Weifeng Jin

UID: 904429456
Section: 4A
MATH 151A HW #4
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Problem 1

(a) Let $x_0 = -1, x_1 = 0, x_2 = 1$.

$$\begin{aligned}
h(x) &= f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \\
&= f(-1)\cdot\frac{x(x-1)}{2} + f(0)\cdot\frac{(x+1)(x-1)}{-1} + f(1)\cdot\frac{(x+1)x}{2} \\
&= \frac{f(-1)}{2}(x^2-x) - f(0)(x^2-1) + \frac{f(1)}{2}(x^2+x)
\end{aligned}$$

(b) The error term

$$\begin{aligned}
E(x) &= \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)(x-x_2) \\
&= \frac{f^3(\xi(x))}{3!}(x+1)x(x-1) \\
&= \frac{f^3(\xi(x))}{6}(x^3-x)dx
\end{aligned}$$

(c) The integral of h(x)

$$\begin{aligned}
\int_{-1}^{1} h(x)dx &= \int_{-1}^{1}[\frac{f(-1)}{2}(x^2-x) - f(0)(x^2-1) + \frac{f(1)}{2}(x^2+x)]dx \\
&= \frac{f(-1)}{2}\int_{-1}^{1}(x^2-x)dx - f(0)\int_{-1}^{1}(x^2-1) + \frac{f(1)}{2}\int_{-1}^{1}(x^2+x)dx \\
&= \frac{f(-1)}{2}\left(\frac{1}{3}x^3 - \frac{1}{2}x^2\Big|_{-1}^{1}\right) - f(0)\left(\frac{1}{3}x^3 - x\Big|_{-1}^{1}\right) + \frac{f(1)}{2}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2\Big|_{-1}^{1}\right) \\
&= \frac{f(-1)}{2}\cdot\frac{2}{3} + f(0)\cdot\frac{4}{3} + \frac{f(1)}{2}\cdot\frac{2}{3} \\
&= \frac{f(-1)}{3} + \frac{4f(0)}{3} + \frac{f(1)}{3}
\end{aligned}$$

(d) Since the degree of the function is 2, $h^{(3)}(x) = 0$. Thus, $E(x) = 0$ for all x, and the approximation is exact.

(e) the error bound is 0.

$$\begin{aligned}
\int_{-1}^{1} E(x)dx &\leq \frac{f^3(\xi(x))}{3!}\int_{-1}^{1}(x-x_0)(x-x_1)(x-x_2)dx \\
&= \frac{f^3(\xi(x))}{3!}\int_{-1}^{1}(x+1)x(x-1)dx \\
&= \frac{f^3(\xi(x))}{6}\int_{-1}^{1}(x^3-x)dx \\
&= \frac{f^3(\xi(x))}{6}\left(\frac{1}{4}x^4-\frac{1}{2}x^2\Big|_{-1}^{1}\right) = 0
\end{aligned}$$

Problem 2

(a) $a = 0, b = 4, \frac{a+b}{2} = 2$

$$\begin{aligned}
\int_0^4 f(x)dx &\approx \frac{4}{6}[f(0)+4f(2)+f(4)] \\
&= \frac{2}{3}(1+4\cdot 1+1) = 4
\end{aligned}$$

(b)

$$\begin{aligned}
\int_0^4 f(x)dx &\approx \int_0^2 f(x)dx + \int_2^4 f(x)dx \\
&= \frac{1}{3}[f(0)+4f(1)+f(2)] + \frac{1}{3}[f(2)+4f(3)+f(4)] \\
&= \frac{1}{3}(1+4\cdot 2+1) + \frac{1}{3}(1+4\cdot 2+1) \\
&= \frac{20}{3}
\end{aligned}$$

Problem 3

```
% number of equal spaces
N = 20;

m_a=trap(N);
disp(m_a);
s_a=simp(N);
disp(s_a);

%function of composite trapezoidal rule
```

```
function r = trap(n)
h = pi/n;
r = 0;
for k = 1:(n-1)
    x = h*k;
    r = r + input(x);
end
r = h*(input(0)+input(pi))/2+h*r;
end

% function of composite simpson's rule
function x = simp(n)
h = pi/n;
q = zeros(1,n+1);
q(1)=0;
q(n+1)=pi;
p1=0; p2=0; p3=0;
for i = 2:n
    q(i) = (i-1)*h;
end
for i = 1:n/2
    p1=p1+input(q(2*i-1));
end
for i = 1:n/2
    p2=p2+input(q(2*i));
end
for i = 1:n/2
    p3=p3+input(q(2*i+1));
end
x=(h/3)*(p1+4*p2+p3);
end

function s = input(x)
s = cos(x);
end
```