**Weifeng Jin**

UID: 904429456
Section: 4A
MATH 151A HW #1
Date: 10/08/2016

# Problem 1

**(a)** For the equation $f(x) := x^2 - 0.7xon[0.5, 1]$, we have $f(0.5) = -0.1$ and $f(1) = 0.3$. Since $f(0.5) \cdot f(1) < 0$, by *Theorem 2.1*, The Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero $p$ of $f$ with

$$|p_n - p| \leq \frac{(1 - 0.5)}{2^n}$$

Thus, the sequence $p_n$ converges with rate of convergence $O(\frac{1}{2}^n)$

**(b)** We will use *Theorem 2.1* to find an integer $N$ that satisfies

$$|p_N - p| \leq \frac{(1 - 0.5)}{2^N} = \frac{1}{2^{N+1}} < 10^{-5}$$

Since $\frac{1}{2^{N+1}} < 10^{-5}$ implies that $log_{10}\frac{1}{2^{N+1}} < log_{10}10^{-5} = -5$, we have

$$-(N + 1)log_{10}2 < -5 \quad and \quad N > \frac{5}{log_{10}2} - 1 \approx 15.6$$

Hence, 16 iterations will ensure an approximation with an absolute error of less than $10^{-5}$.

# Problem 2

**(a)** $p_1 = \frac{p_0^2+3}{2p_0} = \frac{3^2+3}{2\times3} = 2$, $p_2 = \frac{p_1^2+3}{2p_1} = \frac{2^2+3}{2\times2} = 1.75$

**(b)** This is the same as finding a fixed point for $\{p_n\}_{n=0}^{\infty}$. Suppose for some $N$, we have

$$p_N = \frac{p_N^2 + 3}{2p_N}$$
$$2p_N^2 = P_N^2 + 3$$
$$p_N^2 = 3$$
$$p_N = \sqrt{3} \quad or \quad -\sqrt{3}$$

Thus, the possible limits of sequence $\{p_n\}_{n=0}^{\infty}$ are $\sqrt{3}$ and $-\sqrt{3}$.

**(c)** let $f(x) = x^2 - 3$, finding a solution of the equation $x^2 - 3 = 0$ is the same as find the zero of $f(x)$. By the definition of Newton's Method, we have $p_{n+1} = g(p_n)$, for which

$$g(p_n) = p_n - \frac{p_n^2 - 3}{2p_n} = \frac{p_n^2 + 3}{2p_n} = p_{n+1}, \ for \ n \geq 1.$$

Thus, the given sequence is actually generated by Newton's method to find a solution of the equation $x^2 - 3 = 0$.

**(d)** Let $p = \sqrt{3}$, thus we have $f(p) = 0$ and $f'(p) \neq 0$.Also, we have, for $n \geq 1$,

$$g(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^2 - 3}{2x} = \frac{x^2 + 3}{2x}$$

Let $k$ be in (0,1). We want to show that $|g'(x)| \leq k$, for all $x \in [\sqrt{3} - \delta, \sqrt{3} + \delta]$. Hence,

$$g'(x) = 1 - \frac{f'(x)f'(x) - f(x)f''(x)}{[f'(x)]^2} = \frac{f(x)f''(x)}{[f'(x)]^2}$$

Since $f(p) = 0$ and $f'(p) \neq 0$, so we have

$$g'(p) = \frac{f(p)f''(p)}{[f'(p)]^2} = 0$$

Since $g'$ is continuous in the interval and $0 < k < 1$. We have shown that

$$|g'(x)| \leq k, \ for \ all \ x \in [\sqrt{3} - \delta, \sqrt{3} + \delta]$$

Then, by *Theorem 2.4*, for any number $p_0$ in $[\sqrt{3} - \delta, \sqrt{3} + \delta]$, the sequence $p_{n+1} = g(p_n)$ converges to the unique fixed point $p$ in the interval. Since we have shown in part (b) that $\sqrt{3}$ is a fixed point, the sequence converges to $\sqrt{3}$ for $p_0$ in $[\sqrt{3} - \delta, \sqrt{3} + \delta]$.

# Problem 3

**(a) i.**

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})} = p_{n-1} - \frac{({p_{n-1}}^2 - 3)(p_{n-1} - p_{n-2})}{{p_{n-1}}^2 - {p_{n-2}}^2} = p_{n-1} - \frac{{p_{n-1}}^2 - 3}{p_{n-1} + p_{n-2}}$$

