$P(\text{toss}) = \langle \text{head}, \text{tail} \rangle$

Some notation:

false : $\neg$

fix order of domain ele : $\langle x_1, x_2, x_3 \rangle$

Discrete Random variable: — countable number of distinct values

Ex: six-sided fair die

— whole numbers
— finite set of values within a certain range

if range = 0 – 5
values → 0, 1, 2, 3, 4 or 5

continuous এর মতো, 1, 1.1, 1.11 . . . . so on
within a range
but infinite values can be taken

ex: height

Probability axioms (3):

1) $0 \le P(a) \le 1$
2) $P(\text{true}) = 1$, $P(\text{false}) = 0$
3) $P(a \vee b) = P(a) + P(b) - P(a \wedge b)$

probability of disjunction, given by inclusion-exclusion principle

Atomic event: complete specification of the state or outcome of a random variable.

Prior probability: initial probability based on prior knowledge

$\boxed{\text{Ex}}$: 10% chance of raining tomorrow based on historical data

Joint Probability: probability of multiple events occuring simultaneously

– combined probabilities of a set of variables

Ex: $P(B) = 0.1$ → prior ~~pro~~ probability of B occuring

$P(A|B) = 0.8$

↳ if B occurs, then the probility of A occuring

Formulas:

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

conditional probability

$P(b)$ → normalizing constant

Product rule: $P(a \wedge b) = P(a|b)\,P(b)$

Marginal Pro: $P(B) = \sum_a P(B, a)$

Conditional marginal Pro: $\sum_a P(B|a)\,P(a)$

Bayes' Rule:

$$\underbrace{P(a|b)}_{\text{Posterior}} = \frac{\overbrace{P(b|a)}^{\text{Likelihood}} * \overbrace{P(a)}^{\text{Prior}}}{\underbrace{P(b)}_{\text{Normalization}}}$$

Independent [illegible]: $P(A \cap B) = P(A)\,P(B)$

Conditional " " : $P(A \cap B|C) = P(A|C)\,P(B|C)$

$$P(a) = 0.09 + 0.1 = 0.19$$

Math: $P(a|b) = 0.09$

b true হলে a true হওয়ার সম্ভাবনা

	a	¬a
b	0.09	0.01
¬b	0.1	0.8

$$P(b|a) = P(b \wedge a) / P(a)$$
$$= 0.09 / 0.19$$
$$= 0.47$$

$$P(b \wedge a) = 0.09$$

Inference from the joint * $\boxed{P-36}$

smart এ যা আছে সব

1) $P(\text{smart}) = 0.432 + 0.16 + 0.48 + 0.16$

2) $P(\text{study}) = 0.432 + 0.48 + 0.84 + 0.36$

3) $P(\text{prepared} \mid (\text{study} \wedge \text{smart}))$

$$= \frac{P(p \wedge st \wedge sm)}{P(st \wedge sm)} = \frac{0.432}{0.432 + 0.48}$$

3 টাই true

study ও smart true

independence

37 page : 1) smant ind of study ?

$\rightarrow$ if $P(sm \mid st) = P(sm)$

$P(sm \mid st)$

$$= \frac{P(sm \cap st)}{P(st)}$$

$$= \frac{0.432 + 0.048}{0.432 + 0.048 + 0.084 + 0.036}$$

$$= \boxed{0.8}$$

$P(sm)$

$= (\cancel{+0.048}$ $\cancel{+0.36})$

$= 0.432 + 0.16 + 0.048 + 0.16$

$= \boxed{0.8}$

$\boxed{\text{Yes}}$

2) pnepaned ind of st ?

$\rightarrow$ if $P(P \mid st) = P(P)$

$P(P \mid st)$

$$= \frac{P(P \cap st)}{P(st)}$$

$$= \frac{0.432 + 0.084}{0.432 + 0.048 + 0.084 + 0.036}$$

$$= \boxed{0.86}$$

$P(P)$

$= 0.432 + 0.16 + 0.084 + 0.008$

$= \boxed{0.684}$

$\boxed{\text{No}}$

conditional independence $\boxed{38P}$

1) smart conditionally independent of prepared, given study:

$$p(sm \wedge p \mid st) = \frac{p(sm \wedge p \wedge st)}{p(st)}$$

$$= \frac{0.432}{0.432+0.048+0.084+0.036}$$

$$= \boxed{0.72}$$

$$p(sm \mid st) * p(p \mid st)$$

$\frac{p(sm \wedge st)}{p(st)}$ $\frac{p(p \wedge st)}{p(st)}$

$$= \frac{(0.432+0.048)}{0.432+0.048+0.084+0.036} * \frac{0.432+0.084}{0.432+0.048+0.084+0.036}$$

$$= 0.688$$

$\rightarrow$ No

2) $$p(st \wedge p \mid sm) = \frac{p(st \wedge p \wedge sm)}{p(sm)}$$

$$= \frac{0.432}{0.432+0.16+0.048+0.16} = 0.\cancel{3}54$$

$$p(st \mid sm) * p(p \mid sm) = \frac{p(st \wedge sm)}{p(sm)} * \frac{p(p \wedge sm)}{p(sm)}$$

$$= \frac{(0.432+0.048) * (0.432+0.16)}{(0.432+0.16+0.048+0.16)^2}$$

$$= \cancel{0.356} \; 0.444$$