Graph Theory Lecture 1

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1 Graph

A graph is a triple consisting of a vertex set V(G), an edge set E(G) and a relation that associates with each edge, two vertexes(not necessarily distinct) called it's endpoints.

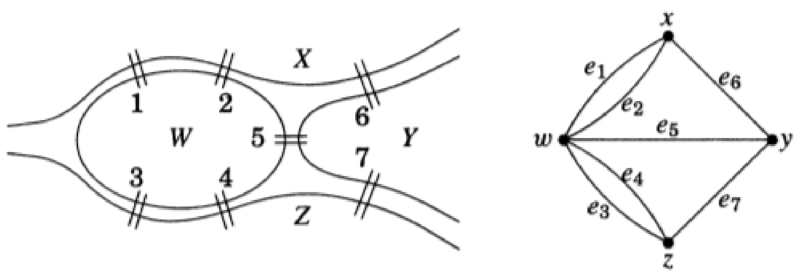

Figure 1: Konigsberg Bridge Problem

2 Loop

A loop is an edge whose endpoints are equal.

3 Multiple Edges

Edges that have the same two endpoints.

4 Simple Graph

A simple graph is a graph that has no loops or multiple edges.

5 Multigraph

A multigraph is a graph that can have loops or multiple edges.

6 Adjaceny and Neighborhood

When u and v are the endpoints of an edge, they are called adjacent and neighbors. We write u $\leftrightarrow$ v for "u is adjacent to v".

7 Finite Graph

A graph $G = (V, E)$ is finite if V and E are both finite.

8 Null Graph

A graph $G = (V, E)$ is null if V and E both are empty.

9 Complement Graph

The complement $\bar{G}$ of a simple graph G is the simple graph with vertex set V defined by $uv \in E(\bar{G})$ if and only if $uv \notin E(G)$.

10 Clique

A clique in a graph is a set of pairwise adjacent vertices.

11 Independent Set

An independent set in a graph is a set of pairwise non-adjacent vertices.

12 Bipartite Graph

A graph G is bipartite if V(G) is the union of two disjoint(possibly empty) independent sets.

13 Meeting Scheduling Problem

Suppose, we must schedule meetings among a population. Each meeting takes a non-zero subset of that population. For any person, we don't want any clash in two of his meetings. How many different time periods do we need?

14 Map coloring

Kenneth and Appel proved that it doesn't require more than 4 colors to color a map.

15 Chromatic Number

The chromatic number of a graph G is the minimum number of colors needed to label the vertices so that adjacent vertices have different labels.

16 Path

A path is a sequence of vertices that do not repeat and consecutive vertices have edge among them.

17 Cycle

A cycle is a closed path.

18 Subgraph

A subgraph of g = (v,e) of G = (V,E) is a graph where v ∈ V and e ∈ E.

19 Connectedness

A graph is connected if for any two vertices u and v there is a path from u to v.

20 Adjacency Matrix

$V \times V$ matrix $a_{ij} = 1$ if vertex i has an edge connecting it to vertex j, $a_{ij} = 0$, otherwise.

21 Incidence Matrix

$E \times V$ matrix $a_{ij} = 1$ if edge i is incident with vertex j, $a_{ij} = 0$ otherwise.

22 Isomorphism

Isomorphism is an equivalence relation(reflexive, symmetric, transitive)

23 Isomorphism Class

A class of graphs, where every graph is pairwise isomorphic.

24 Complete Graph

A simple graph is complete if every pair of vertex has an edge between them.

25 Complete Bipartite Graph

If every vertex from the independent sets have edges with every vertex from the other independent set.

26 Walk

A walk is any sequences of vertex where consecutive vertices have edges between them.

27 Trail

A trail is a walk with no repeating edges.

28 Connected Component

A component that is connected.

29 Cut Edge

If one removes a cut-edge, the number of connected component increases.

30 Cut Vertex

If one removes a cut-vertex, the number of connected component increases.

31 Induced Subgraph

Suppose there is a graph G = (V,E). A subset T of V is chosen. The subgraph induced by T is G[T] = (T, E[T]). Where E[T] is the subset of E such that any edge of E, if it has two of it's endpoints in T, is found in E[T].

32 Lemma 1

Every u-v walk contains a u-v path.

33 Proposition

Every graph with n vertices and k edges has at least n-k components.

34 Theorem 1

An edge is a cut edge if and only if it belongs to no circle.

35 Lemma

Every closed odd walk contains an odd cycle

36 Konig Theorem

A graph is bipartite if and only if it has no odd cycle

37 Eulerian Circuit

A graph is called eulerian if it has a closed trail containing all edges.

38 Even Graph

A graph is even if all of it's vertices are of even degrees.

39 Lemma 2

If every vertex of a graph G has degree at least 2, then G has a cycle.

40 Theorem 2

A graph is eulerian if and only if it has at most one non-trivial component and it's vertices all have even degree.

41 Proposition 2

Every even graph decomposes into cycles.

42 Proposition 3

If G is a simple graph in which every vertex has degree at least k, then contains a path of length at least k. If $k \geq 2$, then G also contains a cycle of length at least k+1.

43 Proposition 4

Every simple graph has at least two vertices that are not cut-vertices.

44 Lemma 3

In an even graph, every maximal trail is closed.

45 Theorem 3

For a connected non-trivial graph with exactly 2k odd vertices, the minimum number of trails that decompose it is maxk,1

46 Algorithm 1

Breadth First Search

47 Algorithm 2

Depth First Search

48 Algorithm 3

Cycle Detection

49 Algorithm 4

Articulation point

50 Algorithm 5

Bridge Detection