ [$\because a \cdot b = ab\cos\theta$]

(a) When $\theta = 0°$

$$U = -p \cdot E \text{ (min)}$$

(b) When $\theta = 90°$

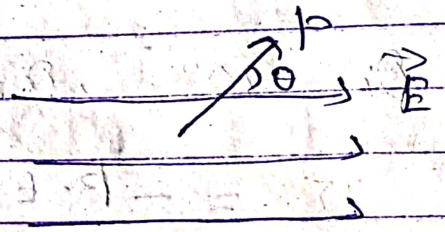
$$U = 0$$

(c) When $\theta = 180°$

$$U = p \cdot E \text{ (max)}$$

Ques:- Prove that the motion of an electric dipole is in S.H.M, when it is slightly displaced from equilibrium in a uniform electric field also find its time period.

Sol^n:- Suppose an electric dipole $p = 2lq$ is placed in a uniform electric field $\vec{E}$



When an electric dipole is placed to an angle θ and left the dipole executes simple harmonically due to restoring torque.

$\therefore$ Restoring Torque.

$$\tau = -pE\sin\theta \quad \text{—(I)}$$ [-ve sign is due to restoring torque]

From, Newton's 2nd law of motion.

$$\tau = I\frac{d^2\theta}{dt^2} \quad \text{—(II)}$$

where I = moment of Inertia of the dipole.

from (i) and (ii)

$$I \cdot \frac{d^2\theta}{dt^2} = -p \cdot E \sin\theta$$

$$\frac{d^2\theta}{dt^2} = \left(\frac{-pE}{I}\right) \cdot \sin\theta$$

$$\frac{d^2\theta}{dt^2} = -\omega^2 \sin\theta \quad \text{where } \omega^2 = \left(\frac{pE}{I}\right)$$

if θ is very small

$$\sin\theta \simeq \theta$$

$$\therefore \frac{d^2\theta}{dt^2} = -\omega^2\theta \qquad \omega^2 = \frac{pE}{I}$$

So, which is the genral eqn of S.H.M.
Here the dipole excutes simple hormonically.

Hence, $T = \frac{2\pi}{\omega}$, $\boxed{T = 2\pi\sqrt{\frac{I}{p \cdot E}}}$

P.T.O.