**(a) ii.**

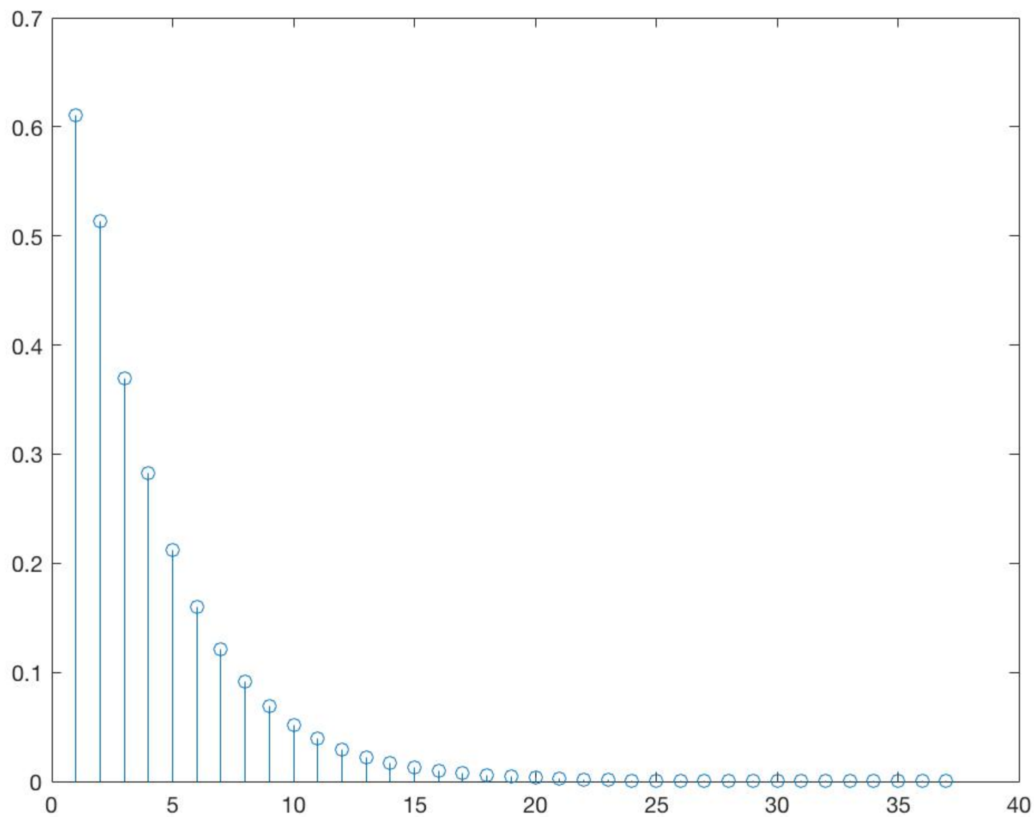
$$p_2 = 3 - \frac{(3^2 - 3)(3 - \frac{1}{2})}{(3^2 - 3) - ({\frac{1}{2}}^2 - 3)} = \frac{9}{7}, \quad p_3 = \frac{9}{7} - \frac{({\frac{9}{7}}^2 - 3)(\frac{9}{7} - 3)}{({\frac{9}{7}}^2 - 3) - (3^2 - 3)} = \frac{8}{5}$$

**(b)** Since $f(p_0) = {\frac{1}{2}}^2 - 3 = -2.75$ and $f(p_1) = 3^2 - 3 = 6$ and $f(p_0) \cdot f(p_1) < 0$, $p_2$ by method of false position is the same as in (a)ii., $p_2 = \frac{9}{7}$. Since $f(p_2) \cdot f(p_1) < 0$, $p_3$ by method of false position is also the same as in (a)ii., $p_3 = \frac{8}{5}$.

