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MATH 155 HW #9
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Problem 1

$$H(u,v) = \int_0^T e^{-2\pi i[u\frac{at}{T}+v\frac{bt}{T}]}dt = -\frac{T}{2\pi i(au+bv)}e^{-2\pi i\frac{au+bv}{T}t}\Big|_0^T = -\frac{T}{2\pi i(au+bv)}\left(e^{-2\pi i(au+bv)}-1\right)$$
$$= -\frac{T}{2\pi i(au+bv)}\left[\cos(2\pi(au+bv)) - i\sin(2\pi(au+bv)) - 1\right]$$
$$= -\frac{T}{2\pi(au+bv)}\left[\frac{1}{i}\cos(2\pi(au+bv)) - \sin(2\pi(au+bv)) - \frac{1}{i}\right]$$
$$= \frac{T}{\pi(au+bv)}\left[\frac{1}{2}\sin(2\pi(au+bv)) + \frac{1}{2i}(1-\cos(2\pi(au+bv))\right]$$
$$= \frac{T}{\pi(au+bv)}\left[\frac{1}{2}\sin(2\pi(au+bv)) - i\sin^2(\pi(au+bv))\right]$$
$$= \frac{T}{\pi(au+bv)}\left[\sin(\pi(au+bv))\cos(\pi(au+bv)) - i\sin^2(\pi(au+bv))\right]$$
$$= \frac{T}{\pi(au+bv)}\sin(\pi(au+bv))\left[\cos(\pi(au+bv)) - i\sin(\pi(au+bv))\right]$$
$$= \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-\pi i(ua+vb)}$$

Problem 2

(a)

We have the Laplacian of Gaussian (LoG) operator:

$$\nabla^2 h(r) = \left[\frac{r^2-\sigma^2}{\sigma^4}\right]e^{-\frac{r^2}{2\sigma^2}}$$

We want to calculate the average value of the LoG operator, $\overline{\nabla^2 h(r)}$, c is some constant of the function representing the "length" of the domain.

$$\overline{\nabla^2 h(r)} = \frac{1}{c}\int_{-\infty}^{+\infty}\left[\frac{r^2-\sigma^2}{\sigma^4}\right]e^{-\frac{r^2}{2\sigma^2}}dr = \frac{1}{c}\frac{1}{\sigma^4}\left(\int_{-\infty}^{+\infty} r^2 e^{-\frac{r^2}{2\sigma^2}}dr - \int_{-\infty}^{+\infty}\sigma^2 e^{-\frac{r^2}{2\sigma^2}}dr\right)$$

From the identities $\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} e^{-r^2/(2\sigma^2)}dr = 1$ and $\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} r^2 e^{-r^2/(2\sigma^2)}dr = \sigma^2$, we know that $\int_{-\infty}^{+\infty} r^2 e^{-r^2/(2\sigma^2)}dr = \sigma^2\sqrt{2\pi\sigma^2}$ and $\int_{-\infty}^{+\infty} e^{-r^2/(2\sigma^2)}dr = \sqrt{2\pi\sigma^2}$. Thus,

$$\overline{\nabla^2 h(r)} = \frac{1}{c}\frac{1}{\sigma^4}(\sigma^2\sqrt{2\pi\sigma^2} - \sigma^2\sqrt{2\pi\sigma^2}) = 0$$

Thus, the average value of the LoG operator is zero.

(b)
From (a) we know that $\int_{-\infty}^{+\infty} \nabla^2 h(r)dr = 0$, thus:

$$\mathcal{F}(\nabla^2 h(r))\Big|_{w=0} = \int_{-\infty}^{+\infty} \nabla^2 h(r)e^{-iwr}dr\Big|_{w=0} = \int_{-\infty}^{+\infty} \nabla^2 h(r)dr = 0$$

We want to know the average value of the function $g(r)$ obtained by an input function $f(r)$ convolved with the operator. Assume the Fourier transform of $g(r)$ and $f(r)$ are $G(w)$ and $F(w)$ respectively.

$$\begin{aligned} g(r) &= f(r) * \nabla^2 h(r) \\ G(w) &= F(w) \cdot \mathcal{F}(\nabla^2 h(r))(w) \\ G(w)\Big|_{w=0} &= F(0) \cdot 0 = 0 \end{aligned}$$

The average value of g is proportional to the value of $G(0)$, thus the average value of g is also 0.

