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MATH 155 HW #7
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Problem 1

(a)

$$f(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}F(u,v)e^{2\pi i(ux+vy)}dudv$$

(b) Assume that $f,\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial x}$ vanish as $x\to\pm\infty$

$$\mathcal{F}\left(\frac{\partial^2 f}{\partial x\partial y}\right)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{-2\pi i(ux+vy)}\frac{\partial^2 f}{\partial x\partial y}dxdy$$

Integration by parts with respect to x, $u=e^{-2\pi i(ux+vy)}$ and $dv=\frac{\partial^2 f}{\partial x\partial y}dxdy$

$$\begin{aligned}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}e^{-2\pi i(ux+vy)}\frac{\partial^2 f}{\partial x\partial y}dxdy &= e^{-2\pi i(ux+vy)}\frac{\partial f}{\partial y}-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(-2\pi iu)e^{-2\pi i(ux+vy)}\frac{\partial f}{\partial y}dxdy\\ &= -\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(-2\pi iu)e^{-2\pi i(ux+vy)}\frac{\partial f}{\partial y}dxdy\end{aligned}$$

Integration by parts with respect to x, $u=(-2\pi iu)e^{-2\pi i(ux+vy)}$ and $dv=\frac{\partial f}{\partial y}dxdy$

$$-\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(-2\pi iu)e^{-2\pi i(ux+vy)}\frac{\partial f}{\partial y}dxdy=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(-2\pi iv)(-2\pi iu)e^{-2\pi i(ux+vy)}f(x,y)dxdy$$

Plug in f(x,y) from (a), we get:

$$\mathcal{F}\left(\frac{\partial^2 f}{\partial x\partial y}\right)=-4\pi^2 uvF(u,v)$$

Problem 2

We know that $\sin\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$

$$\mathcal{F}(\sin(2\pi u_0 x)) = \int_{-\infty}^{+\infty} e^{-2\pi i u x}\sin(2\pi u_0 x)dx = \int_{-\infty}^{+\infty} e^{-2\pi i u x}\frac{e^{2\pi i u_0 x}-e^{-2\pi i u_0 x}}{2i}dx$$
$$= \frac{-i}{2}\int_{-\infty}^{+\infty} e^{-2\pi i u x}e^{2\pi i u_0 x}dx - \frac{-i}{2}\int_{-\infty}^{+\infty} e^{-2\pi i u x}e^{-2\pi i u_0 x}dx$$

The translation property shows that: $\mathcal{F}\left(f(x,y)e^{2\pi i u_0 x}\right) = F(u-u_0)$

$$\mathcal{F}(\sin(2\pi u_0 x)) = \frac{-i}{2}\mathcal{F}(1)(u-u_0) - \frac{-i}{2}\mathcal{F}(1)(u+u_0) = \frac{i}{2}(\delta(u+u_0)-\delta(u-u_0))$$

Problem 3

$$\mathcal{F}(f) = \mathcal{F}(A\sin(2\pi u_0 x + 2\pi v_0 y)) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-2\pi i(ux+vy)}A\sin(2\pi u_0 x + 2\pi v_0 y)dxdy$$
$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-2\pi i(ux+vy)}\frac{A(e^{2\pi i u_0 x + 2\pi i v_0 y} - e^{-2\pi i u_0 x - 2\pi i v_0 y})}{2i}dxdy$$
$$= A\frac{-i}{2}\left(\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-2\pi i(ux+vy)}e^{2\pi i u_0 x + 2\pi i v_0 y}dxdy - \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-2\pi i(ux+vy)}e^{-2\pi i u_0 x - 2\pi i v_0 y}dxdy\right)$$

The translation property shows that: $\mathcal{F}\left(f(x,y)e^{2\pi i(u_0 x + v_0 y)}\right) = F(u-u_0, v-v_0)$

$$\mathcal{F}(A\sin(2\pi u_0 x + 2\pi v_0 y)) = A\frac{-i}{2}\left[\mathcal{F}(1)(u-u_0, v-v_0) - \mathcal{F}(1)(u+u_0, v+v_0)\right]$$
$$= A\frac{i}{2}\left[\delta(u+u_0, v+v_0) - \delta(u-u_0, v-v_0)\right]$$

Problem 4

$$\mathcal{F}(f) = \mathcal{F}(\sin(2\pi u_0 x + 2\pi v_0 y)) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} e^{-2\pi i\left(u\frac{x}{M}+v\frac{y}{N}\right)}\sin(2\pi u_0 x + 2\pi v_0 y)$$
$$= \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} e^{-2\pi i\left(u\frac{x}{M}+v\frac{y}{N}\right)}\frac{(e^{2\pi i u_0 x + 2\pi i v_0 y} - e^{-2\pi i u_0 x - 2\pi i v_0 y})}{2i}$$

$$\mathcal{F}(f) = \frac{-i}{2}\left(\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} e^{-2\pi i\left(u\frac{x}{M}+v\frac{y}{N}\right)} e^{2\pi i\frac{Mu_0 x}{M}+2\pi i\frac{Nv_0 y}{N}} - \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} e^{-2\pi i\left(u\frac{x}{M}+v\frac{y}{N}\right)} e^{-2\pi\frac{Mu_0 x}{M}-2\pi\frac{Nv_0 y}{N}}\right)$$