## Problem 4

**(a)** Root is x = 2.6199e-05, tabular data and graph are given below:

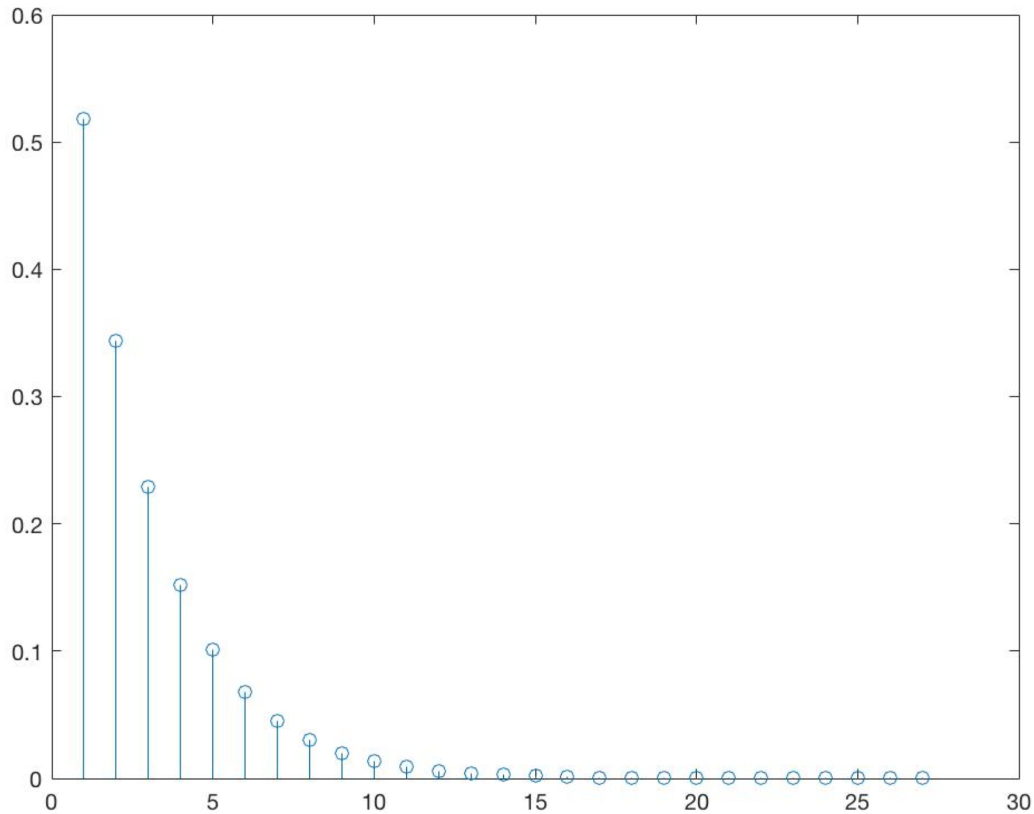
| $n$ | $p_{n-1}$ | $f(p_{n-1})$ | $p_n$ | $f(p_n)$ | $p_{n+1}$ | $f(p_{n+1})$ |
|---|---|---|---|---|---|---|
| 1.0000 | 0.7854 | -0.0783 | 1.1781 | -0.2542 | 0.6106 | -0.0372 |
| 2.0000 | 1.1781 | -0.2542 | 0.6106 | -0.0372 | 0.5132 | -0.0222 |
| 3.0000 | 0.6106 | -0.0372 | 0.5132 | -0.0222 | 0.3689 | -0.0083 |
| 4.0000 | 0.5132 | -0.0222 | 0.3689 | -0.0083 | 0.2828 | -0.0038 |
| 5.0000 | 0.3689 | -0.0083 | 0.2828 | -0.0038 | 0.2118 | -0.0016 |
| 6.0000 | 0.2828 | -0.0038 | 0.2118 | -0.0016 | 0.1602 | -0.0007 |
| 7.0000 | 0.2118 | -0.0016 | 0.1602 | -0.0007 | 0.1208 | -0.0003 |
| 8.0000 | 0.1602 | -0.0007 | 0.1208 | -0.0003 | 0.0912 | -0.0001 |
| 9.0000 | 0.1208 | -0.0003 | 0.0912 | -0.0001 | 0.0688 | -0.0001 |
| 10.0000 | 0.0912 | -0.0001 | 0.0688 | -0.0001 | 0.0520 | 0.0000 |
| 11.0000 | 0.0688 | -0.0001 | 0.0520 | 0.0000 | 0.0392 | 0.0000 |
| 12.0000 | 0.0520 | 0.0000 | 0.0392 | 0.0000 | 0.0296 | 0.0000 |
| 13.0000 | 0.0392 | 0.0000 | 0.0296 | 0.0000 | 0.0223 | 0.0000 |
| 14.0000 | 0.0296 | 0.0000 | 0.0223 | 0.0000 | 0.0169 | 0.0000 |
| 15.0000 | 0.0223 | 0.0000 | 0.0169 | 0.0000 | 0.0127 | 0.0000 |
| 16.0000 | 0.0169 | 0.0000 | 0.0127 | 0.0000 | 0.0096 | 0.0000 |
| 17.0000 | 0.0127 | 0.0000 | 0.0096 | 0.0000 | 0.0073 | 0.0000 |
| 18.0000 | 0.0096 | 0.0000 | 0.0073 | 0.0000 | 0.0055 | 0.0000 |
| 19.0000 | 0.0073 | 0.0000 | 0.0055 | 0.0000 | 0.0041 | 0.0000 |
| 20.0000 | 0.0055 | 0.0000 | 0.0041 | 0.0000 | 0.0031 | 0.0000 |
| 21.0000 | 0.0041 | 0.0000 | 0.0031 | 0.0000 | 0.0024 | 0.0000 |
| 22.0000 | 0.0031 | 0.0000 | 0.0024 | 0.0000 | 0.0018 | 0.0000 |
| 23.0000 | 0.0024 | 0.0000 | 0.0018 | 0.0000 | 0.0013 | 0.0000 |
| 24.0000 | 0.0018 | 0.0000 | 0.0013 | 0.0000 | 0.0010 | 0.0000 |
| 25.0000 | 0.0013 | 0.0000 | 0.0010 | 0.0000 | 0.0008 | 0.0000 |
| 26.0000 | 0.0010 | 0.0000 | 0.0008 | 0.0000 | 0.0006 | 0.0000 |
| 27.0000 | 0.0008 | 0.0000 | 0.0006 | 0.0000 | 0.0004 | 0.0000 |
| 28.0000 | 0.0006 | 0.0000 | 0.0004 | 0.0000 | 0.0003 | 0.0000 |
| 29.0000 | 0.0004 | 0.0000 | 0.0003 | 0.0000 | 0.0002 | 0.0000 |
| 30.0000 | 0.0003 | 0.0000 | 0.0002 | 0.0000 | 0.0002 | 0.0000 |
| 31.0000 | 0.0002 | 0.0000 | 0.0002 | 0.0000 | 0.0001 | 0.0000 |
| 32.0000 | 0.0002 | 0.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 |
| 33.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 |
| 34.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 |
| 35.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 36.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 37.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |




```
TOL = 0.00001; a = pi/4; b = 3*pi/8; fa = sin(a)-a;
fb = sin(b)-b; c = b-(fb*(b-a))/(fb-fa); fc = sin(c)-c;
i = 1;
disp('     n      Pn-1      f(Pn-1)      Pn      f(Pn)      Pn+1
f(Pn+1)');
disp([i a fa b fb c fc]);
c_list = [c];
while abs(c-b) > TOL
    a = b; b = c; fa = sin(a)-a;
    fb = sin(b)-b; c = b-(fb*(b-a))/(fb-fa);
    fc = sin(c)-c;
    i = i + 1;
    disp([i a fa b fb c fc]);
    c_list = [c_list c];
end
display(['Root calculated by Secant Method is x = ' num2str(c)]);
stem(c_list);
```

