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MATH 155 HW #1
Date: 01/19/2017

Problem 1

Algorithm 1 Algorithm for Gray-Level Reduction

Input: image and level
let **n_col** be the number of columns of the **image**, **n_row** be the number of rows of the **image** and **num** be **256 / level**.
create a new matrix **img** to be a matrix with **n_col** columns and **n_row** rows, and set all entries to be zero.
for $c = 1, 2, 3, ...n\_col$ **do**
    for $r = 1, 2, 3, ...n\_row$ **do**
        let the integer part of **image(r,c)/num** to be **a**, set the entry **img(r,c)** to be the integer part of **a * 255 / (level-1)**
    end for
end for
return the new matrix **img**

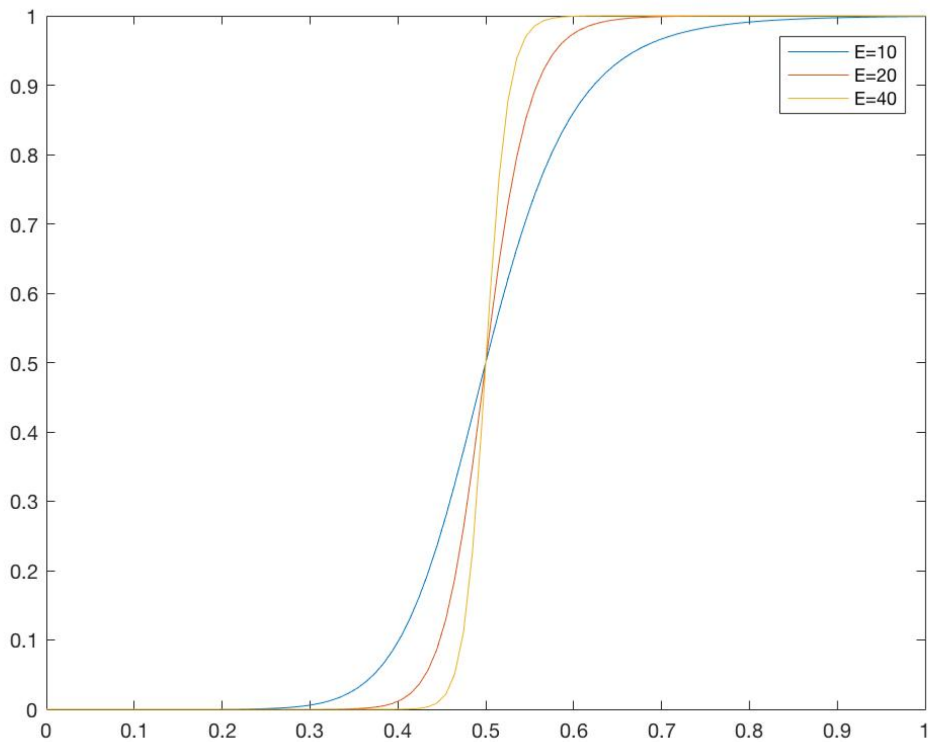
Problem 2

(a). From Fig.3.2(a), we know that:
1. when $r < k$, the slope is increasing
2. when $r > k$, the slope is decreasing
3. when $r = k$, $s = \frac{1}{2}$

Thus, we can guess that the continuous function of s is

$$s(r) = \frac{1}{1 + (\frac{k}{r})^E}$$

(b). The following figure is generated when normalizing the function to $[0, 1]$ and choose $k = 0.5$, $E = 10, 20, 40$ respectively.



Problem 3

Let n be the total number of pixels and let n_rj be the number of pixels in the input image with gray level r_j. By the discrete form of histogram equalization, we have:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k}\frac{n_{rj}}{n} = (L-1)\frac{1}{n}\sum_{j=0}^{k} n_{rj}$$

Since every pixel with value r_k is mapped to value s_k, it follows that $n_{sk} = n_{rk}$. A second pass of histogram equalization would be:

$$v_k = T(s_k) = (L-1)\frac{1}{n}\sum_{j=0}^{k} n_{sj}$$

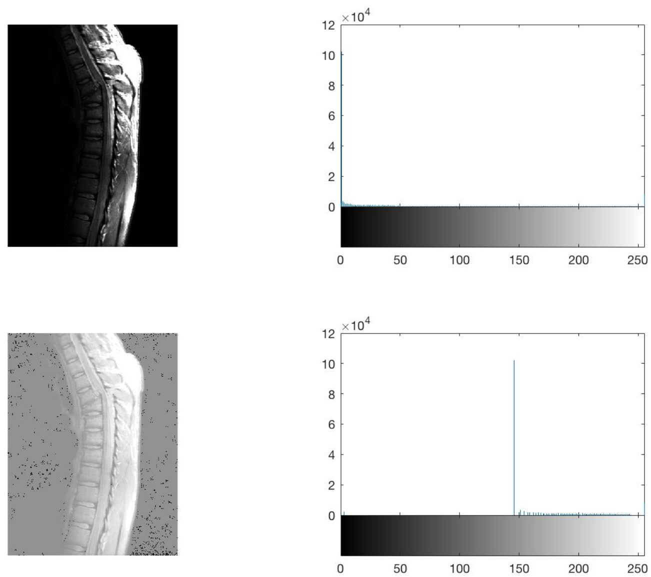
Since $n_sj = n_rk$, we have:

$$v_k = T(s_k) = (L-1)\frac{1}{n}\sum_{j=0}^{k} n_{rj} = s_k$$

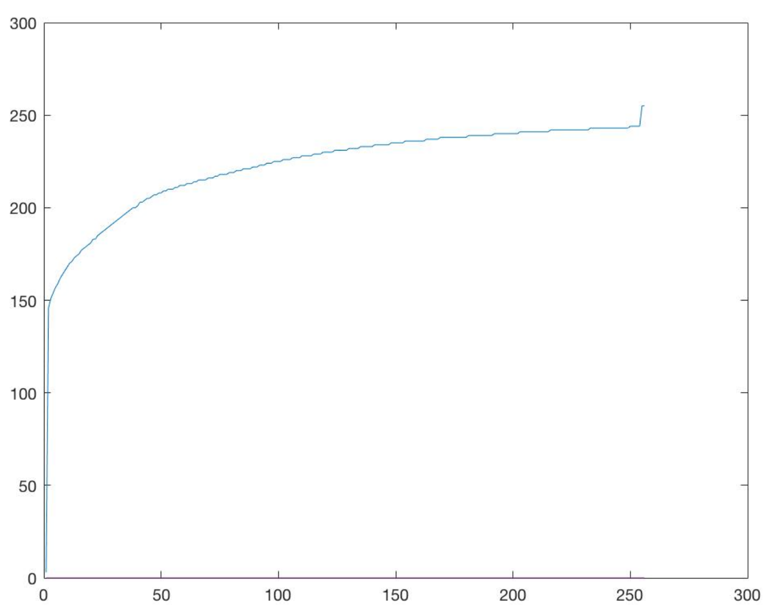
This shows that a second pass of histogram equalization will produce exactly the same result as the first pass.

Problem 4

The original picture with its histogram and the enhanced picture with its histogram is shown below:



The plot for the histogram equalization function is shown below:



In the enhanced image's histogram, the gray levels of the picture are shifted towards the right, which is the brighter sides of the image. Also, the differences between the bars (intensity) are also decreased, which is shown as an enhanced contrast. The code for the project is shown below:

```
I = imread('Fig3.08(a).jpg');
J = hist_eq(I);
subplot(2,2,1);
imshow(I);
subplot(2,2,2);
histim(I)
subplot(2,2,3);
imshow( J );
subplot(2,2,4);
histim(J)

function J = hist_eq(img)
J = uint8(zeros(size(img,1),size(img,2)));
freq = zeros(256);
prob_f = zeros(256);
prob_c = zeros(256);
cum = zeros(256);
output = zeros(256);

for i=1:size(img,1)
    for j=1:size(img,2)
        freq(img(i,j)+1) = freq(img(i,j)+1)+1;
        prob_f(img(i,j)+1) = freq(img(i,j)+1)/(size(img,1)*size(img,2));
    end
end
sum = 0;
for i=1:size(prob_f)
    sum = sum + freq(i);
    cum(i) = sum;
    prob_c(i) = cum(i)/(size(img,1)*size(img,2));
    output(i) = round(prob_c(i)*255);
end

for i=1:size(img,1)
    for j=1:size(img,2)
        J(i,j) = output(img(i,j)+1);
    end
end
```

```
%plot(output)
end

function histim(img)
img = uint8(img);
[count,bin] = hist(img(:),0:255);
stem(bin,count,'Marker','none')

hAx = gca;
set(hAx, 'XLim',[0 255],'XTickLabel',[],'Box','On');

hAx2 = axes('Position',get(hAx,'Position'), 'HitTest','off');
image(0:255, [0 1], repmat(linspace(0,1,256),[1 1 3]), 'Parent',hAx2)
set(hAx2, 'XLim',[0 255], 'YLim',[0 1], 'YTick',[], 'Box','on')

set(hAx, 'Units','pixels')
p = get(hAx, 'Position');
set(hAx, 'Position',[p(1) p(2)+26 p(3) p(4)-26])
set(hAx, 'Units','normalized')

set(hAx2, 'Units','pixels')
p = get(hAx2, 'Position');
set(hAx2, 'Position',[p(1:3) 26])
set(hAx2, 'Units','normalized')

linkaxes([hAx;hAx2], 'x')
set(gcf, 'CurrentAxes',hAx)
end
```