The translation property shows that: $\mathcal{F}\left(f(x,y)e^{2\pi i(\frac{Mu_0 x}{M}+\frac{Nv_0 y}{N})}\right) = F(u - Mu_0, v - Nv_0)$

$$\begin{aligned}\mathcal{F}(f) &= \frac{-i}{2}\left[\mathcal{F}(1)(u - Mu_0, v - Nv_0) - \mathcal{F}(1)(u + Mu_0, v + Nv_0)\right]\\ &= \frac{i}{2}\left[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\right]\end{aligned}$$

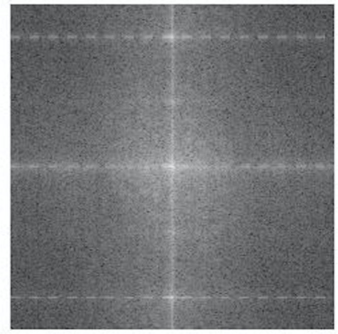
Problem 5

```
img = imread ('Fig5.26a.jpg');
[m,n] = size(img);
f = zeros(m,n);
for i=1:m
    for j=1:n
        f(i,j)=img(i,j)+ (100*sin(2*pi*134.4*i));
    end
end
[M,N] = size(f);
P = 2*M;
Q = 2*N;
fc = zeros(M,N);
for x = 1:1:M
    for y = 1:1:N
        fc(x,y) = f(x,y) * (-1)^(x+y);
    end
end
F = fft2(fc,P,Q);
H_NF = ones(P,Q);
for x = (-P/2):1:(P/2)-1
for y = (-Q/2):1:(Q/2)-1
D = 30;
v_k = 0; u_k = 103;
D_k = ((x+u_k)^2 + (y+v_k)^2)^(0.5);
H_NF(x+(P/2)+1,y+(Q/2)+1) = H_NF(x+(P/2)+1,y+(Q/2)+1) * 1/(1+(D/D_k)^2);
D_k = ((x-u_k)^2 + (y-v_k)^2)^(0.5);
H_NF(x+(P/2)+1,y+(Q/2)+1) = H_NF(x+(P/2)+1,y+(Q/2)+1) * 1/(1+(D/D_k)^2);
```

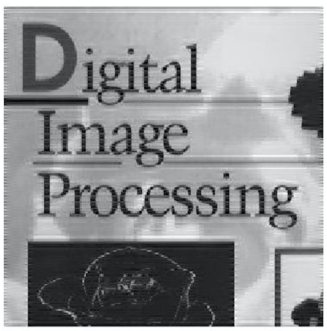
```
v_k = 0; u_k = 205;
D_k = ((x+u_k)^2 + (y+v_k)^2)^(0.5);
H_NF(x+(P/2)+1,y+(Q/2)+1) = H_NF(x+(P/2)+1,y+(Q/2)+1) * 1/(1+(D/D_k)^2);
D_k = ((x-u_k)^2 + (y-v_k)^2)^(0.5);
H_NF(x+(P/2)+1,y+(Q/2)+1) = H_NF(x+(P/2)+1,y+(Q/2)+1) * 1/(1+(D/D_k)^2);
end
end

G_1 = H_NF .* F;
g_1 = real(ifft2(G_1));
g_1 = g_1(1:1:M,1:1:N);
for x = 1:1:M
    for y = 1:1:N
        g_1(x,y) = g_1(x,y) * (-1)^(x+y);
    end
end
figure();
subplot(1,3,1);
imshow(img,[ ]);
xlabel('a).Original Image');
subplot(1,3,2);
imshow(f,[ ]);
xlabel('b).a with Sinusoidal Noise');

subplot(1,3,3);
imshow(log(1 + abs(F)),[ ]);
xlabel('c).Fourier spectrum of b');

figure();
subplot(1,2,2);
imshow(log(1 + abs(G_1)),[ ]);
xlabel('e).Fourier spectrum of d');

subplot(1,2,1);
imshow(g_1,[ ]);
xlabel('d).Result image');
```

a).Original Image

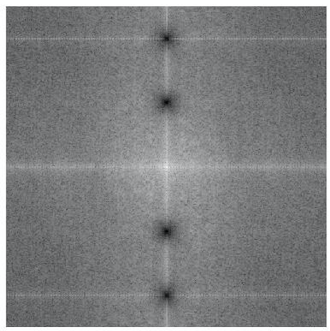
b).a with Sinusoidal Noise



c).Fourier spectrum of b



d).Result image



e).Fourier spectrum of d