Problem 3

```
I = imread('Fig5.26a.jpg');
[m,n] = size(I);
fc = zeros(m,n);
for x = 1:1:m
    for y = 1:1:n
        fc(x,y) = I(x,y) * (-1)^(x+y);
    end
end
F = fft2(fc);
H = ones(m,n);
for u = 1:1:m
    for v = 1:1:n
        H(u,v) = 1/(pi*(0.1*u+0.1*v));
```

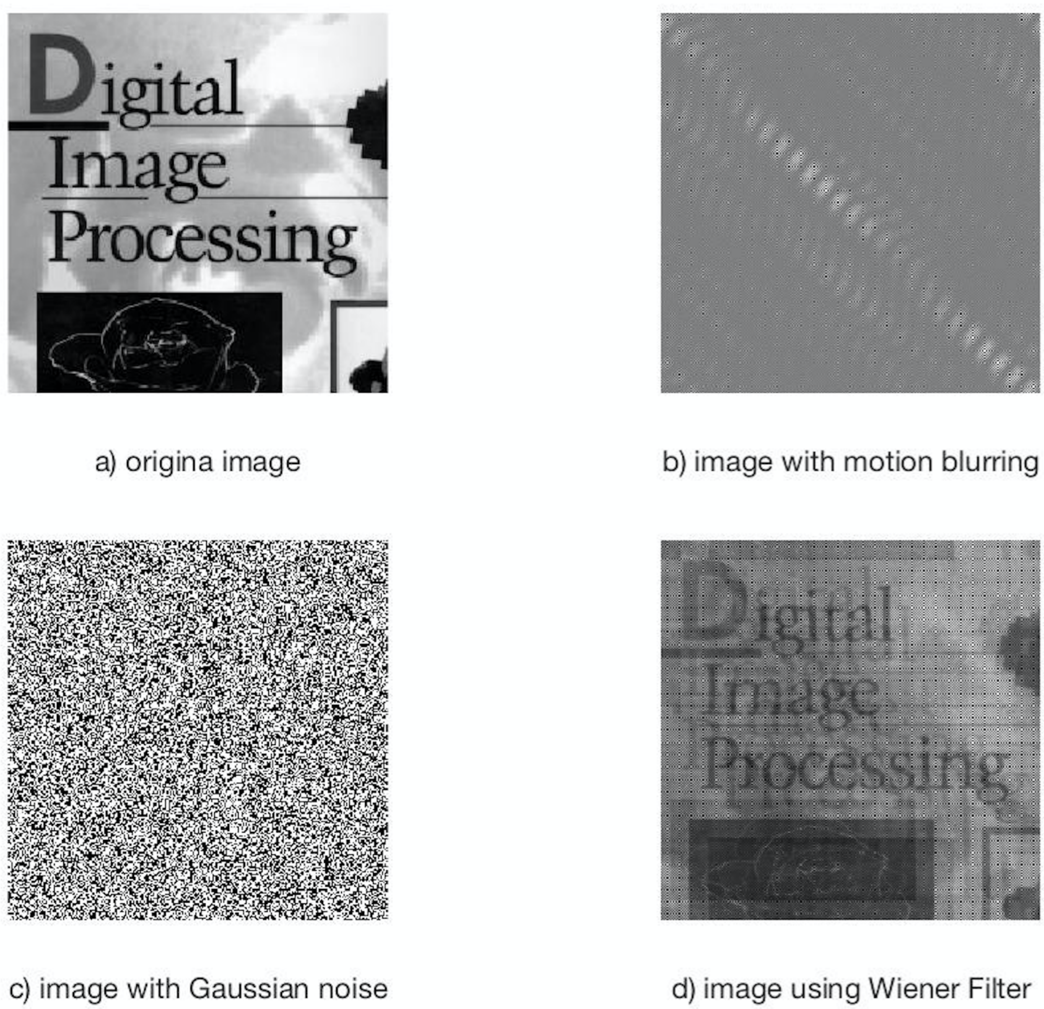
```
        H(u,v) = H(u,v) * sin((pi*(0.1*u+0.1*v))^...
        (exp(-pi*1i*(0.1*u+0.1*v))));
    end
end
G = H .* F;
g = real(ifft2(G));
for x = 1:1:m
    for y = 1:1:n
        g(x,y) = g(x,y) * (-1)^(x+y);
    end
end
J = imnoise(g,'gaussian',0,v);
K = linspace(0.001,0.1,100);
error_a = zeros(1,100);
for i=1:length(K)
   W = conj(H)./(abs(H).^2 + K(i));
    G_f = fft2(J);
    F_f = W.*G_f;
    F_r = uint8(ifft2(F_f));
    error = real(uint8(I)) - real(F_r);
    error_a(i) = mean(error(:))^2;
end
[minErrorValue minErrorPos] = min(error_a);
idealK = K(minErrorPos);
for u = 1:1:m
    for v = 1:1:n
        if H(u,v) == 0
            F_t(u,v) = F_t(u,v);
        else
            F_t(u,v) = ((1/H(u,v))*((abs(H(u,v))^2)/...
            (abs(H(u,v))^2+idealK)));
            F_t(u,v) = F_t(u,v) * G(u,v);
        end
    end
end
f_t = real(ifft2(F_t));
for x = 1:1:m
    for y = 1:1:n
        f_t(x,y) = f_t(x,y) * (-1)^(x+y);
    end
end
subplot(2,2,1);
```

```
imshow(I, [ ]);
xlabel('a) origina image');

subplot(2,2,2);
imshow(g,[ ]);
xlabel('b) image with motion blurring');

subplot(2,2,3);
imshow(J,[ ]);
xlabel('c) image with Gaussian noise');

subplot(2,2,4);
imshow(f_t,[ ]);
xlabel('d) image using Wiener Filter');
```



