Derivatives using chain Rule

(Q1) compute the derivatives df/dx of the following function using chain Rule

$$f(z) = \log_e(1+z)$$

where, $z = x^T x$, $x \in R^D$

Ans :-

Here

$$f(z) = \log_e(1+z)$$

$$= \log_e(1+x^T x)$$

$$= \log_e(u)$$

Let,

$u = 1+z$

$\rightarrow u = 1 + x^T x$

$\therefore z = x^T x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(1 + x^T x)$$

$$= 0 + 2x$$

$$= 2x$$

Again $\dfrac{df}{du} = \dfrac{d}{du}\left(\log_e(u)\right)$

$$\Rightarrow \frac{1}{u}$$

$$\Rightarrow \frac{1}{1+x^T x}$$

So, using chain rule, we get

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{1+x^T x} \cdot 2x$$

$$= \frac{2x}{1+x^T x}$$

$$\therefore \quad \frac{df}{dx} = \frac{2x}{1+x^T x}$$

(Ans)

Q3

compute derivatives df/dx of the following function using chain-Rule :-

$$f(z) = e^{-z/2}$$

$$\therefore z = g(y) = y^T S^{-1} y$$

$$\therefore y = h(x) = x - \mu$$

Ans:-

$$\frac{dz}{dy} = \frac{d(g(y))}{d(y)}$$

$$= \frac{d}{dy}(y^T S^{-1} y)$$

$$= 2S^{-1} y$$

$$\therefore \frac{df}{dz} = \frac{d}{dz}(e^{-z/2})$$

$$= -\frac{1}{2} e^{-z/2}$$

$$\frac{dy}{dx} = \frac{d(h(x))}{d(x)}$$

$$\Rightarrow \frac{d}{dx}(x-\mu)$$

$$= I$$

$\therefore$ Here, I = Identity Matrix

$\therefore$ using chainrule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= -\frac{1}{2}e^{-z/2} \cdot 2S^{-1}y \cdot I$$

$$= -e^{-z/2} \cdot S^{-1} \cdot y \cdot I$$

$$\therefore \frac{df}{dx} = \left(-e^{-z/2}\right) * \left(S^{-1}\right) y * I$$

Ans