**(b)** Root is x = 1.3568e-05, tabular data and graph are given below:

| $n$ | $p_{n-1}$ | $f(p_{n-1})$ | $p_n$ | $f(p_n)$ |
|---|---|---|---|---|
| 1.0000 | 0.7854 | -0.0783 | 0.5181 | -0.0229 |
| 2.0000 | 0.5181 | -0.0229 | 0.3438 | -0.0067 |
| 3.0000 | 0.3438 | -0.0067 | 0.2288 | -0.0020 |
| 4.0000 | 0.2288 | -0.0020 | 0.1524 | -0.0006 |
| 5.0000 | 0.1524 | -0.0006 | 0.1015 | -0.0002 |
| 6.0000 | 0.1015 | -0.0002 | 0.0677 | -0.0001 |
| 7.0000 | 0.0677 | -0.0001 | 0.0451 | 0.0000 |
| 8.0000 | 0.0451 | 0.0000 | 0.0301 | 0.0000 |
| 9.0000 | 0.0301 | 0.0000 | 0.0201 | 0.0000 |
| 10.0000 | 0.0201 | 0.0000 | 0.0134 | 0.0000 |
| 11.0000 | 0.0134 | 0.0000 | 0.0089 | 0.0000 |
| 12.0000 | 0.0089 | 0.0000 | 0.0059 | 0.0000 |
| 13.0000 | 0.0059 | 0.0000 | 0.0040 | 0.0000 |
| 14.0000 | 0.0040 | 0.0000 | 0.0026 | 0.0000 |
| 15.0000 | 0.0026 | 0.0000 | 0.0018 | 0.0000 |
| 16.0000 | 0.0018 | 0.0000 | 0.0012 | 0.0000 |
| 17.0000 | 0.0012 | 0.0000 | 0.0008 | 0.0000 |
| 18.0000 | 0.0008 | 0.0000 | 0.0005 | 0.0000 |
| 19.0000 | 0.0005 | 0.0000 | 0.0003 | 0.0000 |
| 20.0000 | 0.0003 | 0.0000 | 0.0002 | 0.0000 |
| 21.0000 | 0.0002 | 0.0000 | 0.0002 | 0.0000 |
| 22.0000 | 0.0002 | 0.0000 | 0.0001 | 0.0000 |
| 23.0000 | 0.0001 | 0.0000 | 0.0001 | 0.0000 |
| 24.0000 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 25.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 26.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 27.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |




```
a = pi/4; j = 1; fa = sin(a)-a;
ga = cos(a)-1; d = a - fa/ga;
fd = sin(d)-d;
disp('   n       Pn-1       f(Pn-1)       Pn       f(Pn)');
disp([i a fa d fd]);
d_list = [d];
while abs(a-d) > TOL
    a = d; fa = sin(a)-a; ga = cos(a)-1;
    d = a - fa/ga; fd = sin(d)-d;
    j = j+1;
    d_list = [d_list d];
    disp([j a fa d fd]);
end
display(['Root calculated by Newton Method is x = ' num2str(d)]);
stem(d_list);
```