Problem 4

Steps:

(1) Apply the Laplacian of Gaussian filter LoG (shown in the code) to the original image f to obtain $\nabla^2(h_G * f)$

(2) Find the zero crossings of the new image g. If $\nabla^2(h_G * f) \geq 0$, we set g to be white. If $\nabla^2(h_G * f) < 0$, we set g to be black.

```
I=imread('Fig10.15a.jpg');
w=[0 0 1 0 0
   0 1 2 1 0
   1 2 -16 2 1
   0 1 2 1 0
   0 0 1 0 0];
p = [0 0 3 2 2 2 3 0 0
     0 2 3 5 5 5 3 2 0
     3 3 5 3 0 3 5 3 3
     2 5 3 -12 -23 -12 3 5 2
     2 5 0 -23 -40 -23 0 5 2
     2 5 3 -12 -23 -12 3 5 2
     3 3 5 3 0 3 5 3 3
     0 2 3 5 5 5 3 2 0
     0 0 3 2 2 2 3 0 0]
img = conv2(double(I),w,'same');
img2 = conv2(double(I),p,'same');
[m,n] = size(img);
for i = 1:1:m
    for j =1:1:n
        if img(i,j)>=0
            f(i,j)=1;
        else
            f(i,j)=0;
        end
        if img2(i,j)>=0
            f2(i,j)=1;
        else
            f2(i,j)=0;
        end
    end
end
```

```
subplot(2,3,1)
imshow(uint8(I));
xlabel('a) original image');
subplot(2,3,2)
imshow(uint8(img));
xlabel('b) 5x5 LoG Filter')
subplot(2,3,3)
imshow(f,[0 1]);
xlabel('(c) Zero-crossong of b')
subplot(2,3,4)
imshow(uint8(img2));
xlabel('d) 9x9 LoG Filter')
subplot(2,3,5)
imshow(f2,[0 1]);
xlabel('(e) Zero-crossong of d')
```